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Optimization of Controller Parameters for Energy Saving

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Abstract: Among various technologies to tackle the twin challenges of sustainable energy supply and climate change, energy saving through advanced control plays a crucial role in decarbonizing the whole energy system. Modern control technologies, such as optimal control and model predictive control do provide a framework to simultaneously regulate the system performance and limit control energy. However, few have been done so far to exploit the full potential of controller design in reducing the energy consumption while maintaining desirable system performance. This paper investigates the correlations between control energy consumption and system performance using two popular control approaches widely used in the industry, namely the PI control and subspace model predictive control. Our investigation shows that the controller design is a delicate synthesis procedure in achieving better trade-off between system performance and energy saving, and proper choice of values for the control parameters may potentially save a significant amount of energy.

Keywords: Energy saving; system performance; controller parameters optimization; PI controller; subspace model predictive control

1. INTRODUCTION

The Mankind has currently been challenged with two key problems worldwide, namely the sustainable energy supply and climate change. The International Energy Agency (IEA) predicted an over 50 percent increase of the world energy demand in 2030 (IEA (2005)), while dramatic climate change worldwide partly due to greenhouse gas emissions (GGE) requires significant reductions of GGE in the order of 60 percent or more by 2050 (TCG (2005)). With the developments and innovations in control algorithm, advanced control are widely and successfully applied to the industry processes to tackle the twin challenges of the increasing requirement of energy supply and climate change. This is based on the fact that in modern industry, from power generation such as the centrifugal governor to efficient use of energy from the end users such as smart home and advanced process control of large industrial plants, control devices and equipments are widely used to adjust and control most major energy consuming appliances. Therefore, energy saving through advanced control can have profound impact.

In control engineering, the introduction of the feedback control principle into the design of automated devices can be traced back to a few thousand years ago. The first governor designed by James Watt in 1764 using proportional control to adjust the speed of a steam engine marked the beginning of industrial revolution era, and has transformed the living standards of the masses of ordinary people (Lucas (2002)). Control theory has evolved from classic control such as classic PID (Proportional-Integral-Derivative) control, to modern control in dealing with more complex and nonlinear dynamical systems in more complex situations. Among a number of milestone control approaches being proposed so far, PID control has still been widely used in the industry, such as in the power system operation and control as well as many industrial process control (Yamamoto et al. (2009)). Other modern control methods like the optimal control (Teleke et al. (2010)), model predictive control (Zheng et al. (2013); Qin and Badgwell (2003)), and robust control (Pal and Chaudhuri (2005)), etc. have also found many successful applications. Despite enormous progress has been made in modern control theory and application in the last half century, the twin challenges of sustainable energy supply and climate change have thrown down the gauntlet for control engineers to develop a new way of thinking and to exploit the full potential in the controller design to reduce

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the energy consumption while maintaining the system performance.

This paper investigates the correlations between control energy consumption and system performance, and their relations with controller design. It should be noted that control energy and system performance have been both considered in some modern control technologies in order to find the optimal control laws, such as the optimal control and model predictive control where the cost function to optimize the control sequence does consider the both factors, and abundance of results are available for the dual objective optimal control problem (Freudenberg and Looze (1985); Vidyasagar (2011); Chen et al. (2001, 2003)). However, little has been done so far in the control engineering literature to consider how these two factors are related to each other and how to choose a proper control target and a proper controller design strategy to maximize the energy utilization (?) while achieving desirable system performance. This is often referred to as part of the controller synthesis in the literature.

Unlike early work where both control energy consumption and system performance are considered in a weighted cost function based on which a feasible or optimal control law is derived, and weightings are often chose without any specific guidelines, this paper investigates the two criteria separately and investigates the detailed correlations among them for different set of control parameters. To achieve this, a simple first order plant with delay is taken as the example, and the design of PI controller and subspace based model predictive control are investigated. The paper shows that there often exists a nonlinear correlation between the control energy consumption and the system performance, and with this detailed case study, it is demonstrated that the controller design should carefully balance the system performance and control energy consumption through different choice of the control parameters. Further, the paper illustrates that a proper choice of control parameters in these two popular control methods can both reduce energy consumption while maintaining desirable system performance. On the contrary, improper choice of control parameters will only waste energy without any benefit to improve the system performance.

2. OPTIMIZATION OF PI CONTROLLER

2.1 Introduction

In this paper, the controller design for a typical first order single-input single-output (SISO) linear system with a time delay is investigated (Li et al. (2013)). This model represents many typical real systems in power and process industries.

$$G_p(s) = \frac{K}{Ts+1}e^{-\tau s} \tag{1}$$

where K, T and τ are amplification coefficient, time constant and delay time respectively, which are easily obtained from a step response curve. The feedback control is introduced to regulate the system as shown in Fig. 1, where $G_c(s)$ is the controller.

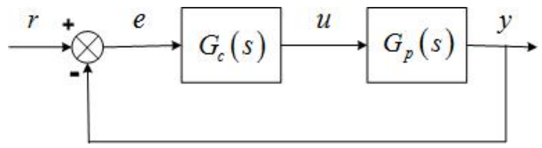

Fig. 1. Feedback control block diagram

A PI controller is first investigated, which can be formulated as

$$G_c(s) = K_p + K_i\frac{1}{s} \tag{2}$$

where K_p and K_i are proportional and integral gains respectively, which need to be carefully chosen to achieve a desired performance.

The desired control purposes or system performance measures are often given in terms of the frequency domain and/or the time domain, such as stability, steady state error, and transient response specified in terms of peak overshoot, rise time, settling time etc. The control energy can be simply defined as $\int_{t_0}^{t_f} u^2(t)dt$.

In some modern control technologies, such as optimal control and model predictive control, both tracking performance (tracking error) and control energy are combined to produce a cost function to optimize the control sequence. For example, in optimal control, the cost function is defined as (Lewis et al. (2012)):

$$J = \Phi[x(t_0), t_0, x(t_f), t_f] + \int_{t_0}^{t_f} L[x(t), u(t), t]dt \tag{3}$$

where $x(t)$ is the state, $u(t)$ is the control, while Φ and L are the end point cost and Lagrangian, respectively.

A particular form of the optimal control is the linear quadratic regulator (LQR) (Kwakernaak and Sivan (1972)), where the cost function is defined as

$$J = \frac{1}{2}\int_0^\infty [x^T(t)Qx(t) + u^T(t)Ru(t)]dt \tag{4}$$

where Q and R are the weighting factors, which are defined by human (engineers).

While the optimal control has provided a design framework to adjust the system performance as well as limit the control signals, there is no systematic means to specify the weighting matrices and to investigate their relations to the input energy consumption and control performance. This is however crucially important for the role of energy efficiency in decarbonizing the whole energy chain.

2.2 PI Controller Synthesis - Stability Issue

The stability is the first design consideration. The control parameters should first ensure the stability of the system for a given control structure. Since there is a delay term in the model (delay often is caused by transport lag and the distribution nature of the process), using the first order Pade approximation, the delay can be approximated by

$$e^{-\tau s} = \frac{-\tau s + 2}{\tau s + 2} \tag{5}$$

It should be noted that the Pade approximation is only used to give an initial ranges of the controller parameters and the ranges will be narrowed down in the simulation, so the first order Pade approximation is sufficient.

Then, the characteristic equation of the feedback control system in Fig. 1 with PI controller is given by:

$$\begin{aligned}\Delta =& T\tau s^3 + (2T + \tau - K\tau K_p)s^2 \\ &+ (2KK_p - K\tau K_i + 2)s + 2KK_i\end{aligned} \tag{6}$$

To make the system stable, according to the Routh Criterion, the following two inequalities should hold.

$$\begin{aligned} 0 < K_p &< \frac{2T+\tau}{K\tau} \\ 0 < K_i &< \frac{2(K_pK+1)(2T+\tau-K_pK\tau)}{K\tau(4T+\tau-K_pK\tau)} \end{aligned} \tag{7}$$

In the following simulation, $K = 12$, $T = 16$ and $\tau = 1$. From (7), the complete relation of K_i and K_p that makes the system stable is illustrated in Fig. 2.

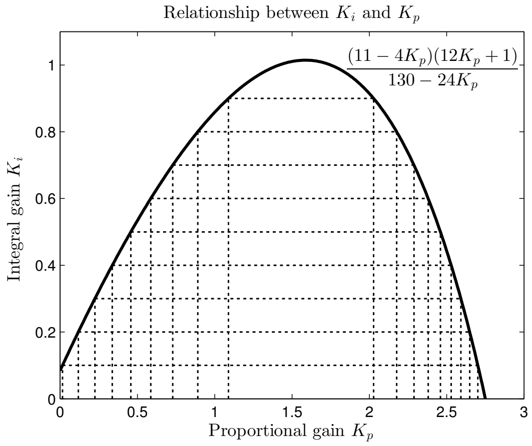

Fig. 2. The relationship of feasible K_i and K_p

2.3 Control Parameters and System Performance

Once the feasible ranges of K_i and K_p that make the system stable are obtained, the tracking performance and control energy consumption for different settings of the control parameters could be investigated for achieving a good trade-off between the control energy reduction and desirable tracking performance.

Note: The ranges of K_p and K_i in Fig. 2 are obtained according to the first order Pade approximation model, and the tracking performance will be quite bad when the proportional gain K_p is close to the boundary of the ranges of K_i satisfying the second equation in (7). In our following simulations, the ranges for K_p and K_i are further reduced in order to obtain better tracking performance. Examples of feasible K_i and K_p are listed in Table 1.

Table 1. Ranges of feasible K_i and K_p

K_i	0.1	0.2	0.3	0.4	0.5
K_p	0.2∼1.9	0.3∼1.8	0.4∼1.7	0.5∼1.6	0.6∼1.5

In this paper, the tracking error is defined as

$$E_y = \sum_{k=1}^{T} (r_k - y_k)^2 \triangle T \tag{8}$$

where y_k is the actual system output and r_k is the reference input, $\triangle T$ is the sampling interval, T is the total simulation time and k is sampling step.

The control energy is defined as

$$E_u = \sum_{k=1}^{T} u_k^2 \triangle T \tag{9}$$

Further, the total cost function (system performance) is defined as E_s, which is the sum of E_y and E_u, i.e. $E_s = E_y + E_u$.

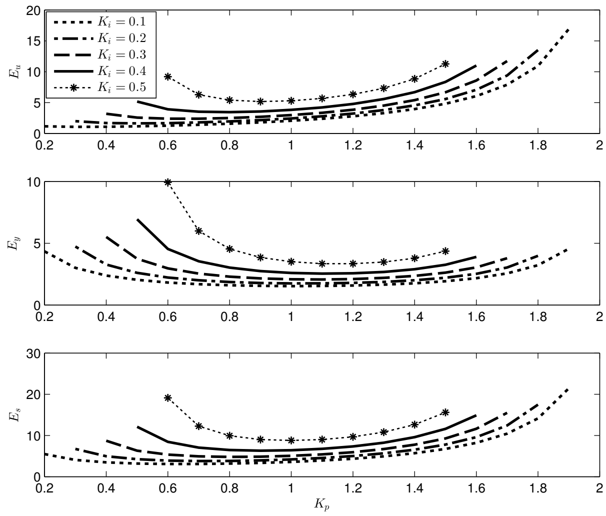

Fig. 3. Relation of system performance with different control parameters

Fig. 3 shows the correlation between the control energy E_u, tracking performance E_y and controller parameters. In the experiments, the two parameters K_i and K_p are incrementally changed with a magnitude of 0.1. The following observations can be obtained.

Observation 1: For the same K_p, the bigger the K_i is, as more energy is consumed, the dynamic tracking performance becomes worse. Further, the steady-state error also needs to be taken into consideration. For this reason, the integral gain K_i can never be set to zero.

Observation 2: For a fixed integral gain K_i, as the proportional gain K_p increases, both the tracking error and control energy decrease firstly and then increase. This implies that the values of the control parameters which lead to the increase of both tracking error and control energy can not be selected.

Observation 3: For this system, when K_p is around 1.1 the tracking errors approach to the minimum and when when K_p is around 0.8 the cost function values get near to the minimum.

2.4 Tracking Performance for Different K_i and K_p

To visualize the tracking performance of the system for different settings of the control parameters K_i and K_p which make E_u, E_y and E_s minimum respectively, Fig.4 gives the details of the system responses. The results are also compared to the response with the Ziegler-Nichols

(Z-N) tuning parameters which can be easily found in many literatures about PID controller design (Ziegler and Nichols (1942); Xue et al. (2002)). The detailed numerical results are summarized in Table 2, where $\sigma\%$ is the overshoot and t_s is the settling time.

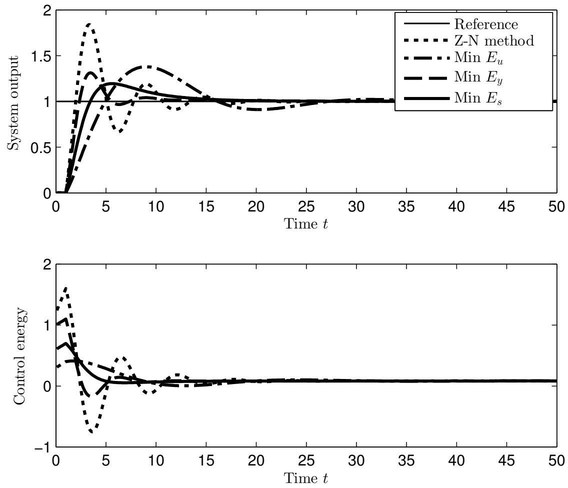

Fig. 4. Tracking performance for different K_i and K_p

Table 2. System performance for different settings of K_i and K_p

$K_i \setminus K_p$	E_u	E_y	E_s	$\sigma\%$	t_s
$0.4 \setminus 1.2$	4.78	2.57	7.36	83.98	17.9
$0.1 \setminus 0.3$	**1.06**	3.02	4.08	37.83	32.4
$0.1 \setminus 1.0$	2.06	**1.54**	3.60	31.19	**10.8**
$0.1 \setminus 0.6$	1.26	1.83	**3.09**	**19.41**	15.8

The first row of data in Table 2 are deduced by the Z-N tuning method. From Fig. 4 and Table 2, it is obvious that the system performances with Z-N tuning parameters are not satisfied with almost every aspect. Settings of 0.1 and 0.6 provide a balance between control energy and tracking performance and both the overshoot and settling time are acceptable. This shows that the controller synthesis is a delicate process in order to reduce the control energy while maintaining desirable system performance.

2.5 Sensitivity Analysis

To further explore the nonlinear correlation between the tracking performance and the control energy, the following measures are defined.

The changes of control energy, tracking performance and cost function are defined as follows:

$$\begin{aligned} D_u(t) &= \delta E_u(t) \\ D_y(t) &= \delta E_y(t) \\ D_s(t) &= \delta E_s(t) \end{aligned} \tag{10}$$

The sensitivity of tracking performance and cost function with respect to the change of control energy are defined as:

$$\begin{aligned} S_{yu}(t) &= \frac{D_y(t)}{D_u(t)} \\ S_{su}(t) &= \frac{D_s(t)}{D_u(t)} \end{aligned} \tag{11}$$

Let $K_i = 0.1$, Fig. 5 shows the correlation between these system measures (control energy, tracking performance) and the proportional gain K_p.

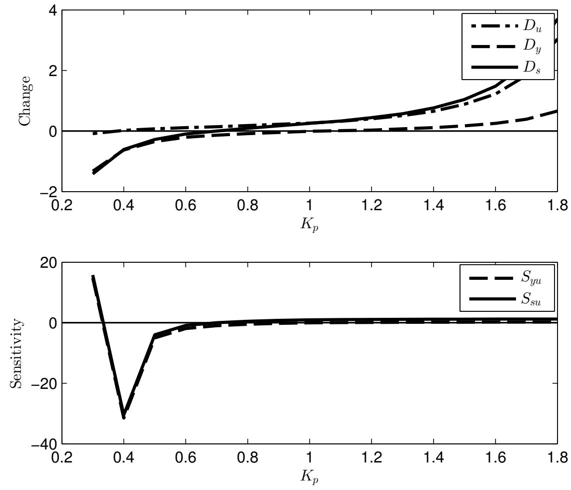

Fig. 5. System performance via control energy

The upper subgraph shows that all three performance indexes decrease first and then increase. The cost function transverses the x-axis and reach the minimum value at around 0.8. Both the energy consumption and tracking error increase when K_p is bigger.

From the bottom subgraph, it shows that at the beginning both the control energy and tracking error decrease as K_p increases and then with the growing of K_p the same energy could only bring an insignificant decrease of the tracking error. In other words, to improve the tracking performance with the same degree, huge amount of energy is required.

3. SUBSPACE MODEL PREDICTIVE CONTROL

As an advanced control method, Subspace Model Predictive Control (SMPC) (Katayama (2005); Huang and Kadali (2008)) has been widely used in the industry. In this method, a number of controller parameters, including prediction horizon, control horizon, tracking error weighting and control weighting, need to be optimized beforehand. For simplicity, only the correlation of the system energy saving and the control weighting will be studied in this paper.

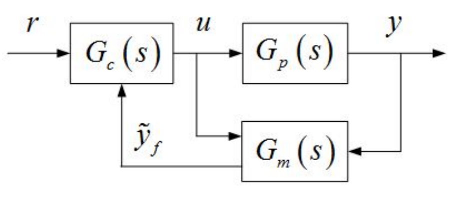

Fig. 6. Framework of model predictive control

3.1 Introduction of SMPC

A basic model predictive control system framework is shown in Fig. 6, where G_m is the subspace prediction model, the future prediction outputs $\tilde{y}_f$ are given by

$$\tilde{y}_f = L_w w_p + L_u u_f \tag{12}$$

In this equation, matrices L_w and L_u are plant model parameters which are identified using the subspace method. At time t, prediction controller can be designed using equation (12). The data column vectors over a horizon for the output and input signals are defined as:

$$\begin{aligned}\tilde{y}_f &= y_{t+1|t+N_2} = [y_{t+1} \cdots y_{t+N_2-1}\ y_{t+N_2}]^T \\ u_f &= u_{t|t+N_u} = [u_t \cdots u_{t+N_u-2}\ u_{t+N_u-1}]^T \\ w_p &= w_{t-N|t} = [y_{t-N} \cdots y_t\ u_{t-N} \cdots u_{t-1}]^T\end{aligned} \tag{13}$$

where N_2 and N_u are the prediction horizon and control horizon respectively, and N is the modelling horizon.

The following objective function is minimized to obtain the optimal control law:

$$J = (r_f - \tilde{y}_f)^T Q_y (r_f - \tilde{y}_f) + u_f^T P_u u_f \tag{14}$$

where Q_y is the tracking error weighting matrix and P_u is the control weighting matrix.

$$Q_y = \begin{bmatrix} q_1 & & & \\ & q_2 & & \\ & & \ddots & \\ & & & q_{N_2} \end{bmatrix} \qquad P_u = \begin{bmatrix} p_1 & & & \\ & p_2 & & \\ & & \ddots & \\ & & & p_{N_u} \end{bmatrix} \tag{15}$$

If the weightings on different time are all the same, the controller parameters could be defined as $Q_y = I_{N_2}$ and $P_u = pI_{N_u}$, where I_{N_2} and I_{N_u} are the identity matrices.

Then the optimal future control is:

$$u_f = (pI_{N_u} + L_u^T L_u)^{-1} L_u^T (r_f - L_w w_p) \tag{16}$$

The input and output data are collected at each sampling time and the model G_m is updated to design predictive controller.

3.2 Control Weighting and System Performance

Considering the following discrete state space model. The state space matrices G, H, C and D are obtained by discretizing equation (1).

$$\begin{aligned} x(k+1) &= Gx(k) + Hu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned} \tag{17}$$

$$\begin{aligned} G &= \begin{bmatrix} 0.8131 & -0.0452 \\ 0.0226 & 0.9994 \end{bmatrix} \quad H = \begin{bmatrix} 0.1807 \\ 0.0023 \end{bmatrix} \\ C &= [-0.3750\ 3] \qquad\qquad D = 0 \end{aligned} \tag{18}$$

Let the control horizon $N_u = 5$ and the prediction horizon $N_2 = 10$, p increases from 0 to 0.2 with an incremental step 0.05. The relations between control weighting p and system performance within a finite simulation time $T = 50s$ is shown in Fig. 7.

According to Fig. 7, it can be observed that as the control weighting p decreases, the tracking error decreases, while

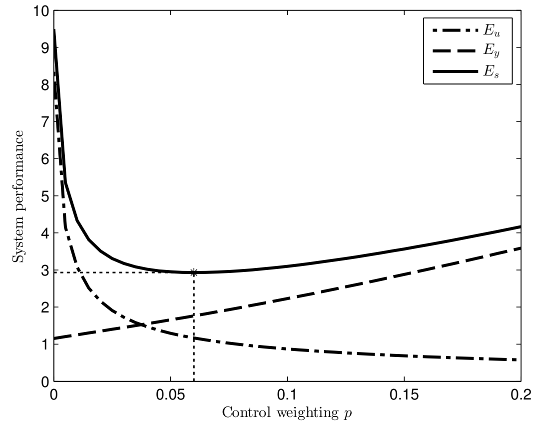

Fig. 7. Control energy and tracking error for different control weightings

the control energy increases and this correlation is non-linear. At a certain point, further increase of the control energy will have much less effect on the improvement of the tracking performance (in reducing the tracking errors). This is well demonstrated in the cost function curve which has a minimum point indicating that there exists a best trade-off between control energy consumption and tracking performance. When p is bigger than 0.1, the change of energy saving becomes insignificant, but the tracking error keeps increasing.

3.3 Tracking Performance and Control Weightings

To further illustrate correlation of tracking performance and control weighting, the system responses are detailed in Fig. 8 and Table 3, where $e_{ss}(\%)$ and u_{ss} are the steady state error and steady state input respectively.

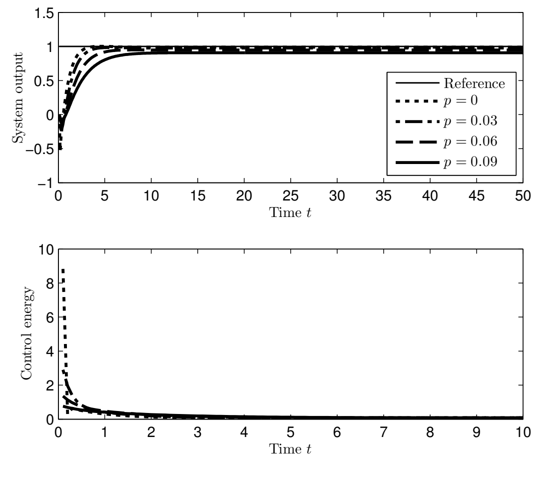

Fig. 8. Tracking performance with different p

Table 3. System performance with different p

p	E_u	E_y	E_s	$e_{ss}(\%)$	u_{ss}
0	8.34	**1.15**	9.49	**0.98**	0.083
0.03	1.73	1.45	3.18	3.02	0.081
0.06	1.17	1.76	**2.93**	7.21	0.077
0.09	**0.93**	2.11	3.03	12.96	**0.073**

For a better recognizable, the time scale of the bottom subgraph is reduced to $10s$. Fig. 8 and Table 3 show the cost function reaches the minimum when p is 0.06 which means the global optimal solution is not the local optimal. However, there exists a steady state tracking error and the difference of e_{ss} is much bigger than the difference of u_{ss} with different control weighting p. This means if the simulation keeps running, the minimum E_s could move to a smaller control weighting p. In this simulation, the energy cost is 1.73 and 1.17 when p is 0.03 and 0.06 respectively. For long term, it could potentially save a lots of energy. For a much more complicated situation when the cost function is defined with the incremental input, the system stability need to be considered and the matrix Q_y can not be simply set to the unit matrix.

4. CONCLUSIONS

This paper has investigated in detail the correlation between control energy and the corresponding system performance under different controller designs. Two popular control strategies that have been widely applied, including PI controller and subspace based model predictive control, are investigated to control a typical first order linear system with a time delay. It has been shown that this correlation is nonlinear for the design of the two controller designs, and improper controller design may only waste the control energy but bring little benefit to the improvement of the system performance. On the other hand, a proper controller design can not only save energy but also maintain desirable system performance.

Our future work is to study from the perspective of control theory to give a direct parameter optimal solution. And SMPC controller parameters optimization with incremental input and the effect of N_2 and N_u on the system performance also need further investigation. The approaches will be applied in the controller design for some real-world energy intensive systems, such as power electronics control in power transmission and distribution, and the integration of renewable resources with the grid, as well energy intensive manufacturing and processing systems.

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REFERENCES

Chen, J., Hara, S., and Chen, G. (2003). Best tracking and regulation performance under control energy constraint. *Automatic Control, IEEE Transactions on*, 48(8), 1320–1336.

Chen, J., Ren, Z., Hara, S., and Qin, L. (2001). Optimal tracking performance: preview control and exponential signals. *Automatic Control, IEEE Transactions on*, 46(10), 1647–1653.

Freudenberg, J. and Looze, D. (1985). Right half plane poles and zeros and design tradeoffs in feedback systems. *Automatic Control, IEEE Transactions on*, 30(6), 555–565.

Huang, B. and Kadali, R. (2008). *Dynamic modeling, predictive control and performance monitoring: A data-driven subspace approach*, volume 374. Springer.

IEA (2005). *World energy outlook*. OECD/IEA Press Releases, International, Energy, Agency (IEA), UK.

Katayama, T. (2005). *Subspace methods for system identification*. Springer.

Kwakernaak, H. and Sivan, R. (1972). *Linear optimal control systems*, volume 172. Wiley-Interscience New York.

Lewis, F.L., Vrabie, D., and Syrmos, V.L. (2012). *Optimal control*. Wiley. com.

Li, K., Wu, Y., Li, S., and Xi, Y. (2013). Energy saving and system performance-an art of trade-off for controller design. In *Systems, Man, and Cybernetics (SMC), 2013 IEEE International Conference on*, 4737–4742. IEEE.

Lucas, R.E. (2002). *Lectures on economic growth*. Harvard University Press.

Pal, B. and Chaudhuri, B. (2005). *Robust control in power systems*. Springer.

Qin, S.J. and Badgwell, T.A. (2003). A survey of industrial model predictive control technology. *Control engineering practice*, 11(7), 733–764.

TCG (2005). Profits up, carbon down. http://www.theclimategroup.org/what-we-do/publications/carbon-down-profits-up-new-edition/,The Climate Group (TCG).

Teleke, S., Baran, M.E., Bhattacharya, S., and Huang, A.Q. (2010). Optimal control of battery energy storage for wind farm dispatching. *Energy Conversion, IEEE Transactions on*, 25(3), 787–794.

Vidyasagar, M. (2011). *Control system synthesis: a factorization approach*. Morgan & Claypool Publishers.

Xue, D., Chen, Y., and Atherton, D.P. (2002). Linear feedback control. *Analysis and design with Matlab, Springer*.

Yamamoto, T., Takao, K., and Yamada, T. (2009). Design of a data-driven pid controller. *Control Systems Technology, IEEE Transactions on*, 17(1), 29–39.

Zheng, Y., Li, S., and Qiu, H. (2013). Networked coordination-based distributed model predictive control for large-scale system. *Control Systems Technology, IEEE Transactions on*, 21(3), 991–998.

Ziegler, J. and Nichols, N. (1942). Optimum settings for automatic controllers. *trans. ASME*, 64(11).