**Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014**

A multi-step robust model predictive control scheme for polytopic uncertain multi-input systems

Langwen Zhang* Jingcheng Wang* Bohui Wang*

* *Department of Automation, Shanghai Jiao Tong University, and Key Laboratory of System Control and Information Processing, Ministry of Education of China, Shanghai 200240*
(Corresponding author: Jingcheng Wang; Email: jcwang@sjtu.edu.cn; Tel: 86-21-34205374.)

Abstract: Model predictive control (MPC) has attracted wide attention in process industries with its ability to handle constrained multivariable processes. Computational complexity can become a limiting factor when MPC is applied to large-scale systems with fast sampling times. In this paper, a control scheme known as multi-step robust MPC is presented for polytopic uncertain multi-input systems. Only one or several state feedback laws are optimized at each time interval to reduce computational complexity. A set invariance condition for polytopic uncertain systems is identified and the invariant set is determined by solving a linear matrix inequality (LMI) optimization problem. Based on the set invariance condition, a min-max multi-step robust MPC scheme is proposed. Numerical simulations show the effectiveness of the proposed scheme.

1. INTRODUCTION

Model predictive control technology has been widely used in complex constrained multivariable control problems including chemical processes, power systems, urban traffic systems and irrigation canals (Qin & Badgwell, 2003). The key idea of MPC technique is to use a linear process model of the system to predict the future states and control inputs. This is achieved by solving an online optimisation problem to obtain the optimal control sequence over a future time horizon. The first control input in the control sequence is then applied to the process. At the next sample, measurements are used to update the optimization problem, and the optimization is repeated (Mayne, Rawlings, Rao, & Scokaert, 2000). This results in demanding online optimization. Computational complexity can become a limiting factor when applying MPC to large-scale systems with fast sampling times. Many works (*i.e.,* distributed MPC, decentralized MPC, and multiplexed MPC) have been proposed to reduce the computational complexity.

A widely investigated scheme is called distributed MPC algorithm, which decomposes the global system into subsystems and designs distributed MPC controllers independently (Giselsson, Doan, Keviczky, Schutter, & Rantzer, 2013; Maestre, Muñoz de la Peña, Camacho, & Alamo, 2011; Scheu & Marquardt, 2011; Zhang, Wang, & Li, 2013; Zheng, Li, & Li, 2011). The iterative distributed MPC requires interactions among subsystems. The performance of closed-loop system is improved with the increase in number of iterations. Nevertheless, the computational time will increase with the increasing number of iterations.

Another approach to reduce the computational complexity is the decentralized MPC, in which each subsystem makes control decision independently (Alessio, Barcelli, & Bemporad, 2011; Rivero, Farina, & Ferrari-Trecate, 2013; Sandell Jr, Varaiya, Athans, & Safonov, 1978). Information exchange including measurements and previous control inputs is allowed only before and after the decision making. There is no coordination between subsystems during the decision process. Nevertheless, it is known that such a decentralized control strategy may result in unacceptable control performance, especially when the couplings between subsystems are strong (Li, Zhang, & Zhu, 2005).

Multiplexed MPC was introduced in (Ling, Maciejowski, Richards, & Wu, 2012; Siva, Maciejowski, & Ling, 2010). The idea of the multiplexed MPC scheme was to solve the MPC optimization problem sequentially and update the control inputs as soon as the solution is available. The multiplexed MPC was applied to the control of large commercial turbofan engine in (Richter, Singaraju, & Litt, 2008). The results showed that the multiplexed MPC reduced the time required for computing the control feedback law with small performance degradation. Nevertheless, it was required that the fresh measurements are available at the reduced update intervals T/m, where T was the sampling time and m was the number of control inputs.

The polytopic uncertain systems are widely investigated in recent years for MPC controller design (Canale, Fagiano, & Signorile, 2012; Huang, Li, Lin, & Xi, 2011; Huang, Li, & Xi, 2013; Yun, Choi, & Park, 2010). The robust MPC controllers are obtained by solving LMIs optimization problem. The computation time is an important issue in on-line solving the LMI constraints. This motivates us to propose a new scheme to reduce the computational complexity in designing the robust MPC for large-scale systems.

In this paper, we propose a scheme known as multi-step robust MPC, in which only subset of control feedback laws are designed sequentially at each time interval. The motivation for this scheme is to facilitate applications of MPC when computation time is critical. The proposed multi-step robust MPC scheme results in reducing computation time in controller design.

The rest of the paper is organized as follows. In Section 2, problem formulations are presented. Multi-step robust MPC algorithm is proposed in Section 3. Numerical example is given in Section 4 and the paper is concluded in Section 5.

Notations: Throughout this paper, we use $\|x\|_Q^2 = x^T Q x$ to define the Q-weighted norm. $|x|$ denotes the absolute value of vector x. The double subscript notation $(k+l\,|\,k)$ denotes a prediction of a variable l steps ahead from time interval k. F_k refers to the control feedback law optimized at time interval k. The operator ' mod ' refers to the modulus after division. The symbol * denotes the matrix transpose.

2. PROBLEM FORMULATIONS

Consider the following discrete-time, linear uncertain system:

$$x_{k+1} = A_k x_k + B_k u_k \tag{1}$$

where $x_{k+1} \in R^n$ denotes the measurable system state. $u_k \in R^m$ denotes the control input, which is assumed to consist of m independent sub-inputs $u_1(k)$, $\cdots$, $u_m(k)$. The state matrix A_k and input matrix B_k are not exactly known, but are affine functions of a time-varying parameter vector $p(k) = [p_1 \quad p_2 \quad \cdots \quad p_L]$, $A_k = \sum_{\alpha=1}^{L} p_\alpha A_{k,\alpha}$, $B_k = \sum_{\alpha=1}^{L} p_\alpha B_{k,\alpha}$ with $0 \le p_\alpha \le 1$ and $\sum_{\alpha=1}^{L} p_\alpha = 1$. That is, the system matrices varies inside a polytope Ω_c whose vertices consist of L local matrices:

$$\Omega_c = Co\left\{ \left(A_{k,\alpha}, B_{k,\alpha} \right), \alpha \in [1, L] \right\} \tag{2}$$

where Co denotes the convex hull.

In conventional centralized MPC scheme, all the control inputs are computed online at each time interval, which optimizes a performance index. The centralized MPC scheme requires online optimization, in which computational burden can become an issue when applying MPC to large-scale systems with fast response sampling times.

Motivated by the deficiency of conventional centralized MPC algorithm, we propose a novel multi-step robust MPC scheme in this work. The structure of robust multi-step MPC is illustrated in Figure 1, which shows that only one or several state feedback laws are optimized at each time interval. We denote $u_i(k+l\,|\,k+l) = F_i(k+l)\,x_{k+l}$, $u_i(k+l+1\,|\,k+l) = F_i(k+l)\,x_{k+l+1}$.

The state feedback laws that are not optimized at the current time interval keep their previous optimized values. m_i state feedback laws are optimized in a single time interval with $B_i \in R^{n \times m_i}$ and $F_{i,k} \in R^{m_i \times n}$. Then, each state feedback law is optimized for every M time intervals, where $M = m / m_i$.

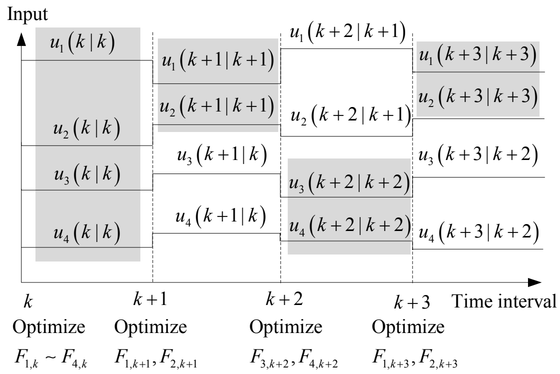

Fig. 1. Structure of multi-step robust MPC

The control input in model (1) is rewritten into several subsets as $u_k = \begin{bmatrix} u_{1,k}^T & u_{2,k}^T & \cdots & u_{M,k}^T \end{bmatrix}^T$, where $u_{i,k} = \begin{bmatrix} u_{(i-1)m_i+1}(k) & \cdots & u_{im_i}(k) \end{bmatrix}$, $u_{i,k} = F_{i,k} x_k$ and i is decided by solving the following indexing function:

$$i = (k \text{ mod } M) + 1 \tag{3}$$

Remark 1: In the proposed multi-step robust MPC scheme, only $F_{i,k}$ is optimized and the rest of the state feedback laws remains the previous one. For example, let $m = 4$ and $m_i = 2$, we have $M = \frac{m}{m_i} = 2$. At $k = 3$, we can compute i by $i = (3 \text{ mod } 2) + 1 = 2$. This means that only $F_{2,k}$ is optimized at time interval $k = 3$ and $F_{1,k}$ keeps the previous optimized state feedback law $F_{1,k-1}$. Then, $u_k = \begin{bmatrix} F_{1,k-1} \\ F_{2,k} \end{bmatrix} x_k$ is applied to the plant at $k = 3$.

System (1) can be rewritten as following distributed form:

$$x_{k+1} = A_k x_k + B_{i,k} u_{i,k} + \sum\nolimits_{j=1(j \ne i)}^{M} B_{j,k} u_{j,k} \tag{4}$$

where $B_i = \begin{bmatrix} B_{i,(i-1)m_i+1} & \cdots & B_{i,im_i} \end{bmatrix}$.

In model (4), $B_{i,(i-1)m_i+1}$ denotes the $((i-1)m_i+1)$th column of B. $u_{i,k} \in R^{m_i}$ denotes the input to be computed at time interval k. $u_{j,k} \in R^{m_j}$ refers to the inputs that are not optimized for the current time interval k. $u_{j,k}$ can be treated as a known input over the prediction horizon $k \ge 0$. The polytope Ω_c can be represented as follows:

$$\Omega = co\left\{ \left(A_{k,\alpha}, B_{i,k,\alpha}, B_{j,k,\alpha} \right), \alpha \in [1, L], j \in [1, M], j \ne i \right\} \tag{5}$$

The constraints on system input and state are represented as:

$$\left|u_j\left(k+l\mid k\right)\right| \le \bar{u}_j,\ \ l\ge 0,\ \ j=1,\cdots,m \tag{6}$$

$$\left|\psi_j x\left(k+l\mid k\right)\right| \le \bar{x}_j,\ \ l\ge 0,\ \ j=1,\cdots,n \tag{7}$$

where $\psi_j \in R^{n\times n}$ denote a matrix that the diagonal element $\left(j,j\right)$ is 1 and the rest elements are 0.

3. MULTI-STEP ROBUST MPC

In this section, we develop a multi-step robust MPC for polytopic uncertain systems. At time interval k, the aim of multi-step robust MPC is to find state feedback laws

$$u_{i,k+l|k} = F_{i,k}x_{k+l|k},\quad l=0,\cdots,\infty \tag{8}$$

by solving following optimization problem:

$$\min_{F_{i,k}} \max_{\left(A_k,B_{i,k},B_{j,k}\right)\in\Omega} J_{i,k|k},\ \ s.t.(4)\text{-}(8) \tag{9}$$

where $J_{i,k|k} = \sum_{l=0}^{\infty}\left(\left\|x_{k+l|k}\right\|_Q^2 + \left\|u_{i,k+l|k}\right\|_{R_i}^2 + \sum_{j=1(j\ne i)}^{M}\left\|u_{j,k+l|t_j}\right\|_{R_j}^2\right)$.

It is noted that the jth ($j\in\left[1,M\right]$, $j\ne i$) input is not optimized at time interval k. The state feedback law F_{j,t_j} is obtained at time interval t_j, where t_j is the previous time interval and decided by the following indexing function:

$$t_j = \begin{cases} k+j-i, & i>j \\ k+j-M-i, & i<j \end{cases} \tag{10}$$

Substituting F_{j,t_j} and (8) into (4), we obtain the following closed-loop system:

$$x_{k+l+1|k} = \left(A_k + B_{i,k}F_{i,k} + \sum\nolimits_{j=1(j\ne i)}^{M} B_{j,k}F_{j,t_j}\right)x_{k+l|k} \tag{11}$$

Considering the Lyapunov function $V_{i,k+l|k}\left(x_{k+l|k}\right) = x_{k+l|k}^T P_{i,k} x_{k+l|k}$ with $P_{i,k}>0$, we have:

$$\begin{aligned}\Delta V_{i,k+l|k} &= V_{i,k+l+1|k} - V_{i,k+l|k} \\ &= x_{k+l+1|k}^T P_{i,k} x_{k+l+1|k} - x_{k+l|k}^T P_{i,k} x_{k+l|k}\end{aligned} \tag{12}$$

Lemma 1: Suppose that the state feedback matrix $F_{i,k}$ is given and let $x_k = x_{k|k}$ be the state measured at time interval k. If there exists a matrix $P_{i,k}\in R^{n\times n}$ satisfying

$$\begin{aligned}&\left(A_{k,\alpha} + B_{i,k,\alpha}F_{i,k} + \sum\nolimits_{j=1(j\ne i)}^{M} B_{j,k,\alpha}F_{j,t_j}\right)^T P_{i,k} \\ &\times\left(A_{k,\alpha} + B_{i,k,\alpha}F_{i,k} + \sum\nolimits_{j=1(j\ne i)}^{M} B_{j,k,\alpha}F_{j,t_j}\right) - \\ &P_{i,k} + Q + F_{i,k}^T R_i F_{i,k} + \sum\nolimits_{j=1(j\ne i)}^{M} F_{j,t_j}^T R_j F_{j,t_j} \\ &\le 0,\quad i=1,\cdots,M,\quad \alpha=1,\cdots,L\end{aligned} \tag{13}$$

then, the ellipsoid $\Omega\left(P_{i,k},\gamma_{i,k}\right)$ is an invariant set of the closed-loop system (11). Moreover, for any initial condition $x_k\in\Omega\left(P_{i,k},\gamma_{i,k}\right)$, the performance index satisfies $J_{i,k}\le x_k^T P_{i,k} x_k \le \gamma_{i,k}$.

Proof: Substituting (11) into (12), we have,

$$\begin{aligned}\Delta V_{i,k+l|k} = x_{k+l|k}^T &\left[\left(A_k + B_{i,k}F_{i,k} + \sum\nolimits_{j=1(j\ne i)}^{M} B_{j,k}F_{j,t_j}\right)^T P_{i,k}\right. \\ &\left.\times\left(A_k + B_{i,k}F_{i,k} + \sum\nolimits_{j=1(j\ne i)}^{M} B_{j,k}F_{j,t_j}\right) - P_{i,k}\right]x_{k+l|k}\end{aligned} \tag{14}$$

We impose an upper bound on the performance index by introducing the following robust stability condition (as in (Huang, et al., 2013; Kothare, Balakrishnan, & Morari, 1996)) on (14), we have:

$$\begin{aligned}\Delta V_{i,k+l|k} \le& -x_{k+l|k}^T Q x_{k+l|k} - u_{i,k+l|k}^T R_i u_{i,k+l|k} \\ &-\sum\nolimits_{j=1(j\ne i)}^{M} u_{j,k+l|t_j}^T R_j u_{j,k+l|t_j}\end{aligned} \tag{15}$$

Summing (15) from $l=0$ to $l=\infty$, we have:

$$J_{i,k|k} \le V_{k|k} = x_{k|k}^T P_{i,k} x_{k|k} \tag{16}$$

Defining an upper bound $V_{i,k|k}\le\gamma_{i,k}$, we have:

$$J_{i,k|k}\le\gamma_{i,k} \tag{17}$$

Then, the ellipsoid $\Omega\left(P_{i,k},\gamma_{i,k}\right)$ is an invariant set of (11) if (15) holds. Furthermore, the robust stability condition (15) can be represented as follows:

$$\begin{aligned}\Delta V_{i,k+l|k} \le -x_{k+l|k}^T &\left(Q + F_{j,k}^T R_j F_{j,k} + \right. \\ &\left.\sum\nolimits_{j=1(j\ne i)}^{m} F_{j,t_j}^T R_j F_{j,t_j}\right)x_{k+l|k}\end{aligned} \tag{18}$$

Substituting (14) into (18), we have:

$$\begin{aligned}x_{k+l|k}^T &\left[\left(A_k + B_{i,k}F_{i,k} + \sum\nolimits_{j=1(j\ne i)}^{M} B_{j,k}F_{j,t_j}\right)^T P_{i,k}\times\right. \\ &\left(A_k + B_{i,k}F_{i,k} + \sum\nolimits_{j=1(j\ne i)}^{M} B_{j,k}F_{j,t_j}\right) - P_{i,k} + Q \\ &\left. + F_{i,k}^T R_k F_{i,k} + \sum\nolimits_{j=1(j\ne i)}^{M} F_{j,t_j}^T R_j F_{j,t_j}\right]x_{k+l|k}\le 0\end{aligned} \tag{19}$$

Then, condition (19) will be satisfied if matrix inequalities (13) hold. This ends the proof. □

A multi-step robust MPC scheme for polytopic uncertain multi-input systems is presented as:

$$\min_{\gamma_{i,k},P_{i,k}>0,F_{i,k}} \gamma_{i,k}\quad s.t.\ \ x_{k|k}^T P_{i,k} x_{k|k}\le\gamma_{i,k},(4)\text{-}(7),(13) \tag{20}$$

It is noted that the presence of uncertainties and constraints makes the optimisation problem (20) intractable. In what follows, (20) will be cast into an LMI optimization problem to solve the state feedback law.

Letting $W_{i,k}=\gamma_{i,k}P_{i,k}^{-1}$, $Y_{i,k}=F_{i,k}X_k$, $x_{k|k}^T P_{i,k} x_{k|k} \le \gamma_{i,k}$ can be represented as:

$$x_{k|k}^T \frac{P_{i,k}}{\gamma_{i,k}} x_{k|k} \le 1 \quad \Leftarrow \quad \begin{bmatrix} 1 & x_{k|k}^T \\ x_{k|k} & W_{i,k} \end{bmatrix} \ge 0 \tag{21}$$

Using the Schur complement, (13) can be represented as:

$$\begin{bmatrix} P_{i,k} & * & * & * \\ A_{k,\alpha}+\sum_{j=1(j\neq i)}^{M} B_{j,k,\alpha}F_{j,t}+B_{i,k,\alpha}F_{i,k} & P_{i,k}^{-1} & * & * \\ \left(Q+\sum_{j=1(j\neq i)}^{M} F_{j,t}^T R_j F_{j,t}\right)^{\frac{1}{2}} & 0 & I & * \\ R_i^{\frac{1}{2}} F_{i,k} & 0 & 0 & I \end{bmatrix} \ge 0,\ \alpha=1,\cdots,L \tag{22}$$

Pre- and post-multiplying (22) with $\mathrm{diag}\{P_{i,k}^{-1},I,I,I\}$ results in:

$$\begin{bmatrix} P_{i,k}^{-1} & * & * & * \\ \left(A_{k,\alpha}+\sum_{j=1(j\neq i)}^{M} B_{j,k,\alpha}F_{j,t}+B_{i,k,\alpha}F_{i,k}\right)P_{i,k}^{-1} & P_{i,k}^{-1} & * & * \\ \left(Q+\sum_{j=1(j\neq i)}^{M} F_{j,t}^T R_j F_{j,t}\right)^{\frac{1}{2}} P_{i,k}^{-1} & 0 & I & * \\ R_i^{\frac{1}{2}} F_{i,k} P_{i,k}^{-1} & 0 & 0 & I \end{bmatrix} \ge 0,\quad \alpha=1,\cdots,L \tag{23}$$

Then, (23) can be simplified by multiplying with $\gamma_{i,k}$:

$$\begin{bmatrix} W_{i,k} & * & * & * \\ \left(A_{k,\alpha}+\sum_{j=1(j\neq i)}^{M} B_{j,k,\alpha}F_{j,t}\right)W_{i,k}+B_{i,k,\alpha}Y_{i,k} & W_{i,k} & * & * \\ \left(Q+\sum_{j=1(j\neq i)}^{M} F_{j,t}^T R_j F_{j,t}\right)^{\frac{1}{2}} W_{i,k} & 0 & \gamma_{i,k} & * \\ R_i^{\frac{1}{2}} Y_{i,k} & 0 & 0 & \gamma_{i,k} \end{bmatrix} \ge 0,\ \alpha=1,\cdots,L \tag{24}$$

To handle the constraints, the following lemma is obtained.

Lemma 2: If there exist symmetric matrices $W_{i,k}$, Z_i, and X_i; any matrix $Y_{i,k}$; and a scalar $\gamma_{i,k}$ satisfying (21), (24) and following LMIs

$$\begin{bmatrix} Z_i & Y_{i,k} \\ Y_{i,k}^T & W_{i,k} \end{bmatrix} \ge 0,\ Z_{i,jj} \le \bar{u}_{i,j}^2, \left(j=1,\cdots,m_i\right) \tag{25}$$

$$\begin{bmatrix} X_i & \bar{\psi} W_{i,k} \\ * & W_{i,k} \end{bmatrix} \ge 0,\ X_{i,jj} \le \bar{x}_j^2, \left(j=1,\cdots,n\right) \tag{26}$$

where $\bar{\psi}=\begin{bmatrix}\psi_1^T & \cdots & \psi_n^T\end{bmatrix}^T$ and $Z_{i,jj}$, $X_{i,jj}$ are the jth diagonal element of Z_i, X_i, respectively, then the input and state constraints (6) and (7) can be satisfied.

Proof: For the input constraint (6), we have:

$$\begin{aligned} \left|u_{i,j,k+l|k}\right|^2 &= \left|\alpha_{i,j}Y_{i,k}W_{i,k}^{-1}x_{k+l|k}\right|^2 \\ &\le \left\|\alpha_{i,j}Y_{i,k}W_{i,k}^{-0.5}\right\|_2^2 \left\|W_{i,k}^{-0.5}x_{k+l|k}\right\|^2 \\ &\le \left\|\alpha_{i,j}Y_{i,k}W_{i,k}^{-0.5}\right\|_2^2 \\ &\le \alpha_{i,j}Y_{i,k}W_{i,k}^{-1}Y_{i,k}^T \\ &\le \bar{u}_{i,j}^2,\quad \left(j=1,\cdots,m_i\right) \end{aligned} \tag{27}$$

where $\alpha_{i,j}$ is the jth row of m_i order identity matrix. By applying the Schur complement, the input constraint (6) is ensured by satisfying (25).

Then, we consider the state constraint (7), we have:

$$\begin{aligned} \left|\psi_j x_{k+l|k}\right|^2 &= \left|\psi_j W_{i,k}^{0.5}W_{i,k}^{-0.5}x_{k+l|k}\right|^2 \\ &\le \left\|\psi_j W_{i,k}^{0.5}\right\|_2^2 \left\|W_{i,k}^{-0.5}x_{k+l|k}\right\|^2 \\ &\le \left\|\psi_j W_{i,k}^{0.5}\right\|_2^2 \\ &\le \bar{x}_j^2,\ \left(j=1,\cdots,n\right) \end{aligned} \tag{28}$$

Similarly, by applying the Schur complement, the state constraint (7) is ensured by satisfying (26). This completes the proof. □

The multi-step robust MPC scheme (20) for polytopic uncertain multi-input systems can be transformed into following optimization problem with LMI constraints:

$$\min_{\gamma_{i,k},W_{i,k}>0,\,Z_i,X_i,Y_{i,k}} \gamma_{i,k} \qquad s.t.\ \ (21),(24)\text{-}(26) \tag{29}$$

Theorem 1: At time interval k, let $x_k = x_{k|k}$ be the state of the uncertain system (4). The state feedback matrix $F_{i,k}$ can be obtained by solving $F_{i,k}=Y_{i,k}W_{i,k}^{-1}$, where $Y_{i,k}$ and $W_{i,k}$ are solutions to (29).

Remark 2: The dimensions of (24) are $\left(3n+m_i\right)\times\left(3n+m_i\right)$ for multi-step robust MPC and $\left(3n+m\right)\times\left(3n+m\right)$ for conventional robust MPC. The dimensions of (25) are $m_i\times m_i$ for multi-step robust MPC and $m\times m$ for conventional MPC. There are $\left(m_i\left(n+1\right)+0.5n(n+3)+1\right)$ variables in solving multi-step robust MPC and $\left(m\left(n+1\right)+0.5n(n+3)+1\right)$ variables for conventional robust MPC. It is noted that $m_i<m$. By comparing with the conventional robust MPC, the multi-step robust MPC has advantages both on the dimensions of the LMIs and the numbers of the optimized variables.

At each time interval, the online optimized state feedback law can be obtained by solving (29). Only subset of control feedback laws are designed sequentially. The solving procedure of the proposed multi-step robust MPC is summarized in Algorithm 1.

Algorithm 1: multi-step robust MPC

Step 1: At time interval k, find and store $F_{\zeta,k}$ with $\zeta \in [1,M]$ by solving (29). Apply the first inputs $u_{\zeta,k|k} = F_{\zeta,k}x_{k|k}$ to the process;

Step 2: Set $k = k+1$. Obtain i by solving (3) to decide the input to be optimized;

Step 3: Compute t by solving (10). Find and store $F_{i,k}$ by solving (29);

Step 4: Apply the optimized input $u_{i,k|k} = F_{i,k}x_{k|k}$, the stored inputs $u_{j,k|t} = F_{j,t}x_{k|k}$ with $j \in [1,M]$, $j \neq i$ into the plant;

Step 5: Go to step 2.

The stability of the closed-loop system with multi-step robust MPC algorithm is studied in the following theorem.

Theorem 2: At time interval k, the state feedback solution $F_{i,k} = Y_{i,k}X_{i,k}^{-1}$, obtained from Algorithm 2, asymptotically stabilizes the closed-loop system (11), where A_k, $B_{i,k}$, $B_{j,k}$ belong to the polytopic description defined in (5).

Proof: This proof is based on the fact that the state feedback law obtained at time interval k is feasible for the subsequent time intervals. The similar proof can be found in (Kothare, et al., 1996; Ling, et al., 2012).

For the uncertain systems (4)-(7), assume that there is a feasible solution at initial time interval k for the current measurement. Suppose that $P_{i,k}$ and $P_{i+1,k+1}$ are optimal for (20) at time interval k and $k+1$, respectively. We have:

$$x_{k+1|k+1}^T P_{i+1,k+1} x_{k+1|k+1} \le x_{k+1|k+1}^T P_{i,k} x_{k+1|k+1} \tag{30}$$

This is because $P_{i+1,k+1}$ is optimal, whereas $P_{i,k}$ is only feasible at time interval $k+1$. We have $x_{k+1|k}^T P_{i,k} x_{k+1|k} \le x_{k|k}^T P_{i,k} x_{k|k}$ for any $\left[A_k, B_{1,k}, \cdots, B_{i,k}, \cdots, B_{M,k}\right] \in \Omega$ if $u_{k+l|k} = F_k x_{k+l|k}$. Since the measured state $x_{k+1|k+1} = x_{k+1}$ equals (4) for some $\left[A_k, B_{1,k}, \cdots, B_{i,k}, \cdots, B_{M,k}\right] \in \Omega$, we have:

$$x_{k+1|k+1}^T P_{i+1,k+1} x_{k+1|k+1} \le x_{k|k}^T P_{i,k} x_{k|k} \tag{31}$$

Thus, $V_{i,k|k} = x_{k|k}^T P_{i,k} x_{k|k}$ is a strictly decreasing Lyapunov function for the closed-loop system (11), which implies that $x_k \to 0$ as $k \to \infty$. This ends the proof. □

4. NUMERICAL EXAMPLE

In this section, the effectiveness of the multi-step robust MPC is studied and compared with that of the conventional MPC. We consider a process with three inputs and three outputs. The real process model is assumed to vary between the following two models:

$$G_1(s) = \begin{bmatrix} \dfrac{4.05}{s+6} & \dfrac{1.77}{2s+1} & \dfrac{5.88}{2s+3} \\ \dfrac{5.39}{s+4} & \dfrac{5.72}{s+2} & \dfrac{6.90}{2s+11} \\ \dfrac{4.30}{3s+8} & \dfrac{4.42}{2s+5} & \dfrac{7.20}{s+7} \end{bmatrix} \tag{32}$$

$$G_2(s) = 0.4G_1(s)$$

State-space models are obtained on a canonical realization of (32) with sampling time of 0.3 seconds and not shown for brevity. The weighting matrices are $Q = \mathrm{diag}\{5,5,5\}$ and $R_1 = R_2 = R_3 = 1$. We consider the set-point regulation and disturbance rejection problem in this simulation.

The set-point regulation problem is considered by regulating $y_1 = 0.5$, $y_2 = 1$, and $y_3 = 0.5$. The disturbance rejection problem is considered by setting $y_1 = 2$, $y_2 = 1$, $y_3 = 3$, and introducing disturbances at $k = 15$. The disturbance is given as $0.5\sin(k)$ and removed after three seconds to avoid the steady-state offset.

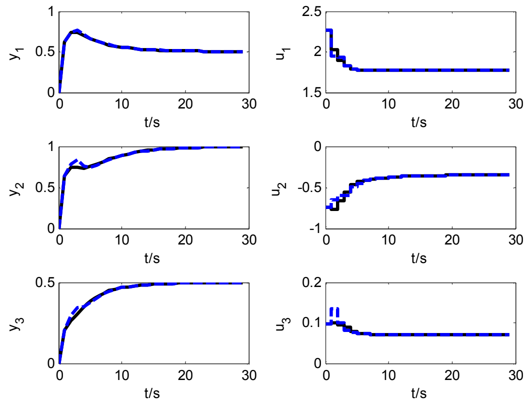

Fig. 2. Dynamic responses for tracking of the conventional MPC (solid line) and the multi-step MPC (dashed line)

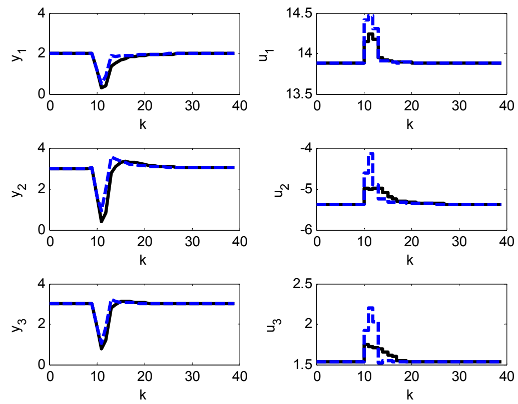

Fig. 3. Dynamic responses for the conventional MPC (solid line) and the multi-step MPC (dashed line)

Fig. 2 shows the dynamic responses of system outputs and computed inputs with the multi-step robust MPC and the conventional robust MPC. The multi-step robust MPC can track the set-point very similar to that of the conventional MPC. Fig. 3 shows that the multi-step robust MPC algorithm performs very similar overshoot and rise time to the conventional MPC with disturbance. Fig. 4 shows the computation time costs with the multi-step robust MPC and the conventional MPC at different simulation horizon. It is noted that all the simulations are carried out on a Core i5 CPU 2.27 GHz computer. The computation time at initial time interval is much longer than the subsequent time interval due to the additional time cost, *i.e.,* initialization time for MATLAB LMI Toolbox. The result illustrates that the multi-step MPC is faster than the conventional MPC.

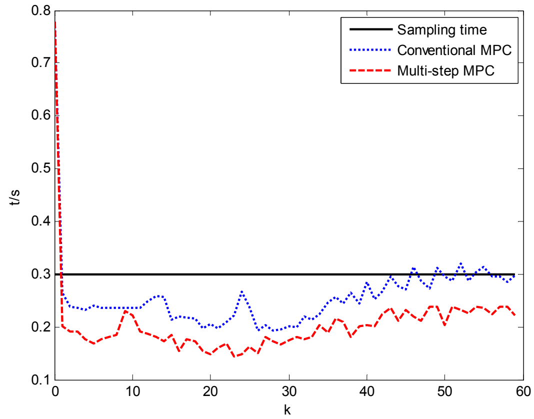

Fig. 4. Computation time for the multi-step robust MPC and the conventional robust MPC

5. CONCLUSIONS

In this paper, we present a multi-step robust MPC to reduce computational complexity. The proposed multi-step robust MPC scheme results in reducing computation time in controller design. The open problems are the investigations of the sub-optimality and conservatism.

ACKNOWLEDGEMENTS

This work was supported by National Natural Science Foundation of China (No. 61174059, 61233004), National 973 Program of China (No. 2013CB035406), Research Project of Shanghai Municipal Economic and Informatization Commission (ZB-ZBYZ-01112634, 12GA-31).

REFERENCES

Alessio, A., D., Barcelli and A. Bemporad (2011). Decentralized model predictive control of dynamically coupled linear systems. *Journal of Process Control,* **21**, 705-714.

Canale, M., L. Fagiano and M. Signorile (2012). Design of robust predictive control laws using set membership identified models. *Asian Journal of Control,* **15**, 1-9.

Giselsson, P., M. D. Doan, T. Keviczky, B. D. Schutter and A. Rantzer (2013). Accelerated gradient methods and dual decomposition in distributed model predictive control. *Automatica,* **49**, 829-833.

Huang, H., D. Li, Z. Lin and Y. Xi (2011). An improved robust model predictive control design in the presence of actuator saturation. *Automatica,* **47**, 861-864.

Huang, H., D. Li and Y. Xi (2013). Mixed H 2/H∞ robust model predictive control with saturated inputs. *International Journal of Systems Science*, 1-11.

Kothare, M. V., V. Balakrishnan and M. Morari (1996). Robust constrained model predictive control using linear matrix inequalities. *Automatica,* **32**, 1361-1379.

Li, S., Y. Zhang and Q. Zhu (2005). Nash-optimization enhanced distributed model predictive control applied to the Shell benchmark problem. *Information Sciences,* **170**, 329-349.

Ling, K. V., J. Maciejowski, A. Richards and B. F. Wu (2012). Multiplexed model predictive control. *Automatica,* **48**, 396-401.

Maestre, J., D. Muñoz de la Peña, E. Camacho and T. Alamo (2011). Distributed model predictive control based on agent negotiation. *Journal of Process Control,* **21**, 685-697.

Mayne, D. Q., J. B. Rawlings, C. V. Rao and P. O. M. Scokaert (2000). Constrained model predictive control: Stability and optimality. *Automatica,* **36**, 789-814.

Qin, S. J. and T. A. Badgwell (2003). A survey of industrial model predictive control technology. *Control engineering practice,* **11**, 733-764.

Richter, H., A. V. Singaraju and J. S. Litt (2008). Multiplexed predictive control of a large commercial turbofan engine. *Journal of Guidance, Control, and Dynamics,* **31**, 273-281.

Rivero, S., M. Farina and G. Ferrari-Trecate (2013). Plug-and-Play Decentralized Model Predictive Control for Linear Systems, *IEEE Transactions on Automatic Control,* **58**, 2608-2614.

Sandell Jr, N., P. Varaiya, M. Athans and M. Safonov (1978). Survey of decentralized control methods for large scale systems. *IEEE Transactions on Automatic Control,* **23**, 108-128.

Scheu, H. and W. Marquardt (2011). Sensitivity-based coordination in distributed model predictive control. *Journal of Process Control,* **21**, 715-728.

Siva, E., J. Maciejowski and K.-V. Ling (2010). Robust multiplexed model predictive control for agent-based conflict resolution. In *Preceedings of 49th IEEE Conference on Decision and Control,* 4335-4340.

Yun, S. W., Y. J. Choi and P. G. Park (2010). State - feedback disturbance attenuation for polytopic LPV systems with input saturation. *International Journal of Robust and Nonlinear Control,* **20**, 899-922.

Zhang, L., J. Wang and C. Li (2013). Distributed model predictive control for polytopic uncertain systems subject to actuator saturation. *Journal of Process Control,* **23**, 1075-1089.

Zheng, Y., S. Li and N. Li (2011). Distributed model predictive control over network information exchange for large-scale systems. *Control engineering practice,* **19**, 757-769.