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Tracking Problem for Linear Systems with Parametric Uncertainties and Unstable Zero Dynamics[1]

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Abstract: The paper deals with tracking problem for single-input single-output linear systems with unstable zero dynamics under parametric and signal uncertainty of control plant and reference model within the framework of the sliding mode techniques for feedback design, state observation and parameters identification in real time.

1. INTRODUCTION

The tracking problem of the given trajectories by the outputs of the control plant is the central problem in the theory of automatic control. The necessary and sufficient conditions for the solvability of tracking problem for linear systems were formulated within the framework of the geometric approach (Wonham, 1979) assuming that the reference signals are generated by known linear dynamic system with unknown initial conditions. For the technical feasibility of the tracking system it is necessary to ensure the stability of the zero dynamics, which requires knowledge of the parameters of plant models and reference signals model (Shtessel Y.B.,Tournes C. 1996).

The present paper offers a solution of the insufficiently explored tracking problem for single-input single-output linear systems under parametric uncertainty under the assumption that only the output variables of the control plant and reference model are measurable without noises. Among the closest papers to the subject we note (Marino et al., 2007), which considers the stabilization problem of linear systems with parametric uncertainty in reference model. The assumption of a parametric uncertainty in reference model significantly expands the class of tracking systems. Note that setting and solving the tracking problem in terms of parametric and signal uncertainties still hasn't been fundamentally explored in control theory.

The tracking problem solution proposed in this paper is based on the presentation of the original parametrically uncertain system in canonical form with the state space expansion by adding the dynamic compensator. This compensator generates the control action derivatives (Utkin, V.A., 2001; Utkin, A.V., 2007). The sliding mode observers (Krasnova et al., 2001) are based on the canonical representation of control plant and reference model. Then the problem of parameter identification can be solved in real time using the estimates of the state vector components (Utkin, V.I., 1992). Within the framework of the block approach (Drakunov et al., 1990; Krasnova et al., 2011), the decomposition procedure of feedback design in tracking problem under an unstable zero dynamics is developed with the use of the estimates.

2. PROBLEM STATEMENT

Let us consider single-input single-output linear system

$$\dot{x} = Ax + bu\,,\ y_1 = d^T x\,, \tag{1}$$

where $x \in R^n$ is the state vector, $y_1 \in R$ is the output (measured and controlled) variable, $u \in R$ is the control, pair (A, b) is controllable, pair (d^T, A) is observable, $v = \min_j [d^T A^{(j-1)} b]_{j=\overline{1,n}} \neq 0$ is known relative degree. The problem of a feedback design providing asymptotic convergence of the output y_1 to the reference signal $\eta_1(t)$ is posed. Ensure

$$\lim_{t\to\infty} e_1(t) = 0,\ e_1(t) = y_1(t) - \eta_1(t) \tag{2}$$

under the assumption that the reference signal is generated by the following dynamic model

$$\dot{w} = Ww,\ \eta_1 = r^T w\,, \tag{3}$$

where $w \in R^l, \eta_1 \in R$, pair (r^T, W) is observable.

Let us give the known solution of tracking problem (2) for system (1), (3) with known parameters and state variables. Let us introduce the nonsingular transformation of variables $\bar{x} = x - R_0 w$, where $\bar{x} \in R^n$, matrix $R_0 \in R^{n\times l}$ satisfies the equation $d^T R_0 = r^T$, and writing system (1) in the form of

$$\dot{\bar{x}} = A\bar{x} + bu + AR_0 w - R_0 Ww,\ e_1 = d^T \bar{x}\,. \tag{4}$$

The tracking problem comes down to stabilization of the output variable of system (4). Select a control in the form

$$u = k_0^T \bar{x} + l_0^T w \tag{5}$$

so that in closed system (4)–(5)

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$$\dot{\bar{x}} = \bar{A}_0\bar{x} + bl_0^T w + (AR_0 - R_0W)w \tag{6}$$

matrix $\bar{A}_0 = (A + bk_0^T)$ to bee Gurvits (with an arbitrarily assignable spectrum due to the controllability of the original system), elements of vector-line l_0^T are defined next.

Represent the state vector of system (6) as a sum of components $\bar{x} = \bar{x}_s + M_0 w$, where $\dot{\bar{x}}_s = \bar{A}_0\bar{x}_s$, matrix M_0 is further defined, which implies

$$\dot{\bar{x}} = \bar{A}_0\bar{x}_s + M_0 W w, \tag{7}$$

and equation (6) becomes as

$$\dot{\bar{x}} = \bar{A}_0(\bar{x}_s + M_0 w) + bl_0^T w + (AR_0 - R_0W)w, \tag{8}$$

$$e_1 = d^T(\bar{x}_s + M_0 w). \tag{9}$$

The selection of matrix M_0 and the row vector l_0^T, satisfying a matrix algebraic equations obtained by equating the terms of the equations (7) and (8) containing the state vector of reference model (3), solves the tracking problem. An additional condition is the identity matrix M_0 of the kernel output display d^T in equation (2.9), namely

$$\bar{A}_0 M_0 + bl_0^T + (AR_0 - R_0W) = M_0 W,\ d^T M_0 = 0. \tag{10}$$

If matrix equations (10) has a solution, it provides an asymptotic solution convergence of tracking problem with arbitrary rate of, defined by the matrix of the proper motions of the closed system (6) $\dot{\bar{x}}_s = \bar{A}_0\bar{x}_s$. Note that the above results are easily extended to controlled system (1), which is the general form of vector inputs and outputs.

This paper proposed a solution of a tracking problem with parametric uncertainty as control plant (1) and reference model (3) under the following assumptions: i) system (1) is assumed controllable and observable with known relative degree and the system (3) relies observed; ii) for measurement are available only output variables y_1 and η_1 in the control plant and in the reference model, respectively.

In general, the feedback design providing (2) requires the preliminary to solve the problem of state observation and parameter identification of control plant (1) and reference model (3). These tasks are not sufficiently studied in control theory. At the same time only for systems in the canonical form of "input-output" the problems solution of observation and parameters identification is known, using the theory of sliding modes. The proposed solution of the tracking problem is based on the representation of the original parametric uncertain system in the canonical form with expansion of the state space due to the compensator, which generates control action derivatives (Utkin, V.A., 2001; Utkin, A.V., 2007) (Section 3). In Section 4 a decomposition procedure of feedback design is developed for tracking in system with the unstable zero dynamics and known parameters and state variables of systems (1), (3) written in the canonical form regarding the output variables. In Section 5 the sliding mode observers and parameters identifiers are designed for control plant and dynamic compensator. Section 6 presents the simulation results confirming the effectiveness of the developed algorithms.

3. TRANSFORMATION TO THE CANONICAL FORM OF CONTROLLABILITY AND OBSERVABILITY

Let us show that any controllable and observable single-input single-output linear system can be represented in the canonical form of input-output with the expansion of the state space by introducing a dynamic compensator

$$u = \xi_1,\ \dot{\xi}_i = \xi_{i+1},\ i = \overline{1, n-\nu-1};\ \dot{\xi}_{n-\nu} = \bar{u}, \tag{11}$$

where $\bar{u}$ is new control, ν is relative degree of system (1).

Statement 1. Systems (1), (11) are controllable by new control $\bar{u}$.

Indeed, let us use a two-level decomposition to stabilize system (1), (11). In the first step the local feedback design in system (1) using control $u = \xi_1 = k^T x$ allows a predetermined range of the matrix of a closed system $(A + bk^T)$. In the second step we solve the stabilization problem of error between the chosen and the real values of a variable that has a solution due to canonical form of system (11).

After differentiating the output variable of system (1) n times using (11) we obtain a canonical representation of the control plant (1) in the form

$$\begin{aligned}
&\dot{y}_1 = d^T A x = y_2, \ldots, \dot{y}_\nu = d^T A^\nu x + d^T A^{\nu-1} b\xi_1 = y_{\nu+1},\\
&\dot{y}_{\nu+1} = d^T A^{\nu+1} x + d^T(A^\nu b\xi_1 + A^{\nu-1} b\xi_2) = y_{\nu+2}, \ldots,\\
&\dot{y}_n = d^T A^n x + d^T(A^{n-\nu-1} b\xi_1 + \ldots + A^\nu b\xi_{n-\nu}) + b_0\bar{u},
\end{aligned} \tag{12}$$

where $y = \mathrm{col}(y_1, \ldots, y_n) \in R^n$ is new coordinate basis received by nonsingular transformation of variables $y = Hx + \Lambda\xi$, $\det H \neq 0$, where $H \in R^{n\times n}$ is observability matrix of system (1), $b_0 = d^T A^{\nu-1} b \neq 0$,

$$H = \begin{pmatrix} d^T \\ d^T A \\ \ldots \\ d^T A^{n-1} \end{pmatrix}, \Lambda = \begin{pmatrix} O_{\nu\times(n-\nu)} \\ \Lambda_{1(n-\nu)\times(n-\nu)} \end{pmatrix}, \Lambda \in R^{n\times(n-\nu)},$$

$$\Lambda_1 = \begin{pmatrix} d^T A^{\nu-1} b & 0 & 0 & \ldots & 0 \\ d^T A^\nu b & d^T A^{\nu-1} b & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ d^T A^{n-2} b & d^T A^{n-3} b & d^T A^{n-4} b & \ldots & d^T A^{\nu-1} b \end{pmatrix}.$$

Note. In the case of $\nu = n$ derivatives with respect to the real control of the system (11) are absent, and entering a dynamic compensator (11) is not necessary.

Let us rewrite system (12) in compact form with dynamic compensator (11) as

$$\dot{y}_i = y_{i+1},\ i = \overline{1, n-1},\ \dot{y}_n = a^T y + g^T\xi + b_0\bar{u}; \tag{13}$$

$$\dot{\xi}_i = \xi_{i+1},\ i = \overline{1, n-\nu-1}; \dot{\xi}_{n-\nu} = b_n\bar{u},\ \xi = \mathrm{col}(\xi_1, \xi_2, \ldots, \xi_{n-\nu}), \tag{14}$$

where $g = \mathrm{col}(g_1, \ldots, g_{n-\nu})$, $g_i = d^T A^{n-i} b$, $i = \overline{1, n-\nu}$.

Statement 2. In system (13) row vectors a^T and g^T are not zero simultaneously.

Indeed, otherwise system (13), (14) is uncontrollable, as is the canonical two uncoupled subsystems without proper motions with the same control, which contradicts statement 1.

The next result allows us to use a two-level decomposition (see statement 1) for solving the tracking and stabilization problem in system (13)–(14).

Theorem 1. System (13), (14) by non-singular transformation of variables of the state vector of the dynamic compensator reduces to

$$\dot{y}_i = y_{i+1},\ i=\overline{1,n-1},\ y_n = \overline{a}^T y + g^T \overline{\xi} + b_0 \overline{u}; \quad (15)$$

$$\dot{\overline{\xi}}_i = \overline{\xi}_{i+1} + c_i y_1,\ i=\overline{1,\ n-v-1};\ \dot{\overline{\xi}}_{n-v} = -g^T \overline{\xi} + c_{n-v} y_1, \quad (16)$$

where $\overline{\xi} = \mathrm{col}(\overline{\xi}_1, \overline{\xi}_2, ..., \overline{\xi}_{n-v})$.

Proof. Let us introduce a nonsingular transformation of variables of the state vector of the dynamic compensator (14) $\overline{\xi} = \xi + Cy$, where $\overline{\xi}_i = \xi_i - \overline{c}_i\ \overline{y},\ i=\overline{1,n-v-1}$, $\overline{\xi}_{n-v} = \xi_{n-v} - \overline{c}_{n-v}\overline{y} - y_n$, $\overline{y} = \mathrm{col}(y_1,...,y_{n-1})$ and by (15) $\dot{\overline{y}} = col(y_2,...,y_n)$, $\overline{c}_i$, $\overline{c}_{n-v}$ are row vectors of the matrix $C \in R^{(n-v)\times n}$ are chosen on. This transformation transforms only the coefficients in the last equation (13), but does not transform its canonical structure that is reflected in (15).

Write (16) in the new variables as

$$\dot{\overline{\xi}}_i = \xi_{i+1} - \overline{c}_i\ \dot{\overline{y}} = \overline{\xi}_{i+1} + \overline{c}_{i+1}\overline{y} - \overline{c}_i\ \dot{\overline{y}},\ i=\overline{1,n-v-1},$$

$$\dot{\overline{\xi}}_{n-v} = b_0\overline{u} - \overline{c}_{n-v}\dot{\overline{y}} - a^T y - g\ \ (\overline{\xi} + Cy) - b_0\overline{u} =$$

$$= -g^T\overline{\xi} + \overline{c}y - \overline{c}_{n-v}\dot{\overline{y}},$$

where $\overline{c} = -a^T - g^T C$. The next choice

$$\overline{c}_{n-v} = (\overline{c}_2,...,\overline{c}_n) \Rightarrow \overline{c}\ \ y - \overline{c}_{n-v}\dot{\overline{y}} = c_{n-v}y_1,$$

$$\overline{c}_i = (\overline{c}_{i+1,2},...,\overline{c}_{i+1,n}) \Rightarrow \overline{c}_{i+1}y - \overline{c}_i\ \dot{\overline{y}} = c_i y_1$$

leads converted subsystem of dynamic compensator (14) to (16), namely,

$$\dot{\overline{\xi}}_i = \overline{\xi}_{i+1} + c_i y_1,\ i=\overline{1,n-v-1},\ \dot{\overline{\xi}}_{n-v} = -g^T\overline{\xi} + c_{n-v}y_1.$$

Controllability of system (16) with respect to virtual control y_1 should controllability of the original system (13), (14). Theorem 1 is proved.

Note that the stabilization problem of system (15), (16) has a solution, due to its handling and, unlike the tracking problem, does not cause any difficulties. Moreover, using a similar procedure to statement 1, the stabilization problem of system (15), (16) can be decomposed into two subproblems of smaller dimension as follows. Let us write system (16) as

$$\dot{\overline{\xi}} = G\overline{\xi} + cy_1 \quad (17)$$

and present the procedure of stabilizing feedback design for system (15), (17). Let us consider a variable y_1 in system (17) as a virtual control, introduce variables transformation $\overline{y}_1 = y_1 + f^T\overline{\xi}$ and rewrite the system (15), (17) in the form

$$\dot{\overline{y}}_i = \overline{y}_{i+1},\ i=\overline{1,n-1},\ \dot{\overline{y}}_n = \overline{a}^T\overline{y} + g^T\overline{\xi} + b_0\overline{u};$$

$$\dot{\overline{\xi}} = (G + cf^T)\overline{\xi} + c\overline{y}_1.$$

The choice of the elements of the row vector f^T is provided the stability of the proper motions in the dynamic compensator, and the subsequent selection of a stabilizing control in the first subsystem solves the problem of stabilizing system (15), (17). Thus, it is shown that the system under parametric uncertainty can be reduced to the form (15)-(16) suitable for further synthesis of tracking system. In the next section a decomposition method for tracking problem (2) for the reference signal generated by the reference model (3) in relation to (15), (17) is designed, assuming that the parameters and the state vectors of the control plant (15), (17) and reference model (3) are known and zero dynamics is unstable.

4. THE SOLUTION FOR THE TRACKING PROBLEM UNDER UNSTABLE ZERO DYNAMICS

Let us write the reference model (3) in the variables η_1 and its derivatives in the canonical form under the assumption of observability of pair (r^T, W)

$$\dot{\eta}_i = \eta_{i+1}, i=\overline{1,l-1}, \dot{\eta}_l = h^T\eta, \quad (18)$$

where $\eta = \mathrm{col}(\eta_1,...,\eta_l) \in R^l$, $h^T = r^T W^l \overline{H}^{-1}$, $\det\overline{H} \neq 0$, $\overline{H}$ is observability matrix of system (3), or in a convenient form for further discussion as

$$\dot{\eta} = \overline{W}\eta,\ \eta_1 = t^T\eta,\ t^T = \mathrm{col}(1,0,...,0). \quad (19)$$

Let us write system (15), (17) in tracking mismatch $e_1 = y_1 - \eta_1$ and its derivatives $e = col(e_1,...,e_n)$ follows

$$\dot{e}_i = e_{i+1}, i=\overline{1,n-1}, \dot{e}_n = \overline{a}^T e + g^T\overline{\xi} + q^T\eta + b_0\overline{u}; \quad (20)$$

$$\dot{\overline{\xi}} = G\overline{\xi} + c(e_1 + \eta_1). \quad (21)$$

The system (21) is interpreted as a subsystem of the internal dynamics relative to the output variable e_1. If it is stable, then the tracking problem is solved directly by choice of a stabilizing control in system (20). Next the decompositional procedure developed by the authors for tracking problem under assumption that the internal dynamics of the system (21) is unstable is proposed. This procedure is using the methodology of block approach (Drakunov et al., 1990), (Krasnova et al., 2011). In the first step, let us introduce a new variable

$$\overline{e}_1 = e_1 - f_0^T\overline{\xi} - n^T\eta, \quad (22)$$

considering in system (21) e_1 as a virtual control. System (20), (21) in the new variables (22) and its derivatives can be represented as

$$\dot{\overline{e}}_i = \overline{e}_{i+1}, i=\overline{1,n-1}, \dot{\overline{e}}_n = \overline{a}_e^T\overline{e} + \overline{g}_e^T\overline{\xi} + \overline{q}_e^T\eta + b_0\overline{u}; \quad (23)$$

$$\dot{\overline{\xi}} = G\overline{\xi} + c[\overline{e}_1 + f_0^T\overline{\xi} + (n^T + t^T)\eta]. \quad (24)$$

In fact of controllability of original system by handling the initial choice of the elements of the row vector f_0^T we can ensure the stability of the proper motions of the dynamic compensator (24). The row vector n^T is determined in the second step.

On the second step the stabilization problem will solve for system (23) and hence of the output variable (22). In this case the tracking problem (2) also will be resolved under the next condition

$$f_0^T \xi + n^T \eta = 0 \,. \qquad (25)$$

To solve the problem of ensuring equality (25) we introduce a structure similar to that in section 2 in relation to the system (24). Let us represent the state of system (24) as the sum of two components

$$\bar{\xi} = \bar{\xi}_s + P\eta \,, \qquad (26)$$

where the first component satisfies $\dot{\bar{\xi}}_s = \bar{G}\bar{\xi}_s \,, \bar{G} = (G + cf_0^T)$. The matrix $\bar{G}$ is Hurwitz, and matrix P ($\dim P = (n-v) \times l$) will be defined.

From relation $f_0^T(\bar{\xi}_s + P\eta) + n^T\eta = 0$ obtained by substituting (26) into (25) with respect $\bar{\xi}_s \to 0$ it can be obtained the first matrix equation $f_0^T P + n^T = 0$, then

$$n^T = -f_0^T P \,. \qquad (27)$$

On the other hand, substituting (26) into (24) we obtain

$$\dot{\bar{\xi}} = \bar{G}\bar{\xi}_s + \bar{G}P\eta + c(n^T + t^T)\eta + c\bar{e}_1. \qquad (28)$$

Let us write the differential equation for the variable (26)

$$\dot{\bar{\xi}} = \bar{G}\bar{\xi}_s + P\bar{W}\eta \,. \qquad (29)$$

By equating the terms with the components of vector η in (28) and (29) we obtain the second matrix equation $\bar{G}P + cn^T = P\bar{W}$. Taking into account $\bar{G} = (G + cf_0^T)$ after the substitution of (27) we have

$$GP + ct^T = P\bar{W} \,. \qquad (30)$$

Thus, the combined solution of matrix equations (27), (30) is reduced to the successive solution of system (30) and substituting it into (27). Given the fact that the choice of the elements of the row vector p provides a stability of the proper motions of dynamic compensator (24), then we can solve the stabilization problem of system (23) for this control, for example, in the class of continuous functions

$$b_0\bar{u} = -\bar{a}^T\bar{e} - \bar{g}^T\bar{\xi} - w^T\bar{\eta} + k^T\bar{e} \,, \qquad (31)$$

where the choice of the elements of the row vector k^T provides the stabilization of closed system (23)

$$\dot{\bar{e}}_i = \bar{e}_{i+1} \,, \; i = \overline{1, n-1}, \; \dot{\bar{e}}_n = k^T\bar{e} \,.$$

Note that when using the discontinuous control type

$$b_0\bar{u} = -M\mathrm{sign}(s) \,, \qquad (32)$$

where $s = \bar{e}_n + p^T\bar{e}_{n-1}, \bar{e}_{n-1} = \mathrm{col}(\bar{e}_1,...,\bar{e}_{n-1})$, in sufficiently large amplitude $M > 0$, choice of vector elements in system (23) provided steady sliding motion on sliding manifold $s = 0$ which is invariant to the unknown parameters and reference signals and their derivatives. As seen from the results of tracking problem design for unstable zero dynamics it is necessary to have information about the components of the state vector and parameters in control plant (20), (21) and reference model (19).

In the following section, the problem of observation and parameters identification is solved in control plant (13) and reference model (18) using the sliding modes theory.

5. THE PROBLEMS OF STATE OBSERVATION AND PARAMETERS IDENTIFICATION

Let us return to tracking problem (2) for system (1), (3) under parametric uncertainties assuming that output $y_1(t)$ of system (1) and output $\eta_1(t)$ of system (3) are measured only.

As shown in section 3 the controllable and observable system (1) can be represented in the form (13), (14). While in the presence of parametric uncertainties the transformation itself is unknown. Regarding to output $y_1(t)$ system (13) is observed, and the state vector of dynamic compensator (14) is known. Therefore, if the observation problem of the state vector of system (13) will be solved, we can obtain the estimates of parameters.

Further, using the estimates of parameters the system can be transformed to (15), (16) and then to the system of mismatches (20)-(21), the results of Step 4 can be used for tracking problem with parameter certainty.

Thus, the successful synthesis of tracking problem under parametric uncertainties requires the solution of following subproblems : 1) to obtain estimates of the components of the state vector of the system (13) and reference model (18); 2) for parameter estimation of (13) and reference model (18) to solve equations (30) and (27) for correlation (25); 3) to transform system (13) and (14) to (15) and (16) using parameter estimates; 4) to rewrite system (15) under new variables (22) and its derivatives in the form (23); 5) to solve the problem of stabilizing of system (4) by the choice continuous control (31) or discontinuous (32) using the estimates of the state vectors and parameters.

1. Let us consider the observation problem in system (13), suggesting that variables y_1 and state vector ξ of dynamic compensator are known. This problem considers with using sliding mode observer, which structure is similar to the structure of system (13):

$$\dot{\bar{y}}_i = \bar{y}_{i+1} + v_i, \; i = \overline{1, n-1}; \; \dot{\bar{y}}_n = \tilde{a}^T\bar{y} + \tilde{g}^T\xi + \tilde{b}_0\bar{u} + v_n, \qquad (33)$$

where $\bar{y} = \mathrm{col}(\bar{y}_1,...,\bar{y}_n) \in R^n$ is the state vector of observer; v_i ($i = \overline{1, n}$) are corrective action of observer, $\tilde{a}, \tilde{d}, \tilde{b}$ are evaluation of parameters obtained using parameters identification subsystem.

Taking into account (13), (33) let us write the system of equations under mismatches $\varepsilon = y - \bar{y}$

$$\dot{\varepsilon}_i = \varepsilon_{i+1} - v_i, i = \overline{1, n-1}; \dot{\varepsilon}_n = a^T\varepsilon + \hat{a}^T\bar{y} + \hat{g}^T\xi + \hat{b}_0\bar{u} - v_n, \; (34)$$

where $\hat{a} = a - \tilde{a}$, $\hat{g} = g - \tilde{g}$, $\hat{b}_0 = b_0 - \tilde{b}_0$ are discrepancy between the true parameters and their estimates which will be obtain further. Let us describe briefly cascade design of discontinuous corrective actions of observer (33) (Krasnova et al., 2001).

In the first equation (34) selecting the discontinuous correcting action $v_1 = M_1\mathrm{sign}\varepsilon_1$, $|\varepsilon_2| < M_1 = \mathrm{const} > 0$ will give rise to appearance of the sliding mode on the manifold $S_1 = \{\varepsilon_1 = 0\} \Rightarrow y_1 = \bar{y}_1$ within a finite time $t_1 > 0$. From the static equation we have the equivalent control $\dot{\varepsilon}_1 = 0 \Rightarrow v_{1\text{eq}} = \varepsilon_2$, whose value we will obtain from the output of the filter $\mu_1\dot{\tau}_1 = -\tau_1 + v_1$, $\lim_{\mu_1\to 0}\tau_1(t) = v_{1\text{eq}}(t) = \varepsilon_2(t)$.

In the second equation (34) selecting the discontinuous correcting action $v_2 = M_2 \text{sign}\tau_1$, $|\varepsilon_3| < M_2 = \text{const} > 0$ will give rise to appearance of the sliding mode on the manifold $S_2 = \{S_1 \cap \varepsilon_2 = 0\} \Rightarrow y_2 = \bar{y}_2$ within theoretically finite time $t_2 > t_1$. From the static equation we have the equivalent control $\dot{\varepsilon}_2 = 0 \Rightarrow v_{2\text{eq}} = \varepsilon_3$, whose value we will obtain from the output of the filter $\mu_2\dot{\tau}_2 = -\tau_2 + v_2$, $\lim_{\mu_2\to 0}\tau_2(t) = v_{2\text{eq}}(t) = $ $= \varepsilon_3(t)$.

Continuing this procedure at the last step in the last equation (34) selecting the discontinuous correcting action $v_n = M_n \text{sign}\tau_{n-1}$, $\left|a^T\varepsilon + \hat{a}^T\bar{y} + \hat{g}^T\xi + \hat{b}_0\bar{u}\right| < M_n = \text{const} > 0$ will give rise to appearance of the sliding mode on the manifold $S_n = \{S_{n-1} \cap \varepsilon_n = 0\} \Rightarrow y_n = \bar{y}_n$ within theoretically finite time $t_n > t_{n-1}$. From the static equation we have the equivalent control $\dot{\varepsilon}_n = 0 \Rightarrow v_{\text{neq}} = \hat{a}^T\bar{y} + \hat{g}^T\xi + \hat{b}_0\bar{u}$, whose value we will obtain from the output of the filter $\mu_n\dot{\tau}_n = -\tau_n + v_n$, $\lim_{\mu_n\to 0}\tau_n(t) = v_{\text{neq}}(t)$. Let us denote

$$v_{neq} = \hat{m}^T z,\ \hat{m}^T = (\hat{a}^T, \hat{g}^T, \hat{b}_0),\ z = \text{col}(y, \xi, \bar{u}). \tag{35}$$

2. Let us construct unknown parameters identifier using (35), the state variables and assuming that $\dot{m}^T = (\dot{a}^T, \dot{g}^T, \dot{b}_0) = 0$, then

$$\dot{\tilde{m}} = \lambda(\hat{m}^T z)z, \tag{36}$$

where $\tilde{m}^T = (\tilde{a}^T, \tilde{g}^T, \tilde{b}_0)$ is the state vector of the identifier, $\lambda = \text{const} > 0$. Then equation $\hat{m} = m - \tilde{m}$ becomes

$$\dot{\hat{m}} = -\lambda(\hat{m}^T z)z. \tag{37}$$

To show the convergence of (37) we consider the Lyapunov function $V = \frac{1}{2}\hat{m}^T\hat{m}$, whose derivative by (37) has the form

$$\dot{V} = -\lambda\hat{m}^T(\hat{m}^T z)z = -\lambda(\hat{m}^T z)^2. \tag{38}$$

In certain assumptions of the linear independence of the components of vector z and on the condition that the integral $\int_0^\infty (\hat{m}^T z)^2 dt = \infty$ diverges, we have $\lim_{t\to\infty} V = 0,\ \lim_{t\to\infty}\hat{m} = 0 \Rightarrow$ $\Rightarrow \lim_{t\to\infty}\hat{a} = 0,\ \lim_{t\to\infty}\hat{g} = 0,\ \lim_{t\to\infty}\hat{b}_0 = 0,$ which proves the asymptotic convergence of the state vector identity (36) $\tilde{m}$ to the real values of the parameter of vector m. Similarly one can solve the problem of observation and parameters identification in reference model (18).

In Steps 3-5 synthesis procedure for an extended model of control plant (23), (24) and reference model (18) used the received estimates. Solution of the tracking problem from section 4 is to solve the matrix equation (30), (27) and control design (31) or (32) using these estimates.

6. THE SIMULATION RESULTS

Let us consider the system under parametric uncertainty

$$\dot{x} = Ax + bu,\ y_1 = d^T x,\ x \in R^2,\ y_1 \in R^1, \tag{39}$$

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, b = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}, d^T = \begin{pmatrix} d_1 & d_2 \end{pmatrix},$$

where pair (A, b) is controllable, pair (d^T, A) is observable, $\nu = 1$ is relative degree. The problem of feedback design providing asymptotic convergence of the output y_1 to the reference signal $t_e^T\eta_e$ is posed. Reference signal is generated by the exogenous system of second-order

$$\dot{\eta}_e = W_e\eta_e,\ \eta_{e1} = t_e^T\eta_e,\ t_e^T = (1{,}0),\ W_e = \begin{pmatrix} 0 & 1 \\ h_{e1} & h_{e2} \end{pmatrix}. \tag{40}$$

Let the test system with unstable zero dynamics has the form

$$\dot{x}_1 = -x_1 - x_2 + u,\ \dot{x}_2 = 2u,\ y_1 = x_1, \tag{41}$$

$$\dot{\eta}_1 = \eta_2, \dot{\eta}_2 = -5\eta_1. \tag{42}$$

1. Let us present system (41)-(42) to the form (11), (12)

$$\dot{y}_1 = y_2, \dot{y}_2 = -y_2 - 2\xi + \bar{u}, \dot{\xi} = \bar{u},\ y_2 = -x_1 - x_2 + \xi. \tag{43}$$

2. With $\bar{\xi} = \xi - y_2 - 3y_1$ system (43) takes the form

$$\dot{y}_1 = y_2,\ \dot{y}_2 = -6y_1 - 3y_2 - 2\bar{\xi} + \bar{u},\ \dot{\bar{\xi}} = 2\bar{\xi} + 6y_1, \tag{44}$$

ie $a_1 = -6, a_2 = -3, g = -2, b_0 = 1$.

3. Let us write system for $e_1 = y_1 - \eta_1$, $e_2 = y_2 - \eta_2$ with respect to (44)

$$\begin{aligned} &\dot{e}_1 = y_2 - \eta_2 = e_2, \dot{e}_2 = -6(e_1 + \eta_1) - 3(e_2 + \eta_2) - \\ &- 2\bar{\xi} + 5\eta_1 + \bar{u}, \dot{\bar{\xi}} = 2\bar{\xi} + 6(e_1 + \eta_1). \end{aligned} \tag{45}$$

4. Let us write system for

$$\bar{e}_1 = e_1 - f_0\bar{\xi} - n^T\eta \tag{46}$$

and its derivate with respect to (45)

$$\begin{aligned} &\dot{\bar{e}}_1 = e_2 - f_0\dot{\bar{\xi}} - (n^T + t_0^T)W_0\eta = \bar{e}_2, \\ &\dot{\bar{e}}_2 = -6[\bar{e}_1 + f_0\bar{\xi} + (n^T + t_0^T)\eta] - 3[\bar{e}_2 + f_0\dot{\bar{\xi}} + \\ &+ (n^T + t_0^T)W_0\eta] - 2\bar{\xi} - t_0^T W_0^2\eta + \bar{u}, \\ &\dot{\bar{\xi}} = 2\bar{\xi} + 6[\bar{e}_1 + f_0\bar{\xi} + (n^T + t_0^T)\eta]. \end{aligned} \tag{47}$$

To stabilize the tracking error (46) is required to provide $f_0\bar{\xi} + n^T\eta = 0$ by choosing a row vector n^T as the solution of the matrix equation

$$P\overline{W} = \overline{G}P + c(n^T + t^T),\ \overline{G} = G + cf_0,\ n^T = -f_0 P.$$

With $c = -\bar{a}_1 = 6$, $G = 2$, $t^T = (0{,}1)$ и $f_0 = -1$ we obtain obtain a numerical solution $n_{1e} = -1{,}333,\ n_{2e} = -0.666$.

5. Let us introduce discontinuous control

$$\bar{u} = -M\text{sign}(s),\ M = \text{const} > 0, \tag{48}$$

where $s = \bar{e}_2 + p_s\bar{e}_1$, $p_s = \text{const} > 0$, $\bar{e}_1 = e_1 + \bar{\xi} - n_{1e}\eta_1 -$ $-n_{2e}\eta_2$, $\bar{e}_2 = \dot{\bar{e}}_1 = e_2 + \dot{\bar{\xi}} - n_{2e}h_1\eta_1 - n_{1e}\eta_2$.

When we use a sufficiently large amplitude $M = \text{const} > 0$ the sliding mode occurs on manifold $s = 0$ that guarantees asymptotic convergence to zero of variables $\bar{e}_1, \bar{e}_2$ and sustainability of variable $\bar{\xi}$ in system (47), namely

$\dot{\bar{\xi}} = -\bar{a}_1 y_1 - g\bar{\xi}$, $g = 2$. Figure 1 shows the plot of the tracking error for parametrically defined test system (43), (48) under $y_1(0) = y_2(0) = \xi(0) = 1, M = 100, p_s = 0, f_0 = -1$.

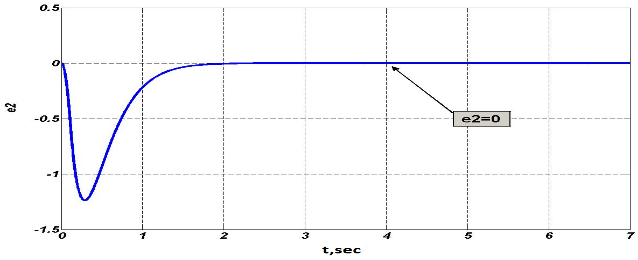

Fig. 1. The plot of tracking error $\varepsilon_{\eta 2}$ for test system (43)

In Figures 2-6 we can see the simulation results of the tracking system (39), (40) with parametric uncertainty under $M = 100,\ y_1(0) = y_2(0) = \xi(0) = 1,\quad p_s = 0, f_0 = -1, M_1 = 30,$ $M_2 = 10, \tau_1 = 10^{-5}, \lambda_e = 5.$ To speed the identifying process we used a high frequency sinusoidal signal $\delta(t) = \sin 15t + 0.5\sin 30t$. The control is modified as $b_e\bar{u}_e = -M\text{sign}(s')$, where

$s' = \dot{\bar{e}}_1' + k_s\bar{e}_1',\ k_s > 0,\ \ \bar{e}_1' = e_1 - f_e\bar{\xi} - n_{1e}\eta_1 - n_{2e}\eta_2 + \delta(t)$,

$\bar{e}_2' = y_2 + f_e(\bar{a}_1 y_1 + \bar{g}\bar{\xi}) - n_{2e}h_1\eta_1 - (n_{1e} + n_{2e}h_2)\eta_2 + \dot{\delta}(t)$.

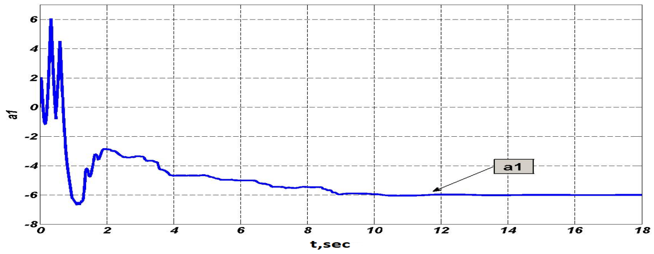

Fig. 2. The plot of identification of parameter a_1

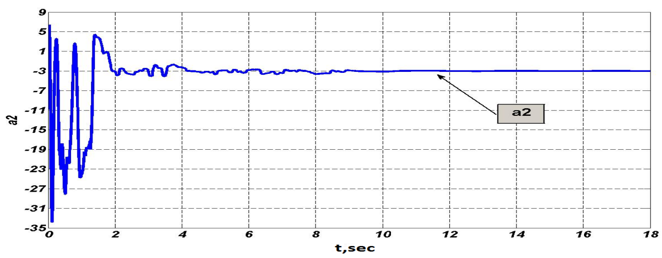

Fig. 3. The plot of identification of parameter a_2

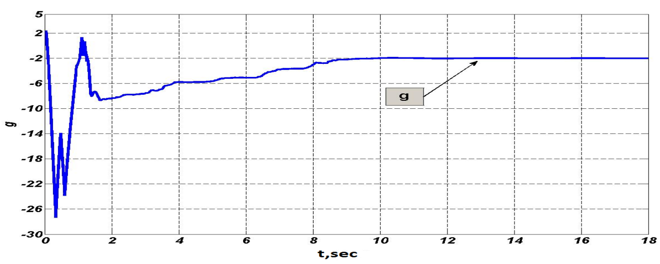

Fig. 4. The plot of identification of parameter g

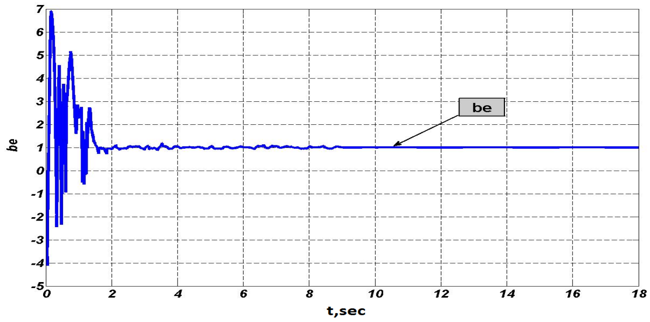

Fig. 5. The plot of identification of parameter b_e

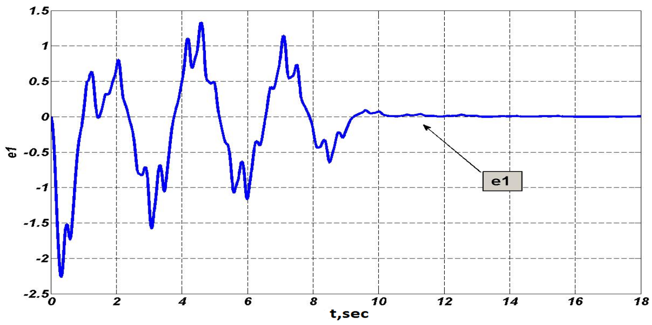

Fig. 6. The plot of tracking error e_1

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