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Identifiability of physical parameters in systems with limited sensors

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Abstract: In this paper, a method for estimating physical parameters using limited sensors is investigated. As a case study, measurements from an IMU are used for estimating the change in mass and the change in center of mass of a ship. The roll motion is studied and an instrumental variable method estimating the parameters of a transfer function from the tangential acceleration to the angular velocity is presented. It is shown that only a subset of the unknown parameters are identifiable simultaneously. A multi-stage identification approach is presented as a remedy for this. A limited simulation study is also presented to show the properties of the estimator. This shows that the method is indeed promising but that more work is needed to reduce the variance of the estimator.

Keywords: Identifiability, Centre of mass, Physical models, Inertial measurement units, Accelerometers, Gyroscopes, Marine systems

1. INTRODUCTION

Many mechanical systems have physical properties that change over time. These properties would cause a model of the system to have time-varying parameters. By estimating these parameters online, a higher model accuracy can be achieved. This is important, for example, in model based control, where the quality of the model governs the controller performance. A low quality model requires the controller to be robust against model errors which can result in sluggish performance (Skogestad and Postlethwaite, 2005). Two examples of this phenomenon are the total mass of an oil tanker that changes over time due to oil being pumped into its tanks and the change of mass due to cargo being shifted onto a container ship in harbors.

Of course, varying physical properties affect the dynamic behavior of the system and some properties might be safety critical. For instance, the roll dynamics of a ship are very sensitive to changes in the loading condition and a worst-case scenario is that the ship will capsize (Fossen, 2011; Tannuri et al., 2003). In Fujiwara and Haraguchi (2005) the authors investigate the dynamic influence of water flooding the car deck of roll–on–roll–off passenger ships, such as the disaster of the ship *Estonia* in 1994. The authors show that both the roll damping and the roll stiffness (metacentric height) change with the amount of water on deck. This is a situation where online estimation of the parameters could be used for a decision support system that could aid the crew to operate the vessel.

In Perez (2005), the author thoroughly investigates methods for ship roll stabilization or ship roll reduction systems using model based control. The model used in the approach is a linear maneuvering model where the parameters are assumed to be known. The effects of model errors are not investigated, but due to the big change in the ship roll dynamics shown in, for instance, Fossen (2011) and (Tannuri et al., 2003), there are reasons to believe that, for example, significant changes in mass will affect the ship dynamics and that online estimation of the mass would make it possible to increase the performance of the controller.

Actually, the mass is one of the most influential parameters in most mechanical systems. However, it is difficult to uniquely determine the mass in many cases. For example, the acceleration a of a particle with mass m, affected by the force F and moving in a straight path is directly proportional to F/m. If a is measured, there are still multiple combinations of F and m that will solve the equation. This ambiguity can be overcome with special experiments where the force acting on the system is known, or by measuring the force acting on the system. However, doing special experiments are in many cases intractable or too expensive.

Instead of focusing on a sensor-rich environment where all possible signals on a ship can be measured, this article investigates the case where only ship roll motion measurements from an inertial measurement unit (IMU) together with the rudder angle are available. The main benefit of this approach is that it can be applied easily without installing any expensive additional sensor systems on the ship. However, the limited measurement setup makes it critical to understand which parts of the roll dynamics can be estimated. Here, these questions will be addressed by doing an identifiability analysis of the approximate dynamical physical model of the ship roll motion. Furthermore, an instrumental variable approach will be used on simulated data to verify the identifiability results.

The remainder of this paper is organized as follows: In Section 2 the problem is formulated. Section 3 describes the structural identifiability concept used in this paper. In Section 4, the theory from Section 3 is applied to the approximate model. Section 5 describes the suggested instrumental variable approach. The method is applied to a simulated data set in Section 6 and finally, in Section 7, the paper is summarized with conclusions and suggestions for future work.

2. PROBLEM FORMULATION

The decoupled roll motion of a turning ship can be approximated as a mass-spring-damper system with an external force acting on its center of rotation (RC). Consider the system in Fig. 1, which is a mass-less inverted pendulum with a mass M attached a distance z_g from the RC and with an inertia I_x. The pendulum is hinged on a mass-less cart that is disturbed by an external force which results in the unknown acceleration a_y. There are two torsional

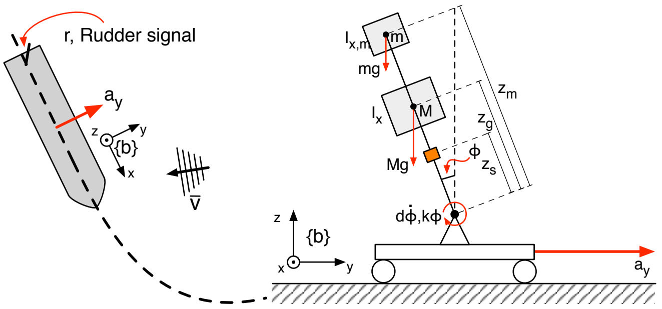

Fig. 1. Left: A ship performing a left turn creates an acceleration a_y. Right: A sketch explaining the approximate model used to describe a ship with additional mass. Note that this depicts the ship from the front, i.e. a positive a_y is equivalent to a left turn and a positive ϕ means that the ship is leaning to the right.

torques acting on the pendulum, one linearly dependent on the angle ϕ corresponding to the hydrostatic restoring forces, and one linearly dependent on the angular velocity $\dot{\phi}$ corresponding to damping due to hydrodynamic effects.

A mass m with inertia $I_{x,m}$ is introduced a distance z_m from the RC, corresponding to an additional load such as cargo. Finally, there is a disturbance torque v acting on the RC.

The system is described by the differential equation

$$\ddot{\phi} = \overbrace{-\frac{(k - Mgz_g - mgz_m)}{I_1}\phi}^{\text{restoring torque}} \overbrace{-\frac{d}{I_1}\dot{\phi}}^{\text{damping}} + \overbrace{\frac{(Mz_g + mz_m)}{I_1}a_y + \frac{v}{I_1}}^{\text{disturbances}} \qquad (1)$$

where $I_1 = I_x + Mz_g^2 + I_{x,m} + mz_m^2$ and it is assumed that ϕ is small, which is a common assumption when modeling ships (Fossen, 2011). Note that all states and disturbances are dependent on time t, but it is dropped to ease the notation.

The torque disturbance v is not assumed to have any specific distribution. The disturbance is an aggregate of neglected coupling, waves, wind and other type of disturbances such as vibration from the engine.

2.1 Sensors – Inertial Measurement Unit (IMU)

It is assumed that the ship motion is measured by a two degrees-of-freedom strap-down inertial measurement unit (IMU) consisting of one accelerometer measuring the tangential acceleration and one gyroscope measuring angular velocity around the RC. Fig. 2 shows the IMU's position in relation to the RC and the accelerations affecting it. The IMU is assumed to sense the angular velocity

$$y_1 = \dot{\phi} + e_1 \qquad (2)$$

where $\dot{\phi}$ is the system's angular velocity and e_1 is white zero-mean measurement noise.

Given small angles, the tangential acceleration sensed by the IMU is

$$y_2 = a_s + e_2 = -z_s\ddot{\phi} + g\phi + a_y + e_2 \qquad (3)$$

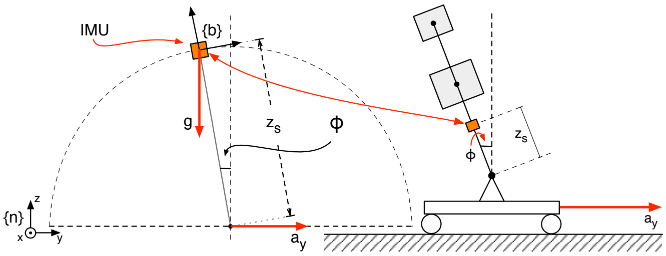

Fig. 2. A conceptual sketch of the IMU's position in the system and the accelerations affecting it.

where the first term is the contribution from the angular acceleration, the second term is due to gravity, the third term is the acceleration of the RC in the world coordinate system and the fourth term is white zero-mean measurement noise. The parameter z_s is the distance from the RC to the IMU coordinate system.

2.2 Identification issues

One of the problems with identifying the system (1) – (3) is the unknown and highly influential acceleration a_y. Several approaches are possible:

(A) a_y could be seen as an input: $x = [\phi,\ \dot{\phi}]$, $y = [y_1,\ y_2]$ and $u = a_y$, i.e. a_y is known or measured. For example, a normal prediction-error method (PEM) or errors-in-variables method could be used (Söderström, 1981).

(B) a_y could be introduced as a state, which implies that a model of a_y is needed:
  - (a) $x = [\phi,\ \dot{\phi},\ a_y]$ and $y = [y_1, y_2]$.
  - (b) If the lateral velocity was measured, for instance using a GPS receiver, then $x = [\phi,\ \dot{\phi},\ v_y,\ a_y]$ and $y = [y_1,\ y_2,\ v_y]$ could be a feasible model since v_y implicitly supplies information about a_y.

(C) a_y is eliminated from the model, similar to a Luenberger reduced-order observer.

Here, alternative C is used since a_y is unknown and no model is known. Equation (3) is solved for a_y and inserted into (1) resulting in

$$\ddot{\phi} = -\frac{k}{I_2}\phi - \frac{d}{I_2}\dot{\phi} + \frac{(Mz_g + mz_m)}{I_2}a_s + \frac{v}{I_2} \qquad (4)$$

where $I_2 = I_x + Mz_g(z_g - z_s) + I_{x,m} + mz_m(z_m - z_s)$ and a_s is now treated as an input.

2.3 State-space representation

By choosing the states $x = \begin{bmatrix}\phi & \dot{\phi}\end{bmatrix}^T$ and inserting (1) into (3), (1), (2) and(3) can be written as

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -\frac{k - Mgz_g - mgz_m}{I_1} & -\frac{d}{I_1} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{Mz_g + mz_m}{I_1} \end{bmatrix} a_y + \begin{bmatrix} 0 \\ \frac{v}{I_1} \end{bmatrix} \qquad (5a)$$

$$y = \begin{bmatrix} 0 & 1 \\ \frac{k - Mgz_g - mgz_m}{\tilde{I}_1} + g & \frac{d}{\tilde{I}_1} \end{bmatrix} x + \begin{bmatrix} e_1 \\ \frac{\tilde{I}_1 - Mz_g - mz_m}{\tilde{I}_1} a_y - \frac{v}{\tilde{I}_1} + e_2 \end{bmatrix} \qquad (5b)$$

where $\tilde{I}_1 = I_1/z_s$. This will be referred to as the *original model*. In the same way, (2) and (4) can be cast into the state-space form

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{I_2} & -\frac{d}{I_2} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{Mz_g + mz_m}{I_2} \end{bmatrix} a_s + \begin{bmatrix} 0 \\ \frac{v}{I_2} \end{bmatrix} \qquad (6a)$$

$$y = [0\ \ 1]x + e_1 \qquad (6b)$$

This will be referred to as the *input model*.

3. STRUCTURAL IDENTIFIABILITY

Structural identifiability concerns whether the parameters in the model can be determined uniquely (Bellman and

Åström, 1970). In the general case, a linear model parameterized with parameter vector $\theta \in \mathbb{R}^{n_\theta}$ can be written in the state space form

$$\begin{aligned} \dot{x} &= A(\theta)x + B(\theta)u \\ y &= C(\theta)x \end{aligned} \tag{7}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the input, $y \in \mathbb{R}^p$ is the output and the matrices are of suitable sizes. Note that the parameterization might be nonlinear even though the system is linear for a given parameterization. Identifiability can be seen as observability of the extended nonlinear system

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix} &= \begin{bmatrix} f(x,u,\theta) \\ 0 \end{bmatrix} = g^0(x,\theta) + g(x,\theta)u \\ y &= h(x,\theta) = [h_1(x,\theta) \ \dots \ h_p(x,\theta)]^T \end{aligned} \tag{8}$$

(Walter, 1982). To show observability, i.e. that it is possible to reconstruct the initial state given the data, for this nonlinear system, the system of equations

$$\begin{aligned} y_1^{(0)} &= h_1^{(0)}(x,\theta) \\ y_1^{(1)} &= h_1^{(1)}(x,\theta,u_1,\dots,u_m) \\ &\vdots \\ y_1^{(K)} &= h_1^{(K)}(x,\theta,u_1,\dots,u_1^{(K-1)},\dots,u_m,\dots,u_m^{(K-1)}) \\ &\vdots \\ y_p^{(0)} &= h_p^{(0)}(x,\theta) \\ y_p^{(1)} &= h_p^{(1)}(x,\theta,u_1,\dots,u_m) \\ &\vdots \\ y_p^{(K)} &= h_p^{(K)}(x,\theta,u_1,\dots,u_1^{(K-1)},\dots,u_m,\dots,u_m^{(K-1)}) \end{aligned} \tag{9}$$

has to be solved for x and θ, where y, u and their time derivatives are assumed to be known (Diop and Fliess, 1991). Here, the notation $(\cdot)^{(k)}$ means the k^{th} derivative of $(\cdot)$ with respect to time. For a general nonlinear system, there is no upper limit on K and it might be infeasible to solve this system of equations or difficult to analyze the result.

Instead of solving the system of equations, it can be linearized around the extended states and an analysis is performed on the Jacobian (Hermann and Krener, 1977)

$$\mathcal{O}(x,\theta,u_1,\dots,u_p^{(K-1)}) = \begin{bmatrix} \frac{\partial h_1^{(0)}(x,\theta)}{\partial(x,\theta)} \\ \frac{\partial h_1^{(1)}(x,\theta,u_1,\dots,u_m)}{\partial(x,\theta)} \\ \vdots \\ \frac{\partial h_1^{(K)}(x,\theta,u_1,\dots,u_1^{(K-1)},\dots,u_m,\dots,u_m^{(K-1)})}{\partial(x,\theta)} \\ \vdots \\ \frac{\partial h_p^{(0)}(x,\theta)}{\partial(x,\theta)} \\ \frac{\partial h_p^{(1)}(x,\theta,u_1,\dots,u_m)}{\partial(x,\theta)} \\ \vdots \\ \frac{\partial h_p^{(K)}(x,\theta,u_1,\dots,u_1^{(K-1)},\dots,u_m,\dots,u_m^{(K-1)})}{\partial(x,\theta)} \end{bmatrix} \tag{10}$$

which hereafter will be called the extended observability matrix (for simplicity denoted $\mathcal{O}$). The system is locally weakly identifiable if $\mathcal{O}$ satisfies an observability rank condition (Hermann and Krener, 1977).

In Section 4 and 5 of Anguelova (2007), the author shows that assuming that the system is a rational function in the states and parameters, it is enough to include the first $K = n + n_\theta - 1$ derivatives of $h_j(x,\theta)$, $j = 1,\dots,p$, with respect to time. Weak local identifiability of the system (8) is tested with a rank condition on $\mathcal{O}$. It is also shown that if $\text{rank}(\mathcal{O}) = q < n + n_\theta$, it is enough to form $\mathcal{O}$ using the first $q-1$ derivatives with respect time. This is useful if the dimension of the extended system is large since the complexity of the rows of $\mathcal{O}$ increases for high values of K. If the rank is less than $n + n_\theta$, one can exclude columns from the observability matrix one-by-one to identify the unobservable parameters or states. Note that excluding a parameter in the extended system is equivalent to dropping the corresponding column in $\mathcal{O}$. See Anguelova (2007) for a thorough discussion about identifiability of systems that are rational in the extended state vector.

4. IDENTIFIABILITY OF APPROXIMATE MODEL

In this section the identifiability analysis of the two systems (5) and (6) will be compared. The analysis is performed using symbolic software. Both systems will be extended according to (8) and the parameters considered are $\theta^T = \begin{bmatrix} I_x & M & z_g & I_{x,m} & m & z_m & k & d \end{bmatrix}$ and the remaining ones are assumed to be known. In both cases, there are $n = 2$ states and $n_\theta = 8$ parameters, i.e. the extended nonlinear system has dimension 10. It is also assumed that all parameters are non-zero unless otherwise stated.

The full $\mathcal{O}$ matrix for the general case is unfortunately not possible to recite due to its size and complexity. Instead, an illustrative special case will be presented in detail to show the typical structure and then, conclusions about the general case for both the original and the input model will be given. The effects of measurement and process noise are neglected in this section, i.e. $e_1 = e_2 = v = 0$.

4.1 Illustrative example – Special case

Here, an example will show identifiability of the parameters m, k and d in the original model (5) when the additional mass has no inertia and is placed at the same distance from RC as the IMU, i.e. $I_{x,m} = 0$ and $z_m = z_s$. The output is $y = \dot{\phi}$ and the input $u = a_y$, which for the analysis is assumed to be *known*. For notational simplicity, the parameters z_g, I_x, g and z_m are assumed to be equal to 1. In this case, the extended observability matrix is

$$\mathcal{O} = \left[\underbrace{\begin{matrix} \frac{\partial h(x,\theta)}{\partial x_1} \\ \vdots \\ \frac{\partial h^{(4)}(x,\theta)}{\partial x_1} \end{matrix}}_{\mathcal{O}_{x_1}} \;\middle|\; \underbrace{\begin{matrix} \frac{\partial h(x,\theta)}{\partial x_2} \\ \vdots \\ \frac{\partial h^{(4)}(x,\theta)}{\partial x_2} \end{matrix}}_{\mathcal{O}_{x_2}} \;\middle|\; \underbrace{\begin{matrix} \frac{\partial h(x,\theta)}{\partial m} \\ \vdots \\ \frac{\partial h^{(4)}(x,\theta)}{\partial m} \end{matrix}}_{\mathcal{O}_{m}} \;\middle|\; \underbrace{\begin{matrix} \frac{\partial h(x,\theta)}{\partial k} \\ \vdots \\ \frac{\partial h^{(4)}(x,\theta)}{\partial k} \end{matrix}}_{\mathcal{O}_{k}} \;\middle|\; \underbrace{\begin{matrix} \frac{\partial h(x,\theta)}{\partial d} \\ \vdots \\ \frac{\partial h^{(4)}(x,\theta)}{\partial d} \end{matrix}}_{\mathcal{O}_{d}} \right] \tag{11}$$

and the corresponding columns are

$$\mathcal{O}_{x_1} = \begin{bmatrix} 0 \\ -\bar{M}_k \\ d\bar{M}_k \\ \bar{M}_k(-d^2 + \bar{M}_k) \\ d\bar{M}_k(d^2 - 2\bar{M}_k) \end{bmatrix} \quad \mathcal{O}_{x_2} = \begin{bmatrix} 1 \\ -d \\ d^2 - \bar{M}_k \\ -d(d^2 - 2\bar{M}_k) \\ d^4 - 3\bar{M}_k d^2 + \bar{M}_k^2 \end{bmatrix} \tag{12a}$$

$$\mathcal{O}_m = \begin{bmatrix} 0 \\ x_1 + U_1(0,0,0,u) \\ -dx_1 + x_2 + U_1(0,0,u,\dot{u}) \\ (d^2 - 2\bar{M}_k)x_1 - 2dx_2 + U_1(0,u,\dot{u},\ddot{u}) \\ (4\bar{M}_k - d^2)dx_1 + (3d^2 - 2\bar{M}_k)x_2 + U_1(u,\dot{u},\ddot{u},u^{(3)}) \end{bmatrix} \tag{12b}$$

$$\mathcal{O}_k = \begin{bmatrix} 0 \\ -x_1 \\ dx_1 - x_2 \\ (-d^2 + 2\bar{M}_k)x_1 + 2dx_2 - \bar{M}u \\ (d^2 - 4\bar{M}_k)dx_1 - (3d^2 - 2\bar{M}_k)x_2 + 2\bar{M}du - \bar{M}\dot{u} \end{bmatrix} \tag{12c}$$

$$\mathcal{O}_d = \begin{bmatrix} 0 \\ -x_2 \\ \bar{M}_k x_1 + 2dx_2 + U_2(0,0,u) \\ -2d\bar{M}_k x_1 - (3d^2 - 2\bar{M}_k)x_2 + U_2(0,u,\dot{u}) \\ \bar{M}_k(3d^2 - 2\bar{M}_k)x_1 + (4d^3 - 6d\bar{M}_k)x_2 + U_2(u,\dot{u},\ddot{u}) \end{bmatrix} \tag{12d}$$

where $\bar{M} = M + m$, $\bar{M}_k = k - \bar{M}$ and

$$\begin{aligned} U_1(u_1,u_2,u_3,u_4) &= -d(d^2 + 4\bar{M} - 2k)u_1 + (d^2 + 2\bar{M} - k)u_2 - du_3 + u_4 \\ U_2(u_1,u_2,u_3) &= -\bar{M}(3d^2 + 4\bar{M} - 2k)u_1 + 2\bar{M}du_2 - \bar{M}u_3 \end{aligned} \tag{13}$$

Note the structure appearing in the columns due to the recursive properties of the matrix.

Non-zero informative input: Assuming that the input is informative enough, the maximum rank of $\mathcal{O}$ is 5 for almost all values of x and hence, it is possible to identify all three parameters.

No input: Assuming that the input is zero but that the output still contains information, then, the maximum rank of $\mathcal{O}$ drops to 4. This occurs, for instance, if the system is excited by perturbing it from its equilibrium and releasing it. This decrease in rank indicates that at most two parameters can be uniquely identified. For instance, the drop in rank can be realized by noting that $\mathcal{O}_m = -\mathcal{O}_k$ when there is no input and it can be seen in the row-reduced (allowing division) form

$$\mathcal{O} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{14}$$

where the columns corresponding to m and k are linearly dependent. Hence, it is only possible to identify two-combinations of parameters consistently, unless these two parameters are m and k.

4.2 Identifiability of the original model – general case

For the original model (5), the output is $y = [\dot{\phi},\ a_s]$ and the input $u = a_y$ which for the analysis is assumed to be *known*. This analysis is important as a basis for comparison in Section 4.3.

Non-zero informative input: Assuming that the input is informative enough, the $\mathcal{O}$ matrix formed for the general case has rank 5, which implies that at most 3 parameters can be uniquely identified. This is an expected result since (5) is a second order system. There are a total of 56 three-combinations and it is important to check if there are certain combinations that create problems with identifiability. To this end, sub-matrices of the form

$$\mathcal{O} = \left[\begin{array}{c|c|c|c|c} \frac{\partial h(x,\theta)}{\partial x_1} & \frac{\partial h(x,\theta)}{\partial x_2} & \frac{\partial h(x,\theta)}{\partial \theta_1} & \frac{\partial h(x,\theta)}{\partial \theta_2} & \frac{\partial h(x,\theta)}{\partial \theta_3} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial h^{(4)}(x,\theta)}{\partial x_1} & \frac{\partial h^{(4)}(x,\theta)}{\partial x_2} & \frac{\partial h^{(4)}(x,\theta)}{\partial \theta_1} & \frac{\partial h^{(4)}(x,\theta)}{\partial \theta_2} & \frac{\partial h^{(4)}(x,\theta)}{\partial \theta_3} \end{array}\right] \tag{15}$$

are created for each three-combination of parameters and the rank condition is checked for each combination.

Firstly, each column corresponding to a parameter is linearly independent of the columns corresponding to the states and hence, if only one parameter is unknown, then it is identifiable.

Secondly, except for combinations including both the parameters I_x and $I_{x,m}$, any two-combination of parameters is identifiable.

Finally, the analysis reveals that any three-combination of parameters is identifiable, except for any three-combination of $[I_x\ M\ z_g\ I_{x,m}\ m\ z_m]$ or any combination of parameters where both I_x and $I_{x,m}$ are in the set. For instance, this means that if the entire system is known except for the additional load, it is not possible to estimate its inertia, mass and its center of mass at the same time.

No input: Assuming that the input is zero but that the output still contains information, the maximum rank of $\mathcal{O}$ drops to 4 and dropping column by column shows that any two-combination of parameters is identifiable except for the combination of I_x and $I_{x,m}$.

4.3 Identifiability of input model – general case

The results for the original model show what is possible to estimate. However, as mentioned in Section 2.2, a_y is unknown and therefore the input model (6) is used. For the input model, the output is $y = \dot{\phi}$ and the input $u = a_s$, i.e. the measured tangential acceleration.

Non-zero informative a_y: As in the previous section, assuming that the input a_y is informative enough, the maximum rank of $\mathcal{O}$ is 5. Also the identifiable one-, two- and three-combinations of the parameters are the same. Hence, the identifiability properties of the original model are retained when using the mathematical reformulation.

No input ($a_y = 0$): An apparent advantage with the reformulation (6) is that the unknown a_y is replaced by the measured acceleration a_s. As defined in Section 2.1, the measured tangential acceleration is

$$a_s = -z_s\ddot{\phi} + g\phi + a_y$$

and if the system is perturbed from its equilibrium, then a_s will be non-zero for almost all t even if a_y is zero. At a first glance, this might seem beneficial since it suggests that the input a_s is informative even though a_y is zero. However, it is the input a_s that is used in the analysis and it is assumed that the input can be chosen arbitrarily. The input is a linear combination of a_y and the states and is thus not free to be chosen if a_y is zero. By inserting the known relation (3), the maximum rank of $\mathcal{O}$ is 4 as in Section 4.2, again retaining the same identifiability properties as the original model.

Hence, in an identifiability sense, nothing is gained or lost by using the input model instead of the original model.

5. IDENTIFICATION PRELIMINARIES

In this section, an extended instrumental variable (IV) method for estimation of the parameters is presented. Firstly, the model will be discretized, then linear constraints will be introduced to ensure that the dependencies amongst the parameters will be obeyed and finally, the suggested extended IV approach will be introduced.

5.1 Discretization of input model

All measurements are taken at discrete time instances and a discrete time model is needed to relate the measurements to the parameters. The input model (6) can be written as a transfer operator

$$H_c(p) = \frac{\beta_1 p^{-1}}{1 + \alpha_1 p^{-1} + \alpha_2 p^{-2}} \tag{16}$$

from the input a_s to the output $\dot{\phi}$, where $\alpha_1 = d/I_2$, $\alpha_2 = k/I_2$, $\beta_1 = (Mz_g + mz_m)/I_2$, I_2 according to Section 2.2 and p is the differentiation operator. The transfer function (16) can be discretized with the bilinear transform $p \approx \frac{2}{T}\frac{q-1}{q+1}$ giving

$$H_d(q) = \frac{\bar{\beta}_0(1 - q^{-2})}{1 + \bar{\alpha}_1 q^{-1} + \bar{\alpha}_2 q^{-2}} \tag{17}$$

where

$$\bar{\alpha}_1 = \frac{2\alpha_2 T^2 - 8}{\bar{I}_2},\ \bar{\alpha}_2 = \frac{4 - 2\alpha_1 T + \alpha_2 T^2}{\bar{I}_2},\ \bar{\beta}_0 = \frac{2\beta_1 T}{\bar{I}_2}, \tag{18}$$

$\bar{I}_2 = 4 + 2\alpha_1 T + \alpha_2 T^2$ and q is the the shift operator.

5.2 Obeying nonlinear parameterization

As shown in Section 4, only a subset of all parameters can be identified. The transfer function (17) has three free parameters, i.e. three degrees of freedom to fit the model to the data. This gives a model that is more flexible than permitted when only one or two parameters are estimated since the parameters in (17) are dependent through the original nonlinear parameterization. Fortunately, for certain parameter combinations, it is possible to use linear constraints together with (17) and still obey the original nonlinear parameterization. Below, two of these linear constraints will be presented for use in the next section.

I_x and d unknown: Both α_2 and β_1 have known numerators and the quotient

$$\frac{Mz_g + mz_m}{k} = \frac{\beta_1}{\alpha_2} = \frac{2T\bar{\beta}_0}{1+\bar{\alpha}_1+\bar{\alpha}_2} \tag{19}$$

is thus known which gives the constraint

$$\left[-\frac{Mz_g+mz_m}{k} \;\; -\frac{Mz_g+mz_m}{k} \;\; 2T\right]\left[\bar{\alpha}_1 \; \bar{\alpha}_2 \; \bar{\beta}_0\right]^T = \frac{Mz_g + mz_m}{k} \tag{20}$$

m and z_m unknown: In the same way, the quotient

$$\frac{d}{k} = \frac{\alpha_1}{\alpha_2} = \frac{T(1-\bar{\alpha}_2)}{1+\bar{\alpha}_1+\bar{\alpha}_2} \tag{21}$$

is known since both numerators of α_1 and α_2 are known which gives the constraint

$$\left[\tfrac{d}{k} \;\; \tfrac{d}{k}+T \;\; 0\right]\left[\bar{\alpha}_1 \; \bar{\alpha}_2 \; \bar{\beta}_0\right]^T = T - \frac{d}{k} \tag{22}$$

5.3 Extended IV – using rudder angle as instrument

The instrumental variable (IV) method uses a sequence of instruments ζ_t where each element is uncorrelated with the process disturbance v_t, and the measurement noises $e_{1,t}$ and $e_{2,t}$. In the general case, this can be achieved by solving

$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}} \|\Phi_N\theta - Y_N\|_2^2 \quad \text{s.t. } B\theta = b \tag{23}$$

where

$$\Phi_N = [\zeta_1 \dots \zeta_N]\begin{bmatrix}\varphi_1^T\\ \vdots \\ \varphi_N^T\end{bmatrix}, \; Y_N = [\zeta_1 \dots \zeta_N]\begin{bmatrix}y_1\\ \vdots \\ y_N\end{bmatrix} \tag{24}$$

and B and b are the matrices in either (20) or (22)(Söderstrom and Stoica, 1989). To decrease the variance of the estimate, an iterative method is used. After each iteration, the new estimate $\hat{\theta}_i$ is used to simulate y and u from the rudder angle r. The rudder angle r and the simulated signals y_s and u_s are then used as instruments in the next iteration. The iteration is aborted if $\|\hat{\theta}_i - \hat{\theta}_{i-1}\|_2$ is below a threshold or after a maximum number of iterations. The instruments are chosen as

$$\zeta_t = [r_t \dots r_{t-n_r} \; y_{s,t} \dots y_{s,t-n_y} \; u_{s,t} \dots u_{s,t-n_u}]^T \tag{25}$$

where r_t is the rudder angle at time t, $y_{s,t}$ and $u_{s,t}$ are the output and the input, respectively, simulated using (1), (3) and the estimate $\hat{\theta}_i$ from the previous iteration. Furthermore, n_r, n_y and n_u are the number of time lags included in ζ_t for the rudder angle, the input and the output, respectively. For the first iteration, solely the rudder angle is used as instrument.

5.4 Multi-stage identification

To estimate the changes in mass and the center of mass on a real system, a multi-stage identification procedure can be used. If no information is known about the system, this is indeed necessary since a maximum of three parameters can be identified simultaneously. The three stages are:

(1) The static parameters M, z_g and k of the unloaded ship are estimated in harbor by moving weight to known locations of the ship using ballasting.
(2) The dynamic parameters I_x and d for the *unloaded* ship are estimated with the method described in Section 5.
(3) The changes in mass m and the CM z_m are estimated again using the method in Section 5.

6. EXPERIMENTAL VERIFICATIONS

To investigate the properties of the suggested method, it has been applied to simulated data of a ship maneuvering on open water without any waves affecting it, i.e. v is zero. The dataset was synthesized with the approximate model (1) – (3) where a_y is formed by feeding the rudder signal r through the transfer function $F(s) = \frac{1}{0.2s+1}$. The true parameters are listed in Table 1. Two data sets were

Table 1. Parameters used in the simulation. Note that the additional mass is less than 1 % of the total mass.

M	z_g	I_x	d	k	m	z_m	z_s
22.04	0.018	0.1385	0.1067	10.39	0.2	0.274	0.218

created, one with nominal mass and one with additional mass. Both have 150 000 data points, corresponding to 25 min of data and a sampling period of 0.01 s. White zero-mean Gaussian measurement noise with a standard deviation of 0.01 was added to the simulated signals, where the amplitude of the noise was chosen to be similar to a real IMU. The noisy signals were filtered through an FIR equiripple low-pass filter of order 20 with a cut-off frequency of 0.5 Hz. Fig. 3 shows the first 110 s of the data set (with no additional mass) and Fig. 4 shows a typical behavior of the system. The black boxes in Fig. 3 corresponds to the data shown in Fig. 4.

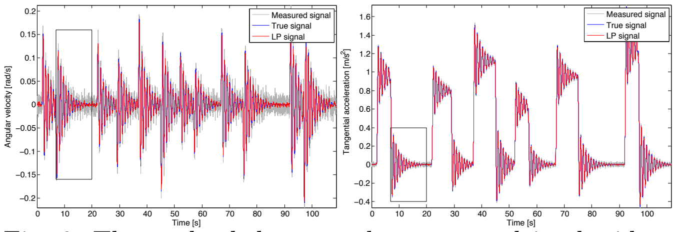

Fig. 3. The no load data set that was used in the identification. The measured signal is shown in gray, the true signal is blue and the measured signal after the LP filter was applied is dashed red. The left plot shows the angular velocity and the right plot shows the acceleration measurement. The black boxes mark the data shown in Fig. 4

The parameters are estimated using both the described IV method and for comparison, a constrained least-squares approach. In the IV method, $F(s) = \frac{1}{0.1s+1}$ is used, i.e. the time constant is different from the true one.

The results from the second stage, i.e. estimating I_x and d can be seen in Fig. 5. The left plot shows $\hat{I}_x$ and the right shows $\hat{d}$. In both cases, the LS estimator has less variance but is biased. However, note that the bias is in the order of 10^{-3} for the I_x estimator and in the order of 10^{-2} for the d estimator. These are both small relative to the true values.

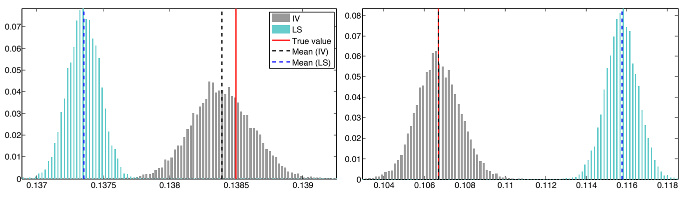

Fig. 5. The histogram of a Monte Carlo experiment with 10 000 runs. The red line corresponds to the true value, the blue bars correspond to the LS method, the dashed blue line corresponds to the mean of the LS estimator, the gray bars correspond to the IV method and the dashed black line corresponds to the mean of the IV estimator. The left plot shows the results for I_x and the right plot shows the result for d.

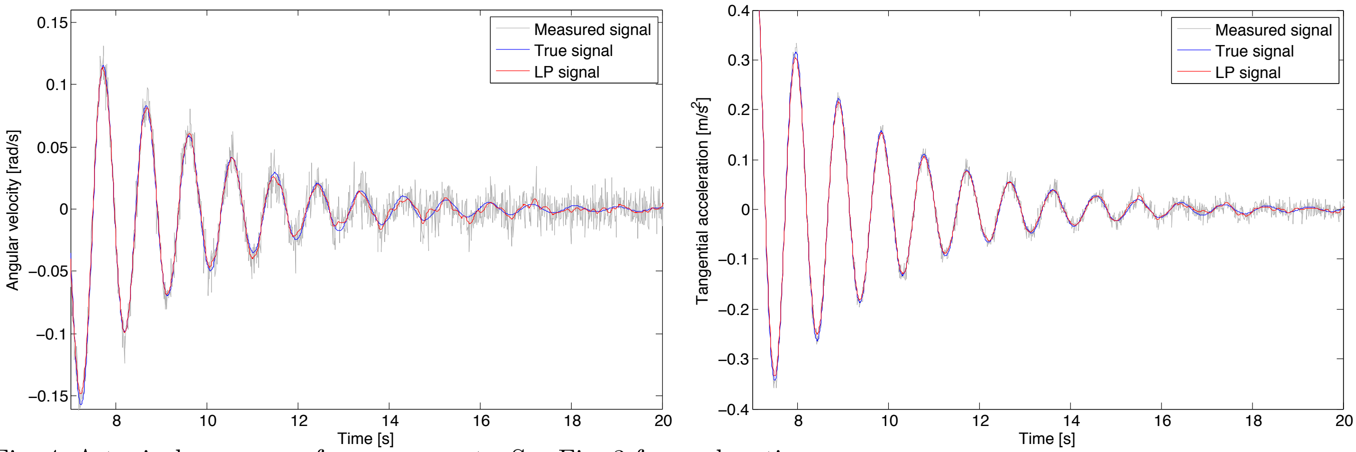

Fig. 4. A typical sequence of measurements. See Fig. 3 for explanation.

The results from the third and final stage, i.e. estimation of m and z_m, can be seen in Fig. 6. The left plot shows the result for the mass m and the right plot shows the result for the distance from the RC z_m. In this stage, the estimates, i.e. $\hat{I}_x$ and $\hat{d}$, from the previous stage are treated as known. Again, the IV estimator has a smaller bias than the LS estimator for both m and z_m but a larger variance.

According to theory, the estimated discrete-time linear parameters in the IV method will tend to the true ones as the number of data points goes to infinity (Söderstrom and Stoica, 1989). With the suggested IV estimator and the data length used, there is an obvious variation in the estimated physical parameters. This is due to the variance properties of the IV estimator and the sensitivity of the nonlinear transformation from the discrete-time linear parameters. This sensitivity and the required variance properties of the IV estimator need further investigation.

It should however be noted that the increase in mass is less than 1 % of the total mass and that the IV estimator actually detects the mass within ± 0.1 kg. It is also interesting to note that even though the LS estimator in the previous stage is biased, the estimator for mass and center of mass has low bias and variance. In this limited experiment, only measurement noise is considered and the real benefit of the IV method is when both the input and output are correlated with the same noise source.

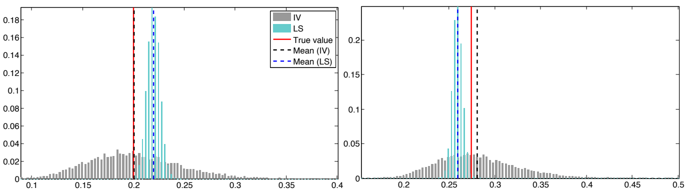

Fig. 6. The left plot shows the results for m and the right plot shows the result for z_m. See figure text of Fig. 5 for explanation.

7. CONCLUSIONS AND FUTURE WORK

In this paper, a method for estimation of physical parameters using measurements from an IMU has been presented. The roll motion has been studied and an instrumental variable method for estimation of the parameters of a transfer function from the tangential acceleration to the angular velocity has been proposed. The identifiability analysis has shown that only a subset of the unknown parameters are identifiable simultaneously. As a remedy, a multi-stage identification approach was suggested.

A limited simulation study was presented to show the applicability of the method and the properties of the estimator. The results are promising but, by no means perfect. Future work includes investigation of better choices of instruments and evaluation on real data. The real challenge lies in getting consistent estimators in situations with severe process disturbances.

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