Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Distributed Concurrent Targeting for Linear Arrays of Point Sources ⋆

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Abstract: Motivated by practical applications in satellite formations and antenna arrays etc., the problem of targeting a linear array of point sources at one common point of interest is formulated and then solved using a novel distributed control strategy. These point sources are located collinearly and the two at the two ends orient readily to the target point. The other point sources have to rely on sensing the changes of the relative orientation angles of their nearest neighbors to adjust their own orientations; we rigorously prove that under our control law using only local information, their orientation lines will intersect at the same point of concurrency as the two point sources at the two ends. The crucial idea behind our designed control law is the intuitive argument from plane geometry that reducing the differences between the distances to the baseline of the pairwise intersection points of the orientation lines helps realizing concurrent targeting. This idea is further utilized to construct the key argument in our analysis about the boundedness and the exponential convergence speed of the orientation angles. Numerical simulations are used to validate the effectiveness of our control strategy.

Keywords: multi-agent systems, point sources, concurrent targeting, distributed coordination

1. INTRODUCTION

Coordinated teams of multiple autonomous agents have been employed in a wide range of applications, and as a result the study on cooperative control of multi-agent systems has been especially active over the past decade {Bai et al. (2011); Cao et al. (2008); Mesbahi and Egerstedt (2010); Murray (2007); Ren and Cao (2011); Qu (2009)}. There are several challenging application scenarios, though coming from different engineering fields, sharing noteworthy common features. In the satellite formation flying project led by the Jet Propulsion Laboratory (JPL) in the US, to be competent for staring-imaging missions, all the formation-flying satellites in the low earth orbit (LEO) are required to "gaze" at one target point, e.g. an aircraft carrier on the earth. In Fig. 1, the scenario is illustrated where a formation of microsatellites in space targets at a common area of interest on the earth. Such cooperative gazing tasks become even more important when synthetic aperture radars (SAR) are more and more often used for remote sensing and mapping the surface of the earth and the other planets {Krieger et al. (2007, 2009, 2013)}. In civilian applications, arrays of sensors are used to capture the changing statuses of a targeting object in order to achieve higher precision in comparison to one single sensor {Blum and Liu (2006)}. Figure 2 shows a linear array

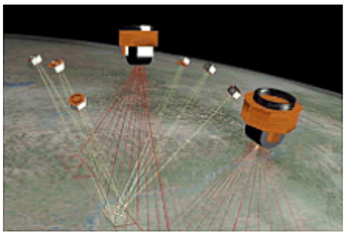


Fig. 1. Satellite formation flying in LEO [1]

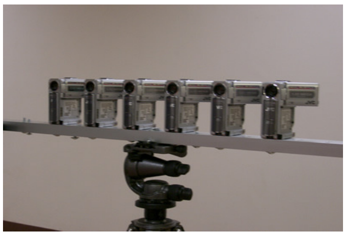


Fig. 2. A linear array of cameras [2]

of such cameras. In communication networks, arrays of directional antennas are frequently used to reduce the influence of noises or to improve energy efficiency {Mas (2011)}.

In all the applications mentioned above, the central element of these tasks is to cooperatively orient arrays of agents such that they target at one common point.

⋆ The work of Zhang was supported in part by the China Scholarship Council (CSC). The work of Ramazi and Cao was supported in part by the European Research Council (ERC- StG-307207) and the EU INTERREG program under the auspices of the SMARTBOT project.

[1] http://dst.jpl.nasa.gov/control
[2] http://www.eee.hku.hk/˜dsp

While various position or attitude coordination strategies using local information have been extensively discussed in the literature {Anderson et al. (2008); Bai et al. (2008); Hatanaka et al. (2012); Ji et al. (2008); Cao et al. (2011); Liu and Jiang (2013); Sarlette et al. (2009); Vig and Adams (2006); Wang et al. (2012); Zhou and Kumar (2012)}, the distributed concurrent targeting problem that we have just identified has several unique features that distinguish itself from most of the well studied cooperative control problems for multi-agent systems. First, when concurrent targeting is accomplished, the orientations of the agents are actually different and so these popular consensus algorithms cannot be directly applied. Second, the sharing of global and local information has special patterns: some information about the target can be manually programmed or remotely communicated to some but not all agents due to cost considerations, and so the control strategy to be designed has to make a smart classification about the agents with or without access to the information about the target. Third, the orientations are usually described by signed angles, and so the notion of the positive direction of rotating angles is a piece of critical information, which might not be able to be shared as a common knowledge among all the agents. We stress that message passing is not allowed in this setup.

While some of the features of the concurrent targeting problem are obvious, some others are subtle. It is the first goal of this paper to formulate clearly the distributed concurrent targeting task for linear arrays of point sources so that interested researchers might attack this challenging problem from different angles. Our second goal is to present our own solution to this problem and prove rigourously its effectiveness. So the main contribution of the present paper is twofold. We formulate an interesting problem arising from several application domains and thus, by attracting attention from researchers to this problem, we promote applications in satellite formations, synthetic sensor networks and directional antenna arrays. Furthermore, the control strategy that we propose carefully uses arguments from plane geometry, which shows the close connection between geometric relationships among agents and the freedom to design distributed control laws. Such exploitation can be useful in other multi-agent coordination problems when geographic or geometric information is inherently embedded in the problem formulation.

The rest of this paper is organized as follows. In Section 2 we formulate the distributed concurrent targeting problem that we are interested in. A distributed control strategy is discussed in Section 3, for which we provide its intuitive motivation and rigourous performance analysis. Numerical simulations and some concluding remarks are given in Sections 4 and 5, respectively.

2. PROBLEM FORMULATION

2.1 General description

We consider a linear array of point sources, each of which is associated with an oriented half-line in the same half-plane. Such a point source is used to model a directional antenna adjusted to transmit or receive signals in one direction, a camera calibrated with narrow field of view to focus on an object at long range, or any source whose size is negligible relative to the distance to a target of interest and whose out- or in-flow is particularly dense in one direction. In the rest of this paper, for the sake of conciseness, we simply call the point sources *agents*. All the agents are initially positioned on an oriented line, which we refer to as the *baseline*, and do *not* share the global information about the positive direction of this line. Each agent, except for the two positioned at the two ends of the linear array, has two nearest neighbors, one on its each side; each of the two agents at the two ends has only one nearest neighbor which is the closest agent in distance. Each agent is equipped with a sensor that is able to measure in its fixed local coordinates (1) the distances to the nearest neighbors, (2) for each nearest neighbor, the magnitude of that unique non-reflex angle determined by the half-line associated with itself and the half line starting from its position and passing through the corresponding nearest neighbor, and (3) for each nearest neighbor, the magnitude of that unique non-reflex angle determined by the half-line associated with the corresponding nearest neighbor and the half-line starting from that neighbor and passing through itself.

Each agent has one degree of freedom that is to rotate its half-line around its current position. The *concurrent targeting problem for linear arrays of point sources* is to design a control protocol for each agent to rotate in such a way that their half-lines intersect at a given target point in the plane. We say a solution to this problem is *distributed* if the location of the target is only known to the two agents at the two ends of the linear array and the other agents have to adjust their orientations using only their own sensed information.

2.2 Mathematical setting

For notational convenience that becomes clear later, we consider there are $n+2$ ($n \geq 1$) agents in the linear array. For analysis purposes, we choose the global coordinate system for the plane by aligning the x-axis with the baseline and positioning the origin at a point on the baseline, which is outside the line segment formed by the positions of the two agents at the two ends of the linear array. We write the coordinates of the target point by (x^r, y^r). We label the agents along the positive direction of the x-axis by $0, 1, \ldots, n+1$. For each agent i, $0 \leq i \leq n+1$, let $(x_i, 0)$ denote its position in the global coordinate system and for a pair of neighboring agents i and j, we use d_{ij} to denote their relative distance. Note that $d_{ij} = d_{ji}$. We use θ_i to denote agent i's orientation angle which is formed by rotating the x-axis counterclockwise until reaching agent i's half-line.

Let α_{ij} be the unique non-reflex angle formed by agent i's half-line and the half-line starting from $(x_i, 0)$ and passing through $(x_j, 0)$. Then for agent i, $1 \leq i \leq n$, the sensed information includes $d_{i,i-1}$, $d_{i,i+1}$, $\alpha_{i,i-1}$ and $\alpha_{i,i+1}$. Let $\alpha_i = \alpha_{i,i+1}$, $1 \leq i \leq n$, be agent i's orientation angle described in its own coordinate system, which, without loss of generality in view of the fact that the agents are in collinear positions, can be taken to be either θ_i or $\pi - \theta_i$ depending on whether the directions are the same for the zero-angle references of the local and global coordinate

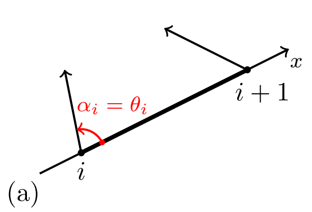


Fig. 3. Relationship between α_i and θ_i. (a) The case when $\alpha_i = \theta_i$; (b) The case when $\alpha_i = \pi - \theta_i$.

systems as illustrated in Fig. 3. We assume that agents 0 and $n+1$ are readily pointing to the target and remain in that direction; otherwise, since agents 0 and $n+1$ know about (x^r, y^r), standard control laws, e.g. PI controllers, can always realize this assumption. So we have

$$\alpha_0 = \bar{\alpha}_0, \qquad \alpha_{n+1} = \bar{\alpha}_{n+1},$$

where without ambiguity $\bar{\alpha}_0$ and $\bar{\alpha}_{n+1}$ are uniquely determined by (x^r, y^r). Assume that the rotational dynamics of each agent i, $1 \le i \le n$, can be described by a single-integrator model

$$\dot{\alpha}_i = u_i, \qquad 1 \le i \le n, \tag{1}$$

where $u_i \in \mathbb{R}$ is agent i's control input that corresponds to a torque. Note that $\dot{\alpha}_i = \dot{\theta}_i$ if agent i's zero-angle direction is the same as that of the global coordinate system and $\dot{\alpha}_i = -\dot{\theta}_i$ otherwise.

So the *distributed concurrent targeting problem* that has been defined in the previous subsection is for each agent i, $1 \le i \le n$, to design u_i using $d_{i,i-1}, d_{i,i+1}, \alpha_{i,i-1}$ and $\alpha_{i,i+1}$ such that the half-lines for all the agents pass through (x^r, y^r).

In the next section, we present our solution to this distributed concurrent targeting problem.

3. MAIN RESULTS

In this section we present our control protocol to solve the distributed concurrent targeting problem and prove that under this protocol, the half-lines of all the agents intersect at the target in the end. We first state the main theorem and then give its intuitive motivation and rigourous mathematical proof.

Theorem 1. Consider the distributed controllers

$$u_i = \frac{1}{d_{i,i^+}} \left(\cot(\alpha_{i,i^+}) + \cot(\alpha_{i^+,i})\right) + \frac{1}{d_{i,i^-}} \left(\cot(\alpha_{i,i^-}) + \cot(\alpha_{i^-,i})\right), 1 \le i \le n, \tag{2}$$

where cot denotes the cotangent function, i^+ is the label of agent i's neighbor who lies in agent i's zero-angle direction and i^- is that of the other neighbor of agent i. When $0 < \theta_i(0) < \pi$, each agent's orientation $\alpha_i(t)$ determined by the closed-loop dynamics (1) and (2) converges exponentially fast to a constant α_i^* as t goes to infinity such that the half-lines determined by $(x_i, 0)$ and α_i^* for each agent i all intersect at (x^r, y^r).

We first remark that although the initial conditions $0 < \theta_i(0) < \pi$, $1 \le i \le n$, are given using θ_i in the global coordinate system, distributed procedures can be constructed to make this initial condition satisfied by all the agents $1, \dots, n$ only using local information. This initial condition can also be automatically satisfied if the agents know the crude information about in which half-plane the target lies, e.g. using the strength of the received signals by a directional antenna, noticing that the agents share the same reference line dividing the two half-planes.

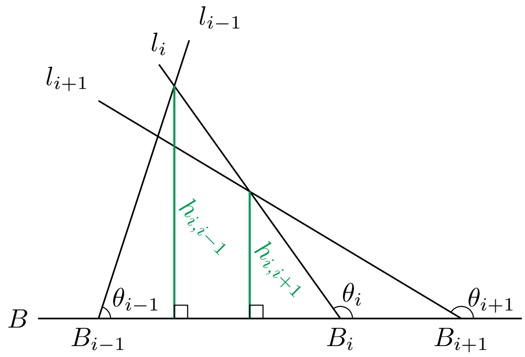


Fig. 4. One example of the geometric relationships among l_{i-1}, l_i, l_{i+1}

We use a specific scenario shown in Fig. 4 to explain intuitively why the control law (2) might work. We use l_i to denote the half-line associated with agent i. Apparently, the half-lines l_{i-1}, l_i and l_{i+1} intersect at a single point if and only if the altitudes $h_{i,i-1}$ and $h_{i,i+1}$ indicated in the figure are equal. Suppose $i^+ = i+1$, then $\frac{1}{d_{i,i^+}}\left(\cot(\alpha_{i,i^+}) + \cot(\alpha_{i^+,i})\right) = \frac{1}{h_{i,i+1}}$ and $\frac{1}{d_{i,i^-}}\left(\cot(\alpha_{i,i^-}) + \cot(\alpha_{i^-,i})\right) = -\frac{1}{h_{i,i-1}}$, and so u_i in (2) becomes $u_i = \frac{1}{h_{i,i+1}} - \frac{1}{h_{i,i-1}}$, which leads to the rotation of l_i making the difference between $h_{i,i+1}$ and $h_{i,i-1}$ become smaller. When $i^+ = i-1$, we have similar arguments.

We now proceed to provide a rigourous proof for Theorem 1. Towards this end, we need first to rewrite (1) and (2) using the global coordinates. The following result makes exactly this attempt.

Lemma 2. If $0 < \theta_i(t) < \pi$ for all $1 \le i \le n$ and $t \ge 0$, then the closed-loop dynamics (1) and (2) written using each agent's local coordinate system can be equivalently written using the global coordinate system for analysis purposes as

$$\dot{\theta}_i = a_{i,i+1} \left(\cot\theta_i - \cot\theta_{i+1}\right) - a_{i,i-1} \left(\cot\theta_{i-1} - \cot\theta_i\right), \tag{3}$$

where $a_{i,i+1} = \frac{1}{d_{i,i+1}}$ and $a_{i,i-1} = \frac{1}{d_{i,i-1}}$.

Proof. From the definition of α_i, we know that $\alpha_i = \theta_i$ if agent i's zero-angle axis agrees with the global x-axis and $\alpha_i = -\theta_i$ otherwise. For the case when $\alpha_i = \theta_i$, it follows that $i^+ = i+1$, $i^- = i-1$ and

$$\dot{\theta}_i = u_i,$$
$$\alpha_{i,i^+} = \theta_i, \qquad \alpha_{i^+,i} = \pi - \theta_{i+1},$$
$$\alpha_{i,i^-} = \pi - \theta_i, \qquad \alpha_{i^-,i} = \theta_{i-1}.$$

Substituting these equalities into (1) and (2) leads to (3). For the other case when $\alpha_i = -\theta_i$, it follows that $i^+ = i-1$, $i^- = i+1$ and

$$\dot{\theta}_i = -u_i,$$
$$\alpha_{i,i^+} = \pi - \theta_i, \qquad \alpha_{i^+,i} = \theta_{i+1},$$
$$\alpha_{i,i^-} = \theta_i, \qquad \alpha_{i^-,i} = \pi - \theta_{i-1}.$$

Again substituting these equalities into (1) and (2) leads to (3). Summarizing the two cases, we arrive at the conclusion. □

Now we prove a weaker version of Theorem 1 assuming that $0 < \theta_i(t) < \pi$ always holds throughout the n-agent system's evolution.

Proposition 1. If $0 < \theta_i(t) < \pi$ for all $1 \le i \le n$ and $t \ge 0$, each agent's orientation $\alpha_i(t)$ determined by the closed-loop dynamics (1) and (2) converges exponentially fast to a constant α_i^* as t goes to infinity such that the half-lines determined by $(x_i, 0)$ and α_i^* for each agent i all intersect at (x^r, y^r).

Proof. Because of Lemma 2, we know that the closed-loop dynamics of the n-agent system is completely described by (3). And so it suffices to study the θ_i-system (3).

Let $\eta_i = \cot\theta_i$, $i = 1, \ldots, n$. Then using the fact that $a_{ij} = a_{ji}$ for all $1 \le i, j \le n$ and $i \ne j$, from (3) we get

$$\dot\eta_i = -\left(1+\eta_i^2\right)\left(-a_{i-1,i}\eta_{i-1} + (a_{i,i-1}+a_{i,i+1})\eta_i - a_{i+1,i}\eta_{i+1}\right),$$

which can be further written into a compact form

$$\dot\eta = -M(A\eta + b), \quad (4)$$

where $\eta = [\eta_1, \cdots, \eta_n]^T$,

$$M = \begin{bmatrix} 1+\eta_1^2 & 0 & \ldots & 0 \\ 0 & 1+\eta_2^2 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 1+\eta_n^2 \end{bmatrix}, b = \begin{bmatrix} -a_{0,1}\eta_0 \\ 0 \\ \vdots \\ 0 \\ -a_{n+1,n}\eta_{n+1} \end{bmatrix},$$

and A is given in $(*)$. In the appendix, we prove that M and A are positive definite. So A^{-1} always exists and is unique. Define

$$e \triangleq \eta + A^{-1}b, \quad (5)$$

and then the dynamics of e are given by

$$\dot e = -MAe. \quad (6)$$

Since M and A are invertible, $e = \mathbf{0}$ is the unique equilibrium of (6). Consider the candidate Lyapunov function

$$V(e) = \frac{1}{2}e^T A^T e. \quad (7)$$

Obviously, it is positive definite and radially unbounded. Its time derivative along the trajectories of (6) is

$$\begin{aligned} \dot V &= e^T A^T \dot e = -e^T A^T M A e \\ &\le -\min_{1\le i\le n}(1+\eta_i^2) e^T A^T A e \\ &\le -\lambda_1^2 e^T e \\ &\le -\frac{\lambda_1^2}{\lambda_n} e^T A^T e = -\frac{2\lambda_1^2}{\lambda_n} V, \end{aligned} \quad (8)$$

where the positive numbers λ_1 and λ_n are the smallest and the largest eigenvalues of the positive definite matrix A, respectively. Hence, we know that system (6) is globally exponentially stable. Because of the one-to-one correspondence between e and η defined in (5), we know that η converges exponentially fast to $\eta^* \triangleq -A^{-1}b$. Consequently, $\cot\theta_i$ converges exponentially to η_i^*. Since $0 < \theta_i(t) < \pi$ for all $t \ge 0$ and the cotangent function is a monotone function on $(0, \pi)$, we know that θ_i converges exponentially to $\text{arccot}\eta_i^*$.

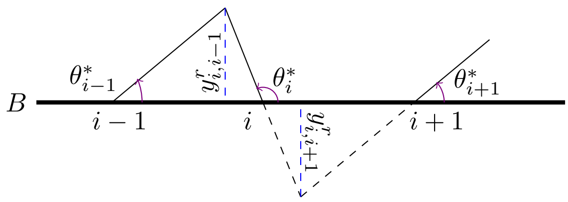


Fig. 5. A hypothetical steady state

Now we only need to prove that when $\theta_i^* = \text{arccot}\eta_i^*$, all the agents' half-lines intersect at (x^r, y^r). Since θ_i^* is the equilibrium of (3), it follows that

$$a_{i,i+1}\left(\cot\theta_i^* - \cot\theta_{i+1}^*\right) - a_{i,i-1}\left(\cot\theta_{i-1}^* - \cot\theta_i^*\right) = 0. \quad (9)$$

For $1 \le i \le n$, we examine the intersection of the half-line determined by $(x_i, 0)$, θ_i^* and the half-line determined by $(x_{i+1}, 0)$, θ_{i+1}^*, and denote its coordinates by $(x_{i,i+1}^r, y_{i,i+1}^r)$, where $y_{i,i+1}^r \ne 0$ since $0 < \theta_i(t) < \pi$, $1 \le i \le n$. We first prove by contradiction that all the intersection points have to live in the same half-plane. Suppose half-line l_i, $1 \le i \le n$, intersects with the half-lines of its neighbors in different half-planes, see Fig. 5. In this case,

$$a_{i,i+1}\left(\cot\theta_i^* - \cot\theta_{i+1}^*\right) < 0,$$

and

$$a_{i,i-1}\left(\cot\theta_{i-1}^* - \cot\theta_1^*\right) > 0,$$

which contradicts (9). So all the intersection points have to be in the same half-plane. In addition, we know $0 < \theta_0 < \theta_{n+1} < \pi$. So all the intersections have to be in the upper half-plane. Thus $y_{i,i+1}^r$, $0 \le i \le n$, are positive. From trigonometry calculation, e.g. the calculation shown in Fig. 4, one can easily show that $a_{i,i+1}\left(\cot\theta_i^* - \cot\theta_{i+1}^*\right) = y_{i,i+1}^r$. Hence, (9) implies that

$$y_{0,1}^r = y_{1,2}^r = \cdots = y_{n,n+1}^r,$$

which, because of the fact that all the points of the half-lines are uniquely determined by their y-coordinates, further implies that all the intersection points $(x_{i,i+1}^r, y_{i,i+1}^r)$ are one and the same. In addition, the two half-lines associated with agents 0 and $n+1$ intersect at (x^r, y^r), so it must be true that all the half lines are concurrent at (x^r, y^r). □

The result stated in Theorem 1 is stronger than that in Proposition 1 since it only stipulates requirements on the initial condition $\theta_i(0)$ instead of the whole evolution of $\theta_i(t)$ for $t \ge 0$. However, the proof of Proposition 1 has helped us to gain insight into the converging process of θ_i; in particular, the construction of the candidate Lyapunov function (7) has shed light on how the trajectories of system (3) might be bounded for all $t \ge 0$. So we are ready to prove the main result of this paper as follows.

Proof of Theorem 1: Because of the result in Proposition 1, it suffices to prove that for system (3), if $0 < \theta_i(0) < \pi$, then $0 < \theta_i(t) < \pi$ holds for all $t \ge 0$. We prove this by contradiction. Suppose $0 < \theta_j(t) < \pi$ can be violated for some $1 \le j \le n$ and we let $T > 0$ to be the first time instant that such a violation takes place. Then since $\theta_j(t)$ changes continuously, we know that either $\theta_j(T) = 0$ or $\theta_j(T) = \pi$. In either case, we have $\lim_{t\to T}\eta_j(t) = \infty$ and so do $e(t)$ and $V(t)$. Since T is the first time that $0 < \theta_i(t) < \pi$, $1 \le i \le n$, is violated, from (8) in the proof of Proposition 1 we know that $\dot V(t) < 0$ for all $0 \le t < T$

$$A = \begin{bmatrix} a_{0,1}+a_{1,2} & -a_{1,2} & & & & \\ -a_{1,2} & a_{1,2}+a_{2,3} & -a_{2,3} & & & \\ & \ddots & \ddots & & & \\ & & -a_{i-1,i} & a_{i-1,i}+a_{i,i+1} & -a_{i,i+1} & \\ & & & \ddots & \ddots & \\ & & & & -a_{n-1,n} & a_{n-1,n}+a_{n,n+1} \end{bmatrix} \quad (*)$$

and thus it must be true that $\lim_{t\to T} V(t) < V(0)$ is finite. W have arrived at a contradiction and the proof is complete. □

Remark 3. In fact, from (8) one can see that as long as θ_0 and θ_{n+1} are fixed, V always decreases and θ_i, $1 \leq i \leq n$, always converges. So if the half-lines of agents 0 and $n+1$ are parallel to each other instead of intersecting at (x^r, y^r), one can check that all the θ_i, $0 \leq i \leq n+1$, become the same exponentially fast. So the distributed control law (2) can also be used to rotate all the half-lines to achieve the same orientation.

In the next section, we will present our simulation results.

4. SIMULATION EXAMPLES

We consider a linear array of 10 agents that are positioned randomly on the baseline. The initial orientations $\theta_i(0)$, $1 \leq i \leq 8$, are randomly chosen from $(0, \pi)$ and the half-lines of agents 0 and 9 intersect at a single point, which is taken to be the target. The converging process of the orientations of these agents under the control law (2) is shown in Fig. 6, where the red half-lines are fixed and associated with the two agents at the two ends, and the blue ones rotate and are for the other agents. As expected, all the half-lines become concurrent at the target.

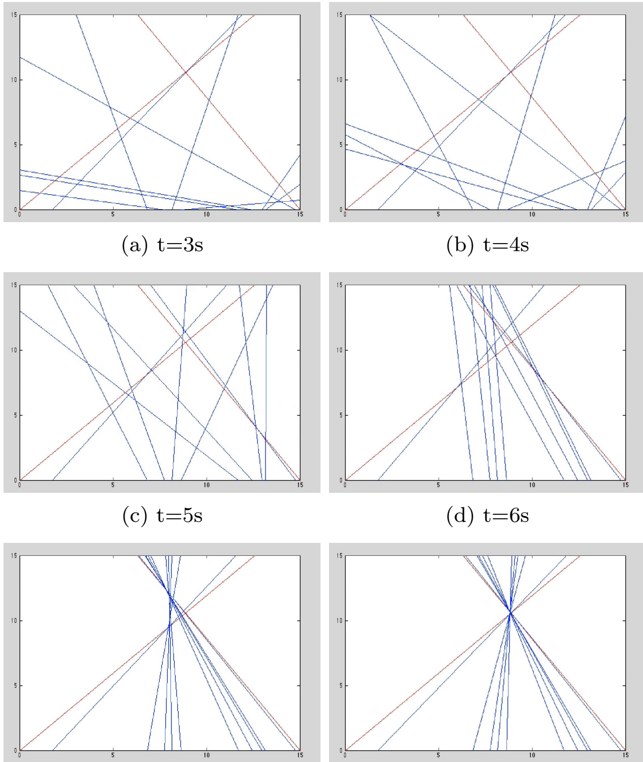
One interesting observation from the simulations that worths further investigation is that some half-lines might become close to being concurrent at a point different than the target before later on they continue to rotate until passing the target. Subgroup "clustering" before group-level synchronizing is common in the rendezvousing behaviors of multiple autonomous robots {Martinez et al. (2007)}, and we will explore the connections to the control laws for solving rendezvous problems in the future.

5. CONCLUSIONS

In this paper we have formulated the distributed concurrent targeting problem for linear arrays of point sources and proposed a control strategy to solve this problem. Both theoretical analysis and numerical simulations have been provided to show the performance of our proposed control law. We are currently working on two generalizations. One is to consider groups of point sources that distribute in the plane instead of on a line. The other is to consider more detailed rotational dynamics of the point sources.

6. ACKNOWLEDGMENT

We thank Prof. Claudio De Persis and Dr. James R. Riehl for their technical discussions.



(a) t=3s (b) t=4s (c) t=5s (d) t=6s (e) t=10s (f) t=40s

Fig. 6. The converging process of the half-lines of the agents

Appendix A. THE POSITIVE DEFINITENESS OF M AND A

The positive definiteness of M follows from the fact that it is a diagonal matrix with positive elements. For the symmetric matrix A, $a_{i-1,i}$, $1 \leq i \leq n+1$, are positive, and

$$A_{ii} \geq \sum_{j=1, j\neq i}^{n} \mid A_{ij} \mid .$$

From the Gershgorin circle theorem {Horn and Johnson (1985)}, we know that all the eigenvalues of A are non-negative. Now we prove by contradiction that they are all positive. Suppose A has a zero eigenvalue. Then there exists a non-zero eigenvector ξ such that $A\xi = \mathbf{0}$. Since the ith row sum of A, $i = 2, \cdots, n-1$, is zero, ξ has to be $\mathbf{1}_n$. However, such an $\xi = \mathbf{1}_n$ cannot be an eigenvector since the first and the last row sums of A are non-zeros. So we have reached a contradiction. Thus 0 is not an eigenvalue of A, and hence, A is positive definite.

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