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Fault diagnosis methodology based on nonlinear system modelling and frequency analysis

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Abstract: In this paper, a new fault diagnosis methodology based on nonlinear system modelling and frequency analysis is developed. The method is derived via resolving several fundamental issues associated with conducting system fault diagnosis using the NARMAX (Nonlinear Auto-Regressive Moving Average with eXougenous inputs) modelling and NOFRFs (Nonlinear Output Frequency Response Functions) based frequency analysis, which was recently proposed and extends the well-known linear system modelling and FRF (Frequency Response Function) based frequency analysis to the nonlinear case. Simulation studies verify the effectiveness of the new method and demonstrate the performance of the method when applied to address a mechanical structural system fault diagnosis problem. The new methodology has the potential to resolve a wide range of input output data based engineering system fault diagnosis problems

Keywords: Fault detection and diagnosis, Nonlinear system identification, Volterra series, Frequency Analysis, Associated linear equations

1. INTRODUCTION

Fault diagnosis can ensure reliable and safe operations of engineering systems and is therefore very important in a wide range of engineering disciplinary areas.

There are many available fault diagnosis strategies including, for example, parity equations Chow and Willsky [1984], state observers Patton et al. [1989], Frank et al. [2000], and system identification Isermann [2005, 2006], etc. In these approaches, faults are either represented as unknown inputs or disturbances to model parameters. The model parameter change based fault representations are capable to represent component wear and sensor failure, and it has been well recognized that system identification approaches are more adequate for dealing with associated fault diagnosis problems Venkatasubramanian et al. [2003]. However, this requires a very detailed physical model of the underlying system, which is often not feasible in practice Venkatasubramanian et al. [2003]. One way to circumvent this difficulty is to use discrete time data based black-box modelling and frequency response function (FRF) based model analysis in which the main features of the FRF are used for the characterization of faults in a system. The well-known modal analysis based fault diagnosis for civil and mechanical structural systems is, for example, a typical and widely applied approach in this category Cunha et al. [2006].The basic principle behind this approach is that any variations in the characteristics of physical systems can be reflected by the changes in the system FRFs. Consequently, the changes in the system FRF can be used to conduct system fault diagnosis. However, in most practical cases where this approach is applied, underlying systems are assumed to be linear. This is basically because the FRF is a well-known linear system concept.

In order to extend the system modelling and frequency analysis based fault diagnosis approach to the nonlinear case, problems with nonlinear system modelling and nonlinear system frequency domain analysis have to be addressed. The black-box modelling for nonlinear systems has been well studied. There are many methods that can be used to identify so-called NARMAX (Nonlinear Auto-Regressive Moving Average with eXogeneous input) models for nonlinear systems which include neural networks and polynomial nonlinear system models Billings [2013]. The frequency domain analysis of nonlinear systems has also been extensively studied. The direct extension of the linear system frequency domain analysis to the nonlinear case is based on the concept of Generalised Frequency Response Functions (GFRFs) proposed by George [1959]. The GFRFs are an extension of the concept of FRF to the nonlinear case under the assumption that the underlying nonlinear systems can be described by a convergent Volterra series. This assumption is valid if so-called fading memory condition - the system response is only dependent on recent inputs but independent from inputs in the remote past - is satisfied by the system Boyd and Chua [1985], which is equivalent to that the nonlinear system is stable about an equilibrium and therefore holds in very general practical situations.

The combination of the black box nonlinear system modelling and GFRF based frequency analysis has been proposed for many years and applied in studies in different science and engineering areas including engineering system fault diagnosis Billings [2013], Tang et al. [2010]. However, due to the multi-dimensional nature of the GFRFs, the GFRFs can only be fully displayed up to the second order. This implies that the well-established Bode diagram based linear system frequency domain analysis cannot be generally extended to the nonlinear case using the concept of GFRFs. Consequently, there is a real need to derive a systematic approach that can be generally applied in practice to analyse an identified model of nonlinear engineering systems in the frequency domain and then conduct the system fault diagnosis using the frequency domain features of the systems.

In order to resolve the difficulties with the practical application of GFRFs and to achieve the objective of developing a frequency analysis method that can be applied to a wide range of nonlinear systems, researchers have made considerable efforts Lang and Billings [2005], Lang et al. [2007], Nuij et al. [2006], Pavlov et al. [2006], Feijoo et al. [2004], Vazquez Feijoo et al. [2010]. Among these, the concept of Nonlinear Output Frequency Response Functions (NOFRFs) proposed by Lang and Billings [2005], has not only provided a new extension of FRF to the nonlinear case but, as having been demonstrated by a wide range of recent studies Peng et al. [2011, 2007a,b], also has potential to profoundly solve problems with the GFRFs etc available nonlinear frequency analysis methods. In addition, a framework that combines the NARMAX modelling and NOFRFs based frequency analysis has been proposed to conduct engineering system fault diagnosis and the performance of this basic idea has been demonstrated by experimental data analysis Peng et al. [2011].

Under the framework of NARMAX modelling and NOFRF based frequency analysis, the fault diagnosis for an engineering system is conducted by analysing the frequency domain features of an identified nonlinear model of the system represented by the systems NOFRFs which represent the frequency characteristics of the system using a series of one-dimensional functions of frequency. There are many fundamental issues that have to be systematically addressed before the ideas behind this framework can be widely applied in engineering system fault diagnosis. These include how to accurately determine the NOFRFs from an identified nonlinear model of the system under inspection, how to extract representative frequency domain features of the system from the NOFRFs, and how to conduct fault diagnosis for the system using the extracted system frequency domain features.

The present study is concerned with the derivation of effective algorithms and methods to resolve these fundamental issues associated with the application of the framework of NARMAX modelling and NOFRFs based frequency analysis to engineering system fault diagnosis, and to demonstrate the effectiveness of these new algorithms and methods in fault diagnosis applications by simulation studies.

The paper is constructed as follows. Section 2 introduces the basic idea of fault diagnosis based on nonlinear system modelling and frequency analysis. Section 3 is dedicated to the derivation of a new algorithm for the determination of NOFRFs, which is the key technique to the new fault diagnosis method proposed in the present study. In Section 4, the detailed procedure of the new method is described and necessary analyses and discussions are provided. Section 5 is concerned with simulation studies. Finally the conclusions are presented in Section 6.

2. FAULT DIAGNOSIS BASED ON NONLINEAR SYSTEM MODELLING AND FREQUENCY ANALYSIS

In engineering practice, faults can often be characterized as disturbances to system physical parameters. In structure health monitoring, for example, the condition of material can be assessed by its Young modulus. However, directly using these physical quantities is often not feasible, as they may not be directly measurable. These problems can often be addressed by evaluating the changes in a representation such as, e,g., FRF of the system characteristics as the changes in the representation can reflect the changes in system parameters. This is why estimating natural frequencies and mode shapes have proved to be more convenient ways to determine the condition of structural systems in mechanical and civil engineering.

Given input-output data of a system, measuring the system FRF can be effectively carried out by linear system modelling, for which a wide variety of techniques are available. Once the FRF has been obtained, an automated procedure, such as, e.g., comparison with a priori known patterns of FRF can be applied to facilitate the assessment if there are any changes in the system parameters.

The method above requires the systems to be linear, which may not be the case in practice. When the effect of nonlinearities cannot be neglected, system modelling can be achieved by using well known approaches such as the NARMAX modelling Leontaritis and Billings [1985]. However, the corresponding frequency analysis is more complicated, as the FRF concept is no longer valid. Extensions of the FRF concept to nonlinear cases have been widely studied, but can only be applied to particular classes of nonlinear systems using approaches such as describing functions Nuij et al. [2006] and generalised frequency response functions (GFRFs) George [1959].

The concept of NOFRFs were recently proposed to overcome the problems with these available nonlinear system frequency analysis approaches. Consider discrete time systems that can be described by the Volterra Series

$$y(t) = \sum_{n} y_n(t) \tag{1}$$

$$y_n(t) = \sum_{k_1=-\infty}^{\infty} \dots \sum_{k_n=-\infty}^{\infty} h_n(\boldsymbol{k}_n) \prod_{i=1}^{n} u(t-k_i) \tag{2}$$

where $y_n(t)$ denotes the n-th order Volterra functional and $h_n(\boldsymbol{k}_n) = h_n(k_1, \dots, k_n)$ is known as the n-th order kernel. Applying the Discrete Fourier Transform (DFT) to (2) yields

$$Y_n(\omega) = \sum \dots \sum H_n(\boldsymbol{\omega}_n) \prod_{i=1}^{n} U(\omega_i) \tag{3}$$

where the summations are computed over all frequencies for which $\omega_1 + \ldots + \omega_n = \omega$. The n-dimensional function $H_n(\boldsymbol{\omega}_n) = H_n(\omega_1, \ldots, \omega_n)$ is the DFT of the n-th order Volterra kernel and called n-th order GFRF.

The concept of NOFRFs was proposed in this context and defined as Lang and Billings [2005]

$$G_n(\omega) = \frac{Y_n(\omega)}{U_n(\omega)}, \qquad |U_n(\omega)| \neq 0 \tag{4}$$

where $U_n(\omega)$ is given by

$$U_n(\omega) = \mathrm{DFT}\{u(t)^n\} = \sum \cdots \sum \prod_{i=1}^{n} U(\omega_i) \tag{5}$$

which can be called the generalised input spectrum where the summations are computed in the same way as in (4).

For a given input, the NOFRFs of nonlinear system (1)-(2) up to the n-th order provide a representation for the system characteristics by a series of one-dimensional functions of frequency. These functions of frequency are of the nature similar to that of the FRF of linear systems because of the following attractive properties, Lang and Billings [2005]

- $G_n(\omega)$ is a one-dimensional function
- The frequency range of $G_n(\omega)$ is the same as that of $Y_n(\omega)$ and $U_n(\omega)$
- $Y_n(\omega)$ can be described similarly to a linear system output frequency response
- $G_n(\omega)$ is input dependent, but is the invariant to gain changes in $U(\omega)$.

Consequently, a framework that combines the NARMAX modelling and NOFRFs based frequency analysis has been proposed to conduct engineering system fault diagnosis Peng et al. [2007a]:

(i) Identifying a NARMAX model for the system under inspection from the input output data
(ii) Determining the NOFRFs of the system from the identified NARX model.
(iii) Extracting the frequency domain features of the system from the determined NOFRFs, and
(iv) Conducting fault diagnosis for the system using the extracted frequency domain features

and the effectiveness of the basic idea in each step has been demonstrated by experimental data analysis.

However, the works reported in Peng et al. [2007a] are preliminary feasibility studies. Comprehensive investigations on the algorithms/methods that can be applied to implement each of the four steps are needed to literally establish a methodology that can be directly applied to the fault diagnosis of a wide range of engineering systems.

The present study is motivated by these needs. The works are mainly concerned with deriving a new and more effective algorithm to determine the NOFRFs from an identified system NARMAX model for Step (ii), and developing a systemic Principle Component Analysis (PCA) based NOFRFs feature extraction method and a Neural Network (NN) based fault diagnosis system for Steps (iii) and (iv). These will be the focuses of Sections 3 and 4, respectively.

3. A METHOD FOR ACCURATE DETERMINATION OF NOFRFS UP TO AN ARBITRARY ORDER

Because nonlinear system identification has been comprehensively studied and there are a wide range of algorithms that can be applied to find the NARMAX model of a nonlinear system, Step (i) in the general framework introduced in Section 2 above can be relatively easily addressed. In Step (ii) of this framework, the NOFRFs of an inspected system need to be determined from an identified system. In Peng et al. [2011], this was achieved by using a Least Squares based numerical approach, which relies on a correct assumption for the maximum order of the system Volterra series representation, easily suffers from many numerical problems and, consequently, often cannot be used to accurately determine the system NOFRF up to any order. In order to address these problems, in this section, a new algorithm for the determination of NOFRFs is derived by exploiting the idea of Associated Linear Equations (ALEs) Feijoo et al. [2004] and extending the basic results of ALEs to the general NARMAX model.

Consider the NARX (Nonlinear Auto-Regressive with eXougenous inputs) model of a nonlinear system described by the following discrete-time polynomial model

$$A\, y(t) = B\, u(t) + \sum_{m=1}^{M} c_m F_m(t) \tag{6}$$

$$F_m(t) = \prod_{l=1}^{L} y(t-l)^{p(m,l)}\, u(t-l)^{q(m,l)} \tag{7}$$

where A and B are linear, time-shifting operators and $p(m,l)$ and $q(m,l)$ are non-negative integers such that $p(m,l) + q(m,l) \geq 2$. Model (6)-(7) can be obtained from step (i) of the general framework by fitting a polynomial NARMAX model to the system input-output data and then setting the error terms to zero.

The basic idea of the new algorithm is to determine the NOFRFs directly using definition (4). This requires decoupling the system n-th order output frequency response $Y_n(\omega)$ from the system output frequency response $Y(\omega)$. For this purpose, the concept of ALEs is exploited. For nonlinear systems described by NARX model (6)-(7), the ALEs are linear difference equations with respect to $y_n(t)$ whose right-hand sides depend only on $u(t)$ and the system nonlinear outputs of orders lower than n. Consequently, $y_n(t)$ can be found by recursively solving these linear deference equations, and the system NOFRFs can then be determined using (4). However, available methods for obtaining ALEs can only deal with a specific class of nonlinear systems Feijoo et al. [2004]. In order to extend the ideas of formulating and solving ALEs to the much more general NARX model (6)-(7), the following proposition is derived.

Proposition 1. The n-th order ALE of system (6) can be described as

$$A\, y_n(t) = B\, u(t) \qquad n = 1 \tag{8}$$

$$A\, y_n(t) = \sum_{m=1}^{M} c_m\, \psi_m(t) \sum_{S_m} \rho_m\, \phi_m(t) \qquad n \geq 2 \tag{9}$$

where

$$\rho_m = \frac{\prod_{l=1}^{L} p(m,l)\,!}{\prod_{l=1}^{L}\prod_{j=1}^{J_m} r(m,l,j)\,!} \tag{10}$$

$$\psi_m(t) = \prod_{l=1}^{L} u(t-l)^{q(m,l)} \tag{11}$$

$$\phi_m(t) = \prod_{l=1}^{L}\prod_{j=1}^{J_m} y_j(t-l)^{r(m,l,j)} \tag{12}$$

$$J_m = n - \sum_{l=1}^{L} q(m,l) + p(m,l) + 1 \tag{13}$$

and S_m is the set of all non-negative integer solutions of the following Diophantine system

$$\sum_{j=1}^{J_m} r(m,l,j) = p(m,l) \qquad 1 \le l \le L \tag{14}$$

$$\sum_{l=1}^{L}\sum_{j=1}^{J_m}(j-1)\,r(m,l,j) = J_m - 1 \tag{15}$$

Proof. Omitted due to page limitation

According to Proposition 1, the n-th order ALE for system (6)-(7) can readily be obtained by using the algorithm as follows.

Algorithm for determining the n-th order ALE

- *Step 1* Write down $\psi_m(t)$, as in (11)
- *Step 2* Build the associated Diophantine system (14)-(15)
- *Step 3* Find all solutions of the Diophantine system
- *Step 4* For each solution found in *Step 3*, write down:
  - · *Step 4.1* The coefficient ρ_m, as in (10)
  - · *Step 4.2* The term $\phi_m(t)$, as in (12)
  - · *Step 4.3* The product $\rho_m\phi_m(t)$
- *Step 5* Obtain the sum of all terms found in *Step 4.3*
- *Step 6* Multiply the results of *Step 5* and *Step 1*
- *Step 7* Multiply the result of *Step 6* by c_m
- *Step 8* Sum all terms found in *Steps 1-7*
- *Step 9* Obtain the left-hand side of the n-th order ALE as $Ay_n(t)$ and the right-hand side as the result of *Step 8*

In order to demonstrate how the procedure works, consider a specific case of system (6)-(7) where A, B and c_1 are arbitrary, $M = 1$, $L = 2$ and $F_1 = y(t-1)^2u(t-1)$.

In this case, for $n = 1$, the ALE can be obtained as

$$A\,y_1(t) = B\,u(t) \tag{16}$$

For $n = 2$, steps 1-4 yield:

- *Step 1* $\psi_1(t) = u(t-1)$
- *Step 2* See Table 1
- *Step 3* See Table 1
- *Step 4* Solutions do not exist, indicating this system does not have a 2-nd order ALE

For $n = 3$:

- *Step 1* $\psi_1(t) = u(t-1)$
- *Step 2* See Table 1
- *Step 3* See Table 1
- *Step 4*
  - · *Step 4.1* $\rho_1 = 1$
  - · *Step 4.2* $\phi_1(t) = y_1(t-1)^2$
  - · *Step 4.3* $\rho_1\phi_1(t) = y_1(t-1)^2$
- *Step 5* $y_1(t-1)^2$
- *Step 6* $y_1(t-1)^2u(t-1)$
- *Step 7* $c_1y_1(t-1)^2u(t-1)$
- *Step 8* $c_1y_1(t-1)^2u(t-1)$
- *Step 9* $A\,y_3(t) = c_1\,y_1(t-1)^2u(t-1)$

Table 1. Diophantine systems of example

n	J_m	System	Number of solutions
2	0	$r(1,1,1) = 2$ $0 = -1$	0
3	1	$r(1,1,1) = 2$ $0 = 0$	1

Once the ALEs up to order n are known, the system NOFRFs can be readily computed for a given input, using the following algorithm

Algorithm for determining the NOFRFs

- *Step 1* Write down all ALEs up to order n
- *Step 2* Solve the ALE system for $y_i(t)$, $1 \le i \le n$
- *Step 3* Determine the i-th order NOFRF $G_i(\omega)$ as follows:
  - · *Step 3.1* Compute $Y_i(\omega) = \mathrm{DFT}\{y_i(t)\}$
  - · *Step 3.2* Compute $U_i(\omega) = \mathrm{DFT}\{u(t)^i\}$
  - · *Step 3.3* Compute $G_i(\omega) = Y_i(\omega)/U_i(\omega)$

It is worth pointing out that this algorithm can be used to accurately determine the NOFRFs of a nonlinear system up to an arbitrary order from the NARX model of the system. Because there is no need to know the maximum order of the system's Volterra series representation, the algorithm overcomes all the problems with the numerical approach originally proposed in Lang and Billings [2005]. In the next section, by using the novel algorithm as the core technique, a new fault diagnosis method under the general framework of nonlinear systems modelling and frequency analysis will be proposed.

4. A NEW FAULT DIAGNOSIS METHOD BASED ON NONLINEAR SYSTEM MODELLING AND FREQUENCY ANALYSIS

The general framework introduced in Section 2 indicates that the fault diagnosis using nonlinear systems modelling and frequency analysis include four steps. In this section, a method will be proposed which includes specific procedures for each of these steps and can therefore be literally applied to conduct fault diagnosis for systems directly from the input and output data.

Step (i) in the general framework can, as already mentioned above, be effectively addressed using well established nonlinear system identification methods. In the proposed method, the PRESS statistics based NARMAX modelling approach Li et al. [2013] will be applied due to its distinctive advantages of being able to integrate modelling and model validation into one single procedure.

Step (ii) is concerned with the determination of NOFRFs from an identified NARX model which will be implemented using the effective algorithm proposed in Section 3 above.

In order to implement Step (iii), the n-th order NOFRF obtained from Step (ii) under a given input with band width $[0, b]$ are described as

$$G_n(\omega) = \alpha_n(\omega)\,\exp\left(j\,\theta_n(\omega)\right) \tag{17}$$

where $\alpha_n(\omega)$ and $\theta_n(\omega)$ are real functions representing the amplitude and phase of the n-th order NOFRF, respectively. Define:

$$\boldsymbol{\alpha}_n^T = [\,\alpha_n(0)\ \ldots\ \alpha_n(nb)\,] \tag{18}$$

$$\boldsymbol{\theta}_n^T = [\,\theta_n(0)\ \ldots\ \theta_n(nb)\,] \tag{19}$$

to represent the information of the underlying system that can be revealed from the nth order NOFRF. Consequently, vector

$$\boldsymbol{x}^T = \left[\,\boldsymbol{\alpha}_1^T\ \ldots\ \boldsymbol{\alpha}_n^T\ \boldsymbol{\theta}_1^T\ \ldots\ \boldsymbol{\theta}_n^T\ \right] \tag{20}$$

provides all the information about the system represented by the system NOFRFs up to the n-th order. The objective of Step (iii) is to extract the features of the system from all the information represented by the NOFRFs which is completely contained in vector $\boldsymbol{x}$. Considering the very large dimension of $\boldsymbol{x}$, the PCA (principal component analysis) approach will be applied in the proposed method to extract a lower dimensional feature vector $\boldsymbol{z}$, such that

$$\boldsymbol{z} = \boldsymbol{x}^T \boldsymbol{P} \tag{21}$$

where $\boldsymbol{P}$ needs to be determined via a training process for the fault diagnosis.

Step (iv) is to conduct fault diagnosis. For this purpose, a neural network will be used as a classifier with $\boldsymbol{z}$ determined from Step (iii) as the input and scores indicating the membership to pre-defined fault classes as the output as illustrated in Fig 1, where vector $\boldsymbol{z}$ is the NN inputs, $\boldsymbol{v} = [v_1\ \ldots\ v_s]$ are the NN outputs, with v_i, $1 \leq i \leq s$ taking either 0 or 1 and s is the number of output classes in the training data set. The neural network classifier needs, as matrix $\boldsymbol{P}$ in Step (iii), to be established via a training process before it can be used for the fault diagnosis. According to the ideas introduced above, the detailed procedures of the new fault diagnosis method using nonlinear systems modelling and frequency analysis can be summarised as follows.

(i) Fit a NARX model to the input and output data from the system to be inspected using the PRESS statistics based NARMAX modelling approach.
(ii) Determine the NOFRFs of the system from the NARX model identified in Step (i) and build the full frequency domain feature vector $\boldsymbol{x}$ of the system as given in (20)
(iii) Extract a lower dimensional feature z of the system by applying a priori trained PCA algorithm (PCA transformation matrix P) to the full frequency domain feature vector $\boldsymbol{x}$ to yield $\boldsymbol{z} = \boldsymbol{x}^T \boldsymbol{P}$
(iv) Conduct the fault diagnosis for the system by applying *a priori* trained neural network classifier to the lower dimensional feature vector $\boldsymbol{z}$ to produce a score vector $\boldsymbol{v} = [v_1\ \ldots\ v_s]$ indicating the status of the system under inspection.

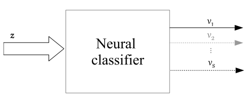

Fig. 1. Neural network fault classifier

A diagram illustrating each step of the new method and the relationships between all the steps is shown in Figure 2. Before this proposed method can be applied, training is needed to determine the PCA transformation matrix $\boldsymbol{P}$ for Step (iii) and to build the neural network classifier for Step (iv).

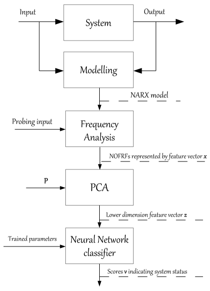

Fig. 2. An illustration of the new method

In order to determine matrix $\boldsymbol{P}$, a set of template systems covering some *a priori* known cases of faulty conditions need to be available. From the input and output data of each of these template systems, the full frequency domain feature vector $\boldsymbol{x}_i$, $i = 1, \ldots, d$, corresponding to these template systems can be obtained by using Steps (i) and (ii) above, where d is the total number of available template systems. Consequently, the matrix $\boldsymbol{P}$ can be determined by applying PCA to a data matrix $\boldsymbol{X}$ whose i-th row is $\boldsymbol{x}_i^T$.

For the purpose of building the neural network classifier, a r-input and s-output neural network needs to be trained by using $\boldsymbol{z}$ as input vector and $\boldsymbol{v} = [v_1\ \ldots\ v_s]^T$ as output vector. Here, r is the dimension of $\boldsymbol{z}$, s is the number of fault classes covered by the template systems, and binary values of $v_1, \ldots, v_s$ are determined by the fault class of the system represented by $\boldsymbol{z}$. For example, if $\boldsymbol{z}$ represents the first of the s fault classes, then $\boldsymbol{v} = [1\ 0\ \ldots\ 0]$; if $\boldsymbol{z}$ represents the second of the s fault classes, then $\boldsymbol{v} = [0\ 1\ \ldots\ 0]$. Thus, by using the mapping between $\boldsymbol{z}_i = \boldsymbol{x}_i^T \boldsymbol{P}$ and $\boldsymbol{v}_i = [v_{1i}\ \ldots\ v_{si}]$, $i = 1, \ldots, d$, the neural network classifier can be found for Step (iv) of the proposed method.

In the next section, a simple simulation example will be used to show how the proposed method works and to demonstrate the effectiveness of the method in conducting fault diagnosis for a system with four different fault classes.

5. SIMULATION STUDY

In this section, the new diagnosis method developed in Section 4 is exemplified, by conducting fault diagnosis for a nonlinear oscillator described as

$$\ddot{y}(t) + \zeta_1 \dot{y}(t) + \zeta_3 \dot{y}(t)^3 + k_1 y(t) + k_3 y(t)^3 = u(t) \tag{22}$$

This type of oscillator is useful for modelling practical situations where moderate nonlinear behaviour arises due to damage or defects in engineering structures. For example, the vibration of cracked structures can be described by a second order oscillator where the stiffness term can be modelled as a polynomial function Peng et al. [2007]. In this context, (22) represents a particular case in which the stiffness has been represented by the first and third order terms only, which is an useful simplification for describing nonlinear effects, while preserving the symmetry of the oscillator restoring force.

System (22) is considered as normal when the parameters take the following nominal values: $\zeta_1 = 2\xi\omega_n$, $k_1 = \omega_n^2$, $\zeta_3 = 0.2\xi\omega_n$, $k_3 = \omega_n^2$, where $\xi = 0.7$ and $\omega_n = 2\pi$. Linear parameters k_1 and ζ_1 are assumed to be fixed, i.e., free of faults.

Training stage

As described in Section 4, a training stage from which matrix $\boldsymbol{P}$ and the NN classifier are obtained, is required prior to diagnosis operation. For this purpose, a simple scenario with 4 fault classes was considered. Each fault class consists of parameter k_3 or ζ_3 taking values different from the ones corresponding to the system normal status. The exact faulty parameter value can be any one within a specific range, as demonstrated in Table 2.

Table 2. Fault classes

Fault Type	Parameter variation			
	k_3		ζ_3	
	Min.	Max.	Min.	Max.
1	$0.4\omega_n^2$	$0.6\omega_n^2$	$0.20\xi\omega_n$	$0.20\xi\omega_n$
2	ω_n^2	ω_n^2	$0.08\xi\omega_n$	$0.12\xi\omega_n$
3	$1.4\omega_n^2$	$1.6\omega_n^2$	$0.20\xi\omega_n$	$0.20\xi\omega_n$
4	ω_n^2	ω_n^2	$0.28\xi\omega_n$	$0.32\xi\omega_n$

The training set consists of $d = 40$ template systems, 10 from each fault class that has been defined. Training data was obtained following steps (i) and (ii) of the procedure in section 4. For step (i), models with maximum delay $L = 2$ and nonlinear terms of degree 2 and 3 were identified. For step (ii), NOFRFs were obtained according to the algorithm presented in Section 3, using a sinc pulse with bandwidth $[0; 3]$ Hz as the probing input. Figures 3-5 show the magnitude of the NOFRFs up to 3-rd order. Figure 3 show that the obtained nonlinear models have different 1-st order NOFRFs, although the faulty states are only related to nonlinearities. This is due to the biased nature of parameter estimates associated with PRESS statistics that was used as the data fitting criterion to build the NARX model in step (i). However, as will be demonstrated later on, this does not compromise the diagnosis system performance, as these models still contain relevant information about each faulty condition.

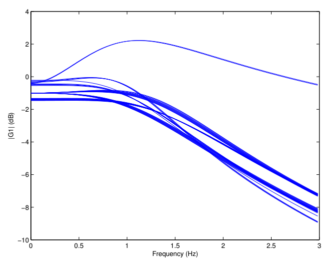

Fig. 3. $|G_1(\omega)|$ for each fault template

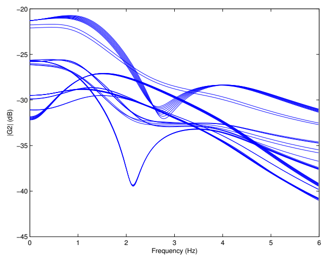

Fig. 4. $|G_2(\omega)|$ for each fault template

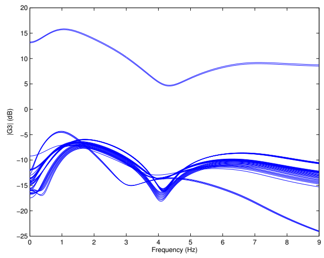

Fig. 5. $|G_3(\omega)|$ for each fault template

After that, NOFRFs are used for building the vectors $\boldsymbol{x}_i$, $1 \leq i \leq 40$, as described in (20) and matrix $\boldsymbol{X}$ as

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{x}_1^T \\ \vdots \\ \boldsymbol{x}_{40}^T \end{bmatrix} \quad (23)$$

Then, PCA is applied to the covariance matrix of $\boldsymbol{X}$ for yielding the transformation matrix $\boldsymbol{P}$. The PCA revealed that five components were sufficient for representing most of the training data variance, therefore, the number of inputs of the NN was set as $r = 5$. In addition, because four fault classes were specified, the number of outputs of the NN, i.e. the dimension of $\boldsymbol{z}$, was set as $s = 4$.

The NN classifier consists of a fully connected MLP (multi-layer perceptron) network with sigmoid activation functions, trained with the standard back-propagation algorithm. The number of hidden layers and neurons was adjusted during the training process by verifying the network performance. Finally, an optimal configuration with one hidden layer of 5 neurons was found. This classifier was able to correctly classify 100% of the training data.

Testing stage

To verify the performance of the diagnosis system, testing data was generated, consisting of 80 fault patterns different from training cases, 20 from each fault type, where the parameter disturbance was randomly chosen within the corresponding parameter interval. Some classification scores are shown in Table 3. The classifier was able to correctly recognize 100% of all the 80 fault patterns. The last row in Table 3 contains the classifier score obtained for when the system is under the normal state, whose template was not present in the training data. The decrease in scores indicates that this is clearly a new pattern and additional training should be carried out for accommodating the corresponding new class.

Table 3. Some classification results

Parameters		Classifier score			
k_3	ζ_3	v_1	v_2	v_3	v_4
$0.4143\omega_n^2$	$0.2000\xi\omega_n$	0.9991	0.0003	0.0000	0.0027
$0.5875\omega_n^2$	$0.2000\xi\omega_n$	0.9996	0.0001	0.0008	0.0000
$1.0000\omega_n^2$	$0.0819\xi\omega_n$	0.0008	0.9988	0.0010	0.0000
$1.0000\omega_n^2$	$0.1197\xi\omega_n$	0.0002	0.9994	0.0000	0.0006
$1.4248\omega_n^2$	$0.2000\xi\omega_n$	0.0002	0.0010	0.9990	0.0012
$1.5868\omega_n^2$	$0.2000\xi\omega_n$	0.0003	0.0004	0.9986	0.0008
$1.0000\omega_n^2$	$0.2803\xi\omega_n$	0.0000	0.0000	0.0006	0.9988
$1.0000\omega_n^2$	$0.3177\xi\omega_n$	0.0005	0.0011	0.0000	0.9998
$1.0000\omega_n^2$	$0.2000\xi\omega_n$	0.0000	0.0272	0.0000	0.8142

It is interesting to note that near optimal results can be found for some NN configuration, including scenarios with data vectors of reduced size. For example, a classifier with 12 neurons and a single hidden layer is able to classify faults with 99% efficiency by using only magnitude data of the NOFRFs. The same is true for a network whose input is only based in phase of the NOFRFs, but with a slightly different structure of 10 neurons. Notice that in all cases, the classifiers had a very simple structure showing that NOFRFs are efficient features for characterizing different fault cases of a system.

6. CONCLUSION

Fault diagnosis for systems directly from input and output data often involves system modelling and model frequency analysis and has been used to address modal analysis based structural health monitoring etc many practical engineering problems. In most of these applications, inspected systems are assumed to be linear, and linear system modelling and FRF based frequency analysis are applied to reveal changes in the system characteristics from the changes in the system FRF.

In order to extend this approach to the more general nonlinear case, issues with nonlinear system modelling and nonlinear system frequency analysis have to be addressed. Although nonlinear system modelling has been comprehensively studied, and many methods can be used to identify NARMAX etc nonlinear system models, existing GFRFs etc concept based methods for nonlinear frequency analysis are difficult to be applied in practice to conduct frequency analysis for the purpose fault diagnosis in a way similar to the FRF based analysis for linear systems. In order to solve this problem, the concept of NOFRFs was recently proposed which provides a representation for the frequency domain properties of nonlinear systems similar to the FRF representation of linear systems. Moreover, a framework that combines the NARMAX modelling and NOFRFs based frequency analysis has been proposed to conduct system fault diagnosis and the performance of this idea has been demonstrated by experimental data analysis.

The present study is concerned with addressing several fundamental issues associated with applying NARMAX modelling and NOFRFs based frequency analysis to the fault diagnosis of practical engineering systems. These involve how to accurately determine the NOFRFs from an identified nonlinear model of the system under inspection, how to extract representative frequency domain features of the system from the NOFRFs, and how to conduct fault diagnosis for the system using the extracted system frequency domain features. Effective algorithms and methods have been derived to address these problems. These include a new algorithm that can accurately determine the NOFRFs up to an arbitrary order from an identified system NARX model, a systemic Principle Component Analysis (PCA) based NOFRFs feature extraction method, and a Neural Network classifier for fault diagnosis. Simulation studies have been conducted. The results verify the effectiveness of the proposed new algorithm and methods and demonstrate the performance of these new techniques when being applied to conduct nonlinear system modelling and frequency analysis based fault diagnosis.

The study has established a comprehensive fault diagnosis methodology based on nonlinear system modelling and frequency analysis, which has the potential to resolve a wide range of input output data based engineering system fault diagnosis problems.

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