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A probabilistic approach for data-driven fault isolation in multimode processes

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Abstract: This paper addresses the problem of fault isolation in processes which are working in different operating points. Due to nonlinearities and set-point changes, the statistical model which is obtained from data is different from one operating point to another. Therefore the classical multivariate statistical process monitoring approaches may not be suitable for monitoring and diagnosis purposes. For that, a data-driven fault isolation method is proposed which splits the process into several local models. Based on the local models, a probabilistic approach is proposed to determine the contribution of each variable to the fault detection index and find the risky variables which are responsible for the fault. Finally, the proposed method is demonstrated through its application on a laboratory setup of continuous stirred tank heater.

Keywords: Fault Isolation, Nonlinear Processes, Multimode Systems, Bayesian Approach, Data-driven Techniques

1. INTRODUCTION

Fault isolation plays a central role in process monitoring and diagnosis and sometimes evolves to a real challenge for process engineers. Basically, fault isolation is the task of gaining process information about the location of the fault in the process, Ding (2008). In a modern large scale process where the number of faults, process components and measurements are huge the fault isolation becomes a severe problem.

For decades, model-based methods have been widely used to design fault diagnosis systems, Ding (2008). These approaches involve rigorous development of process model based on first principles. During last decades, the complexity of process plants has been increased, which imposes great challenges in development of model-based monitoring approaches and it becomes sometimes unrealistic for modern large scale processes. Alternative to model-based approaches, data-driven methods have been developed, which offer powerful tools to extract useful information for design of monitoring systems based on the available process measurements. Multivariate statistical process monitoring (MSPM) approaches were successfully applied for fault detection and diagnosis in many technical processes. Principal component analysis (PCA), Qin (2003), partial least squares (PLS), Wise and Gallagher (1996) and their nonlinear and dynamic variants were studied in academic communities and applied in wide range of industrial applications for fault detection. On the other hand, several methods exist for diagnosis purposes. Most popular ones are contribution analysis methods, Alcala and Qin (2009a); Cherry and Qin (2006); Raich and inar (1996) , which determine the contribution of variables to the fault detection indices. The contribution analysis methods may not explicitly determine the cause of fault alarm, but it can be used as a guideline for process engineers to find the source of fault.

The classical MSPM approaches are based on the assumption that the data follows unimodel multivariate normal distribution. In many industrial applications, the process under consideration operates in different operating regimes due to different product specifications, working environments and economic considerations. In these cases, the above mentioned assumption might not be satisfied due to nonlinearities in the process. Thus the model derived from data may be only valid in one operating point. Moreover, to design a monitoring system and achieve higher level of automation, the statistical model should overlay all possible normal behavior of process and its underlying structure. To cope with this problem, recently multimode process monitoring approaches based on the mixture modeling are developed. The process under consideration is assumed to be linear in each operating point and the data available for each operating point follow multivariate normal distribution with different parameters. The task of mixture modeling is to estimate the parameters of each local model. In Yu and Qin (2008), a method is developed for fault detection in multimode processes based on identification of finite Gaussian mixture model (FGMM). For fault detection purpose, the authors have developed a Bayesian inference strategy which combines the local hypotheses with the posterior probabilities of each local model. In Ge and Song (2010), the authors have extended the probabilistic PCA (PPCA) approach Tipping and Bishop (1999); Kim and Lee (2003) for multimode processes monitoring. Moreover, a Bayesian regularization has been introduced as well to determine the effective dimension of principal subspace.

Although different methods have been developed for data-driven fault detection in multimode processes, the fault diagnosis and isolation aspects have not been taken into consideration extensively. In Chen and Sun (2009), the authors have developed a probabilistic contribution analysis method based on missing variable approach. Once a fault is detected, the monitoring index will be recalculated with one variable being missing. This will be repeated for all variables. The variable corresponding to the smallest recalculated index will be denoted as the risky variable. The proposed idea has been extended to PPCA mixture model for fault detection and diagnosis in multimode processes.

Motivated by the above mentioned works, a new probabilistic approach for fault diagnosis in multimode processes is introduced in this paper. to achieve that, a unified index is used for fault detection. Similar to standard reconstruction based contribution (RBC) analysis, once a fault is detected, the index will be decomposed into two parts, one representing the behavior of the monitoring index in normal operating conditions and the other representing the contribution of the variables to the fault which will be used for diagnosis purpose.

The structure of this paper is as follows. In Section 2, a brief overview of multimode MSPM methods will be presented. Section 3 describes the proposed diagnosis approach for multimode processes. The paper continues with application of the proposed method on a laboratory-scale continuous stirred tank heater (CSTH) in Section 4 and ends with concluding remarks in Section 5.

2. MULTIMODE FAULT DETECTION

PCA is most popular MSPM approach and has been successfully applied for chemical process monitoring. Let $\mathbf{x} \in \mathbb{R}^m$ be a sample of m sensors. The data matrix $\mathbf{X} \in \mathbb{R}^{N\times m}$ can be formed using N samples of measurements. The matrix X is scaled to zero mean and unit variance. The PCA based fault detection system design involves performing an SVD on covariance data matrix X in training step

$$\frac{1}{N-1}\mathbf{X}\mathbf{X}^T = [\mathbf{P}_{pc}\ \ \mathbf{P}_{res}] \begin{bmatrix} \Lambda_{pc} & 0 \\ 0 & \Lambda_{res} \end{bmatrix} [\mathbf{P}_{pc}\ \ \mathbf{P}_{res}]^T, \qquad (1)$$

and using T^2 and SPE indices in monitoring step, where

$$\begin{aligned} T^2 &= \mathbf{x}^T \mathbf{P}_{pc} \Lambda_{pc} \mathbf{P}_{pc}^T x \\ SPE &= \mathbf{x}^T \mathbf{P}_{res} \mathbf{P}_{res}^T \mathbf{x}. \end{aligned} \qquad (2)$$

Modern complex industrial plants are commonly working on multiple operating conditions because of different product specifications and some external restrictions. As a consequence, the plant characteristics change from one operating condition to another due to nonlinearity and set-point changes in the system. Hence, the statistical model obtained from traditional MSPM techniques for an operating mode is not valid anymore for the others and will induce false alarms. This is because the basic assumption in MSPM methods that the data should follow unimodal Gaussian distribution. Under the assumption that the data corresponding to each operating point follows a multivariate Gaussian distribution with various statistical properties, the available historical data can be seen as a mixture of Gaussian components with different mean vectors and covariance matrices.

Recently, some research efforts have been done for data-driven nonlinear process identification and monitoring based on the multiple model assumption with the help of mixture modeling tools. The PCA process monitoring method has been extended to be used for multimode process monitoring under Gaussian mixture model (GMM) assumption (see for example Yu and Qin (2008); Yu (2012a); Chen et al. (2006); Ge and Song (2009); Ge et al. (2010); Haghani et al. (2012a,b)) and their applications in monitoring of batch processes and semiconductor technology have been reported (see for instance Chen and Zhang (2010); Yu and Qin (2009); Yu (2012b, 2011, 2012c)).

After obtaining the GMM, the PCA can be applied on each local model to detect the faults, under the assumption that the monitored sample is generated on the considered local model. In order to generalize the fault detection to the global model, using the results of local fault detection following form of fault detection indicator

$$p(\mathbf{x}(k) \in f) = \sum_{i=1}^{K} p(\mathbf{x}(k) \in f | \mathbf{x} \in \mathscr{M}_i) p(\mathbf{x} \in \mathscr{M}_i), \qquad (3)$$

where $p(\mathbf{x}(k) \in f | \mathbf{x}(k) \in \mathscr{M}_i)$ represents the local fault indicator and $p(\mathbf{x} \in \mathscr{M}_i)$ represents the hypothesis that the sample is generated in i^{th} local model. The application of the global index in fault detection and its derivation have been discussed by Yu and Qin (2008); Yu (2012c); Haghani et al. (2012a,b).

3. PROBABILISTIC FAULT DIAGNOSIS

In practice, fault detection is usually followed by the isolation step where location of the fault is determined. In context of MSPM, isolation is usually accomplished with contribution analysis where the process variables contributing to the fault are determined and a contribution plot is constructed. The statistical model which is built by MSPM methods will be used for analysis of the contribution of process variables or latent variables on the fault detection indices. Methods based on complete and partial decomposition and angle based contribution analysis have been developed and successfully applied for diagnosis purposes. In these approaches, usually the quadratic form of fault detection index is considered:

$$\text{Index}(\mathbf{x}) = \bar{\mathbf{x}}^T \mathbf{D} \bar{\mathbf{x}}, \qquad (4)$$

where $\bar{\mathbf{x}} \in \mathbb{R}^m$ is the normalized process measurement and the matrix $\mathbf{D}$ is constructed based on the monitoring index and the MSPM method, for instance in PCA-based process monitoring $\mathbf{D} = \mathbf{P}_{pc}\Lambda_{pc}^{-1}\mathbf{P}_{pc}^T$ for T^2 (see (2)). The monitoring index can be decomposed as

$$\begin{aligned} \text{Index}(\mathbf{x}) &= \bar{\mathbf{x}}^T \mathbf{D} \bar{\mathbf{x}} = ||\mathbf{D}^{(1/2)} \bar{\mathbf{x}}||^2 \\ &= \sum_{j=1}^{m} \left(\xi_j^T \mathbf{D}^{(1/2)} \bar{\mathbf{x}} \right)^2 = \sum_{j=1}^{m} c_j^{\text{Index}}, \end{aligned} \qquad (5)$$

where c_j^{Index} is the contribution of the variable x_j to $\text{Index}(\mathbf{x})$ and ξ_j is the j^{th} column of identity matrix as proposed by Miller et al. (1998).

Recently, in the work of Alcala and Qin (2009b), it has been revealed that the standard contribution analysis may lead to misdiagnosis of the faults and an alternative method has been proposed. The method is based on the reconstruction of a fault detection index along a variable direction, hence it is called reconstruction based contribution (RBC). When a fault happens in the system with direction ξ_j, the reconstructed measurement vector can be represented as:

$$\mathbf{z}_j = \bar{\mathbf{x}} - \xi_j \mathbf{f}, \qquad (6)$$

where $\mathbf{f}$ is the reconstructed part to be determined and $\mathbf{z}_j$ represents the fault-free behavior of variables and can be constructed by finding the value of $\mathbf{f}$ which minimizes $\text{Index}(\mathbf{z}_j)$

$$\text{Index}(\mathbf{z}_j) = \mathbf{z}_j^T \mathbf{D} \mathbf{z}_j = ||\mathbf{D}^{(1/2)}(\bar{\mathbf{x}} - \xi_j \mathbf{f})||^2. \qquad (7)$$

The optimal value for $\mathbf{f}$ can be obtained by derivation of $\text{Index}(\mathbf{z}_j)$ with respect to $\mathbf{f}$:

$$\frac{\text{d}(\text{Index}(\mathbf{z}_j))}{\text{d}\mathbf{f}} = -2(\bar{\mathbf{x}} - \xi_j\mathbf{f})^T\mathbf{D}\xi_j. \tag{8}$$

Setting (8) to zero yields to:

$$\mathbf{f} = (\xi_j^T\mathbf{D}\xi_j)^{-1}\xi_j^T\mathbf{D}\bar{\mathbf{x}}. \tag{9}$$

The reconstruction-based contribution of the variable x_j to fault detection index $\text{Index}(\mathbf{x})$ can be described by

$$RBC_j^{\text{Index}} = ||\mathbf{D}^{(1/2)}\xi_j\mathbf{f}||^2 \tag{10}$$

or

$$RBC_j^{\text{Index}} = \bar{\mathbf{x}}^T\mathbf{D}\xi_j(\xi_j^T\mathbf{D}\xi_j)^{-1}\xi_j^T\mathbf{D}\bar{\mathbf{x}}. \tag{11}$$

It is interesting to point out that following relation exists between fault detection index, reconstructed index and reconstruction based contribution:

$$\text{Index}(\mathbf{x}) = \text{Index}(\mathbf{z}_j) + RBC_j^{\text{Index}}. \tag{12}$$

Both reconstructed index, $\text{Index}(\mathbf{z}_j)$ and RBC_j^{Index} can be used for diagnosis purpose as shown by Alcala and Qin (2009b); Dunia and Qin (1998).

Based on contribution analysis concept, several methods have been developed and applied to different applications. An overview of these methods and their generalization is given in Alcala and Joe Qin (2011) and references therein.

Contribution analysis methods usually assume a single normal operating mode for the plant. In this context, an abnormal event will form a new operating region and the difference between normal and faulty states is used to identify the variable contribution. In many real applications the process itself works in different operating region and using standard contribution analysis methods would lead to misdiagnosis of the faults. To solve this problem, in this section a new fault isolation method is proposed which follows the multimode fault detection concept introduced in Haghani et al. (2012a) and tries to represent the variable contributions in a probabilistic form.

To extend the fault isolation approaches to multimode cases, the fault detection indices are generalized as an index which represents the probability of fault $p(\mathbf{x}(k) \in f)$ given a sample of measurement $\mathbf{x}(k)$. Using marginalization the above mentioned probability can be represented as:

$$p(\mathbf{x}(k) \in f) = \sum_{i=1}^{K} p(\mathbf{x}(k) \in f|\mathbf{x} \in \mathscr{M}_i)p(\mathbf{x} \in \mathscr{M}_i), \tag{13}$$

where $p(\mathbf{x}(k) \in f|\mathbf{x}(k) \in \mathscr{M}_i)$ can be calculated by integrating the probability density function (pdf) of the local fault detection index up to its current value. In other words:

$$\begin{aligned} p(\mathbf{x}(k) \in f|\mathbf{x}(k) \in \mathscr{M}_i) &= p(\text{Index}(\mathbf{x},i) \leq \text{Index}(\mathbf{x}(k),i)) \\ &= \int_0^{\text{Index}(\mathbf{x}(k),i)} \text{pdf}(\text{Index}(\mathbf{x},i))\text{d}\mathbf{x}, \end{aligned} \tag{14}$$

where $\text{Index}(.,i)$ represents the calculated value of the local index based on the assumption that the sample belongs to $\mathscr{M}_i$. The index in (14) also serves as a local fault indicator for mode $\mathscr{M}_i$. Since $0 \leq p(\mathbf{x}(k) \in f) \leq 1$, a confidence level $(1-\alpha)$ can be specified for fault detection purpose with the following hypothesis:

$$\begin{cases} p(\mathbf{x}(k) \in f) \leq 1-\alpha & \text{fault free} \\ p(\mathbf{x}(k) \in f) > 1-\alpha & \text{faulty} \end{cases} \tag{15}$$

The main idea of this new fault isolation approach is to calculate the contribution of a faulty measurement sample $\mathbf{x}(k)$, assuming that it belongs to mode $\mathscr{M}_i$ and then combine it with the hypothesis that the measurement $\mathbf{x}(k)$ is generated under the mode $\mathscr{M}_i$. The local fault detection index, $\text{Index}(\mathbf{x}(k),i)$ can be decomposed using (12) as

$$\text{Index}(\mathbf{x}(k),i) = \text{Index}(\mathbf{z}_j(k),i) + RBC_{j,i}^{\text{Index}}, \tag{16}$$

where $\text{Index}(\mathbf{z}_j(k),i)$ is the detection index according to the reconstructed measurement along variable $x_j(k)$, assuming $\mathscr{M}_i$ as the current operating mode and $RBC_{j,i}^{\text{Index}}$ is the amount of reconstruction based contribution for that. Using (16), the (14) can be rewritten as

$$\begin{aligned} p(\mathbf{x}(k) \in f|\mathbf{x}(k) \in \mathscr{M}_i) = & \\ & \int_0^{\text{Index}(\mathbf{x}(k),i)-\text{Index}(\mathbf{z}_j(k),i)} \text{pdf}(\text{Index}(\mathbf{x},i))\text{d}\mathbf{x} \\ & + \int_{\text{Index}(\mathbf{x}(k),i)-\text{Index}(\mathbf{z}_j(k),i)}^{\text{Index}(\mathbf{x}(k),i)} \text{pdf}(\text{Index}(\mathbf{x},i))\text{d}\mathbf{x}. \end{aligned} \tag{17}$$

The first term on right hand side of (17) represents the effects of reconstructed contribution $RBC_{j,i}^{\text{Index}}$ into the local fault probability and the second term represents the contribution of reconstructed variable to the local fault probability. That means, the first term can be used to evaluate the effect of the variable x_j on the deviation in local fault detection indicator $p(\mathbf{x}(k) \in f|\mathbf{x} \in \mathscr{M}_i)$. Moreover the first term in (17) can be expressed as

$$\begin{aligned} PRBC_{j,i}^{\text{Index}} &= \int_0^{\text{Index}(\mathbf{x}(k),i)-\text{Index}(\mathbf{z}_j(k),i)} \text{pdf}(\text{Index}(\mathbf{x},i))\text{d}\mathbf{x} \\ &= p(0 \leq \text{Index}(\mathbf{x},i) \leq \text{Index}(\mathbf{x}(k),i) \\ &\quad - \text{Index}(\mathbf{z}_j(k),i)|\mathbf{x}(k) \in \mathscr{M}_i), \end{aligned} \tag{18}$$

where *PRBC* stands for probabilistic *RBC*. Assuming that the sample $\mathbf{x}(k)$ is generated in mode $\mathscr{M}_i$, the variable x_j corresponding to the highest $PRBC_{j,i}^{\text{Index}}$ is the most responsible variable for deviation in the fault detection indicator. This concept is shown in Fig. 1. The pdf of index is known *a priori* (e.g. by considering normal distribution for data in each mode, it can be approximated by χ^2 distribution). The $PRBC_{j,i}^{\text{Index}}$ is calculated by integrating the pdf of local fault detection index up to current value of $RBC_{j,i}^{\text{Index}}$, or by calculating its cdf.

Generalizing it to the multimode processes, a *PRBC* can be defined as:

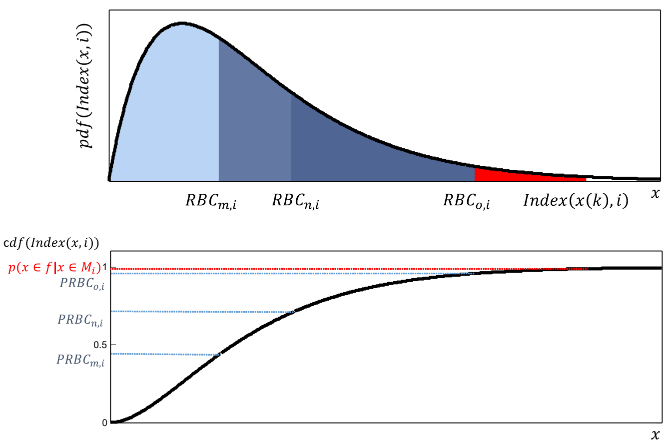

Fig. 1. Probabilistic reconstruction based contribution analysis

$$PRBC_j^{\text{Index}} = \sum_{i=1}^{K} PRBC_{j,i}^{\text{Index}} p(\mathbf{x}(k) \in \mathscr{M}_i)$$
$$= \sum_{i=1}^{K} p(0 \leq \text{Index}(\mathbf{x}, i) \leq \text{Index}(\mathbf{x}(k), i)$$
$$- \text{Index}(\mathbf{z}_j(k), i) | \mathbf{x}(k) \in \mathscr{M}_i) p(\mathbf{x}(k) \in \mathscr{M}_i). \quad (19)$$

The posterior probability $p(\mathbf{x}(k) \in \mathscr{M}_i)$ is used in (19) to incorporate the contribution of each local model to the $PRBC_j^{\text{Index}}$. The $PRBC_j^{\text{Index}}$ in (19) represents the contribution of variable x_j in the global fault detection index in (13). In standard RBC, after detection of a fault the contribution of each variable to the fault detection index, $Index(\mathbf{x}(k))$ is calculated according to (11). In contrast, here when a fault is detected using (13), $p(\mathbf{x}(k) \in f)$ will be decomposed into $PRBC_j^{Index}$ according to (19) using (18), which indicates which process variable has the highest contribution in deviation of $p(\mathbf{x}(k) \in f)$.

It is worth pointing out that $0 \leq PRBC_j^{\text{Index}} \leq 1$ and the variable $x_j(k)$ with the highest $PRBC_j^{\text{Index}}$ has more contribution to the fault and possibly represents the source of the malfunction in the system.

To calculate the probability $p(\mathbf{x}(k) \in \mathscr{M}_i)$ the posterior probability that the reconstructed measurement $\mathbf{z}_j(k)$ belongs to the mode $\mathscr{M}_i$ is used. This is due to the fact that $\mathbf{x}(k)$ is assumed to be faulty measurement, therefore may not provide a correct representation of the actual operating mode of the process. Therefore the reconstructed measurement $\mathbf{z}_j(k)$ which represents the estimation of fault free measurement assuming that the fault happens in the j^{th} sensor, is replaced in marginalization. To this end, the Bayesian inference strategy is used to calculate this posterior probability

$$p(\mathbf{z}_j(k) \in \mathscr{M}_i) = \frac{p(\mathbf{z}_j(k)|\mathscr{M}_i)p(\mathscr{M}_i)}{p(\mathbf{z}_j(k))} = \frac{w_i g(\mathbf{z}_j(k)|\theta_i)}{\sum_{l=1}^{K} w_l g(\mathbf{z}_j(k)|\theta_l)}. \quad (20)$$

where w_i is *a priori* probability that a sample belongs to a mode and $g(\mathbf{z}_j(k)|\theta_i)$ is the pdf of normal distribution with parameters θ_i where w_i and θ_i are obtained in off-line training using GMM tool, as shown by Haghani et al. (2012a).

Once a fault is detected using (13), the reconstructed index $\text{Index}(\mathbf{z}_j(k), i)$ will be calculated using (12) and inserted in (19) together with the $\text{Index}(\mathbf{x}(k), i)$ and the posterior probability $p(\mathbf{z}_j(k) \in \mathscr{M}_i)$, to calculate the probabilistic contribution of variables to the fault, and the variables corresponding to the largest contributions are the risky variables.

4. CASE STUDY

In order to achieve statements about the performance and effectiveness of the proposed algorithm, it has been applied to an industrial benchmark. The used plant was a laboratory setup of a continuous stirred tank heater (CSTH).

These systems are often used in control of chemical processes to ensure optimal conditions for a reaction. Inside the CSTH a certain temperature and level of reactants are being held, that those define the operating point under which an optimal reaction is possible. The considered CSTH plant uses water as reactants. Fig. 2 shows the system structure and components.

The stirred tank is the main component in which a specific preheated amount of reactants, in this case water, is mixed, heated further and held at a given temperature. The tank is surrounded by a water filled heating jacket where its temperature can be set via adjusting the power of a heater. Hence, an increasing of the temperature inside the stirred tank is possible by heating the jacket which leads to an equally distributed steady heat flow from jacket through the tank's wall into the reactant. Another property of the systems it a steady through flow of reactants. Whereas the input is a hand-controlled valve the outflow is controlled by a pneumatic valve and thus can be used for level control. Before leaving the system, the outflowing water is used to preheat the incoming reactant via a heat-exchanger to minimize the temperature difference between inflow and tank content to $5 - 10\,^{\circ}\mathrm{C}$ and save energy.

The dynamics of CSTH plant can be described by mass and energy balance equations:

$$h_{tank}(t_1) = \int_{t_0}^{t_1} \frac{1}{A \cdot \rho} (\dot{m}_{in}(t) - \dot{m}_{out}(t))dt + h_{tank}(t_0), \quad (21)$$

and

$$T_{tank}(t_1) = \int_{t_0}^{t_1} \frac{1}{c_p \cdot m(t)} (Q_{in}(t) - Q_{out}(t))dt + T_{tank}(t_0), \quad (22)$$

whit h_{tank} is the level of water inside the tank [m], $\dot{m}_{in}, \dot{m}_{out}$ are in- and outflowing water mass flow rates [kg/s] , T_{tank} is the temperature of water in the tank [K], Q_{in}, Q_{out} are the rates of heat flow in and out of the tank [W] and A, ρ, c_p, m are the cross section of the cylindrical tank [m^2], density of water [kg/m^3], specific heat capacity of water [J/(kg.K)] and the mass of water in the tank [kg], respectively.

Direct measurement of reactants level and temperature in the main tank as well as the mass flow into the tank $\dot{m}_{in}$ is possible. The mass flow out of the tank $\dot{m}_{out}$ is directly manipulated via a pneumatic valve. The mass of water in the tank m defines the relation between enthalpy and temperature in (22) and can be put in front of the integral since it is assumed to be constant in one operating point.

The heat flow into the tank Q_{in} can be described as a nonlinear function of the controllable power of the heater P_{heater}, which is the controllable power of the heater [W], h_{hj} the water level in the heating jacket [m] and the level of the water in the tank.

Furthermore, the inflowing preheated and the outflowing reactant can be described as additional heat flows in and out of the system, Q_{infl} and Q_{outfl}, respectively. They are depending on the mass flows and the masses temperature which can be assumed to be constant in one operating point. Accordingly, the heat flows can be considered as constant, as well. Another heat flow Q_{out} was introduced to describe all the losses due to non-optimal effects such as insulation. Taking these definitions into account, (21) and (22) become:

$$h_{tank}(t_1) = \frac{1}{A \cdot \rho} \int_{t_0}^{t_1} (\dot{m}_{in}(t) - \dot{m}_{out}(t))dt + h_{tank}(t_0)$$
$$T_{tank}(t_1) = \frac{1}{c_p \cdot m} \int_{t_0}^{t_1} (Q_{in}(P_{heater}(t), h_{hj}, h_{tank}(t))$$
$$- Q_{out}(t) + Q_{infl}(t) - Q_{outfl}(t))dt + T_{tank}(t_0) \quad (23)$$

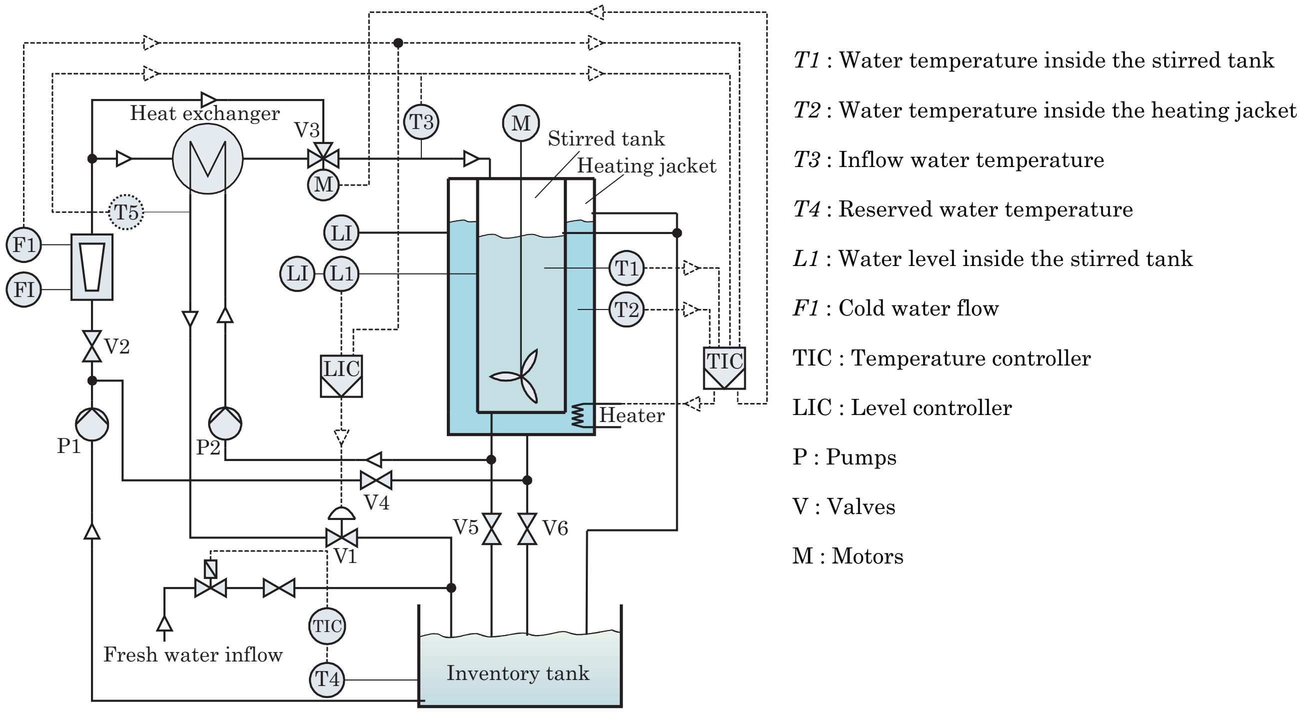

Fig. 2. Piping and instrumentation diagram of CSTH plant

values	$\mathscr{M}_1$	$\mathscr{M}_2$	$\mathscr{M}_3$
level L1 [cm]	10	12	18
level heating jacket [cm]	20	20	20
temperature T1 [°C]	50	45	50
through flow F1 [L/h]	105	105	105

Table 1. Values defining plant operating points

Equation (23) depicts the dynamic behavior of the system for the reactants level on the one hand and its temperature on the other one. Both dependencies on the operating point and nonlinearities are included and can be seen in the model, as well.

The application of multimode technique for fault isolation, proposed in Section 3, is studied in the sequel. To carry out this study, three different operating points, as shown in Table 1, are chosen for the CSTH testbed. For each operating point 500 samples were used to train the model using GMM tool.

For validation purpose, faulty operation is induced by scaling the signal given to level controlling valve $V1$ behind the controller output. Figure 4 shows the run of all measurable plant variables for normal and faulty operation in $\mathscr{M}_1$, $\mathscr{M}_2$ and $\mathscr{M}_3$ each consisting of 100 samples. In on-line monitoring step, the process variables $\mathbf{X}$ are measured and the multimode fault detection method proposed by Haghani et al. (2012a) is implemented for monitoring. The fault detection index $p(\mathbf{x}(k) \in f)$ is

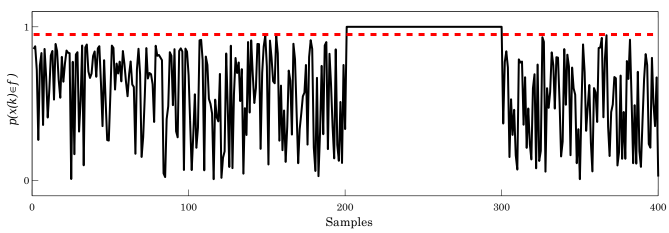

Fig. 3. Result of fault detection using proposed static approach

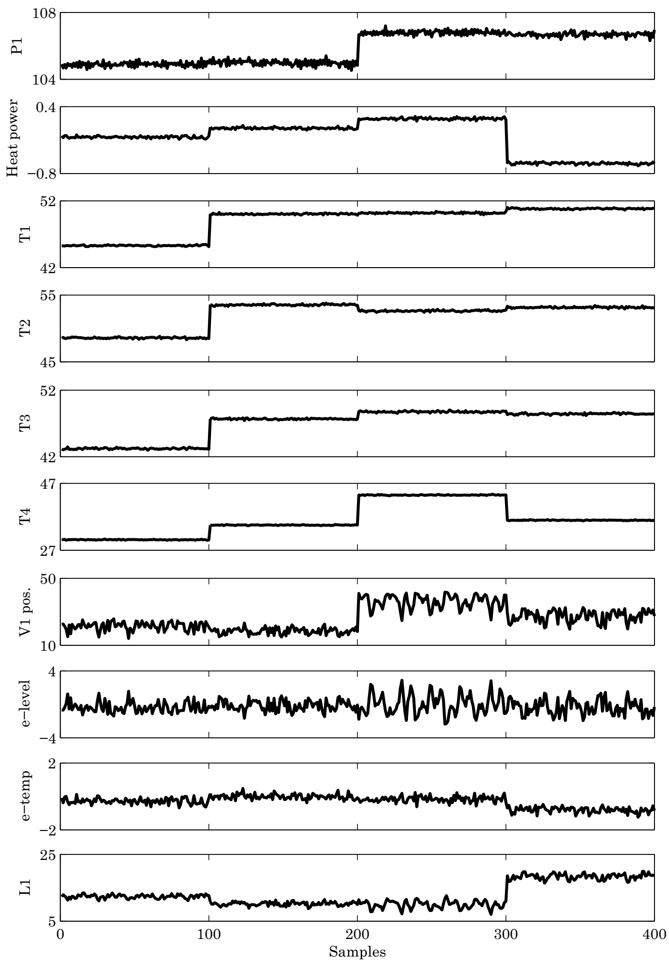

Fig. 4. Process variables for CSTH plant

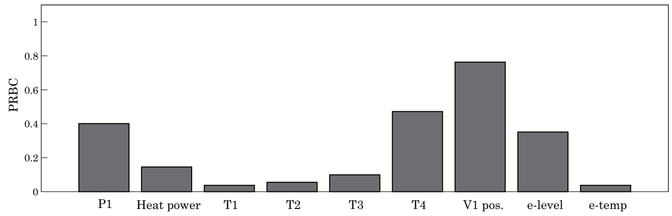

Fig. 5. Probabilistic reconstruction based contribution plot

depicted in Fig. 3 over the faulty and normal operation intervals. The horizontal dashed line represents the threshold with 97% confidence level for $\alpha = 0.03$. It can be seen that the fault is successfully detected within the confidence level.

To identify the possible source of malfunction in the system, the probabilistic contribution analysis method is applied after detection of the fault. The result is shown in Fig. 5. The result shows that the measured signal related to the valve $V1$'s position has the highest contribution to the fault. Moreover, the measured temperature $T4$ signal which is the liquid temperature in reservoir contributes to the fault. As a consequence, it can be concluded that the source of the fault might be the heat exchanger and connecting valve $V1$ or pump, which is confirmed, since in this case the source of the fault was valve $V1$.

5. CONCLUSION

In this paper a probabilistic approach has been developed for fault diagnosis in multimode processes. For that an index has been used for fault detection purpose which represents the probability that a fault happens. After detection of fault the index has been decomposed into two parts which one represents the fault free behavior of the index and the other represents reconstruction based contribution of variables to the fault detection index. A Bayesian strategy has been used to extend the results to multimode processes. For validation purpose, the proposed diagnosis method has been applied on a CSTH testbed and the result shows that fault is successfully isolated.

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