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A Reduced Dantzig-Wolfe Decomposition for a Suboptimal Linear MPC ⋆

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Abstract: Linear Model Predictive Control (MPC) is an efficient control technique that repeatedly solves online constrained linear programs. In this work we propose an economic linear MPC strategy for operation of energy systems consisting of multiple and independent power units. These systems cooperate to meet the supply of power demand by minimizing production costs. The control problem can be formulated as a linear program with block-angular structure. To speed-up the solution of the optimization control problem, we propose a reduced Dantzig-Wolfe decomposition. This decomposition algorithm computes a suboptimal solution to the economic linear MPC control problem and guarantees feasibility and stability. Finally, six scenarios are performed to show the decrease in computation time in comparison with the classic Dantzig-Wolfe algorithm.

1. INTRODUCTION

Recently, energy systems have evolved into highly integrated systems that deliver energy services to our homes and businesses. Electric power networks, also known as smart-grids, connect renewable energy sources (RES) to traditional power plants, cooling networks, as well as to other infrastructures. Increased reliability and performance, cost reduction, and minimized environmental impacts are the main benefits of the new energy systems. However, a major issue is the design of the controllers that coordinate and control the units of these energy systems to ensure that total energy production satisfies customer demand. Uncontrollable availability of renewable energy sources (RES), as well as fluctuations in consumer demand, yield power companies to utilizes dynamic control of energy systems in the view of handling such variabilities.

This paper focuses on the design of a distributed algorithm to compute optimal control sequences for a centralized controller. We propose a Linear Economic Model Predictive Control (MPC) strategy to coordinate and control the independent and controllable units of energy systems in the most economic way. Linear Economic MPC requires repeated online solution of constrained linear optimization problems. Therefore, the computational speed limits the application of such a controller. Energy systems have independent units, so the control problem has a block-angular structure and the Dantzig-Wolfe distributed optimization efficiently solves this class of linear programs. With regard to speeding up the controller, we outline a reduced Dantzig-Wolfe decomposition that reduces computation times and guarantees feasibility and stability. This reduced Dantzig-Wolfe decomposition can be applied to the Linear Economic MPC controller and calculates suboptimal local solutions.

MPC is a well-known control strategy that has been extensively used in several applications. Distributed model predictive control structures have attracted much attention, as shown in Scattolini (2009). Powerful tools to compute robust and efficient optimal control sequences were introduced by Conejo et al. (2006), who described how decomposition techniques can be applied to the control problems by exploiting their structures and efficiently solving the optimization problem. Sokoler et al. (2013) compared the Dantzig-Wolfe decentralized linear MPC with a centralized controller for large-scale systems and Standardi et al. (2013) introduced an early termination strategy to speed up the online computations; however, this approach involved unavoidable extra costs. With the aim of speeding up the control algorithms, suboptimal approaches were developed, guaranteeing feasibility and stability as reported in Scokaert et al. (1999); Zeilinger et al. (2008); Pannocchia et al. (2011). Rawlings et al. (2012) introduced the fundamentals of Economic MPC, the closed-loop properties that can be achieved, such as stability and convergence. However, few studies have addressed computational aspects of the Dantzig-Wolfe decomposition, and most of these works are about mixed integer and binary problems, see Kavinesh et al. (2009); Klein and Young (1999); Rios. and Ross (2014). Little work has been done on speeding up Dantzig-Wolfe decomposition for LPs. Burger et al. (2012) developed a distributed simplex algorithm for degenerate LPs, while Frangioni and Gendron (2013) introduced a stabilized Dantzig-Wolfe decomposition subject to several assumptions.

The outline of this paper is as follows. Section 2 introduces Linear Economic MPC. Dantzig-Wolfe decomposition and its novel reduced version are formulated in Section 3. Suboptimality and stability of the proposed algorithm are

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illustrated in Section 4. We show the performances of our approach using numerical examples in Section 5, while conclusions are in Section 6.

2. LINEAR ECONOMIC MPC COORDINATION OF ENERGY SYSTEMS

The new energy systems are built by connecting individual and controllable power units that need a controller to satisfy the customer demand. The control problem must compute for each power unit the most economic and optimal production plan. We introduce the Economic MPC strategy that balances power supply and demand for such energy systems.

The following stochastic discrete state-space model describes a power unit in energy systems

$$x_{k+1} = Ax_k + Bu_k + Gw_k + Ed_k \tag{1a}$$
$$y_k = Cx_k + v_k \tag{1b}$$
$$z_k = C_z x_k. \tag{1c}$$

x_k denotes the state variable and y_k is the measurement. Moreover, Standardi et al. (2012) includes process and measurement noises, respectively w_k and v_k, being distributed as $\sim N_{iid}(0, R_{ww})$ and $\sim N_{iid}(0, R_{vv})$. Due to the large shares of renewable energy sources (RES), the model needs to consider weather forecasts $d_k \sim N(\bar{d}_k, R_{dd,k})$ predicted by external prognosis systems. The manipulated variable, u_k, denotes the input signal and it is subject to hard constraints

$$u_{min} \leq u_k \leq u_{max} \tag{2a}$$
$$\Delta u_{min} \leq \Delta u_k \leq \Delta u_{max} \tag{2b}$$

z_k indicates the system output and it must be within the interval $[r_{\min,k},\ r_{\max,k}]$; this interval may represent forecast consumer demand, or it can define indoor temperature in a building, or temperatures in a refrigeration system, or state-of-charge of a battery

$$r_{\min,k} \leq z_k \leq r_{\max,k} \tag{3}$$

A decoupled Kalman filter estimates the state and the output variables, while the certainty equivalence principle substitutes all the variables with their mean values as described in Standardi et al. (2012). It is worth noting that this observer works locally for each unit and does not involve the entire energy system.

The control strategy computes the control trajectory in the most economic way, thus minimizing the production costs. For each power unit, the cost of following the production plan u_k is

$$\phi_{i,k} = \sum_{j=0}^{N-1} \hat{c}'_{i,k+j|k} \hat{u}_{i,k+j|k} \tag{4}$$

where $\hat{c}_{i,k+j|k}$ denotes the production costs and is forecast by external systems.

Altogether, the control problem is a linear problem because it applies to linear systems (1) subject to linear constraints (2)-(3) and it minimizes a linear cost function (4). Due to this economic objective, the controller optimizes directly online the economic performances of the energy systems computing the control sequences for each power unit. Therefore, the Economic MPC policy applied to an energy system consisting of P power units (1) can be expressed as

$$\min_{\hat{u}_{i,k+j|k}, \hat{\hat{s}}_{k+j+1|k}} \phi_k = \sum_{i=1}^{P} \phi_{i,k} + \sum_{j=0}^{N-1} \hat{\hat{\rho}}'_{k+j+1|k} \hat{\hat{s}}_{k+j+1|k} \tag{5}$$

subject to the local constraints $\forall i \in P$ and $\forall j \in \mathcal{N}$

$$\hat{x}_{i,k+j+1|k} = A_i \hat{x}_{i,k+j|k} + B_i \hat{u}_{i,k+j|k} + E_i \hat{d}_{i,k+j|k} \tag{6a}$$
$$\hat{z}_{i,k+j+1|k} = C_{z,i} \hat{x}_{i,k+j+1|k} \tag{6b}$$
$$u_{\min,i} \leq \hat{u}_{i,k+j|k} \leq u_{\max,i} \tag{6c}$$
$$\Delta u_{\min,i} \leq \Delta \hat{u}_{i,k+j|k} \leq \Delta u_{\max,i} \tag{6d}$$
$$\hat{r}_{\min,i,k+j+1|k} \leq \hat{z}_{i,k+j+1|k} \leq \hat{r}_{\max,i,k+j+1|k} \tag{6e}$$

and subject to the following connecting constraints $\forall j \in \mathcal{N}$ and $\forall i \in P$

$$\hat{\hat{z}}_{k+j+1|k} = \sum_{i=1}^{P} \tilde{C}_{z,i} \hat{x}_{i,k+j+1|k} \tag{7a}$$
$$\hat{\hat{z}}_{k+j+1|k} + \hat{\hat{s}}_{k+j+1|k} \geq \hat{\hat{r}}_{\min,k+j+1|k} \tag{7b}$$
$$\hat{\hat{z}}_{k+j+1|k} - \hat{\hat{s}}_{k+j+1|k} \leq \hat{\hat{r}}_{\max,k+j+1|k} \tag{7c}$$
$$\hat{\hat{s}}_{k+j+1|k} \geq 0 \tag{7d}$$

where $\hat{\hat{z}}_{k+j+1|k}$ denotes the overall power production, and $\hat{\hat{r}}_{\min,k}$ and $\hat{\hat{r}}_{\max,k}$ define customer demand forecasts. The connecting constraints include slack variables $\hat{\hat{s}}_{k+j+1|k}$; non-zero slack variables involve penalties $\hat{\hat{\rho}}_{k+j+1|k}$ to pay, as expressed in the objective function (5).

For large-scale energy systems consisting of multiple power units, the control problem (5)-(7) includes several variables and constraints; for this reason, decomposition techniques are investigated to efficiently compute the optimal control trajectories. Furthermore, the optimization control problem (5)-(7) consists of two sets of constraints: local constraints (6) for each power unit, and connecting constraints (7) for the overall energy system. This linear programming problem has a block-angular structure tailored for the implementation of the Dantzig-Wolfe decomposition to solve the control linear program. Section 3 introduces the Dantzig-Wolfe decomposition technique.

3. THE REDUCED DANTZIG-WOLFE DECOMPOSITION

The Dantzig-Wolfe decomposition is a specialized version of the Simplex Method to solve linear programming problems that have a block-matrix structure, see Dantzig and Thapa (2003). Among these systems, the block-angular systems have independent blocks defining local constraints and one set of coupling constraints. The linear programming problem (5)-(7) has a block-angular structure that defines local constraints (6) and a set of global constraints (7).

We consider the linear program (8) with the block-angular structure for $i \in \mathcal{M}$, where $\mathcal{M} = \{1, ..., M\}$

$$\min_{q_i} \quad c'_1 q_1 + ... + c'_M q_M \tag{8a}$$
$$s.t. \quad F_1 q_1 + ... + F_M q_M \geq f \tag{8b}$$
$$G_i q_i \geq g_i \tag{8c}$$

This LP has $i \in \mathcal{M}$, with $\mathcal{M} = \{1, ..., M\}$, blocks and each block defines a set of local constraints (8c) coupled through the connecting constraints (8b). Moreover, $q_i \in \mathbb{R}^n$ defines the vector of variables to be determined and $c_i \in \mathbb{R}^n$ is the vector of objective function coefficients. The block-angular constraints matrix consists of $F_i \in \mathbb{R}^{n_f \times n}$, representing the coupling constraints, and $G_i \in \mathbb{R}^{n_{g_i} \times n}$, denoting the local constraints; moreover, $f \in \mathbb{R}^{n_f}$ and $g_i \in \mathbb{R}^{n_{g_i}}$ are involved in the connecting and local constraints, respectively.

We briefly outline the classic Dantzig-Wolfe in Section 3.1 and the novel reduced form is introduced in Section 3.2.

3.1 Dantzig-Wolfe decomposition

The Dantzig-Wolfe algorithm is applied to the block-angular linear program (8), in which each column of coefficients can be freely chosen as any point from a convex set $\mathcal{Q}$ as stated in the Theorem 1 of convex combination.

Theorem 1. Let $\mathcal{Q}_i = \{q_i | G_i q_i \geq g_i\}$, with $i \in \mathcal{M}$ and $\mathcal{M} = \{1, ..., M\}$, be a polyhedral set in $\mathbb{R}^n$. Every point q_i in the polyhedral set $\mathcal{Q}_i$ can be expressed as a convex combination of the finite set $\mathcal{V} = \{1, ..., V\}$ of its extreme points v_i^j and a non-negative linear combination of the finite set $\mathcal{K} = \{1, ..., K\}$ of extreme rays r_i^k

$$q_i = \sum_{j=1}^{V} \alpha_{ij} v_i^j + \sum_{k=1}^{K} \beta_{ik} r_i^k, \quad \sum_{j=1}^{V} \alpha_{ij} = 1 \tag{9}$$

with $\alpha_{ij}, \beta_{ik} \geq 0$.

Proof. See Dantzig and Thapa, 2003.

For the block-angular LP (8), each set of feasible polyhedra $\mathcal{Q}_i$ is bounded, closed and non-empty, thus we only include the extreme points in the problem formulation as in Cheng et al. (2008). However, Dantzig and Thapa (2003) included the extreme rays in the problem formulation.

Substituting the convex combination (9) into the block-angular LP (8) formulates the master problem (MP) or extremal problem. It is worth noting that the MP has fewer rows in the coefficients matrix than the original block-angular program (8). However, the number of columns, and therefore also the number of variables, in the MP is larger, corresponding to all V extreme points of all M polyhedra.

The Dantzig-Wolfe does not solve the impractical full MP and generates at each iteration of the Simplex algorithm only the column of the MP that has been selected to come into basis. As a result, the algorithm formulates the reduced master problem (RMP) (10) for L vertices of the polyhedra, where $L \leq V$

$$\min_{\alpha_{ij}} \quad \gamma = \sum_{i=1}^{M} \sum_{j=1}^{L} p_i^j \alpha_{ij} \tag{10a}$$

$$s.t. \quad \sum_{i=1}^{M} h_i^j \alpha_{ij} \geq f \tag{10b}$$

$$\sum_{j=1}^{L} \alpha_{ij} = 1 \qquad i = 1, ..., M \tag{10c}$$

$$\alpha_{ij} \geq 0 \qquad i = 1, ..., M, j = 1, ..., L \tag{10d}$$

where α_{ij} is the optimization variable, γ is the objective function, and $p_i^j = c_i v_i^j$ and $h_i^j = F_i v_i^j$ denote the cost and the inequality constraints coefficients. However, in order to select which column has to come into basis, the RMP needs an initial basic feasible solution v_i^0. Dantzig and Thapa (2003) proposed an algorithm to obtain such a starting basic solution via Simplex Phase I. Similarly, Standardi et al. (2012) introduced a warm-start strategy specialized to the MPC strategy that provides initial basic feasible solutions without solving any linear problems.

Let us assume that the initial extreme points v_i^0 are available for each polyhedron $i \in \mathcal{M}$. Thus, the RMP provides the dual variables π and μ, respectively, for linking (10b) and convexity constraints (10c). The algorithm utilizes these dual variables to generate only the column having the most negative reduced cost without having to generate all the remaining columns of the MP. This pricing problem is expressed in the following subproblems

$$\min_{q_i} \quad \xi_i = \left[c_i - F_i^{'} \pi\right]^{'} q_i \tag{11a}$$

$$s.t \quad G_i q_i \geq g_i \tag{11b}$$

where ξ_i denotes objective function for the subproblem i. It is evident that each subproblem $i \in \mathcal{M}$ (11) is independent and decoupled; hence, parallel computing techniques can efficiently compute these i optimal solutions. The optimal solution of the subproblem (11) identifies which column has the smallest reduced cost for the MP. Thus, if the optimal objective function value ξ_i^* satisfies the following condition

$$\xi_i^* - \mu_i \geq 0 \quad \forall i \in \mathcal{M} \tag{12}$$

then all the reduced costs for the MP will be non-negative. Hence, the Dantzig-Wolfe algorithm has an optimal solution to the MP and, consequently, to the original block-angular problem (8) through convex combination (9).

In contrast, if $\xi_i^* - \mu_i < 0$, then we augment the columns of the RMP by

$$p_i^{j+1} = c_i v_i^j \qquad h_i^{j+1} = F_i v_i^j \tag{13}$$

where $v_i^j = q_i^j$ is the optimal basic feasible solution of (11).

The classic Dantzig-Wolfe decomposition is illustrated in Algorithm 1.

3.2 Reduced Dantzig-Wolfe decomposition

In this work, we propose a reduced version of the Dantzig-Wolfe decomposition.

At each iteration of the Simplex algorithm, the Dantzig-Wolfe decomposition computes only the column of the RMP (10), which has to come into basis. This column has the most negative reduced cost. Moreover, let us assume that at iteration t, only a set of subproblems $\mathcal{S} \subset \mathcal{M}$ satisfies the optimality condition (12)

$$\xi_s^* - \mu_s \geq 0 \quad s \in \mathcal{S} \subset \mathcal{M} \tag{14}$$

In such a scenario, the classic Dantzig-Wolfe brings variables into basis by adding columns to the RMP (10) for every subproblem $i \in \mathcal{M}$, hence even for the set $\mathcal{S}$ of subproblems.

Algorithm 1 Classic Dantzig-Wolfe

Require: Initial feasible vertex for the RMP (10), see Section 3.1.
  if Any points are found **then**
    Stop.
  else
    L=1
    while $Converged == false$ **do**
      Solve the $L-th$ RMP (10).
      Solve subproblem i (11), $\forall i \in \mathcal{M}$.
      if optimality condition (12) is satisfied $\forall i \in \mathcal{M}$ **then**
        $Converged == true$
      else
        Compute RMP coefficients $\forall i \in \mathcal{M}$ (13).
      end if
      $L = L + 1$
    end while
  end if

In contrast, if condition (14) holds, then the reduced Dantzig-Wolfe does not add columns to the RMP (10) for the set $\mathcal{S}$ of subproblems; this yields to update the coefficients od the RMP as

$$p_i^{t+1} = c_i v_i^t \quad h_i^{t+1} = F_i v_i^t, \qquad i \in \mathcal{M}\backslash\mathcal{S} \tag{15}$$

At iteration $t+1$, the reduced Dantzig-Wolfe solves the following subproblems

$$\min_{q_i} \quad \xi_i = \left[c_i - F_i^{'}\pi\right]^{'} q_i \qquad i \in \mathcal{M}\backslash\mathcal{S} \tag{16a}$$

$$s.t \quad G_i q_i \geq g_i \tag{16b}$$

Consequently, the Dantzig-Wolfe applies the pricing problem on a reduced set of subproblems $\mathcal{S}$. As a result, by applying this reduced Dantzig-Wolfe decomposition, the number of iterations decreases. Algorithm 2 illustrates the reduced Dantzig-Wolfe decomposition.

4. SUBOPTIMALITY AND STABILITY IN LINEAR MPC VIA REDUCED DANTZIG-WOLFE

The reduced Dantzig-Wolfe decomposition computes a solution to the block-angular problem (8) that is not optimal but it is feasible. In this Section we illustrate suboptimality of the reduced Dantzig-Wolfe decomposition; moreover, we demonstrate that this decomposition technique does not affect the convergence, thereby it guarantees feasibility and stability.

Subptimality The reduced Dantzig-Wolfe computes a suboptimal solution to the block-angular problem (8). In order to explain this suboptimality, we introduce Theorem 2 that provides the optimal solution for the MP in the Dantzig-Wolfe decomposition.

Theorem 2. An optimal basic feasible solution of the RMP (10) is also optimal for the MP if

$$\xi_i^* = \mu_i \qquad \forall i \in \mathcal{M} \tag{17}$$

Then the algorithm computes the optimum in a finite number of iterations.

Proof. See Dantzig and Thapa, 2003.

Algorithm 2 Reduced Dantzig-Wolfe

Require: Initial feasible vertex for the RMP (10), see Section 3.1.
  if Any points are found **then**
    Stop.
  else
    $\mathcal{S} = \{\emptyset\}$
    L=1
    while $Converged == false$ **do**
      Solve the $L-th$ RMP (10).
      Solve subproblem i (16), for $i \in \mathcal{M}\backslash\mathcal{S}$.
      if optimality condition (12) is satisfied $\forall i \in \mathcal{M}$ **then**
        $Converged == true$
      else
        if a subproblem s, $s \in \mathcal{M}$, satisfies the optimality condition (12) **then**
          $\mathcal{S} = \{s\}, \mathcal{S} \subset \mathcal{M}$.
          Compute RMP coefficients $\forall i \in \mathcal{M}\backslash\mathcal{S}$.
        else
          Compute RMP coefficients (13) $\forall i \in \mathcal{M}\backslash\mathcal{S}$.
        end if
      end if
      $L = L + 1$
    end while
  end if

Theorem 2 states that the optimal basic feasible solution of the MP is given when every subproblem i, $i \in \mathcal{M}$, satisfies the condition (17). The reduced Dantzig-Wolfe decomposition does not compute the optimality condition (17) for every subproblem at the same time. Instead, this reduced version stops computing the optimal solution for a subproblem when this satisfies the optimality condition (12), even if the other subproblems do not provide an optimal solution. Accordingly, the reduced Dantzig-Wolfe decomposition does not provide an optimal solution, as Theorem 2 is not satisfied; thus, the solution is suboptimal, however feasible.

Feasibility and stability Scokaert et al. (1999); Muske and Rawlings (1993); Mayne et al. (2000); Chisci et al. (1996) demonstrated how feasibility implies stability for a linear MPC strategy. Because of this, the following theorem illustrates the feasibility of the reduced Dantzig-Wolfe decomposition.

Theorem 3. Any α_{ij} that solves the RMP (10) determines a feasible solution q_i for the block-angular program (8) by the convex combination (9). Moreover, if γ has the minimum of the RMP (10) for α_i^*, then the convex combination (9) generates an optimal feasible solution q_i^* to the original problem (8).

Proof. See Dantzig and Thapa, 2003.

Therefore, the reduced Dantzig-Wolfe decomposition guarantees feasibility that suffices for stability.

5. COMPUTATIONAL RESULTS

As mentioned previously, our intention is to show that the novel reduced Dantzig-Wolfe decomposition speeds up the algorithm, guaranteeing feasibility and stability. In this section, we compare the performances of both classic

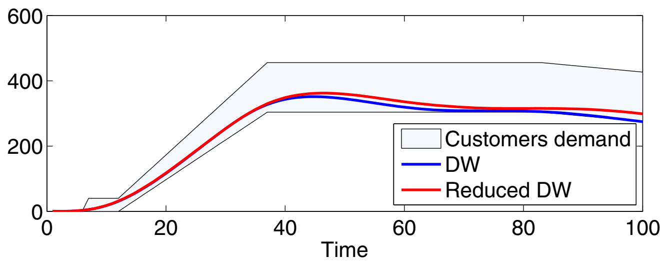

Fig. 1. Overall power production of energy system consisting of 75 power units in closed-loop simulations. The grey area defines the customer demand interval. The control problem is solved applying both classic Dantzig-Wolfe, blue graph, and novel reduced decomposition, red plot.

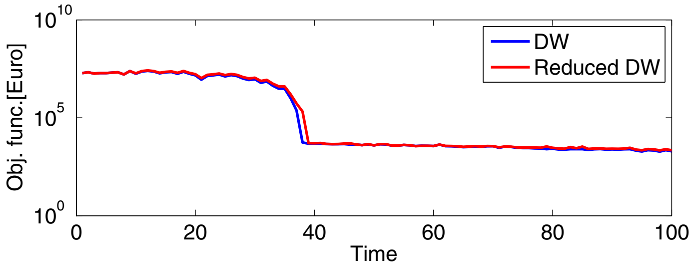

Fig. 2. Objective function (total production costs) of energy system consisting of 75 power units in closed-loop simulations. Simulations run both classic Dantzig-Wolfe, blue graph, and novel reduced decomposition, red plot.

and reduced Dantzig-Wolfe decomposition, as described in the previous section. These are implemented in MATLAB in closed-loop simulations. Section 3.1 introduces the need of initial basic feasible solutions for the RMP. We apply the warm-start technique described in Standardi et al. (2012). Moreover, as mentioned in Section 2, we assume to have the forecasts for weather d_k, costs $\hat{c}_{i,k}$ and penalties $\hat{\hat{\rho}}$. The output bounds $\hat{r}_{\min,k}$ and $\hat{r}_{\max,k}$ represent customer demand interval; these power consumption forecasts are taken from the Nord Pool Spot Power Market and the bounds are derived according to real data from Nord Pool Spot (2012).

Our case studies are energy systems consisting of multiple power units. In particular, these controllable units might represent thermal power plants, gas turbines and diesel generators. We model these units as described in Edlund et al. (2010)

$$Z_i(s) = \frac{1}{(\tau_i s + 1)^3} \left(U_i(s) + D_i(s) \right) \tag{18}$$

where $U_i(s)$ denotes the control signal, $D_i(s)$ is the process noise, and $Z_i(s)$ is the power produced. We consider six energy systems consisting of: 25, 50, 75, 100, 125 and 150 power units. Furthermore, the time horizon is $N = 70$, sampling time is 1 second and time steps are 100. The reduced Dantzig-Wolfe decomposition computes the control trajectories for each power units of the energy system considered. We observe from Figure 1 that the overall power production given by the implementation of the classic Dantzig-Wolfe decomposition satisfies customer demand as well. As expected, the suboptimal control sequence

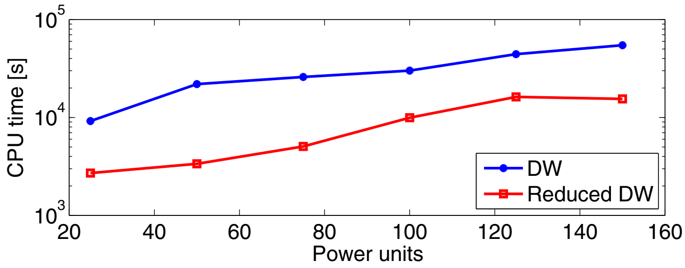

Fig. 3. Computation time for the classic Dantzig-Wolfe, blue graph, and for the reduced version, red plot, Vs. Number of power units in the energy system.

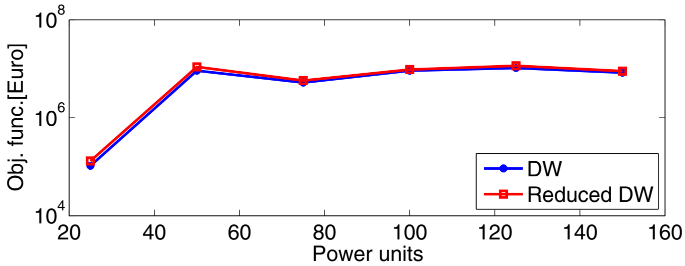

Fig. 4. Objective function optimal values for the classic Dantzig-Wolfe, blue plot, and for the reduced version, red plot, Vs. Number of power units in the energy system.

given by the reduced Dantzig-Wolfe decomposition makes the overall power production meet the customer demand. For the sake of completeness, results show the effect of sub optimality in the deterioration of the objective function. Figure 2 shows the objective function values of this case study including 75 power units. The reduced Dantzig-Wolfe decomposition is more expensive as it has higher costs (red graph in the plot) than the classic Dantzig-Wolfe decomposition (blue graph).

Let us consider all six case studies. Figure 3 shows that the reduced Dantzig-Wolfe decomposition quickens the controller, reducing computation times for all the study cases. Moreover, Figure 4 illustrates the objective function optimal values given by the reduced Dantzig-Wolfe decomposition and the classic algorithm. In order to examine the algorithm performances, Figure 5 shows the percent decrease in the computational time and the percent change in the optimal values of the objective function. The computation times decrease up to 80%, while the deterioration in the objective function optimal value exceeds 20% (upper dashed line) for only 1 case study. Moreover, the percent deterioration of the objective function is often below 10% (lower dashed line), even when the number of power unit in the case study increases.

6. CONCLUSIONS

In this paper we have introduced a reduced Dantzig-Wolfe decomposition for linear Economic MPC controllers. The problem formulation has been formulated as a linear economic MPC strategy to control energy systems consisting of multiple independent units. We have briefly described the classic Dantzig-Wolfe optimization and then derived the reduced version. We have demonstrated how the novel reduced Dantzig-Wolfe decomposition supports

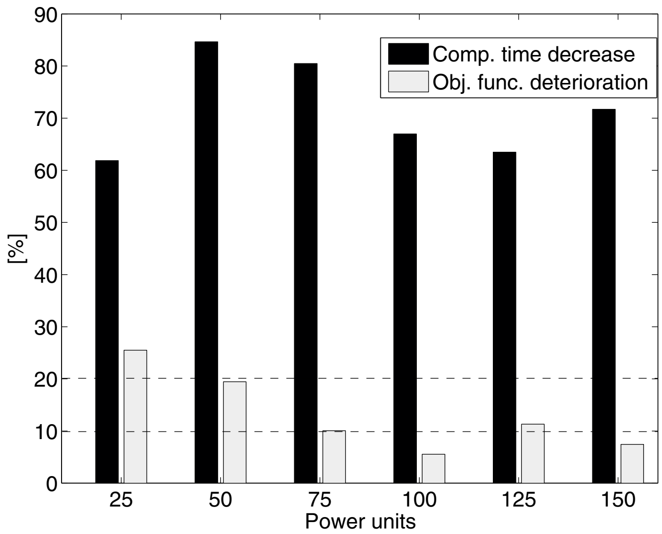

Fig. 5. Percent changes, Vs. Number of power units in the energy system. Black bars denote percent decrease in computation time, while grey bars denote percent change in objective function optimal values. These reductions in computational time and optimal values are the reductions compared with the classic Dantzig-Wolfe algorithm.

suboptimal and feasible solution for LPs; moreover, we have illustrated the stability of the proposed algorithm. We have collected the reduced Dantzig-Wolfe decomposition computation results for six case studies in closed-loop simulations. Results have demonstrated that the proposed reduced Dantzig-Wolfe decomposition speeds up the algorithm. Our study represents a new approach to the solution of linear MPC and improves its applicability. The proposed algorithm guarantees feasibility and stability computing a suboptimal solution. The reduced Dantzig-Wolfe decomposition can be applied to a wide range of systems and it has potential in areas such as independent units building up a larger system.

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