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Choosing an optimal return rate and a reverse logistics policy from the solution of a constrained LQG problem.

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Abstract: Demand for final products has grown significantly worldwide, particularly, due to the increase of emerging countries on the consumer market. The big problem is that consumer of energy and use of raw material increase in parallel. Consequently, an unsustainable environmental situation will be observed on the next future as arguing some researchers. In order to mitigate such a situation, the reuse of resources can be an interesting initial solution. However, reuse products or materials require a somewhat treatment or remanufacturing process that together with the collection process can be highly expensive. The paper tries to identify a return rate that leads to a minimum lower cost for running a reverse logistics. A chance-constrained LQG problem is used to provide both an optimal return rate and optimal production policy for returnable products. A fictitious company is used as an example of application.

1. INTRODUCTION

Due to exaggerated use of raw material, which is extracted from the earth, and the amount of energy spent to produce and distribute products around the world, reverse logistics has become an essential part of an integrated supply chain. The main objective of a reverse system is to shift products from market collectors to places where they can be treated or can be properly disposal. The purpose is that some products can get value along the reverse chain by mean of recycling or remanufacturing processes. Thus, it is possible to contribute positively to improving the environment, reducing waste by mean of remanufacturing or recycling.

A reverse logistics system can be formulated through two integrated mathematical models: one represents the forward channel of the supply chain, where products are manufactured, stored in serviceable inventory units, and then moved to the marketplace in order to meet demand. The other denotes the backward channel, where used-products are recovered or discarded. Authors, like Fleischmann et al. (1997), have provided a typology of quantitative models for reverse logistics. Usually, in the literature, three kinds of problems involving such models are formulated as follows: (i) collecting, packaging, and distributing used products. The collecting process starts at the marketplace, from where customers leave their out-of-use products; (ii) scheduling of remanufacture and recycle process, which allows used products returning to the serviceable inventory of the company; and (iii) planning and controlling of items and products for reusing, without any additional process of remanufacturing.

The forward channel model is a dynamic system that is affected by the stochastic fluctuation of demand. It is assumed that the demand follows a distribution of probability, which can be approximated by a normal distribution, see Graves (1999). As a main consequence, the forward channel system becomes a stochastic process. The problem formulated from this system belongs to the class of stochastic mathematical programming problem, which means that it is more complex to be optimally solved.

It is worth mentioning that stochastic production-inventory systems are usually found in reverse logistics problems, particularly in reason of the return rate not known precisely over the future periods. Fleischmann et al. (1997) have shown that the traditional classification of production and inventory stochastic problems, based on discrete or continuous-time models, can be also applied to model and solve problems of products recovering, see for instance Ouaret, et al. (2013) and Kenné et al. (2012)

In this paper, a discrete-time Linear Quadratic Gaussian (LQG) model with chance-constraints is considered to represent a recovery problem. A two stages procedure is provided to allow managers determining an optimal return rate and optimal manufacturing-remanufacturing policy. In the first stage, different costs are combined in order to identify the optimal return rate. In the sequence, the second stage consists in finding a manufacturing-remanufacturing optimal policy. To solve the chance-constrained LQG problem, an equivalent, but deterministic problem is considered. An Open-Loop Feedback Controller is used as a solution approach. Sensibility analysis, provided from simple variation of some parameters as return rate of used products, delay of return, or both, allows creating production scenarios that help managers to make prospered decisions.

The paper is distributed as follows: section 2 discusses a constrained LQG model for reverse logistics problem; section 3 introduces a two-stage procedure, which allows managers deciding on the optimal return rate and providing an optimal production plan; and the section 4 presents a numerical example to illustrate the application of this procedure

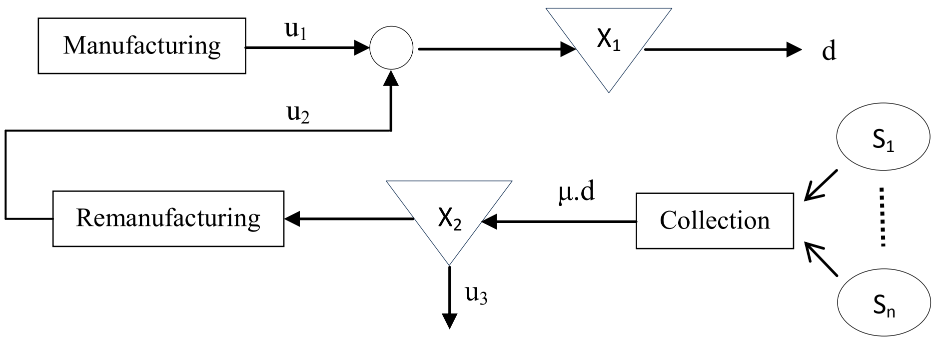

Fig. 1. A forward-backward system under uncertain demand

2. THE REVERSE LOGISTICS MODEL

Figure 1 illustrates the forward and backward channels of a unique product supply chain. Note that there are two stores in this figure: the first one (Store 1) stocks manufactured and remanufactured products to meet the demand, and the second store (Store 2) collects the returned products. These returned products can be remanufactured or disposal properly.

It is worth emphasizing that the demand for products must be fulfilled by the combination between new products (manufactured) and remanufactured products (i.e., used products that are collected from the marketplace and, if possible overhauled). Other features and properties considered for the system exhibited by figure 1 are: a) demand is a random variable that follows a stationary stochastic process while the return process is assumed essentially deterministic; b) both manufacturing and remanufacturing process has infinite capacity; c) similarly, the maximum physical storage capacity of inventory for warehouses 1 and 2 are assumed unlimited; d) there is a constant time-delay associated with return products from the market; and e) used-products may be disposed of after being collected. There are two main reasons to discard used-products: the first has a technical reason, which is related to inappropriately returned products for remanufacturing activities, and the second has a financial reason, in which remanufacturing all products can significantly raise the inventories, and, as a result, such strategy can increase the overall production cost.

2.1. Stochastic inventory-production system

The integrated forward and reverse inventory-production system illustrated in figure 1 can mathematically be described by a discrete-time stochastic control model with two state variables that are related to inventory levels on serviceable and remanufacturable units; and three control variables that are related to manufacturing, remanufacturing, and discard rates.

Note that, such features like dynamic of reverse logistics and uncertainties associated with the process of collecting and, simultaneously, attending the market demand are usually reasonable considerations to justify the use of stochastic control models. In the literature, it is possible to find innumerous contributions related with this kind of model, the majority of them consider the formulation in time-continuous pattern; see, for instance, Dobos (2003) and Miner and Kleber (2001). For reader interest in more generic themes of reverse logistics, it is recommended to visit the site http://www.rltinc.com.

The discrete-time stochastic control model is described by the following two difference equations, which represent, respectively, the inventory balance systems related to forward and reverse channel of the supply chain:

$$x_1(k+1) = x_1(k)+u_1(k)+u_2(k)-d(k) \quad (1)$$

$$x_2(k+1) = x_2(k)-u_2(k)-u_3(k)+r(k) \quad (2)$$

where, for each period k, the notation is given as follows:

$x_1(k)$ = inventory level of serviceable products (store 1);

$x_2(k)$ = inventory level of used product (store 2);

$u_1(k)$ = production rate of manufactured products;

$u_2(k)$ = production rate of treated used-products;

$u_3(k)$ = discard rate of unserviceable products;

$d(k)$ = demand level for serviceable products;

$r(k) = \mu d(k)$ = level of used-products

The demand d(k) is a random variable that follows a normal distribution with mean and variance given by $\hat{d}(k)$ and σ_D^2 finite, respectively. Since d(k) is an independent random variables, the inventory system (1) is a stochastic process. As a consequence, the inventory level $x_1(k)$ is a dependent random variable with mean $\hat{x}_1(k)$ and variance $V_{x_1}(k) \geq 0$. On the other hand, the return rate r(k) is deterministic variable given by $\hat{r}(k)=\mu \cdot \hat{d}(k)$, with $0 \leq \mu \leq 1$ denoting a percentage of the total number of returned units. As a result, the inventory system described by the backward process (2) should be seen as a deterministic process.

2.2. *The stochastic reverse logistics problem*

Based on system (1)-(2), a chance-constrained Linear Quadratic Gaussian (LQG) model is formulated in order to represent a logistic reverse problem as follows:

$$\begin{aligned} \min_{u_1,u_2,u_3} \; & E\left\{ h_1x_1(T)^2 + h_2x_2(T)^2 \right\} + \sum_{k=0}^{T-1}\left\{ E\left[h_1x_1(k)^2 + h_2x_2(k)^2\right] \right. \\ & \left. + c_1u_1(k)^2 + c_2u_2(k)^2 + c_3u_3(k)^2 \right\} \\ \text{s.t.} \; & \\ & x_1(k+1) = x_1(k) + u_1(k) + u_2(k) - d(k) \qquad (3) \\ & x_2(k+1) = x_2(k) - u_2(k) - u_3(k) + r(k) \\ & 1 - \Pr ob.(\underline{x}_1 \le x_1(k) \le \overline{x}_1) \le \alpha_k \\ & 0 \le x_2(k) \le \overline{x}_2 \\ & 0 \le u_1(k) \le \overline{u}_1;\; 0 \le u_2(k) \le \overline{u}_2;\; u_3(k) \ge 0 \end{aligned}$$

It is worth mentioning some characteristics of using a quadratic cost: a) it penalizes equally, both positive (i.e., excess) and negative (i.e., backlogging), variations of decision variables; and b) it induces high penalties for large deviations of the decision variables from the origin, but relatively small penalties for small deviation, see Bertesekas (2000).

Based on problem (3), a two-stage procedure that provides an optimal return rate (μ), and as well an optimal forward and reverse policy for systems (1) and (2) are described next.

3. SOLVING (6) WITH OPTIMAL RETURN RATE (μ)

This section presents a structured procedure that allows the decision maker defining an optimal return rate for a given product. The procedure takes into account the quadratic programming model, which is presented in the formulation (6), to choose the best return rate and to provide an optimal inventory-production for the system given in (1).

It is known that the strategy used for selecting models that help managers to implement a reverse logistics in a given supply chain depends on the costs related to the logistics of returning, which includes costs for collecting, transporting, storing, treating and disposing. Besides there are also indirect costs like energy costs for treating of returnable product and no-measures costs related to pollution in part due to an increase of transportation in urban centers. In fact, these costs are often high and can reduce the profitability of the supply chain. However, the most part of these costs is tangible. This means that the supply chain can obtain intangible gains by associating its image to an environmentally sustainable supply chain, which reduces the extraction of raw material of the earth.

The procedure proposed below takes into account the definition of scenarios to choose the optimal rate of return for a given product, considering the costs in forward and backward (reverse) channels of the supply chain. After setting the best rate, the next step is to identify an optimal production-inventory schedule that considers the projected levels of demand. It is assumed here that the supply chain runs in a make-to-stock pattern.

The procedure is given in two stages:

Stage 1: Determining an optimal return rate

First, it is important to set some basic assumption related to problem (6):

- Collection and transportation costs are included in the inventory cost of the product returnable.
- Demand d(k) is assumed to be equal to the monthly average demand (i.e., $d(k)=\hat{d}(k)$). Note that such average demand $\hat{d}(k)$ is computed from historical data that contain monthly record of sales.

Then, some scenarios can be constructed by relating costs between the forward and reverse channels of the supply chain. This allows comparison of different scenarios that involve costs with inventory and manufacturing of serviceable products (i.e., c1 and h1, respectively) and the costs of inventory, treatment and disposal of collected products (i.e., h2, c2 and c3, respectively). There are several possibilities of combinations of these costs.

Lastly, the problem (6) is solved, but considering the return rate "μ" as being the decision variable that must be found. The follow two steps provide the scheme:

Step 1: Setting demand equal to its average monthly demand, (i.e., $d(k) = \hat{d}(k)$ for k = 1, 2, …, T), the problem (3) is transformed to a Mean Value Problem, which is given as:

$$\begin{aligned} \min_{u_1,u_2,u_3} \; & h_1\hat{x}_1(T)^2 + h_2\hat{x}_2(T)^2 + \sum_{k=0}^{T-1}\left\{ h_1\hat{x}_1(k)^2 + h_2\hat{x}_2(k)^2 \right. \\ & \left. + c_1u_1(k)^2 + c_2u_2(k)^2 + c_3u_3(k)^2 \right\} \\ \text{s.t.} \; & \\ & \hat{x}_1(k+1) = \hat{x}_1(k) + u_1(k) + u_2(k) - \hat{d}(k) \qquad (4) \\ & \hat{x}_2(k+1) = \hat{x}_2(k) - u_2(k) - u_3(k) + \mu\hat{d}(k-\tau) \\ & \underline{x}_1 \le \hat{x}_1(k) \le \overline{x}_1; \quad 0 \le \hat{x}_2(k) \le \overline{x}_2 \\ & 0 \le u_1(k) \le \overline{u}_1;\; 0 \le u_2(k) \le \overline{u}_2;\; u_3(k) \ge 0 \end{aligned}$$

Step 2: Changing the prices in the criterion of 4 (i.e., h_1, h_2, c_1, c_2 and c_3), it is possible to create different scenarios of retuning by solving the problem (4) when μ varies in the range (0, 1]. The best value of μ (i.e., $\mu=\mu^*$) is the one that provides the smaller criterion of (4).

Stage 2: Determining optimum production and inventory levels for serviceable and returnable products, with $\mu=\mu^*$.

Computing a true optimal solution (i.e. an optimal closed-loop solution) for the stochastic problems, like problem (9), is not a simple task. Thus, in the literature, it is possible to find alternative approaches that provide near-optimal solutions. The sequential suboptimal techniques (Bertesekas, 2000) can be interesting approaches to providing revised policies. In this context, Open-Loop Feedback Controller (OLFC) is a suboptimal alternative proposed here for investigation. This procedure permits updating the optimal policy periodically, taking into account the current state of systems (1)-(2). Note that the feedback characteristic of the OLFC makes it an adaptive procedure. This means that the solution provided by the procedure is better than that one given by an open-loop procedure. Note that an open-loop solution is obtained from the classical mean optimal problem.

Note that the problem (3) is a stochastic optimal control problem with perfect state information. So, it is perfectly possible to measure the inventory level (i.e. the state) at the beginning of each new period k. This characteristic allows immediate application of the OLFC procedure since it does not need use state estimators. Basically, the steps of the OLFC procedure are (Bertesekas, 2000):

Step 1. In the beginning of each period k, the exact position of the serviceable ($x_1(t)$) and returnable ($x_2(t)$) inventory levels are measured.

Step 2. With these information (i.e., ($\tilde{x}_1(k)$ = ($x_1(t)$) and ($\tilde{x}_2(t)$ = ($x_2(t)$))), an optimal production policy $\{u_1(k)\ u_2(k)\ u_3(k)\}_{k=t,\ t+1,\ \cdots,\ T-1}$ is computed from solving the following equivalent problem (Silva Filho, 2011):

$$\underset{u_1,u_2,u_3}{\text{Min}} \quad h_1\tilde{x}_1(T)^2 + h_2\tilde{x}_2(T)^2 + \sum_{k=t}^{T-1}\left\{h_1\tilde{x}_1(k)^2 + h_2\tilde{x}_2(k)^2 + c_1u_1(k)^2 + c_2u_2(k)^2 + c_3u_3(k)^2\right\} + K_t$$

s.t.

$$\begin{aligned}
&\tilde{x}_1(k+1) = \tilde{x}_1(k) + u_1(k) + u_2(k) - \tilde{d}(k) \\
&\tilde{x}_2(k+1) = \tilde{x}_2(k) - u_2(k) - u_3(k) + \mu\tilde{d}(k-\tau) \\
&\underline{x}_{1,\alpha}(k) \le \tilde{x}_1(k) \le \overline{x}_{1,\alpha}(k);\ \ 0 \le \tilde{x}_2(k) \le \overline{x}_2 \\
&0 \le u_1(k) \le \overline{u}_1;\ \ 0 \le u_2(k) \le \overline{u}_2;\ u_3(k) \ge 0
\end{aligned} \tag{5}$$

where, the integration constant of the criterion K_t, when it takes in probability, depends of the variance of demand and the period t, it gives by

$$K_t = (h_1 + h_2)\cdot\sigma_d^2\cdot(T-t) \tag{6}$$

The physical constraints of serviceable and collected products are transformed into equivalents, but deterministic inequalities given by:

$$\begin{aligned}
\underline{x}_{1,\alpha}(k) &= \underline{x}_{1,k} - \sqrt{k}\cdot\sigma_d\cdot\Phi^{-1}(\alpha_k) \\
\overline{x}_{1,\alpha}(k) &= \overline{x}_{1,k} + \sqrt{k}\cdot\sigma_d\cdot\Phi^{-1}(\alpha_k)
\end{aligned} \tag{7}$$

Step 3 As a result, it can be established a production policy to the problem (10) that uses the result provided exactly during period t. This means that the rest of policy is completely ignored.

Some comments:

The main characteristics about this procedure are: (a) it requires the solution of N problems in a rolling horizon scheme. This is equivalent to maintain the production plan constantly revised; (b) only the decision of the period k=t (i.e, $u_i(t)$, i=1,2,3) is effectively applied. The rest of the policy (i.e, $\{u_i(t+1), u_i(t+2), ..., u_i(T-1); i=1,2,3\}$) is ignored since it does not take into account the actual level of inventory; (c) there is easy computational implementation of multidimensional problems. The procedure permits the application of different optimisation techniques, and then, user can choose the one that requires less computational effort; (d) fixing t=1 and $d(k)=\hat{d}(k)$ ($\sigma_D^2=0$, $\forall k$), the problem (5) becomes a static optimisation problem, also known as mean problem; (Silva Filho, 2009) and (e) being adaptive, the OLFC provides a better solution than the one provided by the mean optimal solution.

4. NUMERICAL EXAMPLE

A fictitious company makes to stock and distributes a product to the marketplace. Under the pressure of environmental movements, the supply chain department of this company has been seeking alternatives to implement a model of reverse logistics that not causes a strong impact on its profitability. In short, currently, the company is in charge of manufacturing the final product of this supply chain and distributing it into the marketplace. In the brief future, the company will have to collect it back and to treat or dispose of it.

The operational data of the company are given in the Table 1:

Table 1. Company's data

Elements	Description
T=12 months	Planning horizon
α=0.9	Customer satisfaction level
σ=50	Standard deviation of demand
x_{01}=300	Initial inventory level of serviceable
$\underline{x}_1 = 0\ \overline{x}_1 = 300$	Lower and upper bounds of serviceable
$\overline{u}_1 = 750$	Upper production bounds
h_1=2; c_1=3	Current production and inventory costs
\$ 6.570.000	Total cost of the company per year

The company aims to answer some questions: a) assuming different costs, as shown in Table 2, what is the best rate of return for the company?; b) after selecting optimum rate of return, which will be the final cost of the optimal reverse policy?; and c) compared with the current policy model, what difference is verified?

Initially, let's consider the first question: assuming that inventory costs are h1> h2, it becomes possible to evaluate the behavior of rates of return for different situations related to the production and disposal costs, see Table 2.

Table 2. Costs relations with h1>h2

Situations	Production and disposal costs
1	c1 > c2 = c3
2	c2 < c1 = c3
3	c1< c2 = c3
4	c2 < c1 = c3
5	c1> c2 > c3
6	c1< c2 < c3

Setting the mean value of demand to the problem (3) and solving the step 2 of stage 1, the best value for a return rate for the company is promptly identified. The relation among return rates and costs provide different curves that are shown in the figure 2.

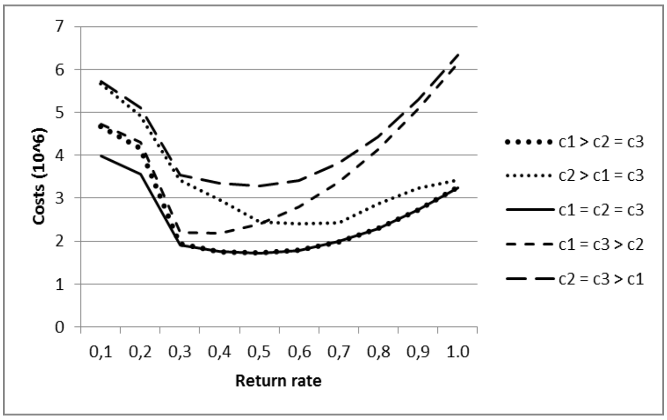

Fig. 2. Return rate versus costs

From figure 2, it is possible to identify two optimal situation for the company that is to consider c1>c2=c3 or c1=c2=c3. The return rate for these two situations is to ½, that is, μ=0,5.

4.1. Scenario without a reverse policy (μ = 0%)

Initially, let's assume that the company does not use a reverse system as the one exhibited by figure 1. The operation policy of the company in terms of serviceable inventory level and manufacturing rate is shown in figure 3.

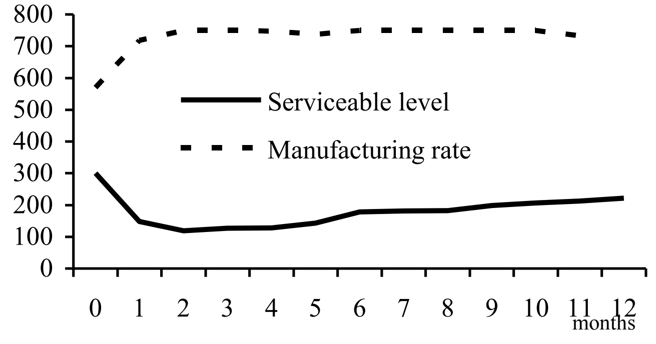

Fig. 3. Inventory and production levels for μ=0

It is worth mentioning that the company is running in its maximum load, which means that $u_1(k) \cong \bar{u}_1 = 750$. About the inventory levels, every product manufactured by the company is practically used to meet the demand at once. However, a small part of products is kept in stock to ready-delivery. In fact, the company uses a performance measure for customer satisfaction that look for guaranteeing 95% of ready-delivery stock to the market place. The inequality given by (7) shows how to keep a save stock for ready-delivery with α_k= 95% of chances of no violation of serviceable inventory constraint.

Table 3 provides the costs of the company to run its production operation without a reverse policy (μ=0). Note that both holding serviceable and manufacturing new product costs are considered in this Table. The idea is to compare these costs with those costs that will obtain with μ = 50%.

4.2, Scenario with the return rate μ = 50%

Figures 4 and 5 illustrate optimal inventory and production trajectories for forward and backward channels. In this scenario, 50% of collected products return from the marketplace. These products are checked before to be sent to remanufacture or to be properly disposed. An interesting aspect to be noted through this situation is the reduction of the production of new products. Practically, new product production rate was reduced to 47% compared to the previous situation (i.e., without reverse policy; figure 3). Note, from figure 5, that remanufacturing level of used-products was close to maximum capacity. Surely, this new feature reduces costs, particularly due to purchase of components to produce new products. Note that this feature explains the reason of the cost for remanufacturing to be less than the cost for manufacturing new products.

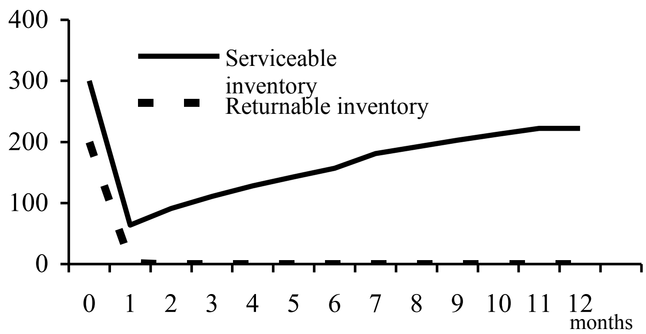

Fig. 4. Inventory-production levels without reverse policy

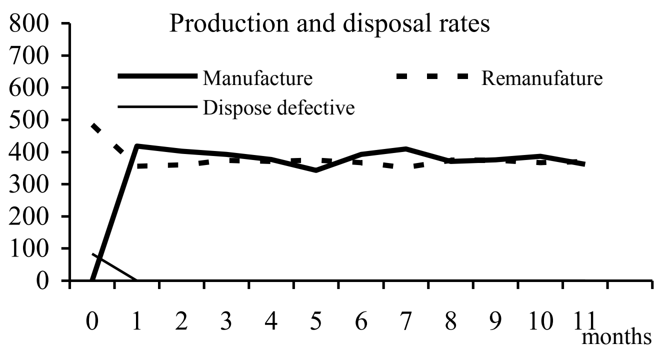

Fig. 5. Optimal production and disposal rates

4.3. Comparing scenario's costs

Table 3 provides the costs related to both scenarios. Note that the total cost of the scenarios 1 is more than the double of scenario 2, which means that the use of remanufactured products can decrease the overall costs of production. It is important to understand here that such a result is only possible because it was assumed that the costs incurred in the reverse channel are less than the costs of the forward channel.

Table 3. Costs of each scenario ($)

Costs (K=1000)	**Scenario 1 (μ=0)**	**Scenario 2 (μ=0,5)**
Serviceable holding	**421,70 K**	**352,45 K**
Returnable holding		**40 K**
Manufacturing	**6.138,20 K**	**1.500 K**
Remanufacturing		**909,39 K**
Disposing		**6.959,30**
Total cost	**6.559,90K**	**2.808,80 K**

4.4. Final comments about scenarios

The use of reverse logistics scheme joint to a good return policy for used products can reduce the holding and production costs of the company. However, it is required that the cost for remanufacturing be at least slightly less than the cost for manufacturing new products. The two scenarios previously analyzed show such a situation. In fact, in scenario one (μ=0), the manufacturing of new products is intense and uses all production capacity. Comparatively, in the scenario two (μ=0,5) returnable products are remanufactured and, as a consequence, the manufacturing process of new products is reduced around 47%, which means less expensive cost for the company. This characteristic is observed in the Table 3.

5. CONCLUSION

The use of raw material extracted from the earth and energy spent to produce and distribute products around the world have done reverse logistics an essential part of an integrated supply chain. The main objective of a reverse system is to move products from market collectors to places where they can be remanufactured or properly disposed. The purpose is that some used-products can get value along the reverse chain by mean of any retreatment. As a consequence, it is possible to improve the environment, reducing waste.

In this paper, reverse logistics channel is added to the forward channel in order to reduce inventory and production costs. A stochastic dynamic problem is introduced to this proposal. The problem is, therefore, defined by two production-inventory systems; one is a forward stochastic system, and the other a reverse system. During the operation of these systems, manufacturing and remanufacturing products are available in a serviceable inventory to meet the demand. After a period, products already used return from the market to the company using a backward channel, which includes a returnable inventory and a remanufacturing process.

A two stage procedure is considered in this study. In them, a stochastic linear quadratic Gaussian (LQG) model with constraints is formulated. In the first stage, the random nature of the problem is eliminated by changing the demand by its monthly average value. Thus, in this first stage, the optimal return rate is provided, taking into account different schemes of costs (as shown in Table 2). In the second stage, the stochastic problem is solved by mean of an Open-Loop Feedback controller, where an estimated demand is used to provide an optimal annual plan for manufacturing, remanufacturing and disposal variables.

Through scenarios analyses, it is possible to compare this optimal plan with a return rate obtained from stage 1, with the scenario where a company does not use a reverse channel. This procedure can help managers to develop decision-making about appropriate policies of returning for the company. In fact, this simple example of a make-to-stock company, it is possible to reach important conclusion regards to the use of returned products to reduce costs for the company.

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