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Design and experimental validation of a hybrid optimal control for DC-DC power converters

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Abstract: In this article, the problem of hybrid optimal control for DC-DC power converters is treated. The designed control is of type bang-bang established from Pontryagin's maximum principle. The control is a state feedback and it is determined using an energy based minimization criterion derived from the power balance of Port-Hamiltonian systems. The developed control has the advantage to be easy to design and simple to implement in real time applications. The proposed control is applied to a SEPIC converter and validated in simulation and experimentation.

1. INTRODUCTION

DC-DC power converters are employed in several applications, including power supplies, electronic devices, DC motor drives, etc. The control of the output voltage of this kind of converter has received a great interest for many years and various control techniques have been proposed such as Lyapunov-based control, deadbeat, sliding modes, predictive control [2][6][9][14]. Most of these techniques are based on a classical approach where the switching behavior of those converters is approximated by an average model. Unfortunately, this approach is only used in a specific range of frequencies and does not take into account the high frequency behavior of the system. While considering these drawbacks, hybrid control theory is suitable to deal with power converters.

Recently, hybrid optimal control of DC-DC converters has been widely investigated but it is still a subject that has not received sufficient interest. Due to the difficulties encountered in the control of switched systems, the design of a state feedback optimal control for DC-DC power converters is not an easy task even though for low dimension systems[15][16]. When using Pontryagin minimum principle to design the optimal control, the main difficulty is the determination of the costate. The dynamic expression of this latter results from the differentiation of the Hamiltonian function with respect to the state. Such differential equation is not simple to solve when the control is restricted to a finite set. Authors in [5] propose an algebraic approach for optimal control using the singular arcs. In this method, the research of the arcs is carried out independently of the costate. Next, a backward integration starting from the singular arcs is used to generate the regular trajectories. Finally, the state feedback obtained with this technique is given by the interpolation of the optimal trajectories using neural networks. Unfortunately, the drawback of this approach is that it can only be used for low dimensional systems (i.e. $2,3$). In [3], a numerical framework for the optimal control is proposed where the technique can be used for higher dimensional systems. The drawback of this numerical method is its implementation in real time. To the best of our knowledge, only [1] proposes a state feedback where the costate is given explicitly. However, this preliminary work allowed to design a control based on a necessary optimal condition but it did not allow to fully design an optimal control strategy. The work presented in the following proposes to solve this problem.

The main contribution of this paper is the synthesis of a state feedback optimal control for switched systems. Even if the approach is ultimately dedicated for the control of DC-DC power converters the technique could be used for other applications. A candidate costate is determined in order to design a control law for power converters, which was the major pitfall of the technique proposed in [5]. The approach consists in using an optimal control derived form Pontryagin's minimum principle and port-Hamiltonian formalism. In contrast with our previous work [1], the minimization criterion is the stored energy of the closed loop system. Finally, the control law is validated in practice on a real SEPIC converter.

The approach has the advantage to be easy to design and to implement for different types of converters.

The paper is organized as follows: the problem statement is presented in Section 2. Notions of optimal control are introduced in Section 3. In section 4, the proposed control approach for DC-DC power converters is detailed. An example of application to a single-ended primary inductor converter (SEPIC) is presented with simulation and experimental results in Section 5. Conclusions and future works are given in Section 6.

2. PROBLEM STATEMENT

Consider the following affine system with one switching input

$$\dot{z}(t) = R(z(t)) + S(z(t))u(t) \tag{1}$$

where $z(t) \in \mathbb{R}^{\nu}$ is the state vector, ν is the state dimension, $R(.)$ is the system dynamic function and $S(.)$ is the input function. The system (1) is a switched system since the control signal $u(t) \in U$ and $U = \{0,1\}$. The switching between the different modes depends only on $u(t)$. Autonomous switching is assumed to be excluded. In the following sections, we will assume that there is only one control signal since various DC-DC power converters use one control signal. However, our approach can be generalized to several control signals.

Let us define the operating points of (1) with respect to the average model of this system. The set of operating points, Z_{ref}, is given by the following equation

$$Z_{ref} = \{z_{ref} \in \mathbb{R}^v : R(z_{ref}) + S(z_{ref})u_{ref} = 0, u_{ref} \in co(U)\} \tag{2}$$

where $co(U)$ is a convex hull of U.

When $u_{ref} \in co(U)\backslash U$, there is no control $u(t)$ which allows to maintain the system on its operating point. Nevertheless, it is possible to approach z_{ref} as close as desired by a fast switching control $u(t)$ between 0 and 1. In this case, $u(t)$ must have as an average value u_{ref}. This is proven by the density theorem given in [12].
The optimal control problem that needs to be solved is:

$$\begin{array}{ll} \min\limits_{u(.)} \displaystyle\int_0^{t_f} \mathscr{L}(z(t)-z_{ref},u)d\tau \\ s.t.\ \dot{z}(t) = R(z(t)) + S(z(t))u(t) \\ z(0) = z_0, \quad u(t) \in U \end{array} \tag{3}$$

where $\mathscr{L} : \mathbb{R}^v \to \mathbb{R}$ is the cost function and t_f is the final time.

For the sake of simplicity, time dependency of the variables is omitted. In order to simplify the equations, we rewrite (3) under Mayer's form by introducing a new state variable which is equal to the optimization criterion:

$$\rho(t) = \int_0^t \mathscr{L}(z(\tau)-z_{ref}(\tau),u)d\tau \tag{4}$$

where

$$\begin{aligned} x &= [z^T, \rho]^T \\ x_0 &= [z_0{}^T, 0]^T \\ f(x) &= [R^T(z), \mathscr{L}(z-z_{ref}, u=0)]^T \\ g(x) &= [S^T(z), \frac{\partial \mathscr{L}(z-z_{ref},u)}{\partial u}]^T \end{aligned}$$

The optimal control problem (3) becomes

$$\begin{array}{ll} \min\limits_{u(.)} \quad [\,0\ 0\ \ldots\ 1\,]x(t_f) \\ s.t.\ \dot{x} = f(x) + g(x)u \\ x(0) = x_0, \quad u(t) \in U \end{array} \tag{5}$$

with $x \in \mathbb{R}^n$ and $n = v+1$.

The objective of the control is to determine a control law $u^*(t) \in U$ for $t \in [0,t_f]$ that minimizes $\rho(t_f)$ for an arbitrary initial state x_0. The following section is devoted to the determination of a solution for (5).

3. OPTIMAL CONTROL

In this section, it is shown that the straightforward application of Pontryagin's minimum principle to (1) provides an admissible solution if the case of the singular arcs is not considered. Since the control is restricted to a finite set, it is not always possible to find a control solution. Indeed, this solution is influenced by the hybrid nature of the system, which makes it pass through a singular arc. Fortunately, DC-DC power converters do not encounter this problem in practice.
In the next subsections, the optimal control problem, the Pontryagin's minimum principle and the singular control solution are presented.

3.1 Minimum principle of Pontryagin

The Hamiltonian function for the problem (5) is given by

$$H(x,\lambda,u) = \lambda^T(f(x)+g(x)u) \tag{6}$$

The dynamics of the state x and the costate λ are given by

$$\dot{x} = \frac{\partial H}{\partial \lambda}, \quad \dot{\lambda} = -\frac{\partial H}{\partial x} \tag{7}$$

The application of the theorem given in [5] to the problem (5) leads to the following corollary:

Corollary 1. Let a pair (x^*,u^*) that solves the problem (5), then there exists an absolutely continuous function $\lambda^* : [0,t_f] \to \mathbb{R}^n$ such as for almost all $t \in [0,t_f]$ the following conditions are verified :

- The minimum condition on the Hamiltonian
$$H^* = H(x^*,\lambda^*,u^*) = \inf_{u \in U} \quad H(x^*,\lambda^*,u) \tag{8}$$
- The first transversality condition For all $t \in [0,t_f]$
$$H(t) = Cte \tag{9}$$
where Cte is a constant and $Cte = 0$ if t_f is not specified.
- The second transversality condition
$$\lambda^*(0) \quad free \quad and \quad \lambda^*(t_f) = [0,\ldots,0,1] \tag{10}$$

This corollary supposes a preexisting solution (x^*,u^*) which it is not always the case. If the set U does not contain any solution, the control domain U must be extended to its convex hull $co(U)$. Therefore, if the relaxed problem (i.e. when $u \in co(U)$) has a bang-bang solution, it solves the original problem (5). Otherwise, if such a solution does not exist, then the control $u(t)$ takes its values in the convex hull $co(U)$. This last solution cannot be applied to the original problem (5) but it can be approximated by a fast switching between the different modes so that the solution of the relaxed problem will be the averaged value of the switching control [7][18].

3.2 Type of solutions

Let us define the following function, called switching function

$$\phi(t) = \frac{\partial H}{\partial u} = \lambda^T g(x) \tag{11}$$

According to (6) and (8), minimizing H with respect to u with $u \in \{0,1\}$ leads to the following control:

$$u = \begin{cases} 0 \ if \ \phi > 0 \\ 1 \ if \ \phi < 0 \\ ? \ if \ \phi = 0 \end{cases} \tag{12}$$

As mentioned before, the case where ϕ vanishes over a time interval (singular arc) is not taken into account. Then, the optimal control is a bang-bang solution and u has the following expression:

$$u = \frac{1-sign(\phi)}{2} \tag{13}$$

4. FEEDBACK CONTROL LAW

Port-Hamiltonian theory provides a geometric description of network models of physical systems. This theory combines the Hamiltonian systems with respect to a power-conserving geometric structure capturing the basic interconnection laws with a Hamiltonian function given by the total stored energy. The port-Hamiltonian modeling provides a unified framework for the physical description of different types of converters [11]. In this section, the properties of the port-Hamiltonian formalism are used to construct a Lyapunov function. The analysis of this function allows to obtain a candidate costate vector. The port-Hamiltonian model for a DC-DC power converter with one switch is given by

$$\dot{z} = P[J_1 + J_2u - R]z + P[B_1 + PB_2u]E \quad (14)$$

J_1 and J_2 are $n \times n$ skew-symmetric matrices, called structure matrices. They correspond to the power interconnections on the system. B_1 and B_2 are the input matrices. P is a diagonal parametric matrix. R is the dissipation matrix. u is the switching input and E is a constant input. It can be proven for power converters that the system (1) is equivalent to system (14).
Consider the tracking error $\tilde{z} = (z - z_{ref})$. We know that z_{ref} is constant then $\dot{z}_{ref} = 0$ and $\dot{z} = \dot{\tilde{z}}$ (See equations (2) and (14)). The tracking error dynamic can be written as

$$\dot{\tilde{z}} = P[J_1 + J_2u - R](\tilde{z} + z_{ref}) + P[B_1 + B_2u]E \quad (15)$$

The Hamiltonian function that represents the stored energy in the system (15) is given by

$$\mathscr{H}(z) = \frac{1}{2}\tilde{z}^T P^{-1}\tilde{z} \quad (16)$$

Recall that R is a symmetric matrix and J_1 and J_2 are skew-symmetric matrices. Then, $\tilde{z}^T J_1 \tilde{z} = 0$ and $\tilde{z}^T J_2 \tilde{z} = 0$. The derivative of (16) is given by

$$\dot{\mathscr{H}}(z) = -\tilde{z}^T R\tilde{z} + \tilde{z}^T(J_1 + J_2u - R)z_{ref} + \tilde{z}^T(B_1 + B_2u)E \quad (17)$$

For this representation, we consider the stored energy as the minimization criterion

$$\int_0^{t_f} \mathscr{L}(\tilde{z}, u) = \frac{1}{2}\tilde{z}^T P^{-1}\tilde{z} \quad (18)$$

where t_f is not specified.

The following cost function could be defined

$$\mathscr{L}(\tilde{z}, u) = \tilde{z}^T(J_1 + J_2u - R)z_{ref} + \tilde{z}^T(B_1 + B_2u)E - \tilde{z}^T R\tilde{z} \quad (19)$$

This function is exactly the power balance of a port-Hamiltonian system. It represents the difference between the supplied power and the dissipation power [11].
Let us write (14) as a Mayer's problem with the cost function (19). From (4) let us consider $\rho = \int_0^{t_f} \mathscr{L}(\tilde{z}, u)d\tau$. Then, (14) becomes

$$\dot{x} = f(x) + g(x)u \quad (20)$$

with

$$x = \begin{bmatrix} z \\ \rho \end{bmatrix}$$

$$f(x) = \begin{bmatrix} P(J_1 - R)(\tilde{z} + z_{ref}) + PB_1E & 0 \\ z_{ref}^T(J_1 - R)^T\tilde{z} - \tilde{z}^T R\tilde{z} + \tilde{z}^T B_1E & 0 \end{bmatrix}$$

$$g(x) = \begin{bmatrix} PJ_2(\tilde{z} + z_{ref}) + PB_2E & 0 \\ z_{ref}^T J_2^T\tilde{z} + \tilde{z}^T B_2E & 0 \end{bmatrix}$$

Now, we propose the following candidate costate and investigate the optimality of the obtained control

$$\lambda^T = [-\tilde{z}^T P^{-1} \quad 1] \quad (21)$$

The control law becomes

$$u = \frac{1 - sign([-\tilde{z}^T P^{-1} \quad 1]g(x))}{2} \quad (22)$$

To check the optimality of the costate candidate, it is primordial to verify the necessary condition given in (7)

$$\dot{\lambda}^T = -\frac{\partial H}{\partial x} \quad (23)$$

The computation of the derivative of the candidate gives the following

$$\begin{aligned} \dot{\lambda}^T &= [-\dot{\tilde{z}}^T P^{-1} \quad 0] \\ &= \left[\, -(\tilde{z} + z_{ref})^T[J_1 + J_2u - R]^T - E^T[B_1 + PB_2u]^T \;\; 0 \,\right] \end{aligned} \quad (24)$$

The expression (23) can be seen as

$$\dot{\lambda}^T = -\lambda^T(f_x(x) + g_x(x)u) \quad (25)$$

where $f_x(x) = \dfrac{\partial f(x)}{\partial x}$ and $g_x(x) = \dfrac{\partial g(x)}{\partial x}$.

The expressions of $f_x(x$ and $g_x(x)$ are given by

$$f_x(x) = \begin{bmatrix} P(J_1 - R) & 0 \\ z_{ref}^T(J_1 - R)^T - 2\tilde{z}^T R + B_1^T E & 0 \end{bmatrix}$$

$$g_x(x) = \begin{bmatrix} PJ_2 & 0 \\ z_{ref}^T J_2^T + B_2^T E & 0 \end{bmatrix}$$

Since the matrices J_1 and J_2 are skew-symmetric, it is possible to write the following

$$-2\tilde{z}^T R\tilde{z} = \tilde{z}^T\left[(J_1 + J_2u - R)^T + (J_1 + J_2u - R)\right]\tilde{z}$$

And the derivative of this last equation with respect to x is

$$-\frac{\partial 2\tilde{z}^T R\tilde{z}}{\partial x} = \left[\tilde{z}^T\left((J_1 + J_2u - R)^T + (J_1 + J_2u - R)\right) \;\; 0\right]$$

Then, $f_x(x)$ becomes

$$f_x(x) = \begin{bmatrix} P(J_1 - R) & 0 \\ z_{ref}^T(J_1 - R)^T + \tilde{z}^T\left[(J_1 + J_2u - R)^T + (J_1 + J_2u - R)\right] + B_1^T E & 0 \end{bmatrix}$$

Straightforward computations of $-\lambda^T(f_x(x) + g_x(x)u)$ shows that

$$\begin{aligned} -\lambda^T(f_x(x) + g_x(x)u) = &-(\tilde{z} + z_{ref})^T[J_1 + J_2u - R]^T \\ &-E^T[B_1 + PB_2u]^T \end{aligned} \quad (26)$$

Equations (24) and (26) show that (23) is satisfied. Furthermore, the verification of the second transversality condition (10) is trivial. Since t_f is not specified, the Hamiltonian $H(t)$ is supposed to be null which is the case with the proposed costate state. In coclusion, all Pontraygin's principle conditions are verified and the control (22) is optimal.

5. EXAMPLE OF APPLICATION

This section is dedicated to the validation, in simulation and in experimentation of the control proposed in the previous section on a SEPIC converter.

5.1 SEPIC converter

A Single-Ended Primary Inductor Converter (SEPIC) is a DC-DC power converter that can have the output voltage either greater than, either less than or equal to the input voltage (see Fig. 1). The SEPIC has the advantage that it can maintain the same polarity and the same ground reference for the input and output. It has a shutdown mode: when the switch is turned off, its output drops to 0 V [8][10].

Circuit modeling The port-Hamiltonian model (14) of the SEPIC is given by

$$\dot{z} = P[J_1 + J_2u - R]z + PB_1E \quad (27)$$

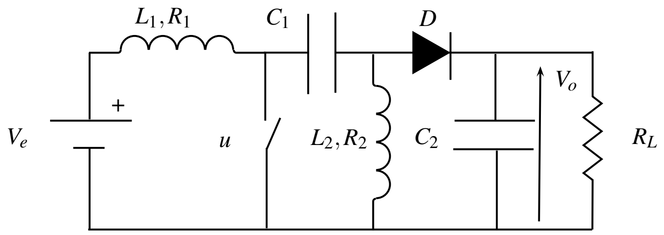

Fig. 1. SEPIC converter

with $J_1 = \begin{bmatrix} 0 & -1 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & -1 & 0 \end{bmatrix}$,

$$J_2 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 \\ 0 & -1 & 0 & 1 \end{bmatrix} R = \begin{bmatrix} R_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & R_2 & 0 \\ 0 & 0 & 0 & \frac{1}{R_o} \end{bmatrix}$$

$$P = \begin{bmatrix} \frac{1}{L_1} & 0 & 0 & 0 \\ 0 & \frac{1}{C_1} & 0 & 0 \\ 0 & 0 & \frac{1}{L_2} & 0 \\ 0 & 0 & 0 & \frac{1}{C_2} \end{bmatrix} B_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

where L_1, L_2, C_1 and C_2 are the input and output inductors and capacitors respectively. R_1 and R_2 are the equivalent internal resistors of the inductances. R_o is the load resistor. I_{L_1} and I_{L_2} denote the inductance currents. V_{C_1} denotes the voltage of capacitor C_1. V_o denotes the output voltage, and E denotes the input voltage. The state vector contains the currents, voltages and the load conductance of the circuit $z = [\, I_{L_1} \ V_{C_1} \ I_{L_2} \ V_o \,]^T$. The converter is controlled via the switching input u.

Control Design Consider the optimal problem (5) of the system (27) and assume that t_f is not specified, the aim of the control is to maintain the output voltage V_o around the chosen reference. Afterward, the control must maintain the state around the reference in steady state. Furthermore, another difficulty is that only one control input is used to derive four states. In practice, the output reaches a limit cycle where the considered reference is the averaged value of this cycle.
The optimal control is determined in such way that the following criterion is minimized

$$\int_0^{t_f} \mathscr{L}(\tilde{z}, u) = \frac{1}{2}L_1 I_{L_1}^2 + \frac{1}{2}C_1 V_{C_1}^2 + \frac{1}{2}L_2 I_{L_2}^2 + \frac{1}{2}C_2 V_o^2 \quad (28)$$

Simulation results The proposed control law is applied on a SEPIC switched model in simulation using MATLAB. The parameters of the circuit are : $L_1 = 2.3 \times 10^{-3}H$, $L_2 = 330 \times 10^{-6}H$, $C_1 = 190 \times 10^{-6}F$, $C_2 = 190 \times 10^{-6}F$, $R_1 = 2.134\Omega$, $R_2 = 0.234\Omega$ and $E = 20V$. The sampling frequency is $20kHz$ and the simulation time is $t = 1s$. The operating points z_{ref} are generated using (2) with two different duty cycles; the first one $u_{ref} = 0.52$ corresponds to an output voltage of $20V$ and the second one $u_{ref} = 0.45$ corresponds to $15V$ as output voltage. The reference variation is operated at the instant $t = 0.4s$. Fig. 3 and Fig. 6 show the evolution of the currents and voltages of the converter. The rising time for the output voltage is $t_s = 0.09s$. The currents and voltages of the SEPIC converter reach a cycle around their references. The ripple of V_o in the positive and negative senses respectively is $+0.1V$ and $-0.1V$.

Experimental results The developed control law is applied on a testing bed Fig.2. The sampling frequency is $20kHz$. A current sensor 'LEM LTSP 25-NP' is used to measure the current I_{L_1}. The control is implemented using 'Real-Time Windows Target Simulink Library' on a 'dSpace-DS1104' board. The output voltage reference is switched from $15V$ to $20V$ at time $t = 0.4s$ to show the reference tracking performance of the control law. Fig. 7 to Fig. 10 show the evolution of the currents I_{L_1} and I_{L_2} and the voltages V_{C_1} and V_o of the converter. The output voltage is driven to a limit cycle around its reference. As expected, one can see that the output voltage (Fig. 10) follows the reference. In steady state, a marginal tracking error of $0.3V$ could be noticed. Beside, the output voltage reaches the new reference after $0.02s$.

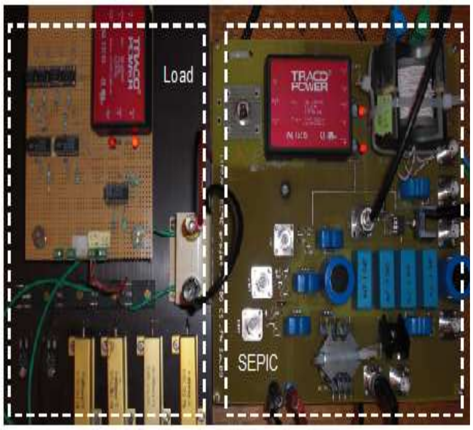

Fig. 2. Test bed

6. CONCLUSION

In this article, we designed an optimal control for DC-DC power converters with a single control input in continuous conduction mode. Pontaryagin's principle is employed to obtain a bang-bang control. Using an energy based minimization criterion, we proposed a candidate costate to design a state feedback and to overcome the problems raised by the approach based on the research of the singular arcs. Then, we investigated the optimality of the proposed control. The technique is validated on a SEPIC converter in simulation and in experimentation. The approach proposed in the paper can be generalized for all DC-DC power converters.
The extension of the technique to multiple control inputs, taking into account the state constraints, the consideration of the discontinuous conduction mode are proposed as perspective works. Also, the results must be compared with classic techniques.

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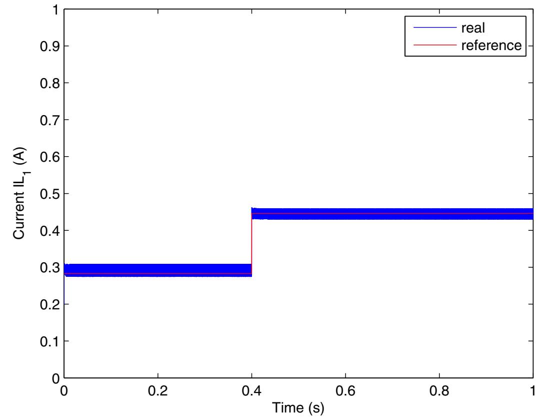

Fig. 3. The current I_{L_1}: Simulation

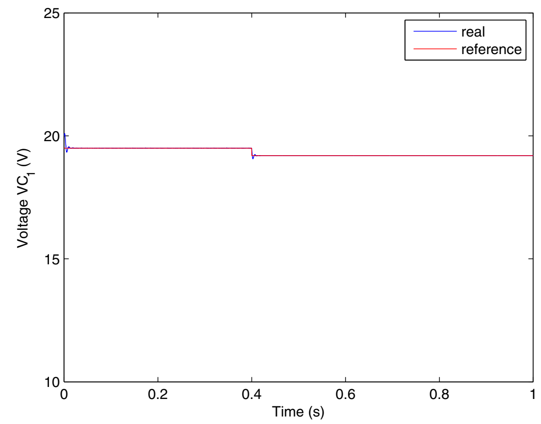

Fig. 4. The voltage V_{c_1}: Simulation

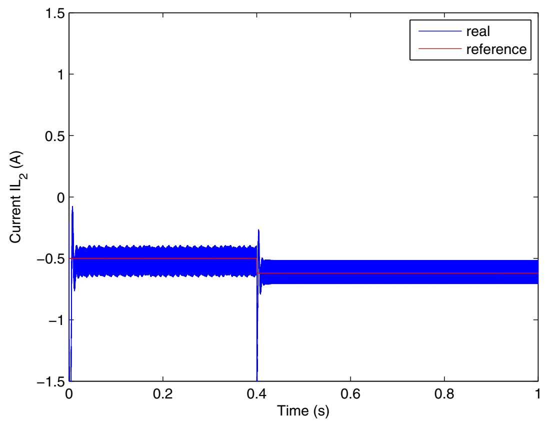

Fig. 5. The current I_{L_2}: Simulation

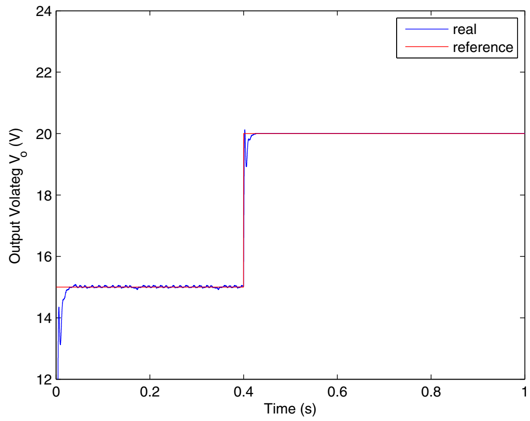

Fig. 6. The output voltage V_o: Simulation

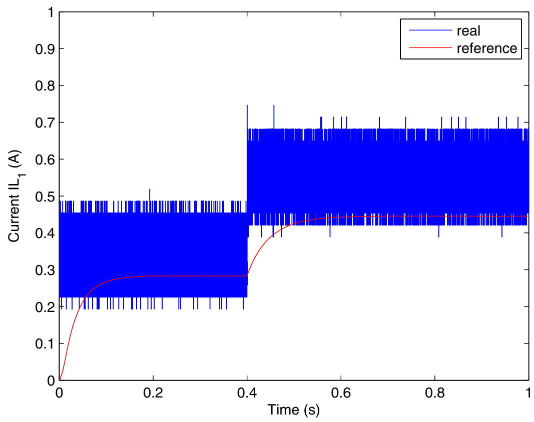

Fig. 7. The current I_{L_1}: Experimentation

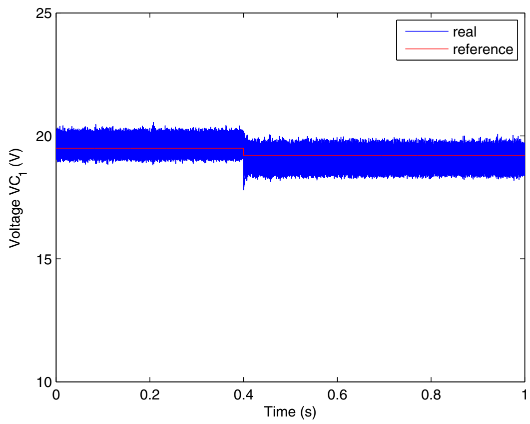

Fig. 8. The output voltage V_{C_1}: Experimentation

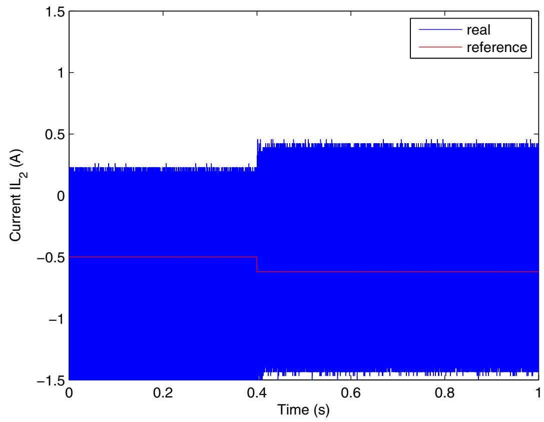

Fig. 9. The current I_{L_2}: Experimentation

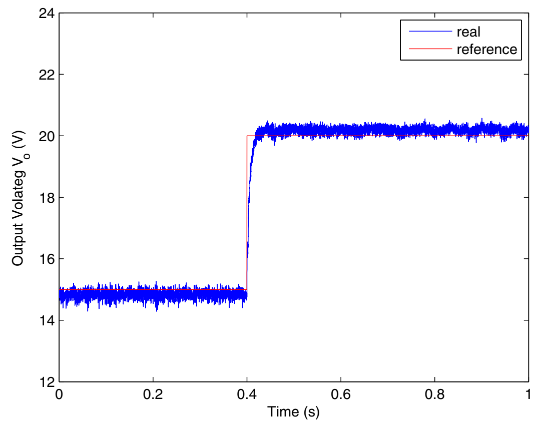

Fig. 10. The output voltage V_o: Experimentation