Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

# Novel Evolutionary Game Based Multi-Objective Optimisation for Dynamic Weapon Target Assignment ⋆

**C. Leboucher** * **H-S. Shin** ** **S. Le Ménec** * **A. Tsourdos** ** **A. Kotenkoff** *
**P. Siarry** *** **R. Chelouah** ****

* *MBDA France, 1 av. Raumur, 92350 Le Plessis Robinson, France (e-mail: {cedric.leboucher ; stephane.le-menec ; alexandre.kotenkoff}@mbda-systems.com).*
** *Cranfield University, College Road, Cranfield, Bedfordshire, MK43 0AL, United Kingdom, (e-mail: {h.shin ; a.tsourdos}@cranfield.ac.uk).*
*** *Université Paris-Est Crteil (UPEC), LISSI (EA 3956), 61, av. du Général de Gaulle, 94010, Créteil (France) (e-mail: siarry@u-pec.fr).*
**** *Ecole Internationale des Sciences du Traitement de lInformation (EISTI), avenue du Parc, 95000, Cergy (France) (e-mail: rachid.chelouah@eisti.eu).*

**Abstract:** This paper develops a novel multi-objective optimisation method based on the Evolutionary Game Theory to solve Weapon Target Assignment problems in real-time. The main research question of this study was how to consider multi-objective functions all together and choose a best solution among many possible non-dominant optimal solutions. The key idea is the best solution can be considered as a solution which best survives in other solution spaces. Therefore, the proposed method first obtains individual solutions for each objective function. Then, Evolutionary Game Theory considers each solution as a player and evaluates them in the solution spaces of other players to check how they can survive in those spaces. The main innovation is that, unlike other multi-objective optimisation approaches, the proposed approach not only considers a set of optimal solutions regarding multi-objective functions, but also finds the best optimal solution in terms of the survivability. The stability and the real-time computation of the proposed algorithm is tested on an adapted and constrained Dynamic Weapon Target Assignment problem matching a real military requirement. The performance of the proposed approach is evaluated via numerical simulations.

*Keywords:* Dynamic Weapon Target Assignment, Multi-Objective optimisation, attainability, particle swarm optimisation, evolutionary game theory.

## 1. INTRODUCTION

The Weapon Target Assignment (WTA) problem has been designed to satisfy Command & Control (C2) requirements in military context. The fundamental goal of WTA is to find an allocation plan which assigns available weapons to oncoming threats under a specific engagement scenario. WTA always considers engagement situations in which weapons try to defend an area or assets from an enemy aiming to destroy them. Because of the uniqueness of each situation, this problem must be solved in real-time and evolve according to the engagement situation. The WTA problem is generally solved by an operator taking all the decisions. However, since the modern warfare is becoming more complex and sophisticated, this approach might not be sustainable. In order to resolve the WTA problem in complex engagement scenarios involving different types of oncoming threats in real time, WTA using the power of computation might be inevitable.

WTA can be classified into two groups: Static WTA (SWTA) and Dynamic WTA (DWTA). In both of these problems, the optimality of one solution is based either on the minimisation of the target survival after the engagement or the maximisation of the survivability of the defended assets. Most of the previous work on the WTA was focused on the resolution of the SWTA. Hosein and Athans was among the first to defined a cost function based on the assets Hosein and Athans (1990a). This model was reused in Bisht (2004) and Malhotra and Jain (2001). Later, a second modelling has been proposed by Karasakal in Karasakal (2008), aiming to maximise the probability of suppression of all the oncoming targets. One other variant of the WTA is to take into account a *threatening* value to each target according to its features and the importance of the protected assets. The research of Johansson and Falkman in Johansson and Falkman (2010) proposed a good overview of all the possible modellings taking the value of the defended assets and the threatening index of the incoming target into account. Kwon *et al.* further explored this principle by associating a value to the weapon in Kwon et al. (2007). Thus, most of the

⋆ The authors would like to thank MBDA France.

WTA resolution consider only one objective which is either the survivability of the defended assets or the hit-kill probability Hosein and Athans (1990a); Bisht (2004); Malhotra and Jain (2001); Karasakal (2008); Johansson and Falkman (2010); Kwon et al. (2007); Grant (1993); Lu et al. (2006); Cullenbine (2000); Blodgett et al. (2003); Hosein and Athans (1990b); Hosein et al. (1988); Hosein and Athans (1990c); Sikanen (2008); Wu et al. (2008); Leboucher et al. (2013).

As WTA is designed to support human operators in most of modern C2, WTA should be able to cope with numerous and heterogeneous objectives which helps the operator to make appropriate decision. This implies that it is required to formulate the multi-objective optimal WTA problem in a robust manner to obtain a consistent optimal solution even with small changes in the tactical situation. Moreover, the optimisation process must be capable of being adapted to any operator's requirement regardless the engagement policy. However, there has been few studies on multi-objective optimisation (MOO) for the DWTA problem. At the authors' best knowledges, only Newman *et al.* Newman et al. (2011) applied MOO based on the Pareto approach and Leboucher *et. al* in Leboucher et al. (2013) used an aggregated approach to solve the DWTA problem. Therefore, this paper focuses on developing an innovative and practical MOO method for complex DWTA problems.

Since this paper is focusing on the WTA problem, it is assumed that the engagement policy is given before the mission. Therefore, only *a priori* articulation of preference and no articulation preference cases will be investigated here.

Considering the real-time applicability requirement of MOO, evaluation of the cost at each step of optimisation should be computationally light and fast enough. From the survey undertaken by Marler and Arora in Marler and Arora (2004), among the numerous possible ways to model a multi-objective problem with *a priori* articulation of preferences and no articulation of preferences, the weighted sum appears one of the lightest in terms of computational load and the easiest to formulate, hence the one the most suited for our WTA problem. In all the aforementioned studies related to the weighted sum, despite of the numerous methods to determine these coefficients, it is pointed out that most of approaches in these studies are unable to guarantee an acceptable solution, i.e., might lead a non-optimal solution Messac (1996). Furthermore, Messac in Messac (1996) also stated that the weights should not be constant in order to accurately model the cost function.

This paper investigates a possible multi-objective method to cope with the WTA problem based on the combination of the Evolutionary Game Theory (EGT) with an Hungarian algorithm. Since the weighted sum approach is the one the best suited for our WTA problem, the proposed approach is based on this weighted sum approach. However, in order to dynamically obtain an appropriated set of weight coefficients, the proposed method uses the EGT with the Hungarian algorithm.

The paper is organised as follows: Section 3 details the proposed approach after providing brief backgrounds of the EGT and the Hungarian method in Section 2. The fourth section states the WTA problem. Section 5 presents and discusses the obtained results. This paper ends with conclusions and perspectives of the presented approach.

## 2. DESCRIPTION OF THE PROPOSED METHOD

Before looking into the details of the application of the designed multi-objective method, the first subsection describes the general principle determining the optimal solution in a multi-objective problem.

Based on a real industrial enquiry to design a method capable of coping with heterogeneous and numerous criteria, the described method proposes an alternative modelling. By using the Evolutionary Game Theory as main support to determine a MOO process, the problem is considered as the principle of survivability of individual solutions in a given environment.

The motivation to use the EGT in a multi-objective approach is determined by the fundamental characteristic of this domain that is that the efficiency of one strategy is not evaluated alone, but in the presence of the others. Thus, this was natural to design an approach comparing criteria based on the evolutionary game theory.

Shapley in Shapley (1953) provides a possible answer to the quantification of the importance of the role played by the players in a cooperation game. Nevertheless, to compute the Shapley value in the case of high number of objectives may requires an important computation load because of the combinatorial aspect of the problem.

The proposed method considers each criterion as a natural environment in which an optimal solution (individual) exists. Thus, there are as many optimal individuals as the number of criteria to optimise. Each of these individuals is compared in the environment of another optimal individual and obtains a performance index in this solution space. From these normalised indexes the payoff matrix is built. This payoff matrix simply represents the ability of one individual to survive in other solution spaces. The used dynamic of evolution is the Replicator Equation (RE) that is an Ordinary Differential Equation expressing the difference between the fitness of a strategy and the average fitness in the population. Lower payoffs (agents are minimizers) bring faster reproduction in accordance with Darwinian natural selection process.

$$\dot{p}_i = -p_i(e_i \cdot Ap^T - p \cdot Ap^T) \quad (1)$$

The replicator equation is the most widely used evolutionary dynamics. It was introduced for matrix games in Taylor and Jonker (1978). In this purpose, it enables to determine what is the ratio of each species surviving when all the criteria are considered together. Figure 1 illustrates the used algorithm.

Thus, the presented method combines many advantages:

- heterogeneous criteria can be compared,
- a natural selection by using the EGT process is used in order to avoid the encountered difficulties to select the coefficients in the aggregated approach,
- the measurement of the survivability of the species enables to consider all the criteria and auto-eliminate the less expressive criteria,
- a very high number of objectives can be compared together,
- operator or expert intervention might not be required to decide which solution to choose.

From the mathematical properties of the EGT, the $\varepsilon$-stability of the obtained solution can be guaranteed Maynard-Smith (1982).

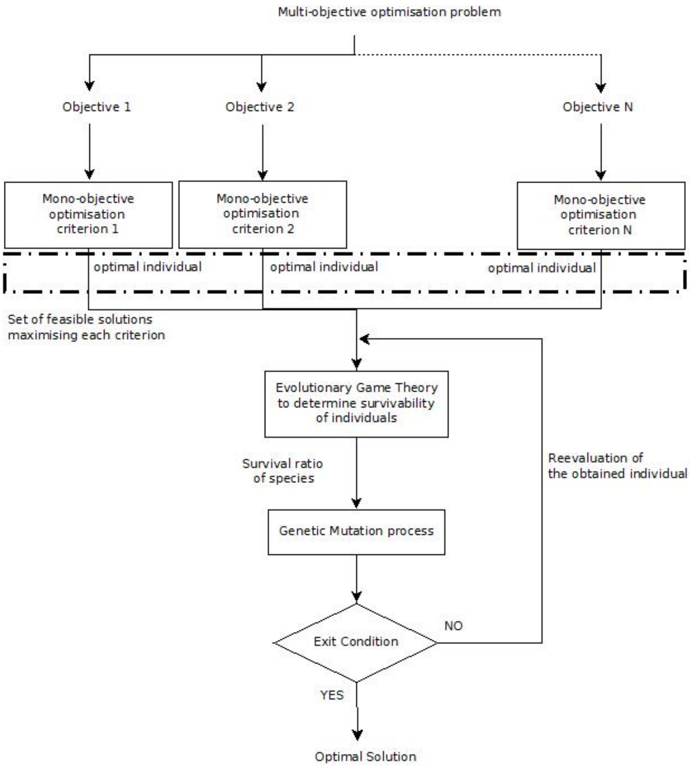


*Figure 1. Description of the proposed multi-objective approach based on the Evolutionary Game Theory.*

## 3. PROBLEM FORMULATION

A common approach to the DWTA problem is based on the capabilities of the defence system to minimise the probability that a target can leak the proposed engagement plan. However, the problem dealt with in this study is slightly different from the classic DWTA. Whereas the classic DWTA is considering a multi-stage approach, the solved problem considers a continuous time where the targets are evolving in the space according to their own objectives and features. The choice of this modelling can be explained by the lack of reality of the previous model.

The weapon system is defending an area from oncoming targets. This area is represented by a circle. All the weapons are disposed over $L$ launchers deployed all around the defended zone. In order to make the problem as general as possible, it is assumed that each weapon has its own velocity and own range. Each target aims one of the $O$ objectives located in the defended zone. To make the approach as realistic as possible, the goals of the targets are not known in advance by the defending side. The trajectories of the targets are designed by Bezier's curves using 4 control points, all randomly drawn on the space, but the last point which is set to one of the defended asset. Thus, this allows the problem considered in this paper to present a high diversity and can test the proposed method in the most trickier cases.

The assignment is computed in real-time in order to validate the reactivity of the studied algorithm when facing real situation. This implies that a timer is set at the beginning of the simulation, and the position of the targets evolves according to this time.

As a "human in the loop" process is required in reality, this paper assumed that an operator is in charge of approving an assignment. The operator intervenes at different stages of the mission. First, the operator provides the objectives of the mission in a mission preparation phase. Then, during the mission, the operator has to approve the engagement plan and proceed the launch of a missile.

**Hungarian algorithm adapted to the WTA:** The assignment of the targets to the weapons is realised by using the Hungarian algorithm Kuhn (1955). All the details of this adapted Hungarian method are available in Leboucher et al. (2013) and Section 3.4.1. Since in real scenarios the number of targets is only rarely the same as the number of weapons, the Hungarian algorithm designed for asymmetric bipartite graphs is used.

### *3.1 The engagement plan*

The engagement plan represents the solution space. An engagement plan is composed of a set of weapon/target assignments. For example, if the following situation involves 3 weapons and 2 targets, a possible engagement plan $EP$ could be:

$$EP(t) = \{(W_1, T_2); (W_3, T_1)\}$$

Where the $W_i$, $i \in \{1,2,3\}$ and $T_j$, $j \in \{1,2\}$ represent the weapon $i$ and the target $j$. The engagement plan evolves according to the situation, and depends on the current simulation time and on the aerial situation. In this application, the engagement plan is recomputed in real time as soon as a plan has been proposed to the human operator in order to deal with the manoeuvres of the targets and the changes in the tactical situation.

### *3.2 The weapon-target assignment*

To assign the available weapons to the targets, the Hungarian algorithm is used. The weapons and the targets are modelled as an asymmetric bipartite graph. In the studied problem, it is assumed that the initial number of weapons is greater than the number of oncoming targets.

The quality of the proposed assignment is evaluated according to four different criteria: the capacity to propose an early fire, the width of the firing time window, the distance of the defended area by our own assets and the probability to successfully suppress an enemy.

These criteria respectively represent:

- the capability of the system to propose an early firing time, and then its ability to cope with a target in the earliest possible time in order to avoid any risk.
- the width of the firing time window represents the time that we have to cope with one target, then the larger is this firing time windows, the more time we have to propose one engagement solution,
- limiting the overfly in our own area enables to cope with security problem in case of material failure.
- the kill probability represents the probability of successful suppression.

### *3.3 The mutation process - The Hungarian algorithm*

In this paper, the selected mutation process is based on Graph Theory, and especially the Hungarian method. Indeed, the Hungarian method is proved to be optimal and has a polynomial complexity Kuhn (1955). Thus, the use of this method for real-time task allocation was a natural choice. Since the modelling of the cost function as a linear weighted sum of the objectives was the more appropriate way to cope with the studied problem (see 1 for the details), it was a key idea to find a convenient weighting method. The proposed method enables to determine these weights in a robust and stable way Maynard-Smith (1982).

### *3.4 Mathematical modelling*

This section describes the mathematical modelling of each step followed to achieve the DWTA. The assignment of the weapons to the targets is computed to achieve in the best possible way destroying all the threatening targets. The weapon-target assignment is done by using the graph theory, especially the Hungarian algorithm.

In the following section, $EFF_{w/t}$ denotes the earliest feasible fire for the weapon $w$ on the target $t$. The latest feasible fire for the weapon $w$ on the target $t$ is denoted by $LFF_{w/t}$. $FTW_{w/t}$ denotes the set of the firing time windows (time windows in which a weapon $w$ can be fired with a given probability to reach the target $t$). Kill probability of the weapon $w$ for the target $t$ is denoted as $PK_{w/t}$. The last overflying criterion is represented by $OF_{w/t}$. $E_{w/t}$ represents the edge linking the weapon $w$ with the target $t$. The average speed of the weapon $w$ is denoted by $S_w$. $R_t$ and $R_w$ denote the state of the target $t$ (respectively the weapon $w$). The states are composed of the position $X_t = (x_t, y_t)$ and the speed $(v_{t_x}, v_{t_y})$ of the target $t$ (respectively position $X_w = (x_w, y_w)$ and the speed $(v_{w_x}, v_{w_y})$ of the weapon $w$) in the $(x, O, y)$ plan, where $O$ denotes the origin of the referential. The entering point of the target $t$ in the capture zone of the weapon $w$ and the entering point of the defended area is computed in the same time as the $FTW_{w/t}$ and they are denoted by $P_{t_{in}}$ and $P_{t_{out}}$. The capture zone can be defined as the area in which the weapon can reach with an associated kill probability for a target. In order to compute the capture zone, from the target position a worst case approach is adopted. That means that a straight line to the center of the defended area is assumed as the trajectory of the target. The initial position of the weapon $w$ is denoted by $P_{w_0} = (x_{w_0}, y_{w_0})$.

*The assignment: Hungarian method* Let $W$ be the set of the available weapons and $T$ the set of the oncoming targets. If $A$ represents the assignments linking the vertices $W$ to the targets $T$. $G = (W, T, A)$ denotes the complete bipartite graph.

The weight of each edge is computed from the linear combination of the four criteria: earliest possible fire, width of the firing time windows, the overflying distance over the defended area and the associated probability to kill the target $t$ at the current time. These criteria are represented as follows:

$$f_1(E_{w/t}) = EFF_{w/t}, (w \in W), (t \in T)$$

As mentioned, $EFF_{w/t}$ denotes the earliest feasible fire for the weapon $w$ on the target $t$.

$$f_2(E_{w/t}) = LFF_{w/t} - EFF_{w/t}, (w \in W), (t \in T)$$

The latest feasible fire for the weapon $w$ on the target $t$ is denoted by $LFF_{w/t}$. The larger this window is, the better the solution is.

$$f_3(E_{w/t}) = d\left(P_{t_{out}}, P_{w_0}\right)$$

Here the function $d(P_1, P_2)$ represents the Euclidean distance function between the point $P_1$ and the point $P_2$.

Then, the last criterion representing the kill probability (PK) of the weapon $w$ for the target $t$ is determined by the distance between the weapon and current location of the target:

$$f_4(E_{w/t}) = \begin{cases} 0 & \text{if target out of the capture zone} \\ 1 - \dfrac{d(X_w, X_t)}{w_{range}} & \text{if target in the capture zone} \end{cases}$$

Then, the global weight of the assignment $E_{w/t}$ is the linear combination of the four functions described above: $H(E_{w/t}) = \alpha_1 f_1(E_{w/t}) + \alpha_2 f_2(E_{w/t}) + \alpha_3 f_3(E_{w/t}) + \alpha_4 f_4(E_{w/t})$, where $H(E_{w/t})$ denotes the weighting function of the assignment $E_{w/t}$ and $(\alpha_1, \alpha_2, \alpha_3, \alpha_4) \in [0,1]^4$, with $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 1$. Note that the $\alpha_i$, $i \in 1,2,3,4$ coefficients are obtained as the result of the EGT computation.

The cost matrix used for the Hungarian algorithm has the following form:

$$H = \begin{pmatrix} E_{1/1} & E_{2/1} & E_{3/1} & \dots & E_{|W|/1} \\ E_{1/2} & E_{2/2} & E_{3/2} & \dots & E_{|W|/2} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ E_{1/|T|} & E_{2/|T|} & E_{3/|T|} & \dots & E_{|W|/|T|} \end{pmatrix}$$

$|T|$ and $|W|$ represent the cardinal of the sets $T$ and $W$.

Note that, since the studied problem is an improvement of one of our previous study, the used mathematical modelling is based on the one used in Leboucher et al. (2013).

## 4. RESULTS AND DISCUSSION

The presented method has been tested on a numerical simulator to investigate the following points: first, the evolution of the obtained coefficient using the EGT to obtain the weighted sum composing the multi-objective cost function; second, the impact of the obtained stability of the coefficients over the optimal assignment is investigated by studying the stability of the proposed approach over the time and the robustness of the approach to small changes in the tactical situation due to the target manoeuvres. In order to represent a classical anti-aerial situation, the simulator parameters are set as follows:

**The aerial space:** Square of 50000 m by 50000 m

**The defended assets:** They are points located in the central area of the space within a radius of 5000 metres of the origin of the map.

**Launchers:** Launchers are located all around the defended assets in order to insure the protection of the central area against the oncoming targets.

**Weapons:** They are randomly assigned over the Launchers The range of each weapon is randomly drawn between 5000 meters and 15000 meters.

**Targets:** The initial position is set up between 30000 m and 50000 m from the centre of the space.The trajectories that the targets are following are modelled in using a Bezier's curve defined by 4 control points. The last control point is automatically set as one of the defended assets.The speed is randomly drawn between 50 m/s and 800 m/s.

**The initial conditions:** 20 weapons vs. 12 targets.

**Condition of engagement success:** The success of an engagement of one weapon on one target is determined in drawing one random number. If this number is greater than a determined value, then the shoot is considered as a success. Otherwise, it is considered that the target avoids the weapon.

Figures 2, 3 and 4 represent an overview of one scenario in 3 different times of the mission. On these graphs, the upper left one shows the evolution of the obtained Evolutionary Stable Strategy (ESS), then the used value to compose the weighted sum of the cost function. The upper right graph represents the optimal found assignment according to the used cost function. The lower graph is a synthetic view of the tactical situation and enables to have an *intuitive* understanding of the possible change in the optimal assignment over the time.

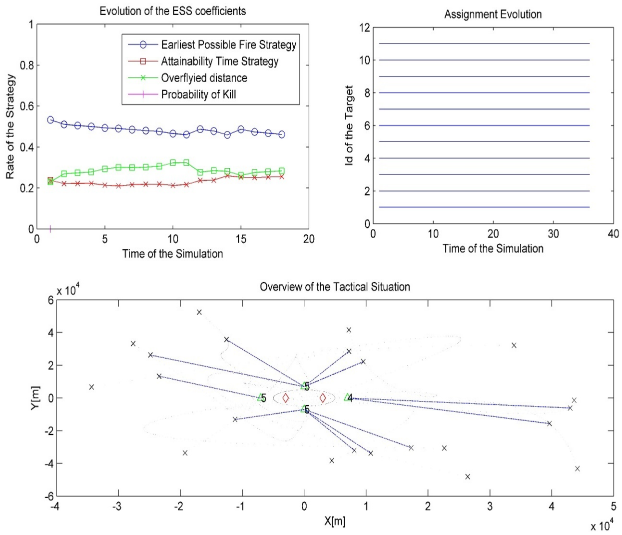


*Figure 2. Tactical situation during earlier stage of the mission: The upper left graph represents the evolution of the ESS coefficients. The upper right graph shows the evolution of the proposed engagement over the time. The lower graph represents a tactical view of the current scenario. This Figure states the situation between the beginning of the mission and a first fire.*

## *4.1 Study on the EGT coefficient evolution*

This subsection investigates the evolution of the 4 computed weights by using EGT. These 4 coefficients represent the weighting coefficients that we assign to each objective in order to model the global cost function, thus the multi-objective problem is reduced to a single-objective by using an aggregated approach.

The three upper left graphs on the Figures 2, 3 and 4 show the evolution of the obtained coefficients by using the EGT process, so the computed ESS at each time step of the simulation. Each curve represents one of the criteria and the survival ratio is represented in function of the simulation time. By analysing the Figure 2 which represents a photography of the tactical situation after 20 seconds of the mission, the Evolution of the ESS coefficients presents some interesting points: first, the obtained ESS is almost the same over this time interval. It is also noticeable that the Earliest Fire strategy is predominant over the others and that the attainability and the overflight distance can cohabit with the same ratio ($\backsim [1/4, 1/4]$). Then, the Probability of Kill (PK) solution does not express at all. This can be explained by the absence of attainability in the current

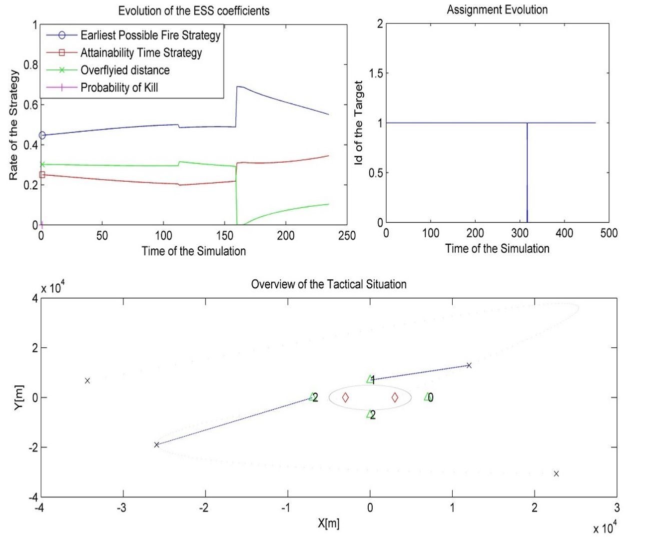


*Figure 3. Tactical situation in the middle stage of the mission: The upper left graph represents the evolution of the ESS coefficients. The upper right graph shows the evolution of the proposed engagement over the time. The lower graph represents a tactical view of the current scenario. This Figure states the situation in the middle of the mission.*

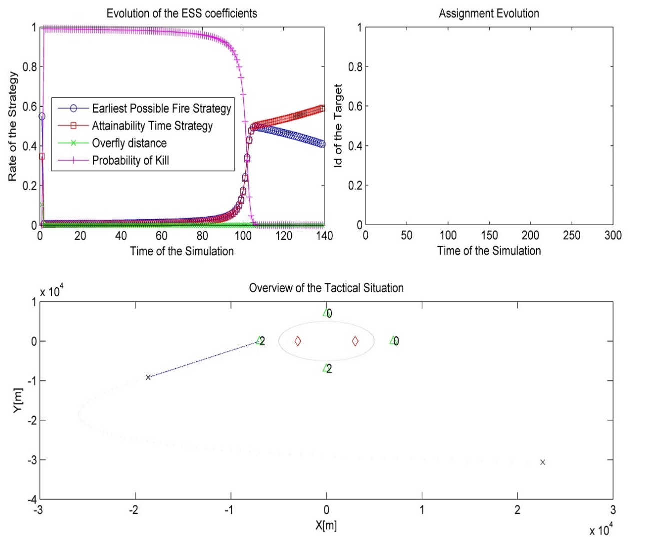


*Figure 4. Tactical situation during later stage of the mission:The upper left graph represents the evolution of the ESS coefficients. The upper right graph shows the evolution of the proposed engagement over the time. The lower graph represents a tactical view of the current scenario. This Figure states the situation at the final stage of the mission*

tactical situation. Indeed, as the tactical view shows it, all the targets are out of the weapon capture zones. From this Figure, the assignment evolution shows that the proposed engagement plan is extremely stable over the time and that no change occurs during this time interval. The analysis of the Figure 3 shows that until the time $t = 159$, the ESS coefficients are stable. Then, the ESS becomes different and leads to a change of the proposed engagement. That phenomenon can be explained by the lost of attainability with the previous assignment. Indeed, as shown on the lower graph, the target is moving from the right to the left. The situation is adapted in order to cover all the targets. To conclude this study, the final phase of the mission is analysed with the Figure 4. The first remark is that the PK is now taken into account, and the optimal solution is based only on this criterion. This sudden change in the global assignment strategy can be explained by the fact that we now have attainability on the target and the PK expresses itself more than the other criteria when the target can be reached.

### *4.2 The stability of the proposed engagement*

As mentioned in the section 3.2, the final decision to fire is the responsibility of a human operator. Therefore, the proposed engagement has to be as stable as possible to make sure that the operator can prepare himself in advance to proceed a fire or to delay it because of operational data that only the operator can obtain during the military operation. By analysing the upper right graphs of the Figures 2, 3 and 4, it is noticeable that there is a good stability in the proposed engagement plan as long as there is no major change over the time. When the algorithm proposes a slightly different engagement, that can always be explained by the tactical situation. Thus, the human operator can prepare himself for the launch of the engagement with high probability that a given engagement plan at the time $t_{mission}$ will still be valid at the time $t_{mission} + \Delta t$. The last analysed point is that the proposed algorithm is stable enough to support small variations in the inputs (target positions) and to provide the same output. The risk usually encountered with this weighted sum approach is therefore corrected.

## 5. CONCLUSION

In this paper, a multi-objective approach to solve the Dynamic Weapon Target Assignment problem was proposed. Based on the ability of the Evolutionary Game Theory to model multi-objective problems and the efficiency of the Hungarian algorithm to solve the task assignment problem, the proposed approach shows reliable results in terms of robustness, stability and real-time computation. The proposed method was tested by using one simulator designed in order to create random scenarii and generate many heterogeneous situations to test the limit of the designed algorithm. The results have shown that this method can be an effective support for a human operator in charge of supervising the mission.

In the future, it will be interesting to try to integrate within only one algorithm the optimisation of the guidance laws as well as the assignment of the targets. Thus, the engagement plan could include the entire engagement chain and be treated as a whole for a global optimisation.

## REFERENCES

Bisht, S. (2004). Hybrid genetic-simulated annealing algorithm for optimal weapon allocation in multilayer defence scenario. *Defence Sci. J.*, 54(3), 395 – 405.

Blodgett, D., Gendreau, M., Guertin, F., and Potvin, J.Y. (2003). A tabu search heuristic for resource management in naval warfare. *J. Heur.*, 9, 145 – 169.

Cullenbine, A.C. (2000). *A taboo search approach to the weapon assignment model*. Master's thesis, Department of Operational Sciences, Air Force Institute of Technology, Hobson Way, WPAFB, OH.

Grant, K.E. (1993). Optimal resource allocation using genetic algorithms. Technical report, Naval Research Laboratory, Washington (USA).

Hosein, P.A. and Athans, M. (1990a). Preferential defense strategies. part i: The static case. Technical report, MIT Laboratory for Information and Decision Systems with partial support, Cambridge (USA).

Hosein, P.A. and Athans, M. (1990b). Preferential defense strategies. part ii: The dynamic case. Technical report, MIT Laboratory for Information and Decision Systems with partial support, Cambridge (USA).

Hosein, P.A. and Athans, M. (1990c). Some analytical results for the dynamic weapon-target allocation problem. Technical report, MIT Laboratory for Information and Decision Systemswith partial support.

Hosein, P.A., Walton, J.T., and Athans, M. (1988). Dynamic weapon-target assignment problems with vulnerable c2 nodes. Technical report, MIT Laboratory for Information and Decision Systems with partial support, Cambridge (USA).

Johansson, F. and Falkman, G. (2010). Sward: System for weapon allocation research & development. *13th Conference on Information Fusion (FUSION)*, 1, 1–7.

Karasakal, O. (2008). Air defense missile-target allocation models for a naval task group. *Comput. Oper. Res.*, 35, 1759 – 1770.

Kuhn, H. (1955). The hungarian method for the assignment problem. *Naval Research Logistics Quarterly*, 2, 83 – 97.

Kwon, O., Lee, K., Kang, D., and Park, S. (2007). A branch-and-price algorithm for a targeting problem. *Naval Res. Log.*, 54, 732 – 741.

Leboucher, C., Shin, H.S., Siarry, P., Chelouah, R., Menec, S.L., and Tsourdos, A. (2013). *A Two-Step Optimisation Method for Dynamic Weapon Target Assignment Problem, Recent Advances on Meta-Heuristics and Their Application to Real Scenarios*. InTech. doi:10.5772/53606.

Lu, H., Zhang, H., Zhang, X., and Han, R. (2006). An improved genetic algorithm for target assignment optimization of naval fleet air defense. In *6th World Cong. Intell. Contr. Autom.*, 3401 – 3405. Dalian (China).

Malhotra, A. and Jain, R.K. (2001). Genetic algorithm for optimal weapon allocation in multilayer defence scenario. *Defence Sci. J.*, 51(3), 285 – 293.

Marler, R. and Arora, J. (2004). Survey of multi-objective optimization methods for engineering. *Structural and Multidisciplinary Optimization*, 26, 369–395.

Maynard-Smith, J. (1982). *Evolution and the theory of games*. Cambridge University Press.

Messac, A. (1996). Physical programming: effective optimization for computational design. *AIAA J.*, 34, 149 – 158.

Newman, A.M., Rosenthal, R.E., Salmern, J., Brown, G.G., Price, W., Rowe, A., Fennemore, C.F., and Taft, R.L. (2011). Optimizing assignment of tomahawk cruise missile missions to firing units. *Naval Research Logistics*, 58, 281 – 295.

Shapley, L.S. (1953). A value for n-person games. *Annals of Mathematical Studies*, 28, 307 – 317.

Sikanen, T. (2008). Solving weapon target assignment problem with dynamic programming. Technical report, Mat-2.4108 Independent research projects in applied mathematics.

Taylor, P. and Jonker, L. (1978). Evolutionary stable strategies and game dynamics. *Mathematical Bioscience*, 40, 145–156.

Wu, L., Xing, C., Lu, F., and Jia, P. (2008). An anytime algorithm applied to dynamic weapon-target allocation problem with decreasing weapons and targets. In *IEEE Congr. Evol. Comput.*, 3755 – 3759. Hong Kong (China).