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A Control Matching-based Predictive Approach to String Stable Vehicle Platooning

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Abstract: A predictive control strategy for vehicle platoons is presented in this paper, accommodating both string stability and constraints (e.g., physical and safety) satisfaction. In the proposed design procedure, the two objectives are achieved by matching a Model Predictive Controller (MPC), enforcing constraints satisfaction, with a linear controller designed to guarantee string stability. The proposed approach neatly combines the straightforward design of a string stable controller in the frequency domain, where a considerable number of approaches have been proposed in literature, with the capability of a MPC-based controller of enforcing state and input constraints.
A controller obtained with the proposed design procedure is validated in simulations, showing how string stability and constraints satisfaction can be simultaneously achieved with a single controller. The operating region that the MPC controller is string stable is characterized by the interior of feasible set of the MPC controller.

Keywords: Model Predictive Control, vehicle platooning, string stability, control matching

1. INTRODUCTION

Automated highways, in particular vehicle platooning, is considered as an appealing solution to contribute alleviating and/or preventing traffic flow problems like congestions. Vehicle platoons consist of chains of vehicles following each other led by a specific vehicle, i.e., the leader in an automated way. The idea of platooning dates back to the eighties, when the California Partner for Advanced Transportation Technology (PATH) program was established to study and develop intelligent vehicle-highway cooperation and communication systems, PATH (1986). To enable vehicle platooning, controllers of the longitudinal vehicle dynamics must be designed to track the platoon speed profile, while maintaining a desired distance/time gap between vehicles. More in details, based on on-board measurements like, e.g., radar and camera, and possibly information from the other vehicles within the platoon sent over wireless communication links, the longitudinal dynamics controllers have to track a desired speed profile while guaranteeing *(i) string stability* and *(ii)* physical and safety constraint satisfaction.

String stability is defined as the vehicle ability to attenuate the propagation of the effects on the inter-vehicle distances of an acceleration disturbance introduced by the leader (or any other preceding vehicle). However, slightly alternative definitions of string stability can be found in the literature, considering different disturbance signals and different norm. Extensive studies on the design of string stable vehicle platoons in the frequency domain are given in, e.g. Eyre et al. (1998), Seiler et al. (2004), Papadimitriou and Tomizuka (2004) and Shaw and Hedrick (2007).

Combining string stability and constraints satisfaction requirements in a single controller is not a trivial control design problem. As shown in previous works, e.g. Bu et al. (2010); Li et al. (2011); Kianfar et al. (2012), control specifications and requirements, e.g., safety, performance and actuators limitations can be formulated as inequality constraints in a model predictive controller. Alternatively, constraints satisfactions and safety can be verified a posteriori using set based approaches as in, e.g. Al Alam et al. (2011) and Kianfar et al. (2013a).

However, in general, guaranteeing constraints satisfaction, is not trivial in the frequency domain approaches. On the other hand translating the frequency domain definition of string stability into time domain settings as MPC, is not trivial either. In Dunbar and Caveney (2012) and Kianfar et al. (2013b), the string stability requirement is translated into inequality constraints in an MPC controller. However, the proposed methods require that each vehicle broadcasts its intended trajectory, which might be impractical.

In this work, we propose a predictive control design procedure for vehicle platoons, accommodating both string stability and constraints (e.g., physical and safety) satisfaction. This is a two-step procedure. In the first step, two controllers are designed, namely, an MPC and a string stable linear controller. In particular, physical, safety and design constraints are embedded in the MPC controller, while the linear controller is designed in order to guarantee string stability. It is important to point out that the latter can be based on any string stability definition and designed by resorting to any design procedure leading to a linear control structure. In this paper, as example, a controller based on $H_{\infty/2}$ is synthesized to guarantee string stability with respect to the acceleration signals. In the second step, the MPC controller is *mathced* Di Cairano and Bemporad (2010) to the linear controller. I.e., the weighting matrices in the cost function of the MPC controller are *tuned* to resemble the string stable behavior of the linear controller.

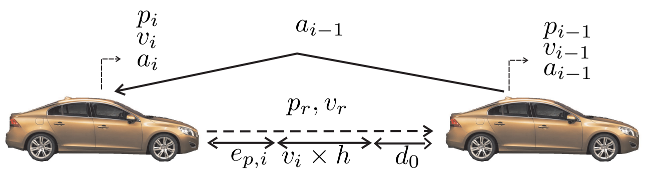

Fig. 1. Two adjacent vehicles in the platoon.

As a result, a controller designed according to the proposed procedure guarantees constraint satisfaction and string stability, as long as constraints are not active.

2. VEHICLE MODELING

Consider two adjacent vehicles, as shown in Fig. 1. Let p_{i-1}, v_{i-1} and a_{i-1} denote the position, velocity and acceleration of the preceding vehicle and p_i, v_i and a_i denote the position, velocity and acceleration of the following vehicle (hereafter also referred to as the ego vehicle), respectively. Denote by $e_{p,i}$ the position error w.r.t. a desired distance from the preceding vehicle, i.e., $e_{p,i} = p_i - p_i - d_0 - v_i h_i$, where d_0 and h_i are a constant safety distance and the constant headway time, respectively. The headway time is the time that the ego vehicle takes to reach the preceding vehicle while traveling at its current speed. The error dynamics are then described by the following set of equations

$$\begin{aligned} \dot{e}_{p,i} &= e_{v,i} - a_i h_i, \\ \dot{e}_{v,i} &= a_{i-1} - a_i. \end{aligned} \tag{1}$$

where $e_{v,i}$ is the relative velocity. The dynamics of low level controller can be described by a first order system,

$$a_i = \frac{K_i}{\tau_i s + 1} e^{-\theta_i s} a_i^{\text{des}}, \tag{2}$$

where K_i, τ_i and θ_i are the steady state gain, the time constant of the actuator (engine and brake) and the actuator delay, respectively and a_i^{des} is the demanded acceleration, Rajamani (2005). The model (1)-(2) can then be written in a state-space form as

$$\dot{x}(t) = Ax(t) + B_u u(t - \theta) + B_\omega \omega(t), \tag{3}$$

where

$$A = \begin{bmatrix} 0 & 1 & -h_i \\ 0 & 0 & -1 \\ 0 & 0 & -1/\tau_i \end{bmatrix}, \tag{4}$$

$$B_u = \begin{bmatrix} 0 \\ 0 \\ \dfrac{K_i}{\tau_i} \end{bmatrix}, B_\omega = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \tag{5}$$

and

$$x = [\, e_{p,i} \;\; e_{v,i} \;\; a_i \,]^{\mathrm{T}}, \tag{6}$$

$$u = a_i^{\text{des}}, \tag{7}$$

$$\omega = a_{i-1}, \tag{8}$$

are the state, the control and the disturbance vectors, respectively. Notice that the acceleration of the preceding vehicle is considered as a measured disturbance which is provided through a communication link.

3. CONTROL OBJECTIVE AND REQUIREMENTS

The control objective is to minimize the position and velocity errors while satisfying a number of requirements described next.

Safety: Safety requirement is introduced to guarantee that a safe *minimum* distance is maintained between vehicles in order to reduce the risk of rear-end collisions. Based on the notation introduced in Section 2, the safety requirement can be written as

$$e_{p,\min} \leq e_{p,i}(t) \leq e_{p,\max}, \qquad \forall t \geq 0, \tag{9}$$

where $e_{p,\max}$ is the maximum allowed distance from the preceding vehicle. We observe that, while $e_{p,\min}$ clearly reflects a safety requirement, $e_{p,\max}$ can be selected according to performance criteria, e.g., to bound the platoon length.

Performance: Since the primary objective of the automated driving system is to regulate vehicle velocity to the platoon velocity, the relative speed between the two adjacent vehicles is constrained as,

$$e_{v,\min} \leq e_{v,i}(t) \leq e_{v,\max}, \qquad \forall t \geq 0. \tag{10}$$

where $e_{v,\min}$ and $e_{v,\max}$ can be chosen based on performance requirements.

Acceleration requirement Typically vehicle motion control systems operate within *comfort zones* expressed by acceleration ranges. We assume that bounds on the longitudinal acceleration are given according to the following inequalities

$$a_{\min} \leq a_i(t) \leq a_{\max}, \qquad \forall t \geq 0, \tag{11}$$

where $a_{\min}$ and $a_{\max}$, are the minimum and maximum allowed accelerations, respectively.

Actuator limitations: The accelerations requested by the longitudinal motion controllers must be within the propulsion and braking systems capabilities, according to the following constraint

$$u_{\min} \leq u_i(t) \leq u_{\max}, \qquad \forall t \geq 0. \tag{12}$$

String stability: Hereafter, we adopt the following string stability definition based on the $\mathcal{L}_2$ norm of the acceleration signals.

Definition 1. (*String stability*): A vehicle platoon is string stable if the following condition holds,

$$\sup_{a_{i-1} \neq 0} \frac{\|a_i(t)\|_{\mathcal{L}_2}}{\|a_{i-1}(t)\|_{\mathcal{L}_2}} < 1, \tag{13}$$

where a_{i-1} and a_i are the acceleration signals of two adjacent vehicles.

Condition (13) states that the total energy of the acceleration signals decreases along the platoon.

Remark 1. An alternative definition for string stability can be defined based on $\mathcal{L}_\infty$, which guarantees that the peaks of the variable of interest decrease along the platoon.

4. CONTROLLERS DESIGN

We recall that the design procedure proposed in this paper consists of two steps. In the first step an MPC and a linear controller are designed. The MPC controller is designed with the objective of minimizing the position and velocity tracking errors while satisfying the constraints (9)-(12). The linear controller is designed with the objective of fulfilling the condition (13). In the second step, the weighting matrices in the MPC controller are tuned in order to mimic the behavior of the linear controller through the control matching method in Di Cairano and Bemporad (2010).

In Sections 4.1 and 4.2, the design of the MPC and the linear controllers (i.e., the first step of the procedure) are presented. The control matching is presented in Sections 5.

4.1 Model Predictive Controller

In order to formulate the position and velocity tracking problem in a receding horizon framework, the longitudinal system dynamics of the i-th vehicle (3) are discretized using the forward Euler method with the sampling time t_s:

$$x_i(t+1) = F_i x_i(t) + G_{1,i} u_i(t) + G_{2,i} \omega_i(t). \quad (14)$$

We assume that the state and the disturbance vectors can be measured every sampling time instant t_s, and solve the following quadratic programming (QP) problem in receding horizon,

$$\min_{\mathcal{U}(t)} \quad \|Px(N|t)\|^2 + \sum_{k=0}^{N-1} \|Qx(t+k|t)\|^2 + \|Ru(t+k|t)\|2 \quad (15)$$

subj. to

$$x(t+k+1|t) = F_i x(t+k|t) + G_{1,i} u(t+k|t) + G_{2,i} \omega(t+k|t) \quad (16)$$

$$\omega(t+k|t) = \omega(t|t) \quad (17)$$

$$u(t-1|t) = u(t-1) \quad (18)$$

$$\omega(t|t) = \omega(t) \quad (19)$$

$$x(t|t) = x(t) \quad (20)$$

$$u_{\min}(t) \le u(t+k|t) \le u_{\max}(t), \quad k = 0, ..., N-1 \quad (21)$$

$$x_{\min}(t) \le x(t+k|t) \le x_{\max}(t), \quad k = 1, ..., N \quad (22)$$

where $\mathcal{U}(t) = [u(t), \ldots, u(t+N-1)] \in \mathbb{R}^N$ is the vector of future input signal, i.e., the vector of optimization variables, N is the prediction horizon length, $Q \succeq 0$, $R \succ 0$ and $P \succeq 0$ are weighting matrices of appropriate dimensions on quadratic state, control signal and final state, respectively. Constraints (22) include the *safety* and the *performance* constraints (9)-(10), respectively, introduced in Section 3, while constraints (21) account for *actuators limitations.* Note that to reject the measured disturbance ω, the prediction model (17) can be written in the augmented form where the augmented state vector is $\tilde{x} = [x, \omega]^{\mathrm{T}}$, while the dynamic of $\omega(t)$ is described by (17). Then, the first element of optimal control sequence $\mathcal{U}^\star(t)$, i.e., $u^\star(t)$ is applied to the plant (vehicle) and the rest of elements in $\mathcal{U}^\star(t)$ are discarded. This procedure is repeated again at the next time step when the new measurements are available.

4.2 Mixed $H_{\infty/2}$ controller synthesis

In the *string stable* linear controller considered in this paper, the requested acceleration a_i^{des} is calculated through the following state feedback/feedforward control law

$$u_i = K_{ss}\tilde{x} = \left[K_{ss}^{\mathrm{FB}} \; K_{ss}^{\mathrm{FF}}\right] \begin{bmatrix} x \\ \omega \end{bmatrix}, \quad (23)$$

where K_{ss}^{FB} and K_{ss}^{FF} are static state feedback and feedforward gains, respectively. The state variable x and the disturbance signal ω are defined as in (6) and (8), respectively.

The gains K_{ss}^{FB}, K_{ss}^{FF} are determined, in order to achieve the following objectives

- *Obj 1* minimize and bound the $\mathcal{L}_2$ gain from acceleration of preceding car a_{i-1} to the acceleration of following vehicle a_i,
- *Obj 2* fulfill string stability condition (13).
- *Obj 3* minimize and bound the $\|.\|_2$ of the transfer function from acceleration of preceding vehicle a_i to position error $e_{p,i}$ and u_i, respectively. This can be seen as minimizing the average gain from $a_{i-1} \rightarrow [e_{p,i}, u_i]$ over the entire frequency.

The gains K_{ss}^{FB}, K_{ss}^{FF} in (23), can be calculated in order to achieve the *Obj 1*, *Obj 2* and *Obj 3*, by solving the following optimization problems Maschuw et al. (2008)

$$\min_{K_{ss}} \quad \alpha\|\Gamma_i\|_\infty + \beta\|H_i\|_2 \quad (24a)$$

subj.to

$$\|\Gamma_i\|_\infty \le \; 1, \quad (24b)$$

where $\|\cdot\|_\infty$ and $\|\cdot\|_2$ denote the ∞- and 2-norm, α and β are tuning parameters and the matrices F_i, H_i are defined as

$$\Gamma_i(s) \triangleq \frac{a_i(s)}{a_{i-1}(s)}, \quad (25a)$$

$$H_i(s) \triangleq \left[\frac{e_{p,i}(s)}{a_{i-1}(s)} \; \frac{u_i(s)}{a_{i-1}(s)} \right]^{\mathrm{T}}, \quad (25b)$$

and calculated as

$$\Gamma_i = C_\Gamma \left(sI - A_{cl}\right)^{-1} B_{cl} + D_\Gamma, \quad (26)$$

$$H_i = C_H \left(sI - A_{cl}\right)^{-1} B_{cl} + D_H \quad (27)$$

with

$$A_{cl} = A + B_u K_{ss}^{FB}, \quad (28)$$

$$B_{cl} = B_u K_{ss}^{FF} + B_\omega, \quad (29)$$

$$C_\Gamma = [\,0 \;\; 0 \;\; 1\,], \quad (30)$$

$$C_H = \begin{bmatrix} 1 & 0 & 0 \\ & K_{ss}^{FB} & \end{bmatrix}, \quad (31)$$

$$D_\Gamma = 0, \quad D_H = K_{ss}^{FF}. \quad (32)$$

In (24a), the first term of the cost function is introduced to penalize the effect of the preceding vehicle acceleration a_i on the ego acceleration a_{i+1} while the constraint (24b) is to guarantee string stability, thus achieving the objective *Obj 1* and *Obj 2*, respectively. The objective *Obj 3* is achieved through the second term of the cost function, which penalizes the effect of a_i on the $e_{p,i}$ and the desired acceleration a_i^{des} (i.e. the control input u_i) in the 2-norm sense. The parameters α and β in the cost function have to be selected in order to trade-off between objectives *Obj 1* and *Obj 3*. Note that the optimization problem (24) can be formulated as an LMI and be solved efficiently.

Remark 2. Considering the string stability criterion (13), the choice of an H_∞ controller seems natural. However, for the proposed control matching approach presented in the next section, any other linear controller which can results in a string stable vehicle platoon can be suitable as well. Hence, the presented controller here can be considered as an example.

5. CONTROL MATCHING PROBLEM

Given the initial conditions, i.e. $x(k)$ and $\omega(k)$, the QP-problem can be written as follows,

$$\min_{\mathcal{U}(k)} \quad \mathcal{U}^{\mathrm{T}} H \mathcal{U} + 2x(k)^{\mathrm{T}} F \mathcal{U} + x(k)^{\mathrm{T}} Y x(k) \tag{33a}$$

subj.to

$$M\mathcal{U}(k) \leq W(k) + Ex(k), \tag{33b}$$

the weighting matrices H, F, Y in the objective function (33a) and the matrices $M \in \mathbb{R}^{r\times N}$, $E \in \mathbb{R}^{r\times n}$ and $W \in \mathbb{R}^r$ in the polyhedral constraints (33b) from the problem (15)-(22) defined as follows,

$$H = (\mathcal{S}^{u'}\bar{Q}\mathcal{S}^u + \bar{R}), \quad F = \mathcal{S}^{x'}\bar{Q}\mathcal{S}^u, \tag{34}$$
$$Y = \mathcal{S}^{x'}\bar{Q}\mathcal{S}^x$$

where $\mathcal{S}^u$ and $\mathcal{S}^x$ are,

$$\mathcal{S}^u = \begin{bmatrix} F & 0 & \cdots & 0 \\ FG1 & G1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ F^{N-1}G1 & F^{N-2}G1 & \cdots & G1 \end{bmatrix}, \mathcal{S}^x = \begin{bmatrix} F \\ F^2 \\ \vdots \\ F^N \end{bmatrix}, \tag{35}$$

and $\bar{Q}$, $\bar{R}$ are block diagonal matrices,

$$\bar{Q} = \begin{bmatrix} Q_1 & 0 & 0 & \cdots & 0 \\ 0 & Q_2 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & Q_{N-1} & 0 \\ 0 & 0 & \cdots & 0 & P \end{bmatrix}, \bar{R} = \begin{bmatrix} R_0 & 0 & \cdots & 0 \\ 0 & R_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & R_{N-1} \end{bmatrix} \tag{36}$$

The idea underlying the control matching approach proposed in Di Cairano and Bemporad (2010) is to tune the weighting matrices in the cost function (15) such that the unconstrained solution of the problem (33a)-(33b) equals the control input calculated through the (23).

The solution of the unconstrained optimization problem (33a) can be calculated as follows,

$$\mathcal{U}^\star(k) = -H^{-1}F^{\mathrm{T}}x(k) \tag{37}$$

In order to match the state feedback control calculated as unconstrained solution of (33a) with the control input calculated by the linear controller, u_i in (23), $\bar{P}$, $\bar{Q}$ and $\bar{R}$ must be determined in order to satisfy the following equation

$$[K_{FB}K_{FF}]\tilde{x}(k) = -\Lambda H^{-1}F^{\mathrm{T}}\tilde{x}(k) \tag{38}$$

where $\Lambda = [\, I \; 0 \cdots 0 \,]$ and $\tilde{x}(k) = [x(k), \omega(k)]^{\mathrm{T}}$ is the augmented state vector. Solving (38) for the matrices Q, P and R requires matrix inversion of the singular matrix Λ. In order to remove Λ from (38), we solve the following equation instead

$$[\, k_0,\, k_1,\, \cdots,\, k_{N-1}\,]^{\mathrm{T}}\tilde{x}(k) = -H^{-1}F^{\mathrm{T}}\tilde{x}(k), \tag{39}$$

where $k_0 = [K_{FB}\; K_{FF}]$, while the gains k_i for $i \in [1, N-1]$ can be chosen freely. Equation (39) can be cast as an optimization (or feasibility) problem. However, considering k_i as free variables results in an optimization problem subject to bilinear matrix inequalities (BMI). Although techniques are available to solve BMI problems, non-convexity dramatically increases their computational complexity. Assuming that the matrix gains k_i are fixed (pre-specified), (39) can be cast as a semi definite programming (SDP), i.e., the minimization of a linear objective function subject to Linear Matrix Inequalities (LMI).

Lemma 1. (Di Cairano and Bemporad (2010)), Consider the following convex optimization problem

$$\mathcal{V} = \min_{Q_i,P,R_i} \|(\bar{R} + \mathcal{S}^{u'}\bar{Q}\mathcal{S}^u)\mathcal{K} + \mathcal{S}^{u'}\bar{Q}\mathcal{S}^x\| \tag{40a}$$

$$\text{subj. to} \quad P \succeq 0, \quad R_i \succeq \sigma I, \quad i = 0, \cdots, N-1 \tag{40b}$$
$$Q_i \succeq 0, \quad i = 1, \cdots, N-1$$

where $\mathcal{K}$ is the matrix of pre-specified gains over the N-step prediction horizon,

$$\mathcal{K} = \begin{bmatrix} K \\ K(A+BK) \\ \vdots \\ K(A+BK)^{N-1} \end{bmatrix} \tag{41}$$

if the objective function (40) becomes zero at the optimal point, i.e. $\mathcal{V}^\star = 0$, then the solution to the optimization problem, i.e. Q_i, P and R_i are also the exact solution to the (39) as well. Hence, if $\mathcal{V}^\star = 0$, the MPC controller (15) will behave identical as the string stable controller presented in Section 4.2, as long as the constraint (33b) are not active.

Note that the MPC controller proposed in Section 4.1 can enforce string stability as long as the constraints are not active, i.e. when the following holds,

$$M\mathcal{U}(k) \prec W(k) + Ex(k) \tag{42}$$

This corresponds to the interior of feasible (admissible) set of the MPC controller. Once the constraints are active, the behavior of MPC controller would be different than the string stable controller in general. However, this is not restrictive since when the constraints are active, e.g. either the safety is endanger or the control signal is saturated. Clearly, if the safety is endanger, then string stability is not the priority of the controller, and if the control signal is saturated the string stability cannot be guaranteed with string stable controller neither.

6. SIMULATIONS AND RESULTS

To evaluate the effectiveness of presented control strategy, two simulation scenarios are considered. In the first scenario the real data from a vehicle is used as the velocity for the leader to evaluate the capability of MPC controllers in terms of string stability. In the second scenario a harsher maneuver is considered to show that an MPC controller can be superior compared to $H_{\infty/2}$ when the constraints are active.

6.1 Scenario 1 (string stability)

As can be seen from Fig. 2(a), the lead vehicle introduces a speed variation disturbance which is used to evaluate string stability. The speed profile of lead vehicle is the velocity measurement collected from a real vehicle. The five followers are controlled by MPC controllers whose their weighting matrices are tuned according to the method presented in Section 5 to guarantee string stability. However, for the sake of better illustration, the simulation results for only three of these vehicles are presented. As can be seen from Fig. 2(b), the MPC controllers show a string stable behavior in terms of attenuation of the acceleration signals. In Fig. 2(c), the position errors between vehicles are depicted. The results indicate string stability w.r.t position error as well. The control signals of second follower for the same scenario is shown in Fig. 3. The control signal presented to the top, is control signal generated by MPC controller and the one to the bottom is control signal generated by the string stable $H_{\infty/2}$ controller. Simulation results indicate that as long as the constraints are not active both controllers, i.e. the MPC controller and $H_{\infty/2}$ controller show identical behaviors. We should note that the MPC controllers are implemented using fast QP solver generated by CVXGEN, Mattingley and Boyd (2012). The sampling time of controllers is $t_s = 0.1sec$ and a prediction horizon $N = 3$ is considered.

6.2 Scenario 2 (constraint satisfaction)

In this case, a harsher maneuver is considered to push the vehicles to the constraints. In particular, a velocity profile according to Fig. 4(a) is considered. The safety constraint, i.e. constraint on the negative position error is set by the MPC controller. As can be seen from Fig. 4(b), a soft constraint on $e_{p,min} = -0.15m$. The soft constraint is used to avoid infeasibility and a maximum relaxation of $s_{max} = -0.5m$ is used as hard constraint. The results show the effectiveness of constraint in limiting the position error compared to string stable $H_{\infty/2}$ controller. Looking at the position error around $t = 80s$ shows that the maximum position error of MPC controller is less than half of the position error introduced by $H_{\infty/2}$. The difference between control signal of MPC controller and $H_{\infty/2}$ is depicted in Fig. 4(c). As can be seen the response of the two controllers match each other except when the safety constraint is hit.

7. CONCLUSION AND FUTURE WORK

In this work a control matching approach is used to combine the benefits of frequency domain controller design, i.e. string stability and benefits of MPC, i.e. constraint satisfaction for a vehicle following application. We showed that by using this approach, the MPC controller can be tuned such that it behaves similar to a string stable $H_{\infty/2}$. Therefore, string stability is guaranteed as long as the constraints are not active. Then it is demonstrated that when the constraints are active the MPC controller is superior. The operating region where the outputs of the two controllers matches, is determined by the interior of feasible set of MPC controller. This can be characterized by a convex polytope.

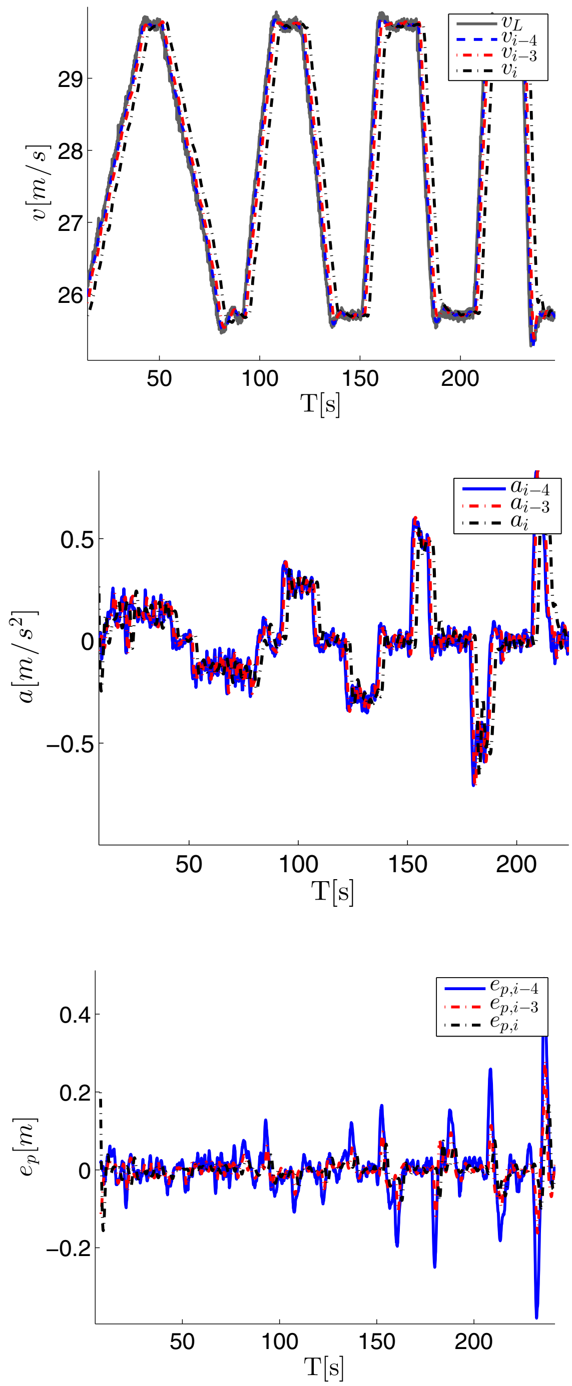

Fig. 2. To the top, velocity profile of four vehicles, i.e., lead vehicle (real data) in solid gray, first follower controlled by an MPC in dashed blue and the last follower controlled by an MPC dashed dotted black. In the middle, acceleration profile, first follower controlled by an MPC in solid blue and the last follower controlled by MPC dashed dotted black. To the bottom, position error between vehicles, same color code as acceleration profile

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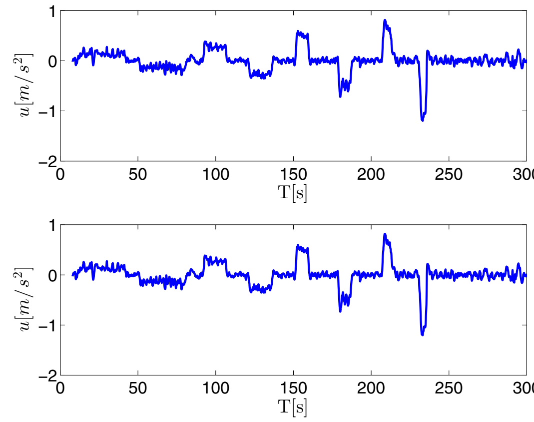

Fig. 3. Control signal generated by MPC controller to the top and control signal generated by the $H_{\infty/2}$

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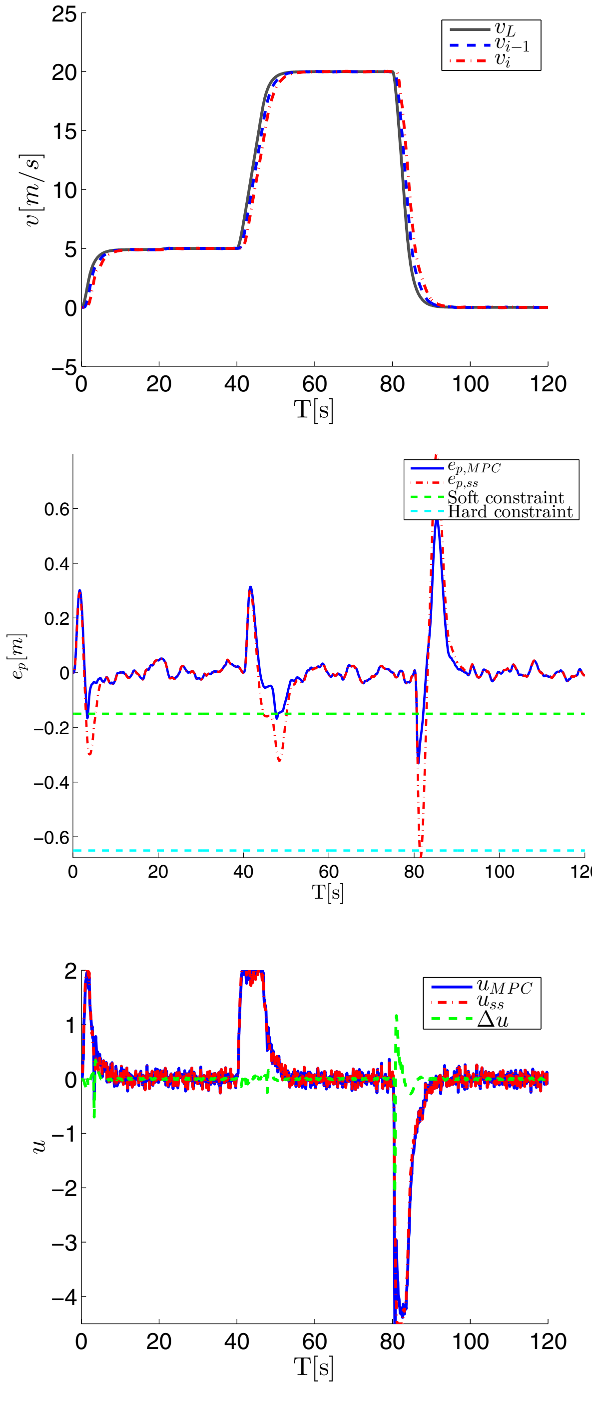

Fig. 4. To the top, velocity profile of three vehicles, i.e. lead vehicle in solid gray, first follower controlled by an MPC in dashed blue and the second follower controlled by MPC dashed dotted red. In the middle, position error, i.e. first follower and second follower for MPC and $H_{\infty/2}$ controllers are shown in solid blue and dashed dotted red, respectively. The soft and hard constraints are shown in dashed green and blue, respectively. To the bottom, control signal of the MPC controller, $H_{\infty/2}$ controllers and the difference between control signals of MPC controller and String stable controller for the first follower