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A control system for the individual route guidance in traffic flow networks

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Abstract: The problem of optimally routing a class of guided vehicles on an urban traffic network is considered in this paper. The guided vehicles give their requests in terms of origin, destination, and time window, and the control system returns to each of them the start time and the path to be followed, as provided by the solution of an original individual route guidance (IRG) problem. The control system integrates the IRG problem with a macroscopic traffic model in order to take into account the evolution of the traffic (in particular, of the congestions) on the network links. The traffic model and the IRG problem are mutually dependent one from the other, and for this reason the ultimate solution of the route guidance problem is iteratively sought. The control system has the objective of minimizing both the impact of the whole traffic and the individual costs of the guided vehicles.

Keywords: Intelligent transportation systems; Traffic control; Optimization problems; Mathematical programming, Discrete-time traffic model.

1. INTRODUCTION

The optimization of traffic flows in urban areas is one of the most considerable challenges faced by systems and transportation engineers. The difficulty of building new infrastructures drives scientists and engineers towards the development of models and methods aimed at the mitigation of congestion phenomena. Moreover, the advances in technology and the availability of personal electronic devices make possible the adoption of specific strategies to induce the drivers to follow certain paths of a traffic network. In this paper, the problem of individually routing a class of users through the network is considered, and an original control system is proposed to solve it.

One of the most significant methods which have been adopted in the past to mitigate traffic congestions is the route guidance, that is, the optimal routing of vehicles through the traffic network. In particular, the *collective route guidance* has been adopted worldwide by means of the use of variable message signs (Pedic and Ezrakhovich [1999]), VMSs, which provide information (traffic conditions, accident warnings, public utility messages, and so on) to the users. Several approaches aiming at optimizing and controlling a traffic network by means of VMSs can be found in the literature (e.g., Mahmassani and Hawas [1997], Park [2005], Zuurbier et al. [2006]). However, the effectiveness of the collective routing to optimize traffic flows is questionable owing to the need of suitably modelling the responsiveness of drivers to the messages. Such an issue has been considered in the past (Mammar et al. [1996], Levinson and Huo [2003]), and some models have been defined (e.g., *logit*- and *probit*-based models, Peeta and Ramos Jr. [2006]). The *individual route guidance* (IRG) is not a recent technique (Case and French [1987], Van Aerde and Case [1988]), but the interest in such methodology has grown in the last decade, due to the availability of personal electronic devices which allow users to communicate the data of their trips and to receive information about the path to be followed (Panou [2012], Pan et al. [2013]). IRG approaches are mainly based on the solution of shortest path problems (Deo and Pang [1984], Fu et al. [2006]), and are obviously directed to the determination, for each single vehicle, of the optimal routes from a given origin to a given destination (as, for example, in Fu [2001]).

In this paper, a traffic control system which includes the solution of an original IRG problem is proposed. The number of guided vehicles is assumed to be considerable, so that the solution of the IRG problem has an impact on the traffic dynamics. For this reason, in the proposed system, the route guidance problem is coupled with a *dynamic traffic model* in order to deal with the effects of the IRG on the traffic. An iterative procedure is proposed: starting from a basic solution of the IRG problem and from a *stochastic user equilibrium* (SUE) traffic assignment (Cascetta [2009]), the traffic model provides the new travel times on the links (considering the flows of guided vehicles in addition to that of unguided ones); next, the IRG problem is solved and its solution provides the new flows of guided vehicles, which are calculated on the basis of the new travel times; next, the traffic model provides new further travel times as a consequence of the new flows, and so on.

The traffic model is a macroscopic network model which explicitly considers the queues in the links, in order to take into account congestion phenomena which usually characterize an urban traffic networks (spillbacks, bottlenecks, etc.). Each link is a discrete-time dynamic system and consists of a running section and of a queue section (the lenghts of the two sections vary with time). The traffic model is fed by an estimation of the flows of unguided vehicles entering the network and by the flows of guided vehicles provided by the solution of the IRG problem.

2. THE CONTROL SYSTEM

The control system combines, in an iterative way, the solution of an IRG problem with a macroscopic dynamic traffic model, with the aim of finding the optimal paths for the guided vehicles. In the following, the term *controlled flow* (resp., *uncontrolled flows*) will be used to denote a flow of vehicles consisting of guided vehicles (resp., unguided vehicles) only.

As sketched in Fig. 1, the traffic model provides the evolution of the traffic flows (both controlled and uncontrolled) on the network, and computes the travel times on the links for the relevant time interval. It is fed from the outside by the uncontrolled flows which enter the network at nodes, and, to compute the travel times, it takes into account the controlled flows provided by the solution S_p of the IRG problem. As a matter of fact, the controlled flows are assumed to be a significant portion of the overall flows on the network and then they affect the performance (i.e., the travel times) of the system. The IRG problem is modeled as a mathematical programming problem whose solution defines the path to be followed by each guided vehicle. It is fed from the outside by the single *travel requests* and it takes into account the travel times $\tau_{i,j}$ on the links, as provided by the traffic model.

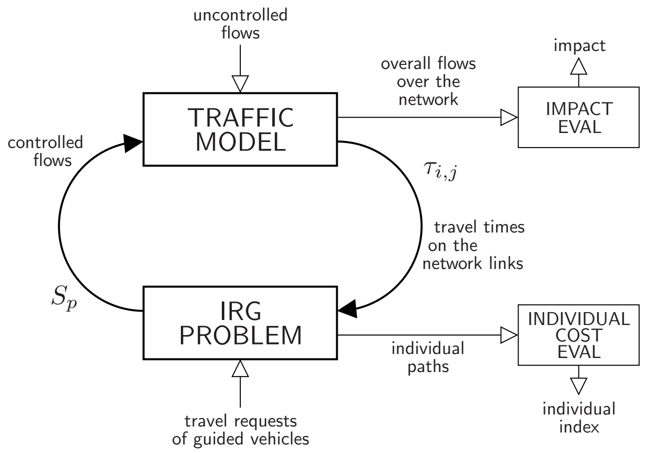

Fig. 1. The proposed control system.

The traffic model is influenced by the solution of the IRG problem and the IRG problem is influenced by the evolution of the traffic model. Then, these two parts of the control system are iteratively executed/solved until an equilibrium point is reached, that is, until the values of the performance indices of the system (impact for the whole network and individual costs for the guided vehicles) change significantly.

As regards the system timing, the traffic model and the IRG problem act at different time scales. The traffic model is a discrete-time model whose sampling interval is Δt (e.g., 30 seconds or 1 minute), whereas the IRG problem optimizes the paths of the guided vehicles within a wider time interval ΔT (e.g., 1 hour), which is assumed to be an integer multiple of Δt, namely, $\Delta T = N\,\Delta t$. In this connection, let:

- $[T_h, T_{h+1}]$, $h = 0, \ldots, Q-1$, be the generic optimization interval of the IRG problem ($[T_0, T_Q]$ is the overall optimization horizon);
- $[t_{k,h}, t_{k+1,h}]$, $k = 0, \ldots, N-1$, be the generic discrete-time interval of the traffic model, within the h-th optimization interval; clearly, $t_{0,h} = T_h$ and $t_{N,h} = T_{h+1}$.

A detailed scheme of the control system is in Fig. 2. The traffic model provides the overall flows on the network links for any discrete-time interval $[t_{k,h}, t_{k+1,h}]$ of the h-th optimization interval, namely, $q(t_{0,h}, S_p), \ldots, q(t_{N-1,h}, S_p)$. The inputs of the model are:

- an estimations of the uncontrolled flows which enter the network in the various intervals, namely $\widehat{q}_{\mathrm{U}}(t_{0,h}), \ldots, \widehat{q}_{\mathrm{U}}(t_{N-1,h})$; such estimations are determined on the basis of:
  (1) the nominal input flows (as provided by a SUE traffic assignment process of the origin-destination matrix $\mathcal{OD}(T_h)$ for the considered interval);
  (2) the percentage of guided vehicles in the considered optimization interval;
  (3) the measurements of actual input flows (of unguided vehicles) in the preceding optimization intervals;
- the controlled flows which are "assigned" to the network in the various intervals, namely $q_{\mathrm{C}}(t_{0,h}, S_p), \ldots, q_{\mathrm{C}}(t_{N-1,h}, S_p)$; such flows are provided (through a suitable user choice model) by the solution S_p of the IRG problem, which is the solution at the p-th step of the iterative procedure;
- the splitting percentages at intersections, namely $\alpha(T_h)$; such values are a-priori determined via the SUE traffic assignment process; it is worth noting that such percentages are assumed fixed within a single optimization horizon (that is, they are not affected by the solution of the IRG problem) since the unguided users do not know in advance the paths that the guided vehicles will follow, and then they take their choices only on the basis of the mean travel times which are attained in analogous past periods.

The IRG problem is solved on the basis of the individual travel requests whose departure time is within the actual optimization interval (they are denoted by $\mathcal{R}(T_h)$), and on the basis of the travel times on the network for the whole interval, namely $\tau(t_{0,h}, S_p), \ldots, \tau(t_{N-1,h}, S_p)$. A solution S_p consists of the optimal paths to be followed by the guided vehicles. At the beginning, the controlled flows are initialized through the solution S_0 of a shortest path problem on the traffic network loaded with the nominal travel times $\tau^{\mathrm{nom}}(T_h)$, as provided by the traffic assignment process. Such a solution is hereafter indicated as $\underline{q}_{\mathrm{C}}^{0}$. Note that the initial solution does not guarantee that the guided vehicles are assigned to their relevant optimal paths. In fact, this solution does not take into account the effects that guided flows have on the travel times, as described in the following section.

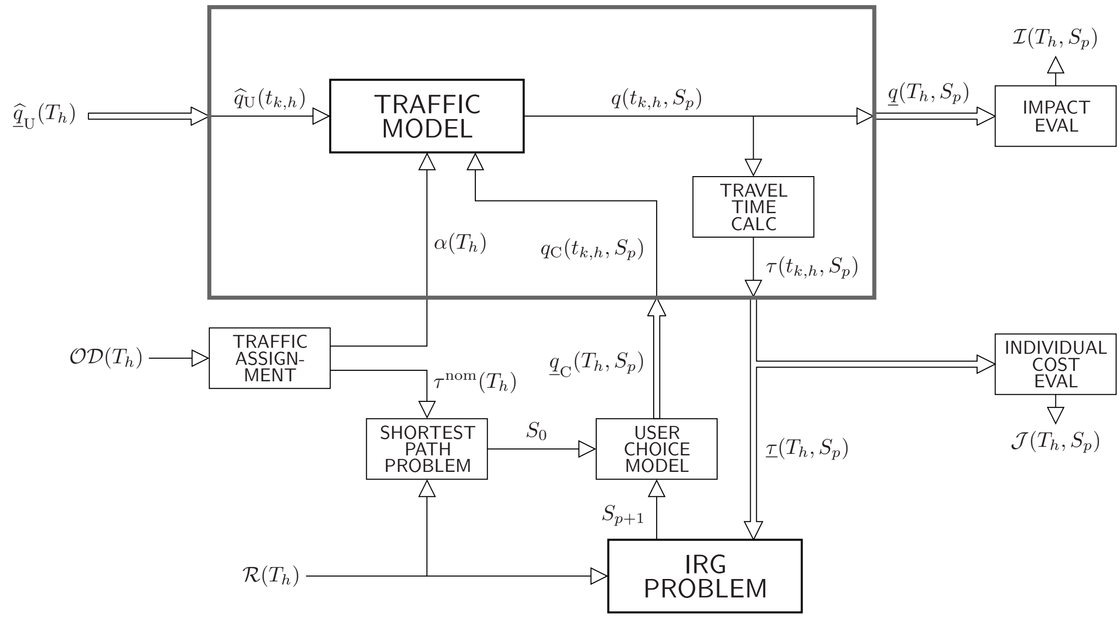

Fig. 2. The proposed control system (detailed scheme).

2.1 Iterative Search of an Equilibrium

The guided vehicles follow the paths that are provided by the IRG problem. Such paths are in general different from the shortest paths which can be determined only on the basis of the nominal (constant) travel times $\tau^{\mathrm{nom}}(T_h)$. As a matter of fact, the IRG problem tries to split users on different alternative paths, so that the phenomenon of creating congestion by sending vehicles to the same shortest path (as sometimes happens in the collective routing when too many users follow the VMS indications, and consequently go to the same links) is avoided.

In this paper, it is assumed that the travel demand of guided vehicles is sufficiently large to influence the travel time on links, that is $\tau_{i,j}(q_{U\,i,j}) < \tau_{i,j}(q_{U\,i,j} + q_{C\,i,j})$ for some link (i, j) of the traffic network. In other words, the optimal paths followed by the guided vehicles increase the flows on the relevant links and this leads to an increase of the travel times on that links. Such an increase of the travel times makes the solution S_p of the IRG problem no longer optimal, as it was determined on the basis of different travel times. For this reason, the IRG problem is again solved with the new actual travel times and a new solution S_{p+1} is provided. Such a new solution provides the new controlled flows, which lead to the new overall flows on the network and to the new travel times. Again, a further solution of the individual routing problem can be sought on the basis of such new travel times.

Such a process can be thought of as the iterative sequence

$$\tau^{\mathrm{nom}} \to \underline{q}_{\mathrm{C}}^{0} \to \underline{\tau}^{0} \to \ldots \to \underline{q}_{\mathrm{C}}^{p} \to \underline{\tau}^{p} \to \underline{q}_{\mathrm{C}}^{p+1} \to \underline{\tau}^{p+1} \to \ldots \tag{1}$$

in which

- $\underline{\tau}^{p} = \underline{\tau}(T_h, S_p)$, $p = 0, 1, \ldots$, is the vector of travel times that are computed on the basis of the overall flows provided by the traffic model loaded with controlled flows provided by the solution S_p of the IRG problem;
- $\underline{q}_{\mathrm{C}}^{p} = \underline{q}_{\mathrm{C}}(T_h, S_p)$, $p = 1, 2, \ldots$, is the vector of the controlled flows provided by the solution S_p of the IRG problem.

The iterative sequence (1) starts from the initial controlled flows $\underline{q}_{\mathrm{C}}^{0}$ and stops when $\underline{q}_{\mathrm{C}}^{p} \simeq \underline{q}_{\mathrm{C}}^{p+1} = \underline{q}_{\mathrm{C}}^{\star}$ and $\underline{\tau}^{p} \simeq \underline{\tau}^{p+1} = \underline{\tau}^{\star}$ for some $p \in \{0, 1, \ldots\}$. The pair $(\underline{q}_{\mathrm{C}}^{\star}, \underline{\tau}^{\star})$ represents the equilibrium.

The iterative search of an equilibrium can be expressed by the fixed point problem

$$\begin{cases} \underline{q}_{\mathrm{C}}(T_h, S_{p+1}) = f\Big[\underline{\tau}(T_h, S_p), \mathcal{R}(T_h)\Big] \\ \underline{\tau}(T_h, S_p) = g\Big[\underline{q}_{\mathrm{C}}(T_h, S_p), \widehat{\underline{q}}_{\mathrm{U}}(T_h)\Big] \end{cases} \tag{2}$$

initialized by the "initial" controlled flows $\underline{q}_{\mathrm{C}}(T_h, S_0) = \underline{q}_{\mathrm{C}}^{0}$. In (2), f represents the activity of solving the IRG problem and g represents the activity of calculating (by means of the traffic model) the travel times from the overall flows on the network. In (1), (2), and Fig. 2, $\underline{q}_{\mathrm{C}}(T_h, S_p)$, $\underline{\tau}(T_h, S_p)$, $\widehat{\underline{q}}_{\mathrm{U}}(T_h)$, and $\underline{q}(T_h, S_p)$ are the vector of all $q_{\mathrm{C}}(t_{k,h}, S_p)$, $\tau(t_{k,h}, S_p)$, $\widehat{q}_{\mathrm{U}}(t_{k,h})$), and $q(t_{k,h}, S_p)$, respectively, $k = 0, \ldots, N-1$.

The performance of the traffic control system can be measured through the *impact* $\mathcal{I}(T_h, S_p)$ and the individual *generalized cost* $\mathcal{J}(T_h, S_p)$. Such indices are determined through the overall flows $\underline{q}(T_h, S_p)$ and the travel times $\underline{\tau}(T_h, S_p)$, respectively. The impact parameters include the total pollutant emissions, the generated noise, and the total delays; it is worth observing that, in the proposed system, the impacts are determined only with the aim of understanding their behavior in different scenarios, although their values do not influence the choices of both guided and unguided vehicles. The individual costs consist of the sum of the travel times attained by the guided vehicles in the execution of their paths (no monetary cost is taken into account). These costs, on the contrary, influence the behaviors of the guided users, which can always chose whether to follow the suggested path or not, although this event is assumed to be infrequent.

3. THE INDIVIDUAL ROUTE GUIDANCE PROBLEM

The IRG problem is now defined for the period $[T_h, T_{h+1}]$, $h \in \{0, \ldots, Q-1\}$, and for the generic iteration p, $p \in \{1, 2, \ldots\}$; in the following, for the sake of simplicity, the two indices h and p will be dropped from all variables.

The traffic network is modeled through a directed graph $G = \{N, L\}$, being N the set of nodes and L the set of links. $(i, j) \in L$ is the directed link from node $i \in N$ to node $j \in N$. Let $\mathcal{P}_i$ (resp., $\mathcal{S}_i$), $i \in N$, the set of predecessor (resp., successor) nodes of node i, that is, the set of nodes $m \in N$ (resp., $j \in N$) such that $(m, i) \in L$ (resp., $(i, j) \in L$). Moreover, let $\mathcal{D}_i$, $i \in N$, be the set of destinations (nodes) which are reachable from node i.

Each link of the traffic network is a discrete-time dynamic system, with discretization interval equal to Δt. As introduced in Section 2, index k denotes the generic discrete-time interval $[t_k, t_{k+1}]$ of the traffic model. Moreover, let $\tau_{i,j}(k)$ be the estimated travel time for vehicles entering link (i, j) at interval k. The calculation of the estimated travel time is provided in Appendix A, where the traffic model is defined.

In the formalization of the IRG problem, the time-varying travel times represent the weights of the graph's links. Moreover, consider the following definitions:

- U is the set of users that ask for an optimal path, that is, the users that generate the demand $\mathcal{R}(T_h)$, and the traffic flows $\underline{q}_{\mathrm{C}}$;
- $o_u \in N$ and $d_u \in N$ are the origin node and the destination node, respectively, of user $u \in U$;
- ψ_u and δ_u are the departure time and the estimated arrival time, respectively, that are determined by the the solution of the IRG problem and communicated to user u; $\underline{\psi}$ (resp., $\underline{\delta}$) is the vector gathering ψ_u (resp., δ_u) $\forall\, u \in U$;
- $[\Psi_u, \Delta_u]$ is the time window required by user $u \in U$ for its travel;
- $x_{i,j,u}$, with $(i, j) \in L$ and $u \in U$, is a binary variable that assume the value 1 if user u travels on the link (i, j) to reach d_u from o_u, and 0 otherwise; $\underline{x}$ is the vector gathering $x_{i,j,u}$, $\forall\, (i, j) \in L$, $\forall\, u \in U$.

The IRG problem is solved by executing an original mathematical programming problem at each discrete-time t_k. Note that, at t_k, some of the guided vehicles that started their trip before t_k are still traveling on the network towards their final destinations. In this connection, consider the following definitions:

- $U^{\mathrm{T}} \subseteq U$ is the subset of guided users that are traveling on the network at t_k;
- $(i^{\mathrm{T}}_u, o^{\mathrm{T}}_u) \in L$ is the link of the traffic network in which user $u \in U^{\mathrm{T}}$ is at t_k;
- ψ^{T}_u is the time that user $u \in U^{\mathrm{T}}$ will take to reach its destination node $o^{\mathrm{T}}_u \in N$;

In the following subsections, the formulation of the mathematical programming problem is first given, and then the optimization algorithm which finds the solution of the IRG problem is introduced.

3.1 The Mathematical Programming Problem

Assume that the travel times are constant, and let $\tau_{i,j}$ be the travel time on link $(i, j) \in L$. The mathematical programming problem, for the generic k-th iteration of the IRG problem, is the following.

Problem 1. Given o_u, d_u, Ψ_u, and Δ_u for each user $u \in U \setminus U^{\mathrm{T}}$, Given i^{T}_u, o^{T}_u, d_u, ψ^{T}_u, and Δ_u for each user $u \in U^{\mathrm{T}}$, given $\tau_{i,j}$ and $Q^{\mathrm{M}}_{i,j}$ for each link $(i, j) \in L$ of the traffic network, find:

$$\min_{\underline{x},\underline{\psi}} \left\{ \max_{u \in U \setminus U^{\mathrm{T}}} \Big\{ |\psi_u - \Psi_u| \Big\} + \max_{u \in U} \Big\{ |\delta_u - \Delta_u| \Big\} \right\} \quad (3)$$

subject to:

$$\delta_u = \psi_u + \sum_{\forall (i,j) \in L} \tau_{i,j} x_{i,j,u} \quad \forall\, u \in U \setminus U^{\mathrm{T}} \quad (4)$$

$$\delta_u = \psi^{\mathrm{T}}_u + \sum_{\forall (i,j) \in L} \tau_{i,j} x_{i,j,u} \quad \forall\, u \in U^{\mathrm{T}} \quad (5)$$

$$\sum_{\forall l \in \mathcal{S}_i} x_{i,l,u} - \sum_{\forall l \in \mathcal{P}_i} x_{l,i,u} = \kappa_{i,u} \quad \forall\, i \in N, \forall\, u \in U \quad (6)$$

$$\kappa_{i,u} = \begin{cases} 1 & i = o_u \\ -1 & i = d_u \\ 0 & \text{otherwise} \end{cases} \quad \forall\, i \in N, \forall\, u \in U \setminus U^{\mathrm{T}} \quad (7)$$

$$\kappa_{i,u} = \begin{cases} -1 & i = d_u \\ 0 & \text{otherwise} \end{cases} \quad \forall\, i \in N, \forall\, u \in U^{\mathrm{T}} \quad (8)$$

$$x_{i^{\mathrm{T}}_u, o^{\mathrm{T}}_u, u} = 1, \quad \forall\, u \in U^{\mathrm{T}} \quad (9)$$

$$\sum_{\forall u \in U} x_{i,j,u} \le Q^{\mathrm{M}}_{i,j} \quad \forall\, (i, j) \in L \quad (10)$$

$$x_{i,j,u} \in \{0, 1\}, \quad \forall (i, j) \in L, \forall u \in U \quad (11)$$

$$\psi_u \in \mathbb{R}_{\ge 0}, \quad \forall\, u \in U \quad (12)$$

The cost function (3) of Problem 1 minimizes the largest difference between the required departure and arrival times and the corresponding optimized ones. Moreover:

- constraints (4)–(5) define the arrival time of users;
- constraints (6)–(9), together with the definition of variables $x_{i,j,u}$ in (11), define the structure of the graph and impose the continuity of the path of users (included those which are traveling at t_k);
- constraint (10) is introduced to avoid an excessive load of vehicles on a link; the parameter $Q^{\mathrm{M}}_{i,j}$ can be determined on the basis of the SUE traffic assignment process (at the beginning) or on the basis of the evolution of the traffic model, and can be considered as the available room which is left by the unguided vehicles;
- constraints (11)–(12) define the decision variables.

Problem 1 is a Mixed Continuous-Integer Linear Problem (MCILP). It can be solved in a negligible time with respect to the optimization period $[T_h, T_{h+1}]$, even for large networks.

3.2 The Optimization Algorithm

As mentioned before, the travel times on links change as time passes; however, they have been considered constant in the formalization of the optimization problem in the previous section. To cope with this problem, a specific

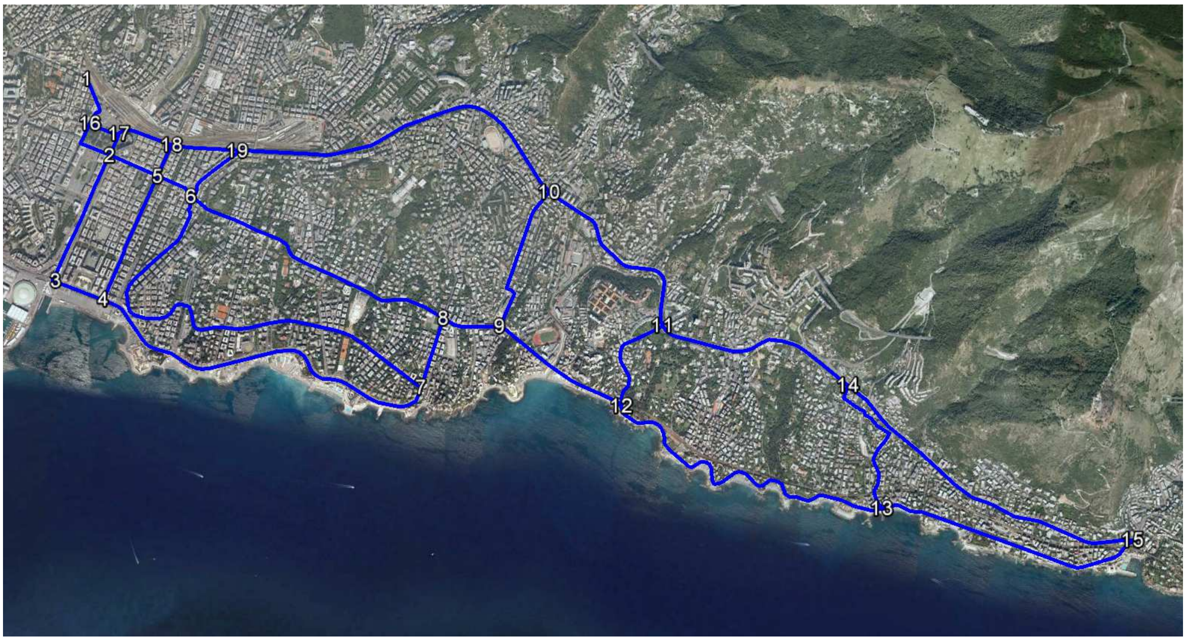

Fig. 3. Case study transportation network.

instance of the optimization problem is solved at each discrete-time instant. More specifically, a new instance of Problem 1 is solved after having updated:

- the travel times on links
- the set of users (both U and U^{T}), because some of them could have started their trip and others could have already reached their destination;
- the positions of the guided vehicles that are traveling on the traffic network.

Then, the optimization algorithm can be written as:

(1) Set $k = 0$ and $U^{\mathrm{T}} = \emptyset$
(2) Set $\tau_{i,j} = \tau_{i,j}(k)$ and $Q^{\mathrm{M}}_{i,j} = Q^{\mathrm{M}}_{i,j}(k)$
(3) Solve Problem 1 and apply the solution
(4) If $k = N - 1$ Stop. Otherwise go to step 5
(5) (a) determine the set of users which end their trip in the interval $[t_k, t_{k+1}]$ and remove such users from U;
  (b) determine the set of users which start their trip in the interval $[t_k, t_{k+1}]$ and add such users to U^{T};
  (c) determine the position at t_{k+1} of the guided vehicles which belong to the (updated) set U^{T} and update values i^{T}_u, o^{T}_u, and ψ^{T}_u, for such vehicles;
(6) Set $k = k + 1$ and go to step 2.

The path for each guided vehicle consists of the set of links associated with the variables $x_{i,j,u} = 1$ that are iteratively computed at different iteration of the procedure. Once determined the path for each controlled vehicle, in connection with each discrete-time interval, the vector of the controlled traffic flows $\underline{q}_C$ can be easily determined by simply adding and gathering all the paths.

Clearly, the solution provided by the proposed algorithm is suboptimal, because it is not guaranteed that the paths determined in t_k and applied till t_{k+1} are the best for the subsequent periods of time. Nevertheless, the whole procedure has the advantages of simplicity. In particular, the optimization problem is relatively simple, and can be solved efficiently in very short time. Moreover, travel times usually do not change too much among different periods of the same macro-period T_h, and then the sub-optimality of the provided solution is expected to be small.

On the other hand, users can refuse to follow the suggested path, or part of it. Such a phenomenon can be instantaneously detected by monitoring the guided vehicle positions, and can be managed by the optimization algorithm by suitably updating the input data and set of Problem 1 at each algorithm iteration. To tackle with this problem, it is possible to reformulate and solve Problem 1 via stochastic programming techniques (Birge and Louveaux [2011]).

4. SYSTEM TEST

An application to a real world case study is described in this section. To this aim, consider the network depicted in Fig. 3, which is relative to a portion of the Italian city of Genoa. The considered portion covers around 11 km^2 along a 7.5 km long coastline, with about 100,000 citizens. The network takes into account the main roads and consists of 19 nodes, 22 bi-directional links, and 6 mono-directional links (the mono-directional links are: (2,5), (5,6), (19,18), (18,17), (17,16), and (16,2)).

For what concerns the behavior of the network users, it is assumed that all of them accept the proposed route. Such an assumption comes from the consideration that the proposed architecture provides personalized information to each single user.

As regards the simulation scenario, a time varying demand is considered. In particular, given the nominal demand and the link costs computed at the initial time, a perturbation in the flows (equivalent to a reduced demand equivalent to 0.75 of the nominal one) is then introduced.

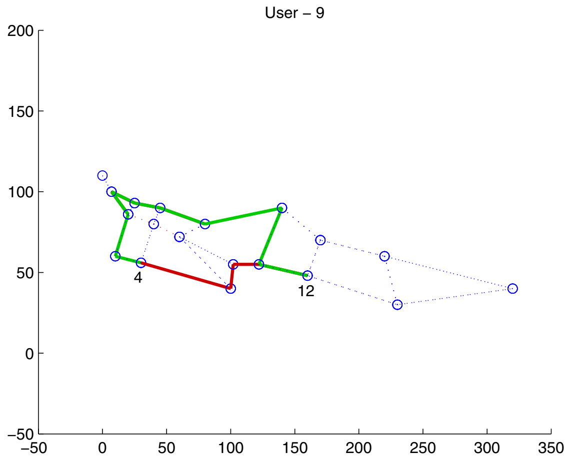

Fig. 4. Nominal path (green) and modified path (red) computed when the travel times are updated.

The considered rolling horizon framework is able to take into account such a change in the costs, as shown in Fig. 4 where, as an example, the optimal paths of the user 9 are reported. In such a figure, the green line, which starts from the origin node 12 and ends in the destination node 4, represents the optimal path computed with the nominal demand, whereas the red one represents the "deviation" computed by the IRG problem when the travel times change due to a traffic flows variation. In the case study, such a new path is computed and communicated to the user while it is traveling on the first link exiting from the node 12, that is, $(i_9^{\mathrm{T}}, o_9^{\mathrm{T}}) = (12, 9)$.

From the travel time point of view, taking into account the traffic congestion due to all the vehicles in the network, the red path requires 20.83 min, whereas the green one requires 17.78 min. This difference is due to the road characteristics of links (with particular reference to the link (7,4)); in effects, although the green path is longer, the relevant roads have greater capacity, and consequently less congestion and travel time. When the traffic is reduced, the red path turns out to be competitive being its travel time equal to 13.96 min with respect of the green one which is 15.44 min.

A second simulation shows the effect of the maximum flow constraint (10). In such a scenario, a set U of 450 users are assumed to travel from node 15 towards node 4 at the same time. The results, depicted in Fig. 5, show the different suggested paths for three generic users, when $Q_{i,j}^M = 270$ vehicles (in this case study, such a parameter is assumed link- and time-invariant for simplicity).

In particular, this solution shows the capability of the control system of avoiding an excessive congestion increase on some links, due to the large number of guided vehicles "routed" on them. Note that, the more path alternatives are considered, the more the constraint (10) can be made stringent by reducing the "capacity" parameter $Q_{i,j}^M$. By the way, the initial value of $Q_{i,j}^M$, $\forall\,(i,j) \in L$, has to be a-priori computed via the assignment process that allows determining the maximum tolerable increase of traffic flows on arcs that does not significantly deteriorate the whole network performance.

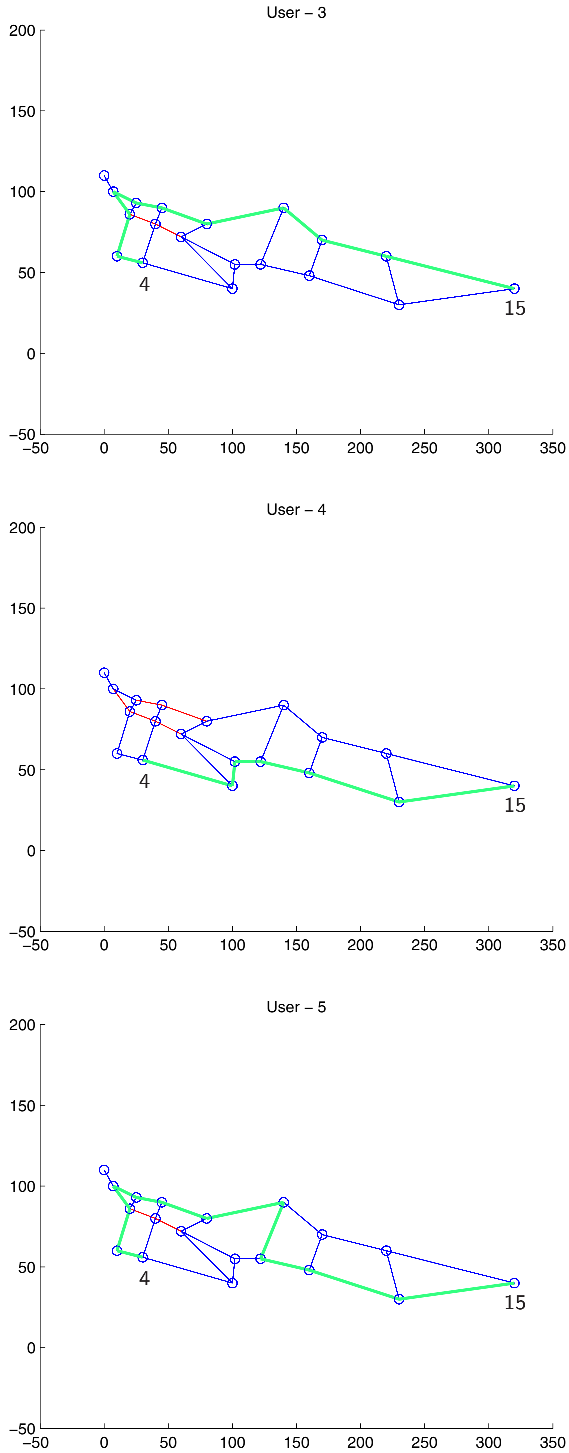

Fig. 5. Different paths fulfilling the capacity constraint in Eq. (10) for users with the same O/D pairs.

To conclude, it is possible to state that the described results show the good capability of the proposed system to provide good performances to the users, also being able to manage the situations in which a large number of users wants to reach the same destination from the same origin. For what concern the computational times, the solution of each instance of Problem 1 is generally less than 1 s.

5. CONCLUSIONS

A control system mainly consisting of a dynamic traffic model and an algorithm which solves an individual route guidance problem has been proposed in this paper. The system is directed to the optimization of the trips of a specific class of "guided" users, which communicate in advance the data of their trip (origin, destination and time window) and are well disposed to follow the routes recommended by the system. The system is defined so that the solution of the IRG problem takes into account the actual dynamic evolution of the traffic network, and distributes the guided vehicles over the network accordingly. Work is in progress for understanding the effects of the presence of guided vehicles on the uncontrolled flows, and for extending the model capabilities so as to be able to represent the stochastic behavior of users.

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Appendix A. THE DYNAMIC TRAFFIC MODEL

As introduced in Section 3, the traffic network is modeled through a directed graph, whose links are discrete-time dynamic systems.

Each link $(i,j) \in L$ is divided into a *running section*, where vehicles flow with a speed $v_{i,j}(k)$ which is function of the density $\sigma_{i,j}(k)$ in the section, and a *queue section*, where the flow of vehicle is extremely slow (at a very slow speed $w_{i,j}(k)$). Each section has its own dynamics, and then the boundary between the two sections moves with time.

The state of the link $(i,j) \in L$, in the generic interval k, is represented by the total number of guided and unguided vehicles moving through the link, namely $n_{\mathrm{C}\,i,j}(k)$ and $n_{\mathrm{U}\,i,j}(k)$ respectively, the total number of guided and unguided vehicles moving through the link and having destination $e \in \mathcal{D}_j$, namely $n^e_{\mathrm{C}\,i,j}(k)$ and $n^e_{\mathrm{U}\,i,j}(k)$ respectively, and the number of guided and unguided vehicles in the queue section, namely $z_{\mathrm{C}\,i,j}(k)$ and $z_{\mathrm{U}\,i,j}(k)$ respectively. These are the state variable of the traffic model. Moreover, let $\rho_{i,j}(k)$ be the total density of link (i,j) and $s_{i,j}(k)$ be the length of its running section. Input variables are the flows which are transmitted to link (i,j) by upstream links (m,i), $m \in \mathcal{P}_i$, and output variables are the flows which are transmitted by (i,j) to downstream links (j,n), $n \in \mathcal{S}_j$. In this connection, let $\varphi_{\mathrm{C}\,i,j}(k)$ (resp., $\varphi_{\mathrm{U}\,i,j}(k)$) and $\Phi_{\mathrm{C}\,i,j}(k)$ (resp., $\Phi_{\mathrm{U}\,i,j}(k)$) be the overall incoming controlled (resp., uncontrolled) flow and outgoing controlled (resp., uncontrolled) flow, respectively, of link (i,j). Other parameters of the generic link (i,j) are the length $l_{i,j}$ and the number of lanes $\lambda_{i,j}$.

Before introducing the state equations which provide values $n_{\mathrm{C}\,i,j}(k+1)$, $n_{\mathrm{U}\,i,j}(k+1)$, $n^e_{\mathrm{C}\,i,j}(k+1)$, $n^e_{\mathrm{U}\,i,j}(k+1)$, $z_{\mathrm{C}\,i,j}(k+1)$, and $z_{\mathrm{U}\,i,j}(k+1)$, the set of equations which provide densities, speeds, travel times, input and output flows and so on, are given.

Let $n_{i,j}(k)$ and $z_{i,j}(k)$ be respectively the total number of vehicles moving through the link and the number of vehicles in the queue section; they are simply given by:

$$n_{i,j}(k) = n_{\mathrm{C}\,i,j}(k) + n_{\mathrm{U}\,i,j}(k) \quad \text{(A.1)}$$

$$z_{i,j}(k) = z_{\mathrm{C}\,i,j}(k) + z_{\mathrm{U}\,i,j}(k) \quad \text{(A.2)}$$

The total density of link the (i,j), the running section length, and the density in the running section, can be easily determined from the state variables:

$$\rho_{i,j}(k) = \frac{n_{i,j}(k)}{\lambda_{i,j} l_{i,j}} \tag{A.3}$$

$$s_{i,j}(k) = l_{i,j} - \frac{z_{i,j}(k)}{\lambda_{i,j}\rho_{i,j}^{\max}} \tag{A.4}$$

$$\sigma_{i,j}(k) = \frac{n_{i,j}(k) - z_{i,j}(k)}{\lambda_{i,j} s_{i,j}(k)} \tag{A.5}$$

being $\rho_{i,j}^{\max}$ the maximum density in the link (i,j) (that is, the density in the queue section).

The mean speed in the running section is

$$v_{i,j}(k) = v_{i,j}^{\text{free}} \tag{A.6}$$

if $\sigma_{i,j}(k) < \rho_{i,j}^{\min}$ (free traffic conditions), or:

$$v_{i,j}(k) = v_{i,j}^{\min} + (v_{i,j}^{\text{free}} - v_{i,j}^{\min})\left[1 - \left(\frac{\sigma_{i,j}(k) - \rho_{i,j}^{\max}}{\rho_{i,j}^{\max} - \rho_{i,j}^{\min}}\right)^{a_{i,j}}\right]^{b_{i,j}} \tag{A.7}$$

if $\rho_{i,j}^{\min} \leq \sigma_{i,j}(k) < \rho_{i,j}^{\max}$; they come from the speed-density fundamental diagram $v[\rho(t)] = v^{\text{free}}[1 - (\rho(t)/\rho^{\max})^a]^b$ which has been proposed in May [1990]. Densities equal to or higher than $\rho_{i,j}^{\max}$ are not allowed since they identify a queue section. Values $v_{i,j}^{\min}$ ("symbolic" speed of the queue platoon), $v_{i,j}^{\text{free}}$ (speed under free traffic conditions, that is, maximum speed allowed), $\rho_{i,j}^{\min}$ (density under which free traffic conditions can be assumed), and $\rho_{i,j}^{\max}$, and parameters $a_{i,j}$ and $b_{i,j}$, are assumed a-priori known for each link of the network (the calibration of such parameters for a specific network is out of the scope of this paper). The mean speed in the queue section is simply:

$$w_{i,j}(k) = v_{i,j}^{\min} \tag{A.8}$$

The estimated travel time employed by vehicles entering link (i,j) at interval k, namely $\tau_{i,j}(k)$ (it corresponds to the variable $\tau(t_{k,h}, S_p)$ considered in Section 2), is defined as:

$$\tau_{i,j}(k) = \frac{s_{i,j}(k)}{v_{i,j}(k)} + \frac{l_{i,j} - s_{i,j}(k)}{w_{i,j}(k)} + d_{i,j}(k)\big[n_{i,j}(k)\big] \tag{A.9}$$

where the first term in the right-hand-side of (A.9) represents the time spent in the running section, the second term expresses the time required to cross the queue section, and $d_{i,j}(k)\big[n_{i,j}(k)\big]$ denotes the delay introduced by the presence of traffic signal (if any) within or at the end of the link (i,j).

Note that, due to the stability condition $l_{i,j}/v_{i,j}^{\text{free}} \geq \Delta t$ (assumed true for each link of the network), the inequality $\tau_{i,j}(k) \geq \Delta t$ holds. This means that, in our congested traffic network, a vehicle can not enter and exit a link within the same sampling interval. Then, in case of traffic networks with short and low-congested links, a sampling interval shorten than 5 minutes is required.

The determination of the outflow of the link (i,j) must take into consideration the fact that no enough room could be available in the downstream links. As a matter of fact, the *actual* outflow of the link is a function of downstream available room. In this connection, let $\Phi_{i,j}^{\text{pot}}(k)$ and $\Phi_{i,j}^{\text{act}}(k)$ be respectively the outflow which "potentially" leaves the link (i,j) during time interval k and the actual outflow of such link in the considered interval. If the whole traffic demand of the links ending in node j is greater than the whole traffic supply of links originating from j, then some links have an actual outflow less than their "potential" value. It is apparent that this situation yields a congestion at some links entering node j. $\Phi_{\text{C}\,i,j}^{\text{act}}(k)$ and $\Phi_{\text{U}\,i,j}^{\text{act}}(k)$ are the fractions of the actual outflow relevant to the guided and the unguided vehicles, respectively; therefore:

$$\Phi_{i,j}^{\text{act}}(k) = \Phi_{\text{C}\,i,j}^{\text{act}}(k) + \Phi_{\text{U}\,i,j}^{\text{act}}(k) \tag{A.10}$$

being:

$$\Phi_{\text{C}\,i,j}^{\text{act}}(k) = \frac{n_{\text{C}\,i,j}(k)}{n_{i,j}(k)}\,\Phi_{i,j}^{\text{act}}(k) \tag{A.11}$$

$$\Phi_{\text{U}\,i,j}^{\text{act}}(k) = \frac{n_{\text{U}\,i,j}(k)}{n_{i,j}(k)}\,\Phi_{i,j}^{\text{act}}(k) \tag{A.12}$$

The *potential* outflow, that is, the flow of vehicles which could leave link (i,j) if enough room would be available in downstream links, is:

$$\Phi_{i,j}^{\text{pot}}(k) = \min\left\{\Phi_{i,j}^{\max}\,,\ \frac{\rho_{i,j}(k) l_{i,j}\lambda_{i,j}}{\tau_{i,j}(k)}\right\} - \Phi_{i,j}^{\text{exit}}(k) \tag{A.13}$$

where $\Phi_{i,j}^{\max}$ is a time-independent physical parameter representing the maximum flow allowed to cross the junction at node j, $\rho_{i,j}(k) l_{i,j}\lambda_{i,j}/\tau_{i,j}(k)$ is the total outflow of vehicles, according to the number of vehicles present in the link and the estimated travel time, and $\Phi_{i,j}^{\text{exit}}(k)$ is the flow of vehicles leaving the network (since node j is their final destination), that is:

$$\Phi_{i,j}^{\text{exit}}(k) = \Phi_{\text{C}\,i,j}^{\text{exit}}(k) + \Phi_{\text{U}\,i,j}^{\text{exit}}(k) \tag{A.14}$$

being:

$$\Phi_{\text{C}\,i,j}^{\text{exit}}(k) = \frac{n_{\text{C}\,i,j}^{j}(k)}{\tau_{i,j}(k)} \tag{A.15}$$

$$\Phi_{\text{U}\,i,j}^{\text{exit}}(k) = \frac{n_{\text{U}\,i,j}^{j}(k)}{\tau_{i,j}(k)} \tag{A.16}$$

The *actual* outflow $\Phi_{i,j}^{\text{act}}(k)$ is determined by solving the following optimization problem.

Problem 2. Given the set of input links (m,j), $m \in \mathcal{P}_j$, which includes the link (i,j), and the set of output links (j,n), $n \in \mathcal{S}_j$, of node j, given link densities $\rho_{m,j}(k)$, potential outflows $\Phi_{m,j}^{\text{pot}}(k)$, flows $\Phi_{m,j}^{\text{exit}}(k)$ of vehicles leaving the network, $\forall\, m \in \mathcal{P}_j$, given the available room

$$\varphi_{j,n}^{\max}(k) = \big(\rho_{j,n}^{\max} - \rho_{j,n}(k)\big)\frac{l_{j,n}\lambda_{j,n}}{\Delta t} \tag{A.17}$$

$\forall\, n \in \mathcal{S}_j$, and given the splitting rates $\beta_{m,j,n}(k)$, $\forall\,(m,n) \in \mathcal{P}_j \times \mathcal{S}_j$, find:

$$\min_{\Phi_{m,j}^{\text{act}}(k)} \left\{ \sum_{m \in \mathcal{P}_j} \rho_{m,j}(k)\Big[\Phi_{m,j}^{\text{pot}}(k) - \Phi_{m,j}^{\text{act}}(k)\Big] \right\} \tag{A.18}$$

subject to:

$$\sum_{m \in \mathcal{P}_j} \Big[\big(\Phi_{\text{C}\,m,j}^{\text{act}}(k) + \Phi_{\text{C}\,m,j}^{\text{exit}}(k)\big)\beta_{\text{C}\,m,j,n}(k) + \\ + \big(\Phi_{\text{U}\,m,j}^{\text{act}}(k) + \Phi_{\text{U}\,m,j}^{\text{exit}}(k)\big)\beta_{\text{U}\,m,j,n}(k)\Big] \leq \varphi_{j,n}^{\max}(k) \\ \forall\, n \in \mathcal{S}_j \tag{A.19}$$

$$\Phi_{m,j}^{\text{pot}}(k) - \Phi_{m,j}^{\text{act}}(k) \geq 0 \qquad \forall\, m \in \mathcal{P}_j \tag{A.20}$$

Such a problem has the objective of satisfying as far as possible traffic demand $\Phi^{\text{pot}}_{m,j}(k)$. The difference $\Phi^{\text{pot}}_{m,j}(k) - \Phi^{\text{act}}_{m,j}(k)$ is weighted by $\rho_{m,j}(k)$ in order to prioritize links with higher densities. Constraints defined in (A.19) express outflow limitations due to the available room of each output link, whereas constraints defined in (A.20) simply prevent actual outflows to be greater than potential outflows. The splitting rates are determined on the basis of some destination-oriented variables which affect the dynamics of the traffic networks, as introduced in Papageorgiou [1990]. These variables are the *composition rates* $\gamma^e_{\mathrm{C}\,m,j}(k)$ and $\gamma^e_{\mathrm{U}\,m,j}(k)$, and the *destination-oriented splitting rates* $\beta^e_{\mathrm{C}\,m,j,n}(k)$ and $\beta^e_{\mathrm{U}\,m,j,n}(k)$, being $e \in \mathcal{D}_n$ the generic destination of vehicles moving through link (j,n). Composition rates are determined on the basis of the system state:

$$\gamma^e_{\mathrm{C}\,m,j}(k) = \frac{n^e_{\mathrm{C}\,m,j}(k)}{n_{\mathrm{C}\,m,j}(k)} \tag{A.21}$$

$$\gamma^e_{\mathrm{U}\,m,j}(k) = \frac{n^e_{\mathrm{U}\,m,j}(k)}{n_{\mathrm{U}\,m,j}(k)} \tag{A.22}$$

and destination-oriented splitting rates are:

$$\beta^e_{\mathrm{C}\,m,j,n}(k) = \gamma^e_{\mathrm{C}\,m,j}(k)\eta_{m,j,n} \tag{A.23}$$

$$\beta^e_{\mathrm{U}\,m,j,n}(k) = \gamma^e_{\mathrm{U}\,m,j}(k)\alpha_{m,j,n} \tag{A.24}$$

where $\eta_{m,j,n}$ are the splitting percentages at nodes, for the guided vehicles, that can be derived from the solution of the IRG problem, and $\alpha_{m,j,n}$ are the splitting percentages at nodes, for the unguided vehicles, that are provided by the traffic assignment process, as discussed in the Section 2. Finally, splitting rates $\beta_{\mathrm{C}\,m,\mu}(k)$ and $\beta_{\mathrm{U}\,m,\mu}(k)$ are simply:

$$\beta_{\mathrm{C}\,m,j,n}(k) = \sum_{e \in \mathcal{D}_n} \beta^e_{\mathrm{C}\,m,j,n}(k) \tag{A.25}$$

$$\beta_{\mathrm{U}\,m,j,n}(k) = \sum_{e \in \mathcal{D}_n} \beta^e_{\mathrm{U}\,m,j,n}(k) \tag{A.26}$$

The solution of Problem 2 provides the actual outflow of the link (i,j). The overall outflow and the outflows of guided and unguided vehicles are respectively

$$\Phi_{i,j}(k) = \Phi^{\text{act}}_{i,j}(k) + \Phi^{\text{exit}}_{i,j}(k) \tag{A.27}$$

$$\Phi_{\mathrm{C}\,i,j}(k) = \Phi^{\text{act}}_{\mathrm{C}\,i,j}(k) + \Phi^{\text{exit}}_{\mathrm{C}\,i,j}(k) \tag{A.28}$$

$$\Phi_{\mathrm{U}\,i,j}(k) = \Phi^{\text{act}}_{\mathrm{U}\,i,j}(k) + \Phi^{\text{exit}}_{\mathrm{U}\,i,j}(k) \tag{A.29}$$

whereas the partial outflows taking into account the vehicles with final destination e are:

$$\Phi^e_{\mathrm{C}\,i,j}(k) = \gamma^e_{\mathrm{C}\,i,j}(k)\Phi_{\mathrm{C}\,i,j}(k) \tag{A.30}$$

$$\Phi^e_{\mathrm{U}\,i,j}(k) = \gamma^e_{\mathrm{U}\,i,j}(k)\Phi_{\mathrm{U}\,i,j}(k) \tag{A.31}$$

$$\Phi^e_{i,j}(k) = \Phi^e_{\mathrm{C}\,i,j}(k) + \Phi^e_{\mathrm{U}\,i,j}(k) \tag{A.32}$$

The flows of vehicles entering link (i,j) from all upstream links, and directed toward final destination e (partial inflows), are given by:

$$\varphi^e_{\mathrm{C}\,i,j}(k) = \sum_{m \in \mathcal{P}_i} \Phi_{\mathrm{C}\,m,i}(k)\beta^e_{\mathrm{C}\,m,i,j}(k) + q^e_{\mathrm{C}\,i,j}(k) \tag{A.33}$$

$$\varphi^e_{\mathrm{U}\,i,j}(k) = \sum_{m \in \mathcal{P}_i} \Phi_{\mathrm{U}\,m,i}(k)\beta^e_{\mathrm{U}\,m,i,j}(k) + \widehat{q}^e_{\mathrm{U}\,i,j}(k) \tag{A.34}$$

$$\varphi^e_{i,j}(k) = \varphi^e_{\mathrm{C}\,i,j}(k) + \varphi^e_{\mathrm{U}\,i,j}(k) \tag{A.35}$$

where the first term in the r.h.s. of (A.33) (resp., (A.34)) is the total upstream traffic demand for the guided (resp., unguided) vehicles; $q^e_{\mathrm{C}\,i,j}(k)$ and $\widehat{q}^e_{\mathrm{U}\,i,j}(k)$ represent the controlled flow and the estimation of the uncontrolled flow, respectively, which are generated from node i to link (i,j) with final destination j. The (partial) controlled flow $q^e_{\mathrm{C}\,i,j}(k)$ is obtained from the solution of the IRG problem, taking into account the individual requests. It is assumed that there is always enough room for the flows generated from node i, that is, the constraint $\sum_{e \in \mathcal{D}_j}\left(q^e_{\mathrm{C}\,i,j}(k) + \widehat{q}^e_{\mathrm{U}\,i,j}(k)\right) \leq \varphi^{\max}_{i,j}(k) - \sum_{e \in \mathcal{D}_j}\sum_{m \in \mathcal{P}_i}\left(\Phi_{\mathrm{C}\,m,i}(k)\beta^e_{\mathrm{C}\,m,i,j}(k) + \Phi_{\mathrm{U}\,m,i}(k)\beta^e_{\mathrm{U}\,m,i,j}(k)\right)$ always hold. The total incoming flow for link (i,j) is

$$\varphi_{i,j}(k) = \sum_{e \in \mathcal{D}_j} \varphi^e_{i,j}(k) \tag{A.36}$$

Finally, the state equations which define the dynamics of the proposed traffic model are:

- total number of guided/unguided vehicles moving through link (i,j):

$$n_{\mathrm{C}\,i,j}(k+1) = n_{\mathrm{C}\,i,j}(k) + \Delta t\big(\varphi_{\mathrm{C}\,i,j}(k) - \Phi_{\mathrm{C}\,i,j}(k)\big) \tag{A.37}$$

$$n_{\mathrm{U}\,i,j}(k+1) = n_{\mathrm{U}\,i,j}(k) + \Delta t\big(\varphi_{\mathrm{U}\,i,j}(k) - \Phi_{\mathrm{U}\,i,j}(k)\big) \tag{A.38}$$

- total number of guided/unguided vehicles moving through link (i,j) with destination e:

$$n^e_{\mathrm{C}\,i,j}(k+1) = n^e_{\mathrm{C}\,i,j}(k) + \Delta t\big(\varphi^e_{\mathrm{C}\,i,j}(k) - \Phi^e_{\mathrm{C}\,i,j}(k)\big) \tag{A.39}$$

$$n^e_{\mathrm{U}\,i,j}(k+1) = n^e_{\mathrm{U}\,i,j}(k) + \Delta t\big(\varphi^e_{\mathrm{U}\,i,j}(k) - \Phi^e_{\mathrm{U}\,i,j}(k)\big) \tag{A.40}$$

- number of guided/unguided vehicles in the queue section of link (i,j):

$$z_{\mathrm{C}\,i,j}(k+1) = \frac{n_{\mathrm{C}\,i,j}(k)}{n_{i,j}(k)} \cdot \Delta t\left(\frac{\rho_{i,j}(k) l_{i,j} \lambda_{i,j}}{\tau_{i,j}(k)} - \Phi_{i,j}(k)\right) \tag{A.41}$$

$$z_{\mathrm{U}\,i,j}(k+1) = \frac{n_{\mathrm{U}\,i,j}(k)}{n_{i,j}(k)} \cdot \Delta t\left(\frac{\rho_{i,j}(k) l_{i,j} \lambda_{i,j}}{\tau_{i,j}(k)} - \Phi_{i,j}(k)\right) \tag{A.42}$$

when $z_{i,j}(k) = 0$, or:

$$z_{\mathrm{C}\,i,j}(k+1) = \frac{n_{\mathrm{C}\,i,j}(k)}{n_{i,j}(k)} \min\Big\{z_{i,j}(k) + \Delta t\big[\sigma_{i,j}(k)v_{i,j}(k)\lambda_{i,j} - \Phi_{i,j}(k)\big], n_{i,j}(k+1)\Big\} \tag{A.43}$$

$$z_{\mathrm{U}\,i,j}(k+1) = \frac{n_{\mathrm{U}\,i,j}(k)}{n_{i,j}(k)} \min\Big\{z_{i,j}(k) + \Delta t\big[\sigma_{i,j}(k)v_{i,j}(k)\lambda_{i,j} - \Phi_{i,j}(k)\big], n_{i,j}(k+1)\Big\} \tag{A.44}$$

when $z_{i,j}(k) > 0$.