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# Train Scheduling Networks under Time Duration Uncertainty

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**Abstract:** In this paper, we consider a railway traffic scheduling problem. We aim to find a schedule for a train scheduling networks where time duration uncertainties are considered. This problem was intensively studied with mixed linear models where trains moving duration are deterministic. In this paper, we formulate the problem as a classical one with scenario-based stochastic programming taking expected values as objective functions. Then, a new criterion is proposed to quantify scheduling robustness in the face of uncertainty. Besides, additional constraints were introduced to make the model feasible when unexpected event occurs during scheduling execution.

*Keywords:* Stochastic programming, optimization, robustness, railway traffic scheduling, time duration uncertainty.

## 1. INTRODUCTION

This paper deals with the train scheduling networks problems. It consists in finding the arrival and the departure times of the lines at certain stages of the network. Depending on required objectives, these stages can be referred to public station and/or switches.

Since 1871s, and more specifically since the first train schedule conference in Germany, train scheduling planning problems have been widely studied (Carey and Lockwood (1995); Cordeau et al. (1998)) and several programming model have been proposed despite their NP-had characteristic (Assad, (1980); Nachtigall and Voget (1997)).

This category of scheduling problems can be shared into two classes, as described as follows:

- *Static or Predictive problem*

It consists firstly on allocating resources to all trains in all routes. Then, the train sequencing entrains the pre-specification of the arrival and departure order of trains at stations. Finally, a time-table is then resulted. This class aims on the minimization of the makespan or the cycle time or on the maximization of the traffic frequency (Harrod (2011); Higgins et al, (1996); Kraay and Harker (1995)).

- *Dynamic or Reactive problem*

It involves when the train planned schedule cannot be respected due to a disturbance handling activity. In this case a new timetable should be found while all the problem constraints are respected. Generally, the objective function consists on the minimization of train delays (Narayanaswami and Rangaraj (2013)).

The focus of this paper is to present a new model which can be useful for the two problem classes simultaneously. The originality's of this approach consist on the following:

- Due to train travelling duration's uncertainties, we present a handling scenarios approach. Where additional criterion is considered to find the most robust schedule.
- We define new variables and constraints to control the train speed and the train waiting time on stations in order to remove a disruption, if any.

Most previous models are handling either predictive train scheduling problem or reactive one. If unexpected event happens, a first schedule solution can be determined using this first problem class models. This can be reached by instantiating known decision variables. Nevertheless, this solution is very simple and cannot, at any way, guarantee the solution performance.
Besides, weather conditions can require on trains to reduce their speeds on some tracks. In fact, a wheels sliding or a wheels skating can happen due to the snow or to the tree leaves on the rail in autumn generally. So, this could lead to several perturbations in arrival and departure times of the timetabling passenger trains. This problem has become recurrent in Europe at the approach of winter holidays and Christmas while a large number of people take the train to travel. Which make rail transport less competitive compared to other means of travel (air and ground transportation).

Following to these introductory remarks, Section 2 is devoted to the problem statement. Section 3 discusses the mathematical problem formulation. The problem solving methodology as well as the new model extensions are presented in Section 4 and 5, respectively.

## 2. PROBLEM STATEMENT

In this study, we consider the single track, bi-directional railway traffic. Trains have to travel in two directions: from right to left (*RtoL*) and from left to right (*LtoR*) (called also nominal direction in literature (Abbas-Turki et al, 2012)). Each right to left direction train is travelling, as soon as possible from the starting station (*station 1*), then it is visiting successively *m-1* stations, numbered from *1* to *m-1*, before arriving to the end station (*station m*). While each left to right direction train has to start by the final station, and then it is visiting the stations: *m-1*, *m-2*, …and *2* successively, before reaching the starting station. We call single track (or segment) the slice of the line confined between any two stations. In general, on each train station several tracks (called block) are available to allow overtakes and crossings.

One of the main specificities of such a system is that the average train travelling durations values are known and any delay can make a network disruption. Moreover, tracks are the most critical resource of such lines. Besides, there are no multiple-tracks between stations and each station can receive more than one train at the same time.

Fig. 1 shows an example of a line layout with single track and bi-directional train movements.

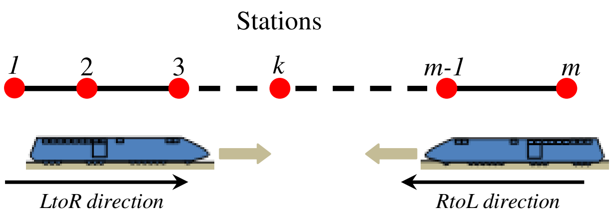


**Fig. 1.** Example of a single track, bidirectional railway

This problem can be considered as a job-shop scheduling problem with very specific constraints, where each segment is considered as a machine and each train as a job.

The constraints we consider here are the following ones:

(C1) Each track can receive simultaneously either RtoL trains or LtoR trains.

(C2) In each station, trains must remain at least a lower duration and at most an upper duration. These durations can vary from one station to another due to station passengers' frequency.

(C3) Between two successive trains moving on the same direction, a minimum safety time duration is required.

(C4) In each meeting station, minimum meeting time duration has to be ensured between the arrival and the departure of trains moving in different direction. Definitely, passages have to be allowed to change from trains.

The studied problem requires two distinct but dependent decisions to be made: (1) scheduling decision-sequence, in which trains have to move (priority to move), and (2) station waiting decision (real waiting time on stations). The strong dependence between these two decisions makes the problem very hard to model. Yet, getting motivated by previous researches using mixed integer linear programming methods, we elaborate a new model; which can solve this train scheduling problem to optimality. In this programming model, we are looking for optimizing the simultaneous travelling durations of several trains moving in different directions through a single line. Besides, it can be also useful to find a solution when a disturbance occurs.

In general, the problem solution is presented using graphic timetable as shown in Fig. 2.

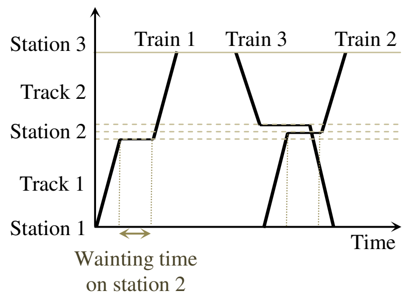


**Fig. 2.** Graphic timetable

This figure illustrates an example of a single track line layout with three stations and three trains (two moving from left to right and one from right to left). Slash lines represents the train moves while horizontal lines show the waiting times on stations. In this example, only one train meeting is carried out on station 2 between train 3 and train 2. After the passage of the train 1, train 3 reaches station 2, wait there until the track 1 becomes available and the minimum meeting time takes in. Then, it leaves station 2 to station 1. While train 2 spends the lower bound of its required waiting time on station 2 and go to station 3.

In the following section, we formally describe the problem and formulate it as a mixed integer linear programming model.

## 3. MATHEMATICAL FORMULATION

A definition of notations used in this paper is necessary in order to describe the elaborated models. So, let's define the following parameters and variables:

*Problem Parameters*

1. $n$ total number of trains.
2. $n_1$ trains that move in the *LtoR* direction.
3. $n_2$ trains that move in the *RtoL* direction.
4. $m$ total number of stations.

**NOTE 1:**

- Stations are indexed from *1* to *m*.
- To simplify the following notation, we denote by $\vec{i}$ a train moving in *LtoR* direction and by $\overleftarrow{i}$ a train moving in *RtoL* direction.

5. $Uptd_{\vec{i}}^k, Lwtd_{\vec{i}}^k$ — The upper and lower bounds travelling time of a *LtoR* train *i* to run through the track between the stations *k* and *k+1*.

6. $Uptd_{\overleftarrow{i}}^k, Lwtd_{\overleftarrow{i}}^k$ — The upper and lower bounds travelling time of a *RtoL* train *i* to run through the track between the stations *k* and *k-1*.

7. $Upwt_{\vec{i}}^k, Lwwt_{\vec{i}}^k$ — The upper and lower bounds waiting time of *LtoR* train *i* on station *k*.

8. $Upwt_{\overleftarrow{i}}^k, Lwwt_{\overleftarrow{i}}^k$ — The upper and lower bounds waiting duration of *RtoL* train *i* on station *k*.

9. $Sfte_{\vec{i},\vec{j}}^k, Sfte_{\overleftarrow{i},\overleftarrow{j}}^k$ — The safety time durations between the arrival of two trains (*i* and *j*) of the same direction to station *k:*

$$Sfte_{\vec{i},\vec{j}}^k = \left|e_{\vec{i}}^k - e_{\vec{j}}^k\right| ; \; Sfte_{\overleftarrow{i},\overleftarrow{j}}^k = \left|e_{\overleftarrow{i}}^k - e_{\overleftarrow{j}}^k\right|$$

10. $Sfts_{\vec{i},\vec{j}}^k, Sfts_{\overleftarrow{i},\overleftarrow{j}}^k$ — The safety time durations between the departure of two trains (*i* and *j*) of the same direction from station *k:*

$$Sfts_{\vec{i},\vec{j}}^k = \left|s_{\vec{i}}^k - s_{\vec{j}}^k\right| ; \; Sfts_{\overleftarrow{i},\overleftarrow{j}}^k = \left|s_{\overleftarrow{i}}^k - s_{\overleftarrow{j}}^k\right|$$

11. $Mmt_{\vec{i},\vec{j}}^k, Mmt_{\overleftarrow{i},\overleftarrow{j}}^k$, $Mmt_{\vec{i},\overleftarrow{j}}^k, Mmt_{\overleftarrow{i},\vec{j}}^k$ — The minimum durations for the meeting of two trains (*i* and *j*) on station *k*.

12. $M$ — Very big number (+∞).

13. $Dd_{\vec{i}}^k, Dd_{\overleftarrow{i}}^k$ — Date when a disturbance occurs in the network for train *i* on or after visiting station *k-1*.

14. $\delta_{\vec{i}}^k, \delta_{\overleftarrow{i}}^k$ — Disturbance duration.

*Problem Decision Variables*

15. $Rtd_{\vec{i}}^k$, $Rtd_{\overleftarrow{i}}^k$ — Real travelling time of a *LtoR* and a *RtoL* train *i* from station *k* to the following one.

16. $Rwt_{\vec{i}}^k$, $Rwt_{\overleftarrow{i}}^k$ — Real waiting time of a *LtoR* and a *RtoL* train *i* on station *k*.

17. $s_{\vec{i}}^k$ — Start moving time of a train *i* from station *k* to station *k+1*.

18. $s_{\overleftarrow{i}}^k$ — Start moving time of a train i from station *k* to station *k-1*.

19. $e_{\vec{i}}^k, e_{\overleftarrow{i}}^k$ — End moving time of a train *i* from station *k*.

20. $Tgs_{\vec{i}}^k$, $Tgs_{\overleftarrow{i}}^k$ — Term gain speed from station *k-1*.

21. $Tgw_{\vec{i}}^k$, $Tgw_{\overleftarrow{i}}^k$ — Term gain waiting on station *k*.

22. $S_{\vec{i},\vec{j}}^k = \begin{cases} 1 & \text{if } (s_{\vec{i}}^k < s_{\vec{j}}^k) \\ 0 & \text{otherwise} \end{cases}$

23. $E_{\vec{i},\vec{j}}^k = \begin{cases} 1 & \text{if } (e_{\vec{i}}^k < e_{\vec{j}}^k) \\ 0 & \text{otherwise} \end{cases}$

24. $S_{\overleftarrow{i},\overleftarrow{j}}^k = \begin{cases} 1 & \text{if } (s_{\overleftarrow{i}}^k < s_{\overleftarrow{j}}^k) \\ 0 & \text{otherwise} \end{cases}$

25. $E_{\overleftarrow{i},\overleftarrow{j}}^k = \begin{cases} 1 & \text{if } (e_{\overleftarrow{i}}^k < e_{\overleftarrow{j}}^k) \\ 0 & \text{otherwise} \end{cases}$

26. $S_{\vec{i},\overleftarrow{j}}^k = \begin{cases} 1 & \text{if } (s_{\vec{i}}^k < s_{\overleftarrow{j}}^k) \\ 0 & \text{otherwise} \end{cases}$

27. $E_{\vec{i},\overleftarrow{j}}^k = \begin{cases} 1 & \text{if } (e_{\vec{i}}^k < e_{\overleftarrow{j}}^k) \\ 0 & \text{otherwise} \end{cases}$

**NOTE 2:** - We assume that trains travel on a single track line layout in bi-directional movements. In addition, all trains have to pass through all stations. Moreover, re-routing is not allowed and safeties as well as lower time durations have to be respected.
- It is important to mention that the following mathematical formulation is a train movement's model: where decision variables define the starting dates of all the train moves from each station *k* and their end dates ( $s_{\vec{i}}^k, s_{\overleftarrow{j}}^k, e_{\vec{i}}^{k+1}$ and $e_{\overleftarrow{j}}^{k-1}$ ).

**NOTE 3:** We denote by $\tilde{x}$ the new decision variable value of $x$ when a regulation is performed after a disturbance. In other words $\tilde{x}$ is the new value of $x$ after a new scheduling.

*Problem Formulation*

$$Minimize: \; C_{max} = \sum_{i \in [\![1,n_1]\!]} e_{\vec{i}}^m + \sum_{j \in [\![1,n_2]\!]} e_{\overleftarrow{j}}^1 \tag{1}$$

$\forall k \in [\![1,m]\!], \vec{i} \in [\![1,n_1]\!] \; and \; \overleftarrow{j} \in [\![1,n_2]\!]$

$$s_{\vec{i}}^{k-1} + Rtd_{\vec{i}}^k = e_{\vec{i}}^k \tag{2}$$

$$s_{\overleftarrow{j}}^{k+1} + Rtd_{\overleftarrow{j}}^k = e_{\overleftarrow{j}}^k \tag{3}$$

$$Lwtd_{\vec{i}}^k \le Rtd_{\vec{i}}^k \le Uptd_{\vec{i}}^k \tag{4}$$

$$Lwtd_{\overleftarrow{j}}^k \le Rtd_{\overleftarrow{j}}^k \le Uptd_{\overleftarrow{j}}^k \tag{5}$$

$$e_{\vec{i}}^k + Rwt_{\vec{i}}^k \le s_{\vec{i}}^k \tag{6}$$

$$Lwwt_{\vec{i}}^k \le Rwt_{\vec{i}}^k \le Upwt_{\vec{i}}^k \tag{7}$$

$$Lwwt_{\overleftarrow{j}}^k \le Rwt_{\overleftarrow{j}}^k \le Upwt_{\overleftarrow{j}}^k \tag{8}$$

$$s_{\vec{j}}^k + Sfts_{\vec{i},\vec{j}}^k \le s_{\vec{i}}^k + S_{\vec{i},\vec{j}}^k.M \tag{9}$$

$$s_{\overrightarrow{i}}^{k} + Sfts_{\overrightarrow{i},\overrightarrow{j}}^{k} \leq s_{\overrightarrow{j}}^{k} + (1 - S_{\overrightarrow{i},\overrightarrow{j}}^{k}).M \tag{10}$$

$$e_{\overrightarrow{j}}^{k} + Sfte_{\overrightarrow{i},\overrightarrow{j}}^{k} \leq e_{\overrightarrow{i}}^{k} + E_{\overrightarrow{i},\overrightarrow{j}}^{k}.M \tag{11}$$

$$e_{\overrightarrow{i}}^{k} + Sfte_{\overrightarrow{i},\overrightarrow{j}}^{k} \leq e_{\overrightarrow{j}}^{k} + (1 - E_{\overrightarrow{i},\overrightarrow{j}}^{k}).M \tag{12}$$

$$s_{\overleftarrow{j}}^{k} + Sfts_{\overleftarrow{i},\overleftarrow{j}}^{k} \leq s_{\overleftarrow{i}}^{k} + S_{\overleftarrow{i},\overleftarrow{j}}^{k}.M \tag{13}$$

$$s_{\overleftarrow{i}}^{k} + Sfts_{\overleftarrow{i},\overleftarrow{j}}^{k} \leq s_{\overleftarrow{j}}^{k} + (1 - S_{\overleftarrow{i},\overleftarrow{j}}^{k}).M \tag{14}$$

$$e_{\overleftarrow{j}}^{k} + Sfte_{\overleftarrow{i},\overleftarrow{j}}^{k} \leq e_{\overleftarrow{i}}^{k-1} + E_{\overleftarrow{i},\overleftarrow{j}}^{k}.M \tag{15}$$

$$e_{\overleftarrow{i}}^{k} + Sfte_{\overleftarrow{i},\overleftarrow{j}}^{k} \leq e_{\overleftarrow{j}}^{k-1} + (1 - E_{\overleftarrow{i},\overleftarrow{j}}^{k}).M \tag{16}$$

$$s_{\overleftarrow{j}}^{k} + Mmt_{\overrightarrow{i},\overleftarrow{j}}^{k} \leq e_{\overrightarrow{i}}^{k} + S_{\overrightarrow{i},\overleftarrow{j}}^{k}.M \tag{17}$$

$$e_{\overrightarrow{i}}^{k} + Mmt_{\overrightarrow{i},\overleftarrow{j}}^{k} \leq s_{\overleftarrow{j}}^{k} + (1 - S_{\overrightarrow{i},\overleftarrow{j}}^{k}).M \tag{18}$$

$$e_{\overleftarrow{j}}^{k} + Mmt_{\overrightarrow{i},\overleftarrow{j}}^{k} \leq s_{\overrightarrow{i}}^{k} + E_{\overrightarrow{i},\overleftarrow{j}}^{k}.M \tag{19}$$

$$s_{\overrightarrow{i}}^{k} + Mmt_{\overrightarrow{i},\overleftarrow{j}}^{k} \leq e_{\overleftarrow{j}}^{k} + (1 - E_{\overrightarrow{i},\overleftarrow{j}}^{k}).M \tag{20}$$

$$S_{\overrightarrow{i},\overrightarrow{j}}^{k} = E_{\overrightarrow{i},\overrightarrow{j}}^{k} \tag{21}$$

$$S_{\overleftarrow{i},\overleftarrow{j}}^{k} = E_{\overleftarrow{i},\overleftarrow{j}}^{k} \tag{22}$$

$$S_{\overrightarrow{i},\overleftarrow{j}}^{k} = E_{\overrightarrow{i},\overleftarrow{j}}^{k} \tag{23}$$

$$e_{\overrightarrow{i}}^{m} = s_{\overrightarrow{i}}^{m} \tag{24}$$

$$s_{\overleftarrow{j}}^{1} = e_{\overleftarrow{j}}^{1} \tag{25}$$

$$S_{\overrightarrow{i},\overrightarrow{j}}^{k}; E_{\overrightarrow{i},\overrightarrow{j}}^{k}; S_{\overleftarrow{i},\overleftarrow{j}}^{k}; E_{\overleftarrow{i},\overleftarrow{j}}^{k}; S_{\overrightarrow{i},\overleftarrow{j}}^{k}; E_{\overrightarrow{i},\overleftarrow{j}}^{k} \in [\![0,1]\!] \tag{26}$$

In this model, the objective function consists in minimizing the makespan $C_{max}$. This variable, as given here, can be defined as the total time needed for all trains to reach their terminals. Constraints (2)-(3) guarantee the fact that: before arriving to the destination station each train has to spend its required travelling time (i.e. the most frequent duration). This duration must be confined within lower and upper bounds as traduced by constraints (4)-(5). Constraint (6) defines the stations waiting dates that must be respected. These durations have to be bounded, as defined by constraints (7)-(8). Before starting to move, each train must be sure that the minimum safety time duration between him and the previous one using the same track is maintained. This statement is translated by constraints (9)-(12) for *LtoR* trains and by constraints (13)-(16) for *RtoL* trains. While constraints (17)-(20) are used to ensure the minimum required time duration at a station between the arrival and the departure of two trains moving in different directions. Constraints (21)-(23) define the precedence rule: if a train $i$ leaves first a station $k$, it must reach first the destination station (before train $i+1$). Consistency constraints are given by (24) and (25), while binary decision variables are defined by (26).

*Problem Complexity*

The complexity of this problem is related to the number of trains, directions and stations (or/and tracks). For this proposed scheduling model, there are a total of $m(27n_1n_2 + 4n_1 + 4n_2) + n_1 + n_2$ constraints. Besides, there are four types of decisions variables: a total of $4m(n_1 + n_2) + 1$ integer variables and $6mn_1n_2$ binary variables to find.

*Problem Formulation limits*

This model can be used in general to find a timetabling for a single track line layout, with bi-directional train movements problem. Nevertheless, due to some unexpected events, train traffic can be disturbed. Consequently, a new, quick and robust schedule solution should be found.

For this aim, we propose: firstly, a new solving methodology for the problem on hand, using metrics that takes into account the scheduling characteristics under train transportation time uncertainty and we illustrate it by an example. Secondly, an evolution of the previous model is performed to make it able to find efficient schedule solution, when an unforeseen event happens. This aim can be achieved using two flexibilities provided by such problems: speed train control and train waiting time control. These studies will be presented consecutively, in the following sections.

## 4. PROBLEM SOLVING METHODOLOGY

Getting motivated by such transportation scheduling problems and previous robustness studies in optimization literature and more precisely in job-shop scheduling problems (Harding and Floudas (1997); Vin and Ierapetritou (2001)), we propose a scenario based stochastic problems methodology. The robustness, as defined for same problems, measures the resilience of the schedule objective to vary under uncertain parameters and disruptive events. Moreover, as explained above, the weather can have a big impact on the train time travelling durations and consequently on the disturbance network.

Therefore, the train travelling durations can be assumed to vary by *P%* about their nominal values. Realistically, this probability may be calculated for each line track and each period of the year, on the basis of the historic of railways transportation companies like SNCF in France. Hence, the performance of the schedule can be evaluated in terms of the makespan required while random transportation durations are satisfied.

The proposed robustness evaluation methodology can be resumed in the following steps:

**Step 1.** Consider a random of the train transportation times in the freedom degree interval.

**Step 2.** Solve the deterministic problem, for all the considered scenarios, with the following new objective function:

$$\sum_{s \in S} p_s.Cmax_s \tag{27}$$

Where: - $p_s$ is the probability of a scenario $s$.

- $Cmax_s$ is the makespan of a scenario $s$.

**Step 3.** Solve the same problem as in Step 2, using the following function as objective:

$$\sum_{s\in S} p_s.Cmax_s + \sqrt{\sum_{s\in S} p_s.\left(Cmax_s\right)^2 - \sum_{s\in S}\left(p_s.Cmax_s\right)^2} \qquad (28)$$

Step 1 and Step 2 can be considered enough to find the best solution under the considered uncertainties while solution stability is not guaranteed. That's why, on step 3, we use the standard deviation, as a metric for robustness evaluation, to determine the more resilient schedule.

*Illustrative Example*

In this example three trains must travel on a line with 4 stations including the starting and the ending stations. Train 1 and 2 travel on *LtoR* direction, while train 3 travels on *RtoL* direction. The travelling time durations are assumed to vary from *0%* to *25%* about their nominal values. These distributions are given in table 1 (see step 1).

**Table 1. Train travelling time durations distribution**

| Train | Track | Travelling time / Probability | | |
|---|---|---|---|---|
| $\vec{1}$ | 1 | 45 / 0.40 | 46 / 0.60 | - |
| | 2 | 31 / 0.45 | 28 / 0.55 | - |
| | 3 | 50 / 1.00 | - | - |
| $\vec{2}$ | 1 | 47 / 1.00 | - | - |
| | 2 | 27 / 0.35 | 32 / 0.65 | - |
| | 3 | 49 / 0.65 | 52 / 0.35 | - |
| $\overleftarrow{3}$ | 1 | 45 / 1.00 | - | - |
| | 2 | 26 / 0.25 | 30 / 0.30 | 34 / 0.45 |
| | 3 | 40 / 0.80 | 43 / 0.20 | - |

The parameter *Probability* in this table could define the frequency of spending these time durations over a period of time. This probability can vary from one season to another due for example to weather conditions. The number of considered scenarios is 96 (2*2*1*1*2*2*1*3*2). And the remaining problem parameters used for simulations are given as follows: $Lwwt_i^k = 5$; $Sfte_{i,j}^k = 1$; $Sfts_{i,j}^k = 2$; $Mmt_{i,j}^k = 3$.

By adopting (27) as objective function of the model, the makespan of all scenarios varies from 428 to 453 t.u. and the most expected values of the makespan is 445 t.u. This makespan is defined in literature as the deterministic value of the makespan (Vin and Ierapetritou (2001)). It determines the most likely schedule to be followed, where probabilistic travelling time durations are considered.

Besides, 441,5 t.u. is the average makespan over all scenarios.

To quantify robustness, we use the standard deviation metric and we integrate it in our objective function as given in (28). Simulation results show that the most robust schedule is for the makespan of 446 t.u.

Graphic timetable of the makespans 445 t.u. and 446 t.u. are reported on figures 3 and 4, respectively. Moreover, we use direct graph models to characterize the difference between slight and heavy robustness (see Fig. 5).

In figure 5, we model priority moving constraints for the above two schedules. As we can notice, schedule 1 presents more precedence constraints than schedule 2. In fact, in schedule 2, train 1 leaves station 3 before the arrival of train 2. Moreover, the safety time duration for the starting move of train 2 is achieved before it arrives to the departure station which makes it free to move at any time. Thus, schedule 1 is a slight robust schedule whereas schedule 2 is a heavy robust one.

## 5. NEW MODEL EXTENSIONS

Such as several previous approaches (Cordeau et al. (1998)), the proposed linear programming model of sections 3 is incapable to face disruptive events when there is a perturbation on the train network. Thus, in this section we propose to make it able to find new feasible and performing schedules if conflicts occur due to one or several delays. For this aim, we propose to add new constraints to solve these inconsistencies.

Two alternatives solutions are possible:

- The first one consists on the propagation of the delay on all the over train travelling dates. For example, if a disturbance $\delta_i^k$ happens at $Dd$ for a train *i*. All the transportation movements scheduled before $Dd$ are not altered. Despite the others, they have to be delayed by $\delta_i^k$.

This solution can be applied but the optimality is not guaranteed. Moreover, the propagation of the delay can have a snowball effect. Therefore, the train delays and the refund fees of railway companies will increase drastically.

- The second solution consists on applying a regulation policy, where all the starting move dates of all the trains are updated. To get this solution, we apply a train speed control and a train waiting time control strategies. These strategies can be traduced in our linear programming model by the following constraints:

$$\widetilde{s_i^{k-1}} + Rtd_i^k + \delta_i^k - Tgs_i^k = \widetilde{e_i^k} \qquad (29)$$

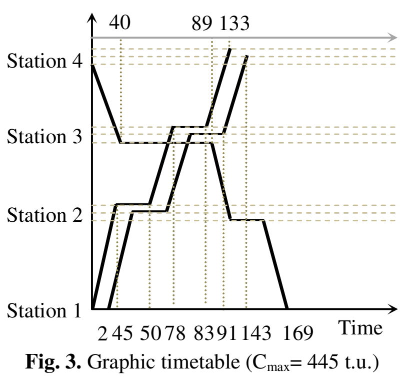


**Fig. 3.** Graphic timetable ($C_{max}$= 445 t.u.)

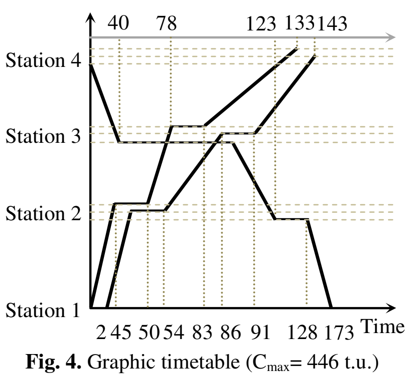


**Fig. 4.** Graphic timetable ($C_{max}$= 446 t.u.)

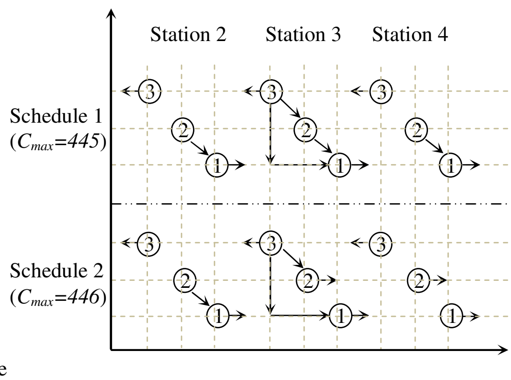


**Fig. 5.** Schedules robustness analysis

$$0 \le Tgs_{\bar{i}}^{k} \le Rtd_{\bar{i}}^{k} - Lwtd_{\bar{i}}^{k} \quad (30)$$

$$\widetilde{s_{\bar{j}}^{k+1}} + Rtd_{\bar{j}}^{k} - Tgs_{\bar{j}}^{k} = \widetilde{e_{\bar{j}}^{k}} \quad (31)$$

$$0 \le Tgs_{\bar{j}}^{k} \le Rtd_{\bar{j}}^{k} - Lwtd_{\bar{j}}^{k} \quad (32)$$

$$\widetilde{e_{\bar{i}}^{k}} + Rwt_{\bar{i}}^{k} - Tgw_{\bar{i}}^{k} \le \widetilde{s_{\bar{i}}^{k}} \quad (33)$$

$$0 \le Tgw_{\bar{i}}^{k} \le Rwt_{\bar{i}}^{k} - Lwwt_{\bar{i}}^{k} \quad (34)$$

Constraints (29), (31) and (33) have to be used instead of constraints (2), (3) and (6). And new time windows constraints ((30), (32) and (34)) have to be added.

Let's detail and analysis these new constraints.
We assume that a disruption happens when a train *i* leaves a station *k-1*, then the arrival date of this train at station *k* will be equal to: $\widetilde{s_{\bar{i}}^{k-1}} + Rtd_{\bar{i}}^{k} + \delta_{\bar{i}}^{k} = \widetilde{e_{\bar{i}}^{k}}$ in spite of equality (2).
Moreover, due to ground topology and thanks to train technology evolution, a first control strategy can be applied. In fact, each train can increase its speed on some segments of its course. Generally, this speed is not deterministic but confined on a speed interval: limited by a lower and an upper bound. So, using this problem specificity, we introduce a new parameter called term gain speed ($Tgs_{\bar{i}}^{k}$) to determine the gain that it can be reached when the train speeds are well controlled. Consequently, equalities (29) and (31) have to be used instead of constraints (2) and (3).
Besides, train speed limits have to be respected. For this aim, we introduce the new constraints (30) and (32).
The second control strategy proposed in our linear programming model is the waiting time control technique. In fact, for this class of scheduling problem, a waiting time is defined on each train station. This time duration can be decreased, to absorb the time disruption, provided that a minimum waiting duration on each station is respected. These statements are traduced by constraints (33) and (34).
To illustrate the effectiveness of our new model, we propose the following example.

*Illustrative Example*
Let's consider the same example as in section 3. However, the following parameters are considered:
$10 \le Rtd_{\bar{i}}^{k} \le 15$; $5 \le Rwt_{\bar{i}}^{k} \le 15$; $Sfte_{\bar{i},\bar{j}}^{k} = 1$; $Sfts_{\bar{i},\bar{j}}^{k} = 2$;
$Mmt_{\bar{i},\bar{j}}^{k} = 3$; $Rtd_{\bar{j}}^{k} = 15$; $10 \le Rtd_{\bar{j}}^{k} \le 15$; $5 \le Rwt_{\bar{j}}^{k} \le 15$;
$Sfte_{\bar{i},\bar{j}}^{k} = 1$; $Sfts_{\bar{i},\bar{j}}^{k} = 2$; $Mmt_{\bar{i},\bar{j}}^{k} = 3$; $0 \le Tgs_{\bar{i}}^{k} \le 2$; $0 \le Tgs_{\bar{j}}^{k} \le 2$
Let's define the following cases study.
*Case 1. No disturbance*. Using the elaborated linear programming model, while disruption and gains are omitted, the optimal solution is obtained for a $C_{max}$ of 147 t.u. as reported on figure 6.
*Case 2. A disturbance of 5 t.u. for train 3 at time Dd=10.* Thanks to the speed and waiting time control strategies, the optimal solution remains the same (see Fig. 6)
*Case 3. Moreover the departure of train 2 is delayed to 25.* As illustrated on figure 6, only the train where disturbance has occurred was delayed. But, we gain about 2,3% on the $C_{max}$, which cannot be neglected, compared to the solution where control strategies are not applied.

In conclusion, on these cases study, we illustrate by a scholar example the effectiveness of the proposed control strategies to find a performing schedule solution when an expected event happens.

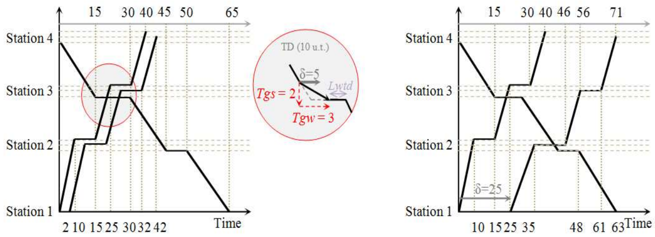


**Fig. 6.** Graphic timetable of cases 1, 2 and 3

## 6. CONCLUSION

In the aim to optimize a train traffic scheduling problem under time travelling uncertainty, we have proposed a scenario based stochastic problems methodology to solve the problem. Moreover we have defined new objective function criteria to quantify the schedule robustness. Besides, we have extended the proposed linear programming model to solve the problem on hand where unexpected event happens. For this aim, we have developed two control strategies based on train speed and waiting time duration on stations.
Future work is to extend the elaborated model to more complex line configurations'.

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