Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

An Improved Detection Statistic for Systems with Unsteady Trend ⋆

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Abstract: The object of this paper is to address data-driven fault detection design for systems with unsteady trend, which shows cyclicity, monotonicity and non-zero mean. Firstly, mean theorem and covariance theorem are proposed and proved. The former is the mean property of projection matrix, and the latter is the recursive formula for covariance matrix of regression residual. Secondly, an improved fault detection statistic, called Least Square T^2 (LST^2), is proposed. It can partly solve the detection problem for systems with unsteady trend. The improvement can also partly cope with the limitations of the traditional multivariate detection methods, such as Principal Component Analysis (PCA). Thirdly, based on the two theorems, a recursive algorithm and a moving window algorithm of LST^2 are given, thus both time and space complexity are greatly reduced for online detection. The effectiveness of the presented detection statistic is evaluated with an application of monitoring satellite attitude control system. The case study result shows that the false alarm rate of LST^2 is much lower than that of T^2 based on PCA, while LST^2 is more sensitive to fault.

Keywords: Fault detection and diagnosis, time-varying systems, recursive identification, time series modelling, estimation and filtering.

1. INTRODUCTION

It is costly to obtain well-established mathematical models for large-scale complex stochastic systems, thus the model-based fault diagnosis methods in (Ding, 2008), such as diagnostic observer (DO) and parity space (PS), sometimes can not be applied directly in practice. However, a complex system is often equipped with various sensors, which record the real-time information of the monitored system. How to utilize the monitored data to estimate the system model and realize the data-driven fault detection is now a hot research topic. Basic data-driven and model-based process monitoring (PM) as well as fault detection and isolation (FDI) methods are surveyed from the application viewpoint in (Qin, 2003; Ding, 2011 a). Without a prior of the system matrices and order, Ding (2012) proposed several design schemes for parity subspace and observer-based FDI systems, respectively. Many traditional methods can handle fault diagnosis task for systems with steady trend, however, control systems in practice are not always steady. Data from unsteady systems are usually cyclical, monotone and with non-zero mean, such as an cyclical industrial data in Fig. 1 and satellite attitude data in Fig. 5, which show non-stationary mean and monotone trend. Multivariate statistics (MS) detection methods such as principal component analysis (PCA), Fisher discriminant analysis (FDA), partial least square (PLS) and canonical variant analysis (CVA) in (Russell, 2000), will be with high false alarm rate and high missed detection rate in such unsteady cases.

To solve the unsteady problems, some dynamic and adaptive methods are constructed. Such methods include multi-way principal component analysis (M-PCA) in (Nomikos, 1994), Recursive PCA (R-PCA) in (Li, 2000), moving window PCA (MW-PCA) in (Wang, 2005; He, 2008) and dynamic PCA (DPCA) in (Ku, 1995). M-PCA is based on the assumption that the monitored system can run for many times beforehand and store all data to compute the system unsteady trend. This assumption for M-PCA is too harsh and require large storage capacity. Both R-PCA and MW-PCA are techniques that make adaptation of the PCA model to accommodate the model unsteady trend. R-PCA is incremental, while MW-PCA is both incremental and decremental. They can partly cope with the unsteady trend problem, but they are still not satisfactory because of two limitations, slow adaptation and large condition number (CN), which will be explained more detailed in Section 3. DPCA is desirable due to its simplicity for interpreting the time-correlation, but it is usually constrained in linear series correlation.

Time series modeling (TSM) is a tool in system identification. The task for TSM is to fit the time series by the

⋆ Resrach supported by National Basic Research Program of China (Grant No. 613156030103), National Natural Science Foundation of China (Grant No. 61304199).

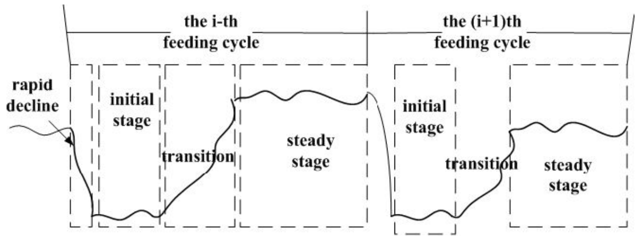

Fig. 1. Unsteady industrial process(Ding, 2011 a)

given function bases, ex. polynomial basis or Trigonometric function bases.

The core idea of this paper is to fit the observed data by selected function bases, then the prediction residual and the covariance of least square regression residual are used to compose an improved detection statistic, which has a similar form of Hostelling T^2 statistic, so it is named Least Square T^2 (LST^2). When recursive identification techniques are used in LST^2, we obtain the recusive LST^2(R-LST^2) and moving window LST^2(MW-LST^2). It reveals that PCA-T^2 is a special case of LST^2 when functions bases are constant, What is more, The idea of LST^2 is similar to that of DPCA-T^2, when functions bases are just polynomials.

The paper is structured as follows. In Section 2, some preliminaries related to regression are given, based on which mean theorem and covariance theorem are proposed originally. In Section 3, two limitations of traditional PCA are analyzed. Also an improved detection statistic, named LST^2, is composed to cope with the limitations. In Section 4, R-LST^2 algorithm and MW-LST^2 algorithm are included. Case study, conclusion and appendix are made in following Sections, respectively.

2. TWO KEY THEOREMS

2.1 Function Bases, Design Matrix and Regression

Function bases are

$$(f_1(t), f_2(t) \dots, f_n(t)), \tag{1}$$

which is designed for regression. Usually polynomial functions are fit for monotone data while trigonometric functions for cyclical data. Number of polynomial bases, n, can be selected by correlation plots in (Lennart, 1999). Personal prior knowledge and visual insight about the monitored data are also important for selecting the type and the number of functions.

Design matrix corresponding to the function bases is

$$X_k \triangleq \begin{pmatrix} f_1(1) & f_2(1) & \cdots & f_n(1) \\ f_1(2) & f_2(2) & \cdots & f_n(2) \\ \vdots & \vdots & \vdots & \vdots \\ f_1(k) & f_2(k) & \cdots & f_n(k) \end{pmatrix}. \tag{2}$$

For example, if $f_i(t) = t^{i-1}, i = 1, \cdots, n$, then $x_{ij} = i^{(j-1)}$, with $i = 1, \dots, k$ and $j = 1, \dots, n$. Block form of the design matrix is

$$X_k = \begin{pmatrix} X_{k-1} \\ x_k^{\mathrm{T}} \end{pmatrix}, \tag{3}$$

where $x_k^{\mathrm{T}} = (f_1(k), f_2(k) \dots, f_n(k))$.

Projection matrix is defined by design matrix as

$$H_k \triangleq X_k \left(X_k^{\mathrm{T}} X_k\right)^{-1} X_k^{\mathrm{T}} \tag{4}$$

It is easy to verify that both the projection matrix H_k and its complement matrix $(I - H_k)$ are symmetrical, idempotent and positive semi-definite.

Monitored k-successive data are denoted as $Y_k \in R^{k\times m}$, and the regression model by X_k is

$$Y_k = X_k \beta_k + E_k. \tag{5}$$

where E_k is regression residual and X_k is defined in (2). The least square estimation of β_k is

$$\hat{\beta}_k = \left(X_k^{\mathrm{T}} X_k\right)^{-1} X_k^{\mathrm{T}} Y_k, \tag{6}$$

The *regression residual* is

$$E_k = Y_k - X_k \hat{\beta}_k = (I_k - H_k) Y_k. \tag{7}$$

The *prediction residual* for y_k is

$$e_{k|k-1} = y_k - \left(x_k^{\mathrm{T}} \hat{\beta}_{k-1}\right)^{\mathrm{T}}. \tag{8}$$

2.2 Mean vector and Covariance Matrix

Suppose

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{k1} & a_{k2} & \cdots & a_{kn} \end{pmatrix} \in R^{k\times n}.$$

The sample mean and covariance based on a collection of instances of the random vector is used in the sequel and their definition is for convenience given below.

Mean vector for A is defined as

$$\overline{A} \triangleq \left(\frac{1}{k}\sum_{i=1}^{k} a_{i1}, \frac{1}{k}\sum_{i=1}^{k} a_{i2} \dots, \frac{1}{k}\sum_{i=1}^{k} a_{in} \right) \in R^{1\times n}. \tag{9}$$

Covariance matrix for A is defined as

$$Cov(A) \triangleq \frac{1}{k}(A - repmat(\overline{A}, k))^{\mathrm{T}}(A - repmat(\overline{A}, k)), \tag{10}$$

where $repmat(\overline{A}, m)$ is a notation from Matlab, denoting m-rows of which are copies of $\overline{A}$. Note that $\overline{A} \in R^{1\times n}$ is a row vector.
Properties from (11) to (15) will are useful for proving the theorems in the Section 2.4.
Basic properties of mean vector are

$$\overline{A_1 A_2} = \overline{A_1} A_2, \tag{11}$$

$$\overline{A_1 + A_2} = \overline{A_1} + \overline{A_2}, \tag{12}$$

$$\overline{\begin{pmatrix} A_1 \\ A_2 \end{pmatrix}} = \frac{1}{k_1 + k_2}\left(k_1 \overline{A_1} + k_2 \overline{A_2}\right). \tag{13}$$

where k_1 and k_2 are the row numbers. Note that $\overline{A_1 A_2} \neq A_1 \overline{A_2}$.
Basic properties of covariance matrix

$$Cov(A) = \frac{1}{k} A^{\mathrm{T}} A - \overline{A}^{\mathrm{T}} \overline{A}, \tag{14}$$

$$Cov\,(A_1A_2) = A_2^{\mathrm{T}} Cov\,(A_1)\,A_2. \tag{15}$$

Note that $Cov\,(A_1A_2) \neq A_1^{\mathrm{T}} Cov\,(A_2)\,A_1$.

2.3 Two Necessary Equations of Inverse Matrix

are two equations of inverse matrix, which can be inferred in (Peigorsch, 1989),

$$\begin{aligned}&(A+BCD)^{-1}\\&= A^{-1} - A^{-1}B\big(DA^{-1}B + C^{-1}\big)^{-1}DA^{-1},\end{aligned} \tag{16}$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}^{-1} = \begin{pmatrix} \tilde{A}^{-1} & -A^{-1}B\tilde{D}^{-1} \\ -\tilde{D}^{-1}CA^{-1} & \tilde{D}^{-1} \end{pmatrix}, \tag{17}$$

where $\tilde{A} = A - BD^{-1}C$ and $\tilde{D} = D - CA^{-1}B$. (16) and (17) are used to prove the following two theorems.

2.4 Mean Theorem and Covariance Theorem

Mean theorem and covariance theorem are the foundation for R-LST^2 and MW-LST^2 algorithm in Section 4.

Theorem 1. **(Mean Theorem)** X_k, H_k and mean vector are defined in (2), (4) and (9), suppose that one of the basis functions is a nonzero constant function, then

$$\overline{(I - H_k)} = 0. \tag{18}$$

According to (18) and (11), we have

$$\overline{E_k} = \overline{(I_k - H_k)\,Y_k} = \overline{(I_k - H_k)}Y_k = 0, \tag{19}$$

which shows that the mean vector for least square estimation is zero. According to (10), (7), (15), (14) and (19), we have

$$Cov(E_k) = \frac{1}{k} Y_k^T\,(I - H_k)\,Y_k. \tag{20}$$

When new data are added, i.e. Y_{k-1} is replaced by Y_k, the regression parameter as well as regression residual covariance should be updated. The recursive formula for $Cov\,(E_k)$ is shown in following theorem.

Theorem 2. **(Covariance Theorem)** E_k, $e_{k|k-1}$ and $Cov(E_k)$ are defined in (6), (7) and (18), respectively, and $\lambda_k \triangleq x_k^{\mathrm{T}}\big(X_{k-1}^{\mathrm{T}}X_{k-1}\big)^{-1}x_k$, then following equation holds.

$$\begin{aligned}&Cov(E_k)\\&= \frac{(k-1)}{k}Cov(E_{k-1}) + \frac{1}{k\,(1+\lambda_k)}e_{k|k-1}e_{k|k-1}^{\mathrm{T}}.\end{aligned} \tag{21}$$

According to (21) and (16), we obtain the recursive formula for $Cov^{-1}\,(E_k)$ as following

$$\begin{aligned}Cov^{-1}(E_k) &= \frac{k}{k-1}[Cov^{-1}(E_{k-1})\\&- \frac{Cov^{-1}(E_{k-1})e_{k|k-1}e_{k|k-1}^{\mathrm{T}}Cov^{-1}(E_{k-1})}{(k-1)\,(1+\lambda_k)\left(1 + e_{k|k-1}^{\mathrm{T}}Cov^{-1}(E_{k-1})e_{k|k-1}\right)}].\end{aligned} \tag{22}$$

Due to space constraints, the proof is omitted. The basic techniques for the proof are inductive reasoning, based on (16) and (17).

3. THE IMPROVED DETECTION STATISTIC

In this section, an improved detection statistic is given and named least square T^2 (LST^2), which can cope with two limitations of PCA. T^2 statistic based on PCA is commonly used statistics for fault detection, and it has following form

$$T^2\,(y) = y^T P_a \Lambda_a^{-1} P_a^T y, \tag{23}$$

where a is the number of principal component and $P_a^T\Lambda_a P_a$ is the first a-component of $P_m\Lambda_m P_m^T$, and there are m columns in Y. The singular value decomposition of $Cov\,(Y)$. In this paper we suppose that $a = m$, i.e.

$$T^2\,(y) = y^T P_m \Lambda_m^{-1} P_m^T y = y^T Cov^{-1}\,(Y)\,y. \tag{24}$$

3.1 Two Limitations of T^2 based on PCA

Limitation 1: Slow Adaptation

If Y and y in (24) are respectively replaced by Y_{k-1} and y_k, (24) will turn into

$$T^2\,(y_k) = y_k^T Cov(Y_{k-1})^{-1} y_k. \tag{25}$$

Because Y_{k-1} and y_k are centered and scaled, thus the original form of (25) is

$$\begin{aligned}T^2\,(y_k) &= \left(y_k - \overline{Y}_{k-1}^T\right)^T Cov(Y_{k-1})^{-1}\left(y_k - \overline{Y}_{k-1}^T\right)\\&\triangleq e_{k|k-1}^T Cov(Y_{k-1})^{-1} e_{k|k-1}.\end{aligned} \tag{26}$$

In the view of prediction, (26) considers $\overline{Y}_{k-1}$ as the prediction of the y_k and the detection statistic, PCA-T^2, is composed by the prediction residual $e_{k|k-1}$ and the train covariance $Cov\,(Y_{k-1})$. This strategy works only if the monitored data are steady. When data are monotone, cyclical and with non-zero mean, PCA-T^2 will be too slow to adapt the normal change. In the view of fitting, multivariate statistics is an under-fitting model for the unsteady process, thus both train and test residuals are very large. Large train residual indicates high lost detection rate and large test residual indicates high false alarm rate.

Limitation 2: Large Condition Number

Condition number (CN) of the data matrix is an index of the relativity of the variables (columns). Large CN means large relativity. CN is defined as

$$CN\,(Y) = \lambda_{max}/\lambda_{min},$$

where λ_{max} is the maximum eigen-value of $Cov\,(Y)$ and λ_{min} the minimum. Because PCA-T^2 neglects the unsteady trend of output variables and predicts y_k by $\overline{Y}_{k-1}$, the slow adaption will result in extremely large CN of $Cov\,(Y_{k-1})$, see *Example 2.*

3.2 An Improved Detection Statistic

Although data Y_k are usually not steady, when the trend, $X_k\hat{\beta}_k$, are eliminated, E_k and $e_{k|k-1}$ will be steady and with distribution close to normal. Suppose that Y_{k-1} are fault-free train data, X_{k-1} is design matrix defined in (2) and y_k is to be detected, then the improved statistic, with a similar form of (26), is

$$LST^2\,(y_k) = e_{k|k-1}^T Cov^{-1}(E_{k-1}) e_{k|k-1}, \tag{27}$$

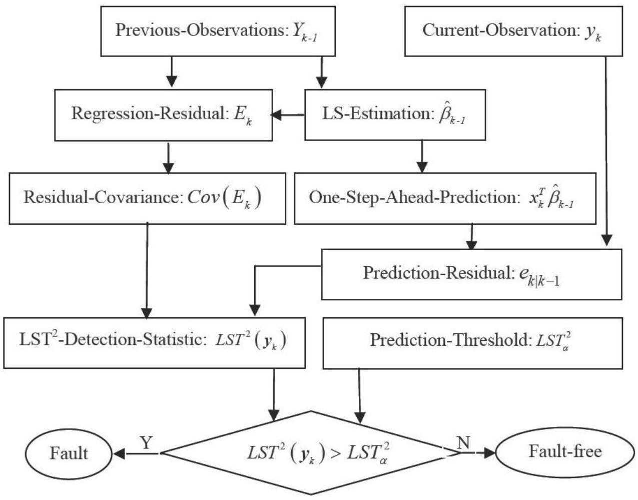

Fig. 2. Flow chart of constructing LST^2

where the regression residual E_{k-1} is from (7), $Cov(E_{k-1})$ is the covariance and $e_{k|k-1} = y_k - \left(x_k^{\mathrm{T}}\hat{\beta}_{k-1}\right)^{\mathrm{T}}$ is the prediction residual, with $\hat{\beta}_{k-1} = \left(X_{k-1}^T X_{k-1}\right)^{-1} X_{k-1}^T Y_{k-1}$. For every selected significance level α, the threshold for detection statistic LST^2 is

$$LST^2_\alpha = \frac{mk\,(k-2)}{k(k-1-m)} F_{(1-\alpha)}\,(m, k-1-m)\,, \qquad (28)$$

where $(k-1)$ and m are respectively the row and column number of Y_{k-1}. In practice, the prediction residual is often larger than the regression residual, thus a modified threshold may be used instead in order to reduce the false alarm rate

$$LST^2_\alpha = \gamma\frac{m\,(k-1)\,(k+1)}{k(k-m)} F_{(1-\alpha)}\,(m, k-m)\,, \gamma \geq 1. \quad (29)$$

The flow chart of constructing LST^2 is shown in Fig.2.

3.3 Two Improvements of LST^2

Improvement 1: Faster Adaptation than PCA-T^2 In fact PCA-T^2 is a special case of LST^2. When $n = 1$ and $(f_1(t), f_2(t) \ldots, f_n(t)) = f_1(t) \equiv 1$, i.e. the function bases are constant, then $\left(x_k^T\hat{\beta}_{k-1}\right)^T = \overline{Y}_{k-1}$ and LST^2 turns into PCA-T^2. What is more, when $n > 1$, usually $\left(x_k^{\mathrm{T}}\hat{\beta}_{k-1}\right)^{\mathrm{T}} \neq \overline{Y}_{k-1}$, where $\left(x_k^{\mathrm{T}}\hat{\beta}_{k-1}\right)^{\mathrm{T}}$ acts as the trend of the system. LST^2 does not neglect the unsteady trend of the data, thus it will ensure smaller regression residual and faster adaptation, which explains why it wins a lower false detection rate and higher detection rate, see Fig.3 and Fig.4 in *Example 1,* which shows that LST^2 ensures faster adaptation and lower false alarm rate.

Example 1 Fault-Free Data With Monotone Trend
Data are with single variable and 100 samples. They are monotone, fault-free, generated by

$$y(t) = 0.1 * t + e(t),$$

where t = 1,2,...,100, and $e(t) \sim N(0,1)$ is independent normal noise with 0 mean and 1 variance. The first 60 samples are treated as train data, and the left 40 are validation data. We can find from Fig.3 and Fig.4 that the residual for PCA-T^2 is much larger than that of LST^2, while the validation false alarm rate for LST^2 is much lower.

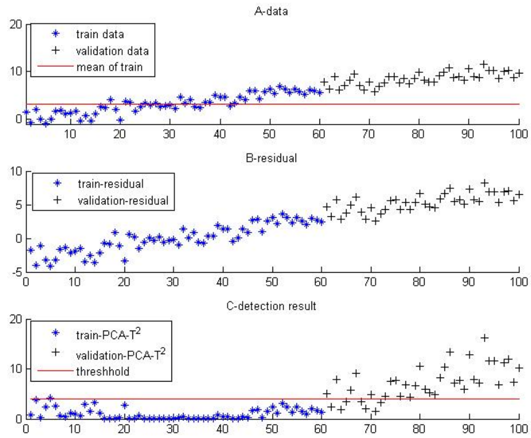

Fig. 3. Performance of PCA-T^2 in *Example 1*

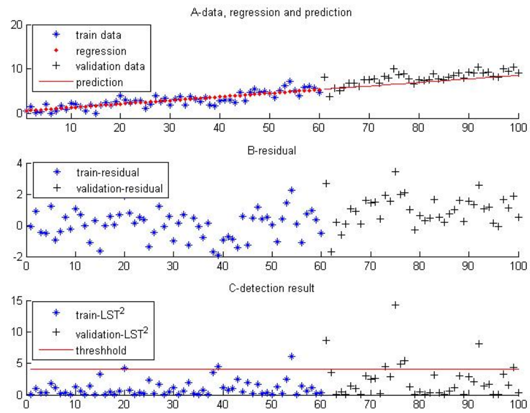

Fig. 4. Performance of LST^2 for data in *Example 1*

Improvement 2: Smaller Condition Number

Example 2 Linear and Relative Data.
Data are with 5 variables and 100 samples. They are monotone, false-free and generated by

$$y_i(t) = 0.1 * i * t + e_i(t),$$

where $e_i(t) \sim N(0,1)$, i=1,...,5 and t=1,2,...,100. The 5-dimension data are linear relative. The condition numbers (CN) of PCA-T^2 and LST^2 are shown in Table 1.

Table 1 shows that the CN of LST^2 is much smaller than that of PCA-T^2. When $n > 1$ and the condition number of LST^2 is always smaller than that of PCA-T^2. Small CN means the computation is stable, thus LST^2 needs no dimension reduction any more, which will bring great convenient to following recursive algorithms in Section 4.

Table 1. Different Condition Numbers

Statistic	Covariance	λ_{max}	λ_{min}	Condition Number
PCA-T^2	$Cov\,(Y_{k-1})$	45776	88	520
LST^2	$Cov\,(E_{k-1})$	129	104	1.24

4. R-LST^2 AND MW-LST^2 ALGORITHMS

In this section, formulae for updating LST^2 are given. They are recursive LST^2 (R-LST^2) algorithm and moving window LST^2 (MW-LST^2) algorithm, which ensures low computation complexity for online detection.

4.1 R-LST^2

When new data arrives, LST^2 should be updated recursively. The core idea of recursive algorithm is to reduce the computation complexity by recursive formulae. In (27), we can see that $LST^2\,(y_k)$ relies on two part, $e_{k|k-1}$ and $Cov^{-1}(E_{k-1})$. The first part, $e_{k|k-1}$, can be updated as follows in (Lennart, 1999)

$$\hat{\beta}_k = \hat{\beta}_{k-1} + K_{k|k-1}e^{\mathrm{T}}_{k|k-1} \tag{30}$$

where

$$K_{k|k-1} = \frac{Q_{k-1}x_k}{1 + x_k^T Q_{k-1}x_k}, \tag{31}$$

$$Q_{k-1} = \left(X_{k-1}^T X_{k-1}\right)^{-1}, \tag{32}$$

$$Q_k = Q_{k-1} - K_{k|k-1}x_k^T Q_{k-1}. \tag{33}$$

The second part, $Cov^{-1}(E_{k-1})$, can be updated according to (22). We obtain the complete algorithm as below.

Algorithm 1 *Recursive least square T^2 (R-LST^2)*

Input:
Previous train data Y_{k-1} and current validation data y_k.
Output:
Detection result and Updated Parameters.
Initialization:
step 1: Compute $\hat{\beta}_{k-1}$ according to (6);
step 2: Compute $Cov(E_{k-1})$ according to (20).
Prediction and Detection:
step 3: Compute $e_{k|k-1}$ according to (8);
step 4: Compute $T^2\,(y_k)$ and T^2_α according to (27) and (28). If $T^2\,(y_k) > T^2_\alpha$, alarm, otherwise go to step 5.
Update:
step 5: Update $\hat{\beta}_k$ according to (30);
step 6: Update $Cov^{-1}(E_k)$ according to (22);
step 7: Update Q_k according to (33).

4.2 MW-LST^2

Recursive algorithm is to update the detection statistic by adding the effect of the new normal data, y_k. The opposite question is how to update the detection detection by subtracting the effect of the old when the old data, y_1, is outdated. This strategy is named moving-window algorithm. Because the moving-window is a dual process to recursive, thus we will give the moving window LST^2 (MW-LST^2) without proof. R-LST^2 is only part of MW-LST^2, i.e. step 1-7 are totally the same to Algorithm 1.

Algorithm 2 *Moving window least square T^2 (MW-LST^2)*

Input: the same as that in Algorithm 1.
Output: the same as that in Algorithm 1.
Initialization:as step 1-2 in Algorithm 1.
Prediction and Detection:as step 3-4 in Algorithm 1.
Update-Incremental: as step 5-7 in Algorithm 1.
Update-Decremental:
step 8: Compute $e_{1|k} = y_1 - \left(x_1^{\mathrm{T}}\hat{\beta}_k\right)^{\mathrm{T}}$;
step 9: Update $\hat{\beta}_{k-1} = \hat{\beta}_k - K_{1|k}e^{\mathrm{T}}_{1|k}$, $K_{1|k} = \frac{Q_k x_1}{1-x_1^T Q_k x_1}$;
step 10:Compute $\lambda_k = x_1^{\mathrm{T}} Q_{k-1}x_1$, update $Cov^{-1}(E_{k-1})$ $= \frac{k-1}{k}\left[Cov^{-1}(E_k) + \frac{Cov^{-1}(E_k)e_{1|k}e^T_{1|k}Cov^{-1}(E_k)}{k\left(1+\lambda_k\right)\left(1-e^T_{1|k}Cov^{-1}(E_k)e_{1|k}\right)}\right]$.

5. CASE STUDY ON SATELLITE ATTITUDE CONTROL SYSTEM

5.1 Data Description

Data of satellite attitude control system (SACS) is provided by CASA (China Aerospace Science and Technology Corporation). There are 7 sensors/variables, see Table 2. The monitored data have 500 samples. We can see that the SACS data is highly non-stationary with monotone trend. Fault happens at time point 353, which is caused by sun sensor on pitching axis, i.e. the 4-th variable at time point 353, see Fig.5.

Table 2. Data of SACS

code	sensor	acronym
1	earth sensor on rolling axis	EarthPhi
2	earth sensor on pitching axis	EarthTheta
3	sun sensor on rolling axis	SunPhi
4	sun sensor on pitching axis	SunTheta
5	gyroscope on rolling axis	GeoPhi
6	gyroscope on pitching axis	GeoTheta
7	gyroscope on yawing axis	GeoPsi

5.2 Train Residual and Condition number comparision

The first 320 fault-free samples are used as train data, then the residual of PCA-T^2 is the blue curve in Fig.6. The dimension of polynomial function base for LST^2 is 3, i.e. n=3 and $(f_1\,(t)\,, f_2\,(t)\ldots, f_n\,(t)) = \left(1, t, t^2\right)$. As is explained in Section 3.3 that PCA-T^2 is a special case of LST^2, i.e. n=1 and $(f_1\,(t)\,, f_2\,(t)\ldots, f_n\,(t)) \equiv 1$. We can find in Fig.6 that the residual of PCA-T^2, points of 'green *', is larger than that of LST^2, the points of 'black +'. This phenomenon is more obvious for the 3rd and 4th variable, which are monotone. What is more, the condition numbers of PCA-T^2 and LST^2 are respectively $CN_{PCA-T^2} = 8.8 \times 10^8$ and $CN_{LST^2} = 4.8 * 10^4$, which means $CN_{PCA-T^2} \gg CN_{LST^2}$.

5.3 Detection Result comparision

The detection statistic and threshold for PCA-T^2 can be seen in the upper sub-figure of Fig.7. It can be found that although the data between 335 and 352 are fault-free, PCA-T^2 is with high false alarm rate, while is not LST^2 is with much lower false alarm rate. What is more,

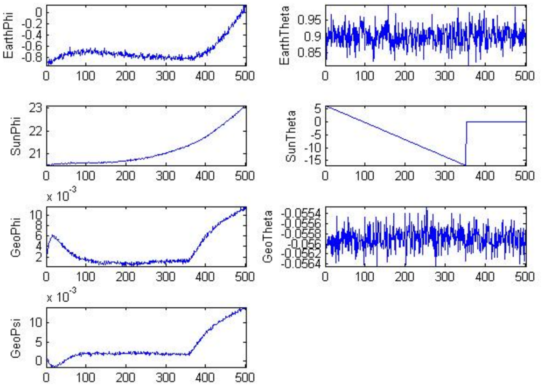

Fig. 5. Fault begins at the 353rd sample of SACS

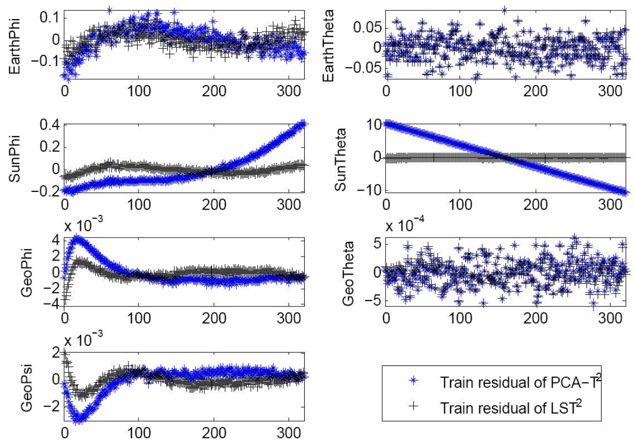

Fig. 6. Train residual for PCA-T^2 and LST^2

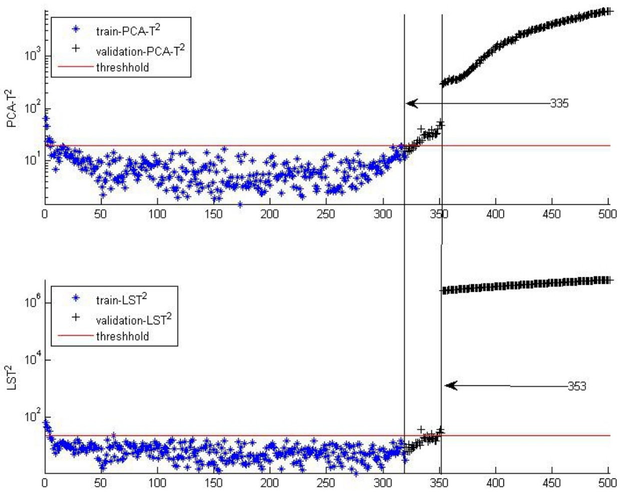

Fig. 7. Detection result for PCA-T^2 and LST^2

the LST^2 is larger in fault time (from 353 to 500), which means that LST^2 is more sensitive to fault than PCA-T^2.

6. CONCLUSIONS

We propose an improved detection statistic, LST^2, which catches the unsteady trend of fault-free data and has two important improvements, faster adaptation and smaller condition number. LST^2 can partly cope with the limitations of the standard detection statistic PCA-T^2. All superiority of LST^2 is verified in the examples and the case study. We propose and prove two useful theorems, mean theorem and covariance theorem. They are the theoretical foundation of R-LST^2 and MV-LST^2 algorithms, which can greatly reduce the computation complexity for on-line detection.

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