Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Generation of New Robot Skills from Learned Skills ⋆

Daniel Hernández García, Concepción A. Monje, Carlos Balaguer *

* *Universidad Carlos III de Madrid. Robotics Lab. Department of Systems and Automation Engineering. Av. de la Universidad 30, Leganés, Madrid, Spain. 28911. (e-mail: {dhgarcia, cmonje, balaguer}@ing.uc3m.es*

Abstract:
Future humanoid robots, working beside humans in complex dynamic environments, would be required to perform a wide repertoire of task. To this end traditional methods for deriving a control policy won't succeed, learning approaches are fundamental. Learning from demonstrations *(LfD)* techniques have appear as a means for developing intuitive control methods and generalize a skill based on a set of demonstrations. However, learning appropriate skills for every conceivable scenario the robot may encounter would still be a daunting undertaking. In this work we propose a framework for the generation and adaptation of new robot skills from previously learned skill models. Using previously learned models of a robot skill, and knowledge of the current task, the models of a skill is adapted to generate a new task by a merger or combination operation over the given robot skill models.

1. INTRODUCTION

One major goal in robotics research is to develop human-like robotic systems capable of interacting and collaborating with humans in the same unstructured working environments. Humanoid robots are particularly suitable for these duties because they are able to interact with the environment using the same tools designed for humans, and can collaborate with humans in several ways (Ambrose et al. [2000]). Also, it is believed that the most human-like of robots will be best equipped for reciprocal relationships with human beings. Since humanoid robots are designed to resemble a human shape and to poses human capabilities, they would be ideally suitable for performing tasks and to safely share the same space and activities with people without the need to adapt the environments and with a higher level of acceptance and a more intuitive way for interaction between human operators and the robotic agents (Monje et al. [2008]). We envision a world, in a no too distance future, where humanoid robots and humans would work, collaborate and interact together sharing the same space, tools, and activities.

For robots, working alongside humans means dealing with continuously changing environments and a huge variability of tasks which they are expected to perform, thus the robots should have the ability to continuously learn new skills and adapt the existing skills to new contexts. For humanoid robots to work with humans in unstructured environments the robot must be able to perform dynamically changing tasks that require great adaptations to react to new constraints. Programming specialized controllers for every single task and situation that could be encounter would not be practical. To develop the capacities expected from the future humanoid robots flexible and generic control methods that can adapt to various tasks and robot's constraints are necessary. *Learning from demonstration (LfD)* or *Programming by Demonstration (PbD)* (Billard et al. [2008]), has appeared as one way to respond to this growing need for intuitive control methods. *LfD* formulates user-friendly methods by which a human user can teach to a robot how to accomplish a specific task.

Learning from demonstrations techniques have appear as a means for developing intuitive control methods. However, learning appropriate skills for every conceivable scenario the robot may encounter would still be a daunting undertaking. In this work we propose a framework for the generation and adaptation of new robot skills from the previously learned skill models.

The rest of this paper is structured as follows. In the next section we present the framework use for learning skills models. Section 3 presents the propose framework for generation and adaptation of robot skills from previously learned skill models. Section 4 describes the preliminary experimental validation. Finally conclusions are presented in section 5.

2. LEARNING THE SKILLS MODELS FROM DEMONSTRATION

The *Imitation Learning* approaches are focused on the development of algorithms that are generic in their representation of the skills and in the way they are generated. One common approach creates models of the skills based on sets of demonstrations performed in slightly different conditions generalizing overt the inherent variability to

⋆ This work was supported by the ARCADIA project DPI2010-21047-C02-01, funded by CICYT project grant on behalf of Spanish Ministry of Economy and Competitiveness and from the RoboCity2030-II-CM project (S2009/DPI-1559), funded by Programas de Actividades I+D en la Comunidad de Madrid and cofunded by Structural Funds of the EU.

extract the essential components of the skills (Calinon [2009]). Employing statistical learning techniques is a popular trend for dealing with the high variability inherent to the demonstrations.

Robot skills should follow certain likeable properties such as autonomous behaviour without explicit time dependency and adaptation of its parameters, flexible learning, basic stability, coupling phenomena of perception and action, compact representation and ease of categorization of movement trajectories, reusable for similar and related task, modifiable to new tasks and context not seen during demonstrations, robustness against both temporal and spatial disturbances of movement in dynamic environments.

Encapsulating the dynamics of the movement into a dynamical system encoding is a promising approach to learning movement trajectories (Billard et al. [2008]). Adopting non-linear dynamics systems theory has applications in several branches of sciences, like physics, mechanics, chemistry, electromagnetism, biology, engineering, etc.

A *Dynamical Systems (DS)* approach to skill learning can offer a fast, simple and powerful formulation for representing and generating movement plans, learned from demonstration. The *DS* framework allows to comply with the attractor dynamics of the desired behaviour, modulating it with a set of non-linear dynamic systems that form an autonomous control policy for motor control. Statistical learning techniques can be used to arbitrarily shape the attractor landscape of the control policy for encoding within the desired trajectory, going from an initial state to an end state driven by the attractor dynamics. *DS* are intrinsically robust and can adapt its trajectories instantly in the face of spatio-temporal perturbations (Khansari-Zadeh and Billard [2010]). The *DS* do not explicitly depend on time indexing and provide closed loop control and are able to model arbitrary non-linear dynamics (Gribovskaya and Billard [2009]). The *DS* can also be easily modulated to generate new trajectories that have similar dynamics, performing in areas that where not covered during demonstrations (Khansari-Zadeh and Billard [2011]). Use of *DS* with statistical approaches permit to develop a representation of movements, encapsulating the relationships between variables and variations of the task into the dynamical systems parameters (Calinon et al. [2012]).

The dynamic system can be generally express as a differential equation,

$$\dot{x} = f(x, \theta), \tag{1}$$

The *DS* is conceive as a 'kinematic policy' which generates target values, in kinematic varibles, e.g., position, velocity, acceleration (Schaal et al. [2007]), appropriate controllers are needed to subsequently convert them to motor commands. Explicit time dependency is removed from the formulation of the *DS* such that the control policy becomes an autonomous dynamic system, this is advantageous as maintaining timing counter or signal adds a burdensome level of complexity to control, additionally support for such clocking signal in biological systems is disputed (Schaal et al. [2007]). Autonomous non-linear dynamical systems is a powerful mechanism to modulate the control policies by learning the model of the skill building a stable estimate $\hat{f}$ of f based on the set of demonstrations.

Ijspeert et al. [2001] was the first work to emphasize this approach, by designing a motor representation based on dynamical systems in order to encode movements and for later replaying them in various conditions. The approach conceive the motions as movement primitives and named it *Dynamic Movement Primitives (DMP)* (Ijspeert et al. [2003]).

The original *DMP* approach operated in a single dimension using a pre-defined dynamical system as a motion primitive, where the trajectory of every single DOF was modulated by its own non-linear function and transformation system separately. (Gribovskaya and Billard [2009]) investigated a method whereby *Gaussian Mixture Models (GMM)* could directly embed the multi-variate dynamics of a motion. Their work presented a generic framework that combined *DS* movement control with *LfD* to teach a robot. An iterative procedure was employed to learn a statistical estimate of an arbitrary multivariate autonomous dynamical system, through the encoding of the demonstrated data with Gaussian Mixtures.

2.1 Stable Estimator of Dynamical Systems

A probabilistic framework is employed to build an estimate $\hat{f}$, of the non-linear state transition map f, based on the set of demonstrations. *Gaussian Mixture Models (GMM)* are used to directly embed the multi-variate dynamics through the encoding of the demonstrated data. *GMM* define a joint probability distribution $p(\xi^i, \dot{\xi}^i)$ of the demonstrations as a mixture of the $\mathbf{K}$ Gaussian multivariate distributions $\mathcal{N}^k$, with π^k, μ^k, and Σ^k, respectively the prior, mean and covariance matrix, parameters of the Gaussian component k. The joint probability distribution, $p(\xi, \dot{\xi})$, for the *GMM* is given by,

$$\begin{gathered} p(\xi, \dot{\xi}; \theta) = \frac{1}{K} \sum_{k=1}^{\mathbf{K}} \pi^k \mathcal{N}^k(\xi, \dot{\xi}; \mu^k, \Sigma^k) \\ \text{with} \quad \mu^k = \{\mu_\xi^k; \mu_{\dot{\xi}}^k\} \quad \text{and} \quad \Sigma^k = \begin{bmatrix} \Sigma_\xi^k & \Sigma_{\xi\dot{\xi}}^k \\ \Sigma_{\dot{\xi}\xi}^k & \Sigma_{\dot{\xi}}^k \end{bmatrix} \end{gathered} \tag{2}$$

To recover the expected output variable $\hat{\dot{\xi}}$, given the observed input in ξ^*. one then can sample from the probability distribution function $p(\xi, \dot{\xi})$ in Eq. 2. This process is called *Gaussian Mixture Regression (GMR)*, more details can be found on (Calinon [2009]). The *GMR* can be express as a non-linear sum of linear dynamical systems,

$$\dot{\xi} = \hat{f}(\xi) = \sum_{k=1}^{K} h^k(\xi)(\mathbf{A}^k \xi + \mathbf{b}^k) \tag{3}$$

$$\begin{cases} \mathbf{A}^k = \Sigma_{\dot{\xi}\xi}^k (\Sigma_\xi^k)^{-1} \\ \mathbf{b}^k = \mu_{\dot{\xi}}^k - \mathbf{A}^k \mu_\xi^k \\ h^k(\xi) = \dfrac{p(\xi; \mu_\xi^k, \Sigma_\xi^k)}{\sum_{k=1}^{K} P(\xi; \mu_\xi^k, \Sigma_\xi^k)} \end{cases} \tag{4}$$

Guaranteeing the estimates $\hat{f}$ result in an asymptotically stable trajectory is one key requirement to provide useful robot skills. To build a globally asymptotically stable *DS* (Khansari-Zadeh and Billard [2010]) proposed a learning method, called *Stable Estimator of Dynamical Systems (SEDS)*, establishing a set of sufficient stability conditions.

$$\begin{cases} \mathbf{b}^k = -\mathbf{A}^k \bar{\xi} \\ \mathbf{A}^k + (\mathbf{A}^k)^\top \prec 0 \end{cases} \quad \forall k = 1 \ldots \mathbf{K} \tag{5}$$

where $\mathbf{A}^k$ and $\mathbf{b}^k$ are defined according to Eq. 4, and $\prec 0$ refers to the negative definiteness of a matrix.

Learning the parameters of the *GMM* proceeds as a constraint optimization problem, ensuring that the model satisfy global asymptotic stability of the *DS* at the target (Khansari-Zadeh and Billard [2011]). For the optimization objective a function based in the *log-likelihood*,

$$\min_{\theta} J(\theta) = -\frac{1}{\mathcal{T}} \sum_{i=1}^{\mathcal{D}} \sum_{t=o}^{\mathcal{T}^i} \ln p((\xi^{t,i}; \dot{\xi}^{t,i}) \mid \theta) \tag{6}$$

where $p((\xi^{t,i}; \dot{\xi}^{t,i}) \mid \theta)$ is given by Eq. 2 and $\mathcal{T} = \sum_{i=1}^{\mathcal{D}} \mathbf{T}^i$ are the total number of points in the demonstration dataset. ξ is a state variable that can unambiguously described the motion, this could represent positions, velocities, accelerations, forces, etc. Here the choice is made to represent the motions in kinematic coordinates, the Cartesian space, with the assumption that appropriate controllers are available to convert the kinematic variables to motor commands. ξ is chosen as the translation component of a motion of the end-effector, a vector of Cartesian coordinates $x \in \mathbb{R}^3$.

By being time-invariant and globally asymptotically stable at the target, the *DS* estimated with *SEDS* are able to respond immediately and appropriately to perturbations that could be encountered during reproduction of the motion (Khansari-Zadeh and Billard [2011]).

3. ROBOT SKILL ADAPTATION AND GENERATION OF NEW MODELS

Despite the *LfD* approaches clear advantages, it would still be impractical for the human operator to teach the robot the skills for every needed task and for every foreseen situation, since the number of demonstrations the human must provide to the robot to generate a new model of a skill could turn it into a tiresome and time-consuming process and it wouldn't be possible to cover every needed task and every situation. Hence, it is important to be able to enhance the *LfD* with the capacity to adapt and generate new skill models. It is necessary to extend the classical *LfD* approach of learning a skill model in a way that allows the adaptation of a robot previously learned motion skills to new unseen contexts.

To reproduce a task adapted for an unseen context the robot must be given knowledge of the state of the environment and the constraints of the task. Using both, the already learned model of a skill, and the extracted constraints information of the current task, the model of the skill can be adapted to reproduce the task. Figure 1, illustrates the process for enhancing classical *LfD* approach to generalize a skill to allow adapting a robot previously learn skills models.

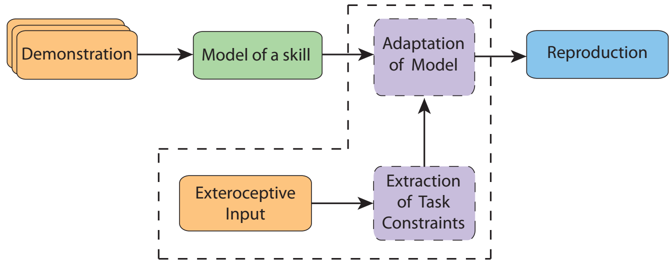

Fig. 1. Augmenting the *LfD* approach for the generalization of a skill to allow adapting a robot previously learn skills models.

The robot skills learned with the methodology described in Section 2 would present stable trajectories that accurately reproduce the demonstrated motion dynamics. The robot skills models were learned by employing a *DS* approach. The learned robot skills models would form a set of basic primitives of action. An approach based on movement primitives relies on possessing available sequences of motor commands, executed in a certain order, to accomplish a given motor task. Evidence exist from human and animal experiments supporting the believe that sets of motor primitives are used to build a basis for voluntary motor control (Schaal [1999]).

To generate complex motions from a learned set of basic primitive skills and be able to reproduce various complex task behaviours, methods for operating and manipulating upon the primitives are needed. The robot skills must be adaptable to conditions of its operating environment. Also, the action primitives approach must be able to generate new skills by merging two or more primitives into a new skill, multiple desired robot skills may be composed from superposition of various primitives. Another important property is the combination of the robot skills models to generate new models that encompass a larger spectrum of the attractor dynamics.

3.1 Merger of Robot Skills Models

The learned *DS* models encode specific motion skills, which can be seen as building blocks used to generate more complex motions. Multiple desired robot skills may be composed from sequencing or superposition of various primitives skills. The modularity of the *DS* approach is essential as it would allow to control a wider repertoire of movements from a smaller set of basic skills (Schaal [1999]).

Intuitively one could consider an approach to merging two or more models of a skill simple by adding and averaging together their learned parameters $\theta = (\pi, \mu, \Sigma)$ in order to obtained a new skill model. While this approach may work for some cases it is important to note that direct superposition of the skills does not allow to control the manner in which the new model is generated and it would not guarantee its stability.

Muelling et al. [2013] presented a framework to generalize learned motor primitives to a wider range of situations using a mixture of motor primitives approach. First a set of elementary movements were learned from a human teacher by kinaesthetic teaching. Subsequently, the system generalizes these movements to a wider range of situations using a mixture of motor primitives approach. Their resulting policy enabled the robot to select appropriate motor primitives as well as to generalize between them.

In order to generate a new skill base on the merger of several robot skills models previously learned, we first review a coupled of useful mathematical properties from the *SEDS* (Khansari-Zadeh and Billard [2011]) formulation chosen to learned the skills,

$$\begin{gathered} \text{if} \quad f(\xi) \quad \text{is } \textit{SEDS}\text{, and} \quad \alpha > 0 \in \mathbb{R} \\ \dot{\xi} = \alpha f(\xi) \quad \text{is } \textit{SEDS} \\ \text{consider} \quad \mathbf{M} \quad \textit{SEDS} \text{ models} \quad f^i(\xi), i \in 1..\mathbf{M} \\ \dot{\xi} = \sum_{i=1}^{\mathbf{M}} \alpha^i f^i(\xi); \alpha^i > 0 \quad \text{is } \textit{SEDS} \end{gathered} \tag{7}$$

Intuitively one could consider an approach to merging two or more models of a skill simple by adding and averaging together their learned parameters $\theta = (\pi, \mu, \Sigma)$ in order to obtained a new skill model through a linear superposition. The models would represent the distributions $f^1(\xi)$ and $f^2(\xi)$ respectively as from Eq. 2. A weighted sum of these densities would give the merged model,

$$f(\xi) = \alpha f^1(\xi) + \beta f^2(\xi) \tag{8}$$

The weights α and β scale the prior of the components to give the new *GMM*.

The merger of the robot skills models can be carry out with a model combination approach expressed in mixtures of experts model,

$$p(t|x) = \sum_{k=1}^{\mathbf{K}} \pi_k(x) p_k(t|x) \tag{9}$$

The *SEDS* models encoded into a *GMM* is already a form of model combination approach. Here, recalling the expression of the non-linear weighting function $h^k(\xi)$, as in Eq. 4, it can be found it shares a similar formulation with the expression of the weights for the gating function as from Eq. 9, in which the mixing coefficients $\pi_k(x)$ of the gating function is given by the non-linear weighting function $h^k(\xi)$, and the $p_k(t|x)$ density are given by the linear *DS* $\mathbf{A}^k\xi + \mathbf{b}^k$ from Eq. 3.

The process for the merging of robot skills would first joined the *GMM* of the robot skills into a single model. Then a new weighting function $\tilde{h}(\xi)$ for the single model must be build out of the original weighting terms $h^k(\xi)$ from the merged models, ensuring the Gaussian with the biggest weight in every region of the trajectory provide the largest influence over the new *GMM* model in that region and that the new weighting function $\tilde{h}(\xi)$ still meets the constraints $0 > h^k(\xi) > 1$ and $\sum h^k(\xi) = 1$. Figure 2 illustrate the result of merging two robot skills to generate a new skill model. Then a new weighting function $\tilde{h}^k(\xi)$ would be given by $\tilde{h}^k(\xi) = \alpha^k(\xi, h) h^k(\xi)$ where $\alpha^k(\xi, h)$ is a scalar function that weight the original $h^k(\xi)$ of the models, and ensures the constraints of $\tilde{h}(\xi)$.

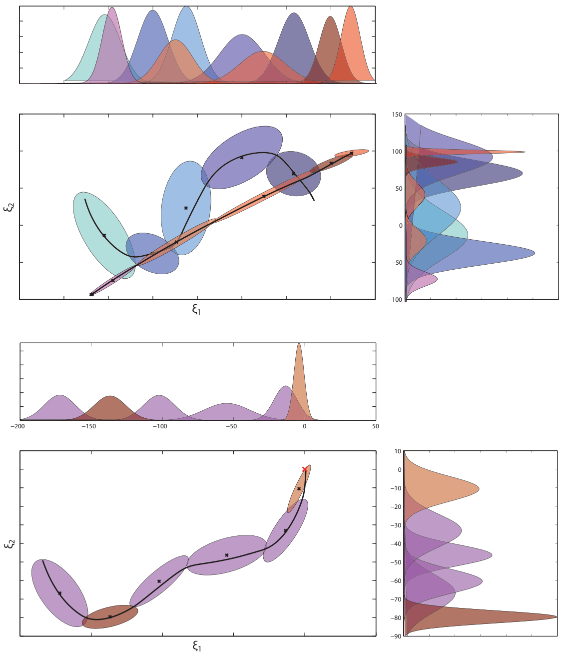

Fig. 2. Generation of a new model of a skill by merging previously learned skill models. Two or more basic models of a skill can be merge (top) to generate a new complex model (bottom).

3.2 Combination of Robot Skills Models

One important gain from the combination of robot skills comes from increasing the accuracy of the generalize behaviour. The convergence of the motion to the target is ensured, yet, due to the lack of information for points far from demonstrations a model may reproduce some trajectories that are not consistent with the usual way of doing the task. The generation of a model by combining robot skills is necessary in order to improve the task execution.

The more direct and intuitive approach would relied on providing the robot with more demonstrations over regions not covered before. By showing the robot more demonstrations and re-training the model with the new data, the robot should be able to successfully accomplish the task (Khansari-Zadeh and Billard [2011]). However, this approach would not seem to be the most flexible and general and the robots performing task in the real world cannot be expected to relied on available teacher to provide them with more demonstrations whenever their knowledge of a task don't suffice.

The work of Shukla and Billard [2012] focused on combining several learned *DS*, with distinct attractors, resulting in a multi-stable *DS*. Their work presented an *Augmented-SVM* model, which inherits region partitioning ability of well know *Support Vector Machine (SVM)* classifiers and is augmented with novel constraints derived from the individual *DS*.

In this work in order to generate a new skill made of the combination of several *Robot Skills Models* previously learned, we developed a method by which combine different skill models. Two different *SEDS* models, $\bar{\mathcal{M}}^1_{RS}, \bar{\mathcal{M}}^2_{RS}$

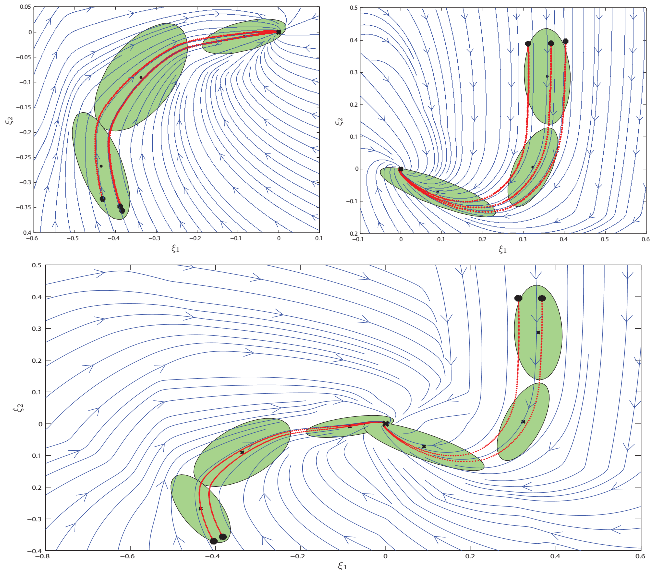

Fig. 3. Combining the dynamics of several skills into a single task model.

can be combine just by concatenating their parameters, such that the parameter of the new model can defined as $\pi = \dfrac{[\pi^1;\pi^2]}{(\pi^1+\pi^2)}$, $\mu = [\mu^1\mu^2]$ and $\Sigma = [\Sigma^1\Sigma^2]$. Them, an area of influence for the *DS* attractor is defined based on the non-linear weighting function $h^k(\xi)$ of the *SEDS* models expressed as a non-linear sum of linear dynamical systems as in Eq. 3. A new weighting function $\tilde{h}(\xi) = \alpha^k(\xi,h)h^k(\xi)$ for the single model must be build out of the original weighting terms $h^k(\xi)$, as in the merging of the models, however in this case the influence of the $h^k(\xi)$ terms over the trajectory must come at any time from only one model, therefore the $\alpha^k(\xi,h)$ function must have a completely different form that for merging the robot skill models. Figure 3 illustrate the result of combining two robot skills to generate a new skill model.

4. PRELIMINARY EXPERIMENTS

To validate the proposed methods for generating new skills from previously learned models by merging or combining robot skills models we choose a very simple scenario in which the robot is required to grasp a plastic cup, see Figure 5.

The robot skills models were learned in a *LfD* framework using the *SEDS* algorithm from Khansari-Zadeh and Billard [2011]. The demonstrations were recorded from a human teacher by employing the Microsoft Kinect sensor, see Figure 4.

At first two skills were taught to the robot for learning to grasp the cup place at its right and at its left side, see Figure 5. Then an additional skill is taught to grasp the cup, in front of it, place in a top shelf, Figure 6. The contemplated task request for the robot to grasp the cup at any possible placement in the top shelf, as long as it is inside the robot's arm workspace.

This task would be unachievable with robot skills learned so far, since the skill to grasp the cup at the left and right of the robot are taught for a placement of the cup

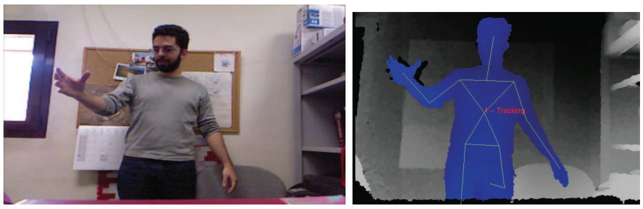

Fig. 4. Visual Demonstrations Teaching of a Skill: (left) A human teacher performs a demonstration. (right) The generated skeleton of the human recorded demonstration.

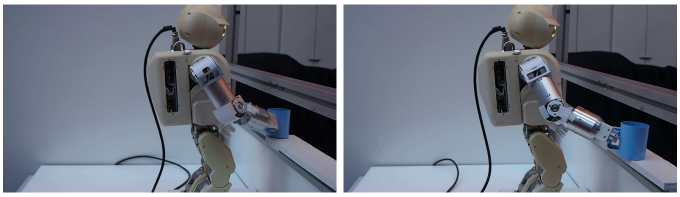

Fig. 5. Robot grasp learned skill: (left) Cup place at the left of the robot. (right) Cup place at the right of the robot.

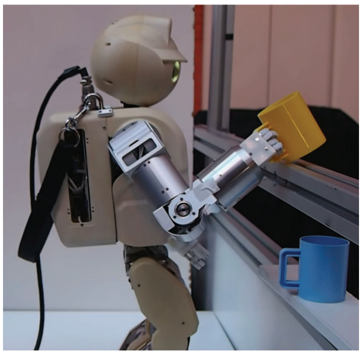

Fig. 6. Robot grasp learned skill: Cup placed centred in top shelf.

in the bottom shelf and the skill reproduction would not generalize well to the target new position. To grasp the cup, placed in the top shelf, at either side of the robot the skill to grasp the cup at the top shelf center must be merge with either the grasp cup left or grasp cup right, respectively, to generate the required new robot skill model. Being able of expanding a robot set of learned skills is clearly an important issue as robots will be asked to perform an increasingly number of activities and learning and programming every possible skill into the robot is infeasible.

Finally, to generalize across the whole working space of the top shelf the three models of the robot skill, for right, left and center, top shelf grasping, are combined into a single model of the attractor dynamics. In order to expand the robot skill set and increase its range of action to encompass a larger spectrum of the attractor dynamics the *Robot Skills Models* must be combinable into new models. This allows to carry out more complex task than those presented during demonstrations, generalizing the models of the skills to regions outside their original demonstrations. To generalize the skill across the whole working space of the shelves in the "cupboard" the three models of the robot skill, for right, left and center, grasping

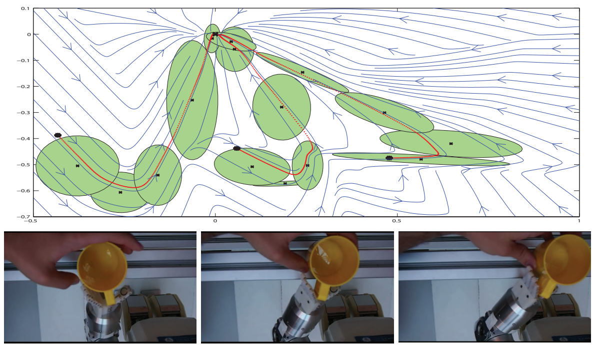

Fig. 7. Robot grasp learned skill: (top) The combine skill model allows a more complex attractor dynamics. (left) The cup placed at the right of the robot. (center) The cup is place at the front of the robot. (right) The cup is place at the left of the robot.

motion on a shelf, are combined into a single model of the attractor dynamics. Figure 7 illustrates the complete behaviour out of the generate new skill models.

5. DISCUSSION

This work described the process by which, using the already learned model of a skill, a robot skill can be adapted to reproduce a new task. Modes are presented for the merger and combination of the robot skill models.

Humanoid robots are required to perform a wide repertoire of task working beside humans in complex dynamic environments. Efforts to generate robotic skills can only have a real implementation value for developing humanoid robotic systems if the models of the skill can be operated upon to generate new behaviours of increasing levels of complexity.

Multiple desired robot skills may be composed from superposition of various models. Section 3.1 presented the generation of a task model by merging robot skills. Skills can be generated by merging two or more models into a new skill.

Section 3.2 presented the generation of a task model by combining robot skills. Models of a skill can be combined to generate new models that encompass a larger spectrum of the attractor dynamics.

New skills can be generated by merging two or more models in order to expand the robot skill set and increase its range of action, multiple desired robot skills may be composed from superposition of various models. The combination of skill models allows to carry out more complex task than those presented during demonstrations, generalizing the models of the skills to regions outside their original demonstrations. One important gain from the combination of robot skills comes from increasing the accuracy of the generalize behaviour.

The experiment described in 4 provide a indication of the value the methods developed in this work can have to generate and adapt new robot skills from previously learned skill models, and expand the applicability of the learned models of the skill, and the robot skill learning framework.

REFERENCES

R.O. Ambrose, H. Aldridge, R.S. Askew, R.R. Burridge, W. Bluethmann, M. Diftler, C. Lovchik, D. Magruder, and F. Rehnmark. Robonaut: Nasa's space humanoid. *Intelligent Systems and their Applications, IEEE*, 15(4):57–63, Jul/Aug 2000. ISSN 1094-7167. doi: 10.1109/5254.867913.

A. Billard, S. Calinon, R. Dillmann, and S. Schaal. Robot programming by demonstration. In B. Siciliano and O. Khatib, editors, *Handbook of Robotics*, pages 1371–1394. Springer, Secaucus, NJ, USA, 2008.

S. Calinon. *Robot Programming by Demonstration: A Probabilistic Approach.* EPFL/CRC Press, 2009.

Sylvain Calinon, Zhibin Li, Tohid Alizadeh, Nikos G Tsagarakis, and Darwin G Caldwell. Statistical dynamical systems for skills acquisition in humanoids. In *Proc. IEEE Intl Conf. on Humanoid Robots (Humanoids)*, 2012.

Elena Gribovskaya and Aude Billard. Learning Nonlinear Multi-Variate Motion Dynamics for Real- Time Position and Orientation Control of Robotic Manipulators. In *Proceedings of 9th IEEE-RAS International Conference on Humanoid Robots*, 2009.

A. Ijspeert, J. Nakanishi, and S. Schaal. Trajectory formation for imitation with nonlinear dynamical systems. In *ieee international conference on intelligent robots and systems (iros 2001)*, pages 752–757, 2001.

Auke Jan Ijspeert, Jun Nakanishi, and Stefan Schaal. Learning attractor landscapes for learning motor primitives. In *Advances in Neural Information Processing Systems*, pages 1523–1530. MIT Press, 2003.

S.M. Khansari-Zadeh and A. Billard. Imitation learning of globally stable non-linear point-to-point robot motions using nonlinear programming. In *Intelligent Robots and Systems (IROS), 2010 IEEE/RSJ International Conference on*, pages 2676–2683, 2010. doi: 10.1109/IROS.2010.5651259.

S.M. Khansari-Zadeh and A. Billard. Learning stable nonlinear dynamical systems with gaussian mixture models. *Robotics, IEEE Transactions on*, 27(5):943 –957, oct. 2011. ISSN 1552-3098. doi: 10.1109/TRO.2011.2159412.

Concepcin A. Monje, Paolo Pierro, and Carlos Balaguer. Humanoid robot rh-1 for collaborative tasks: a control architecture for human-robot cooperation. *Applied Bionics and Biomechanics*, 5(4):225–234, 2008.

K. Muelling, J. Kober, O. Kroemer, and J. Peters. Learning to select and generalize striking movements in robot table tennis. (3):263–279, 2013.

S. Schaal. is imitation learning the route to humanoid robots? In *trends in cognitive sciences*, number 6, pages 233–242, 1999.

Stefan Schaal, Peyman Mohajerian, and Auke Ijspeert. Dynamics systems vs. optimal control - a unifying view. In Trevor Drew Paul Cisek and John F. Kalaska, editors, *Computational Neuroscience: Theoretical Insights into Brain Function*, volume 165 of *Progress in Brain Research*, pages 425 – 445. Elsevier, 2007.

Ashwini Shukla and Aude Billard. Augmented-svm: Automatic space partitioning for combining multiple nonlinear dynamics. In *NIPS*, pages 1025–1033, 2012.