Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

A Design Approach for Insensitivity to Disturbance Period Fluctuations Using Higher Order Repetitive Control

H.J. Guo*. R.W. Longman. T. Ishihara*****

** Department of Electrical and Information Engineering, Tohoku Gakuin University, Sendai, 985-8537 Japan (e-mail: kaku@tjcc.tohoku-gakuin.ac.jp).*
***Department of Mechanical Engineering, Columbia University, New York, NY 10027 USA (e-mail: rwl4@columbia.edu)*
**** Faculty of Science and Technology, Fukushima University, Fukushima, Japan, (e-mail: ishihara@sss.fukushima-u.ac.jp)*

Abstract: Repetitive control (RC) is well known as a method for tracking a periodic command as well as eliminating the influence of a periodic disturbance. An issue that can be encountered in applications is that the period of the disturbance can vary. In some cases one can monitor a drift in the disturbance period, but in other cases the period can fluctuate sufficiently fast that one wants an RC law that is robust to period fluctuations. Several studies address this problem using higher order RC incorporating a negative weight on errors from some previous period. This paper presents a systematic design procedure that can be used to improve RC robustness to disturbance period fluctuations. Methods are given to present the information needed to make the necessary tradeoffs when tuning the set of design parameters.

Keywords: Repetitive control, Higher order RC, Insensitivity, Period fluctuation, Disturbance.

1. INTRODUCTION

Repetitive control (RC) is an effective control method that can produce zero error in a control system tracking a periodic command, or cancel the influence of a periodic disturbance. If the purpose is to eliminate the influence of an external periodic disturbance, it is necessary for the RC system to stay synchronized with the disturbance, and RC is typically rather sensitive to accurate knowledge of the period. Several situations can occur. In some applications, the disturbance is related to a periodic command being executed, in which case one can easily stay synchronized. In Ahn et al. (2013) the disturbance period is the period of rotation of a control moment gyro (CMG) on a spacecraft run using three phase motors, and one knows the phase from the commands to the motors. A little more generally, one can try to measure the period in real time. Tsao et al. (2000) analyze this kind of application. RC can be effective provided that the change in period is sufficiently slow that the RC convergence time keeps the system close to zero error. Sometimes the period fluctuates, and in this situation one wants an RC design that is robust to uncertainty in the disturbance period. It is this situation that motivated Steinbuch (2002) and Steinbuch et al. (2004). The approach there develops higher order RC to create improved robustness to the disturbance period. Higher order RC makes use of measured errors not only from the previous period, but also from one or more earlier periods. Lo and Longman (2005) and (2006) develop an understanding of how this approach can improve period robustness, from both a frequency response point of view and a root locus analysis approach.

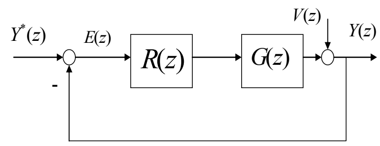

Fig. 1. Basic RC system

We comment that in experiments reported in Ahn et al. (2013), using knowledge of the 3 phase motor voltage inputs to CMGs, the discretization of the period based on the sample time interval used in measurements was sufficiently large that robustification to period uncertainties was preferable. Ahn et al. (to appear), presents an alternative approach to robustification to period uncertainties. It makes use of RC designed for multiple periods, and makes the periods identical.

In this paper we develop a systematic procedure to use to design higher order RC systems with improved robustness to disturbance period fluctuations. The parameters to be adjusted during the design process are listed, and then various kinds of graphical plots can be used to know the influence of each parameter so that one can make intelligent trade-off decisions. In Section 2, we review the design of the higher order RC. The mathematical formulation for the design method is presented in Section 3. Then the detailed design procedure is presented in Section 4. A simple design simulation is discussed in Section 5. Finally, in Section 6 we summarize the main contributions.

2. FORMULATION OF HIGHER ORDER REPETITIVE CONTROL

2.1 Basic Repetitive Control (RC) -- First Order RC

We consider the block diagram in Fig.1 as the repetitive control (RC) system, where $R(z)$ is the repetitive controller, and $G(z)$ is a closed loop transfer function of a feedback control system. Before going to the higher order RC, consider first order repetitive control formulations. The most basic form makes use of the concept of the discrete time equivalent of a separate integral control operating on each time step of the disturbance period. Suppose that the period of the disturbance (or command, or both when both are present) is pT, where T is the sample time interval, and p is the number of time steps per period. Then the simplest form of RC is

$$u(k) = u(k-p) + \phi e(k-p+1); \quad R(z) = z\phi/(z^p - 1) . \qquad (1)$$

The command at time step k is equal to the command one period back, plus a repetitive control gain ϕ times the error one time step ahead of one period back (the T dependence in the arguments is dropped for notational simplicity). The one step ahead compensates for the usual one step delay going through $G(z)$. Note that stability of the repetitive control system is determined by the characteristic polynomial $R(z)G(z) = -1$. And the root locus produced by varying the repetitive control gain ϕ has the zeros of the original feedback control system, and in addition it has a new zero at the origin. The poles are the poles of the original feedback control system, which are assumed to be inside the unit circle, and in addition there are p poles on the unit circle introduced by the repetitive controller. In practice, it is convenient to design the RC law in the frequency domain, using the form

$$\begin{aligned} z^p U(z) &= F(z)[U(z) + \Phi(z)E(z)]; \\ R(z) &= \frac{F(z)\Phi(z)}{z^p - F(z)}. \end{aligned} \qquad (2)$$

This generalizes equation (1) by replacing ϕ z with $\Phi(z)$ that can include not only a gain, but also a compensator. Methods of designing the compensator include those in Longman (2010). Also, $F(z)$ is introduced which can be a zero phase low pass filter used to make a frequency cutoff of the learning process. Such a cutoff makes it easier to obtain a convergent process, but this is done at the expense of not trying to learn to eliminate periodic errors above the cutoff. From Fig. 1, one can write $[1 + G(z)R(z)]E(z) = Y^*(z) - V(z)$, and using equation (2) one obtains the difference equation whose solution is the tracking error of the control system

$$\left\{ z^p - F(z)\left[1 - \Phi(z)G(z)\right] \right\} E(z) = \left[z^p - F(z) \right]\left[Y^*(z) - V(z) \right]. \quad (3)$$

When there is no filter, i.e. $F(z) = 1$ as in equation (1), then the forcing function on the right side of (3) becomes zero because it is the difference of the value of periodic functions at the present time and shifted one period ahead. Hence, in this case the error converges to zero provided the characteristic polynomial in curly brackets on the left has all roots inside the unit circle.

2.2 Higher Order RC

Higher order RC refers to RC laws that make use of error information from not just the previous period, but additional previous periods as well (Longman (2010)). Generally, it has been difficult to determine any clear benefits for higher order RC. But Steinbuch (2002), (2004) suggests using higher order RC with negative coefficients on the measurements for some previous period errors, in order to make RC less sensitive to accurate knowledge of the period, or to period fluctuations. The form for *N*th order RC corresponding to (1) is

$$u(k) = \sum_{j=1}^{N} \alpha_j [u(k - jp) + \phi e(k - jp + 1)] , \qquad (4)$$

where N is the number of periods one wishes to include, and the $\alpha_1, \alpha_2, ..., \alpha_N$ are weights to be chosen by the designer. One may think of this as creating a weighted average, in which case each weight should be non-negative. However, as pointed out in Steinbuch (2002), there is no need to restrict the weights to be positive. The generalizations of equations (2) and (3) to include a compensator and zero-phase low-pass filter are

$$R_N(z) = \frac{F(z)\Phi(z)\left[\alpha_1 z^{(N-1)p} + \alpha_2 z^{(N-2)p} + \cdots + \alpha_N \right]}{z^{Np} - F(z)\left[\alpha_1 z^{(N-1)p} + \alpha_2 z^{(N-2)p} + \cdots + \alpha_N \right]} \qquad (5)$$

$$\begin{aligned} &\left\{ z^{Np} - F(z)[1 - \Phi(z)G(z)]\left[\alpha_1 z^{(N-1)p} + \alpha_2 z^{(N-2)p} + \cdots + \alpha_N \right] \right\} E(z) \\ &= \left\{ z^{Np} - F(z)\left[\alpha_1 z^{(N-1)p} + \alpha_2 z^{(N-2)p} + \cdots + \alpha_N \right] \right\} [Y^*(z) - V(z)] \end{aligned} \qquad (6)$$

and the characteristic equation $R(z)G(z) = -1$ is again given by setting the curly bracket on the left to zero. The conditions needed to converge to zero error again require that all roots of the characteristic polynomial lie inside the unit circle, and that the forcing function on the right of difference equation (6) is zero. The latter occurs when no cutoff filter is used, so that $F(z) = 1$, and in addition it is necessary to restrict the choice of the α_j to satisfy

$$\alpha_1 + \alpha_2 + \cdots + \alpha_N = 1 . \qquad (7)$$

3. MATHEMATICAL FORMULATION FOR ROBUSTIFICATION

Steinbuch (2002) demonstrated that higher order RC can be effective at minimizing the influence of a fluctuating disturbance. Steinbuch (2002) and Lo (2006) show how higher order RC can widen the notches around the fundamental frequency and its harmonics in the sensitivity transfer function from disturbance (as an equivalent output disturbance on the feedback control system output) to resulting error. We will propose a systematic design procedure to use for this purpose.

3.1 Repeating the p Poles on the Unit Circle

The first order repetitive controllers in equations (1) and (2) with the cutoff filter removed, places p poles on the unit circle at the fundamental frequency and its harmonics. This produces a notch at each of these frequencies in the form of a cusp in the magnitude frequency response of the sensitivity transfer function from disturbance to error. Disturbances with the frequency at the bottom of the cusp produce zero error, but because the cusp has steep sides, small deviations from this addressed frequency can result in poor disturbance rejection. Repeating these poles eliminates the cusp producing a zero derivative at the zero minimum in the plot, and widens the notches. Using (5) with $F(z)=1$, one produces a higher order RC law repeating each pole *N* times according to

$$z^{Np}-(\alpha_1 z^{(N-1)p}+\alpha_2 z^{(N-2)p}+\cdots+\alpha_N)=(z^p-1)^N \quad . \qquad (8)$$

Lemma 1: Define the following equations

$$\begin{aligned}(z^p-1)^{k+1}&=[z^{kp}-(\alpha_1^k z^{(k-1)p}+\alpha_2^k z^{(k-1)p}+\cdots+\alpha_k^k)](z^p-1)\\&=z^{(k+1)p}-[(\alpha_1^k+1)z^{kp}+(\alpha_2^k-\alpha_1^k)z^{(k-1)p}+\cdots\\&\quad+(\alpha_k^k-\alpha_{k-1}^k)z^p-\alpha_k^k]\\&=z^{(k+1)p}-[\alpha_1^{k+1}z^{kp}+\alpha_2^{k+1}z^{(k-1)p}+\alpha_k^{k+1}z^p+\alpha_k^{k+1}].\end{aligned} \qquad (9)$$

Then the coefficients satisfying (9) can be determined from the following recursive formula

$$\begin{aligned}\alpha_1^{k+1}&=\alpha_1^k+1\\ \alpha_i^{k+1}&=\alpha_i^k-\alpha_{i-1}^{k+1},\ \ (i=2,\cdots,k)\\ \alpha_{k+1}^{k+1}&=-\alpha_k^k.\end{aligned} \qquad (10)$$

Proof: After establishing equation (9), it is straightforward to calculate (10). Note that relation (7) is easily verified

$$\sum_{i=2}^{k}\alpha_i^{k+1}=\sum_{i=2}^{k}(\alpha_i^k-\alpha_{i-1}^k)=-\alpha_1^k+\alpha_k^k\,, \qquad (11)$$

which results in

$$\sum_{i=1}^{k+1}\alpha_i^{k+1}=\alpha_1^{k+1}+\sum_{i=2}^{k}\alpha_i^{k+1}+\alpha_{k+1}^{k+1}=1 \quad . \qquad (12)$$

3.2 Placing the Additional Poles from Higher Order RC Near but not at the Original p Poles

Repeating the roots produced one or more zero derivatives at the bottom of each notch, eliminating the cusp and locally widening the notch. Consider starting with third order RC with three roots at each frequency of the given disturbance period. Then we can easily widen each notch further by separating the two additional roots, moving one to a neighbouring frequency below the original frequency, and the other to a corresponding neighbouring frequency above the original frequency. If we go to still higher order RC, then one can easily consider including additional neighbouring frequencies, or decide to repeat each of the roots. Both will have beneficial effects on how separated the walls are of the frequency notch at the addressed fundamental frequency and its harmonics. The following equation expresses all such possibilities, with *p* frequencies having the addressed period, *k* additional different fundamental frequencies and harmonics to place in the neighbourhood of the original frequencies, each of which is repeated n_j times

$$\begin{aligned}&z^{Np}-(\alpha_1 z^{(N-1)p}+\alpha_2 z^{(N-2)p}+\cdots+\alpha_N)\\&=(z^p-1)^{n_0}(z^p-q_1)^{n_1}\cdots(z^p-q_k)^{n_k}\,,\end{aligned} \qquad (13)$$

where $n_0+n_1+\cdots+n_k=N$. For simplicity of exposition, we illustrate the design process using the third order RC case as described above, which corresponds to $N=3$, $n_0=n_1=n_2=1$, and from (13) we have

$$z^{3p}-(\alpha_1 z^{2p}+\alpha_2 z^p+\alpha_3)=(z^p-1)(z^p-q_1)(z^p-q_2) \qquad (14)$$

The relations between $\alpha_1,\alpha_2,\alpha_3$ and q_1,q_2 can be directly calculated as follows and satisfy $\alpha_1+\alpha_2+\alpha_3=1$.

$$\begin{aligned}\alpha_1&=1+q_1+q_2\\ \alpha_2&=-(q_1q_2+q_1+q_2)\\ \alpha_3&=q_1q_2.\end{aligned} \qquad (15)$$

Consider $z=r(\cos\theta+i\sin\theta)$, $z_0=r_0(\cos\theta_0+i\sin\theta_0)$ and make use of De Moivre's theorem to write the solution of the equation $z^n=z_0$ as

$$\begin{aligned}&z=z_0^{1/n}=r_0^{1/n}\left\{\cos\left(\frac{\theta_0+2k\pi}{n}\right)\right\}+i\sin\left(\frac{\theta_0+2k\pi}{n}\right)\Bigg\},\\&r=r_0^{1/n},\ \ \theta=\frac{\theta_0+2k\pi}{n},\ \ (k=0,1,\ldots,n-1).\end{aligned} \qquad (16)$$

Use the result to design the poles in (14). First, for the factor (z^p-1) , because $z_0=1,(r_0=1,\ \theta_0=0)$ we have

$$r=1,\ \ \theta=\frac{2k\pi}{p},\ (k=0,1,\ldots,p-1),\ \ \Delta\theta=\frac{2\pi}{p}\ , \qquad (17)$$

where $\Delta\theta$ is the angel interval between the *p* values of θ for these *p* roots on the unit circle. Let the sampling time period be *T*, and considering $\theta=\omega T=2\pi fT$, we have $f=k/pT$, $(k=0,1,\ldots,p-1)$, $\Delta f=1/pT$. Here *k*=1 is the fundamental frequency of the reference signal and the disturbance signal. Δf is the interval between the frequencies. These pole placements ensure the output of the closed loop system perfectly tracks the reference signal and eliminates the influence of the disturbance signal. The sensitivity transfer function of the closed loop system will be zero at the above fundamental frequency and all harmonics up to Nyquist frequency. Between these frequencies, the sensitivity transfer function will not be zero and instead will be amplified because of the Bode integral theorem also known as the waterbed effect. Continuing with the third order RC problem, choose q_1,q_2 as the following conjugate form

$$q_{1,2}=(\cos\theta_1\pm i\sin\theta_1) \qquad (18)$$

De Moivre's theorem applied to the factor (z^p-q_1) produces the following poles

$$\begin{aligned}&z_q=q_1^{1/p}=\left\{\cos\left(\frac{\theta_1+2k\pi}{p}\right)+i\sin\left(\frac{\theta_1+2k\pi}{p}\right)\right\},\\&r_q=1,\ \ \theta_q=\frac{\theta_1+2k\pi}{p},\ \ (k=0,1,\ldots,p-1)\end{aligned} \qquad (19)$$

We introduce parameter *c* to indicate how much the extra poles for this term have been moved from the *p* original poles. The value of *c* indicates how the root is moved as a fraction of the distance $\Delta\theta$ from one of the original pole to the next pole. Select the first pole (k=0) as

$$\frac{\theta_1}{p} = c \times \Delta\theta, \quad -\frac{1}{2} < c < \frac{1}{2} \tag{20}$$

If $c = 0.1$ the new poles are placed at

$$r_q = 1, \ \theta_q = \frac{2k\pi}{p} + 0.1 \times \frac{2\pi}{p}, \ (k = 0,1,\ldots,p-1), \tag{21}$$

and similarly with a minus sign when c is negative for moving the q_2 of the third set of poles in the opposite direction from the original p poles.

4. PROCEDURE AND TOOLS FOR ROBUSTIFIED RC DESIGN

This section discusses the tradeoffs to be made by the designer when generating an RC system robustified to period fluctuations. The design parameters to be chosen and tuned include: (1) The number of repeats of the original p poles, n_0. (2) The number of additional poles k and how many times they are repeated, $n_1, n_2, \cdots, n_k$. These poles must be in conjugate pairs with each pole in a pair repeated the same number of times. Together, (1) and (2) pick the order of the RC, N. (3) For each pair, pick the location as the fractional distance from one of the original p poles to the next, c_1 and c_2, etc. (4) Adjust the overall RC gain ϕ. (5) Design a zero-phase cutoff filter $F(z)$, and in the process consider adjustment of the sample time interval T.

Initial Assumptions: For clarity of exposition, we make some assumptions about the problem to be addressed. We consider that the RC adjusts the command going through a zero order hold feeding a continuous time feedback control system $G(s)$, and we know the poles and zeros of the equivalent digital system $G(z)$. As in the previous section we restrict ourselves to $n_0 = n_1 = n_2 = 1$ so that $N = 3$ and we adjust one c. It will be clear how to generalize. The initial discussion considers the case that all zeros of $G(z)$ are inside the unit circle. Then the compensator is chosen to cancel the poles and zeros of the system and include an overall gain $\Phi(z) = \phi G^{-1}(z)$. The sensitivity transfer function from command minus output disturbance, $Y^* - V$, to error E is $S(z) = 1/[1 + R(z)G(z)]$. The $F(z)$ is set to unity for the first phase of the design process. The magnitude plot of the sensitivity transfer function and the root locus plot of its poles as a function of gain ϕ contain the fundamental information needed.

The Objective: The design objective is to widen the notches in the magnitude plot of the sensitivity transfer function. This plot is now a standard plot for all systems under consideration. Analogous to the definitions of Q factor or bandwidth, we can pick a magnitude, e.g. 0.5, and at this magnitude on the plot, record the notch width. This width is the same for all notches. Hence, the first design tool is a plot of the notch width as a function of the choice of c, and for several choices of overall gain ϕ. This plot allows us to find values of these two design variable for a chosen desired notch width.

Tradeoff 1: The chosen value of c widens notches, but introduces small peaks at the bottom of the notches, between the original pole locations and the newly introduced pole locations. The height of these peaks deteriorates the disturbance cancellation at these peaks. The second design tool is a plot of the height of these peaks as a function of the values of c for several choices of ϕ. Again the same plots apply to all systems under consideration.

Tradeoff 2: The repetitive control system is a feedback control loop and hence is subject to the Bode integral theorem, also known as the waterbed effect. Widening notches attenuates disturbances at more frequencies, and this must be paid for by the increase of disturbance amplification at the peaks occurring between the notches. The third design tool is then a plot of the height of these peaks as a function of the choice of c for several choices of ϕ. Again this plot applies to all systems under consideration.

After an initial assessment using this plot, one can do a more precise evaluation. The sensitivity transfer function goes from an output disturbance to error. The physical disturbance normally occurs before the plant in the feedback control system, and there is a transfer function from the actual disturbance to its equivalent output disturbance. One multiplies this transfer function times the sensitivity transfer function to assess how broadband disturbances that are not periodic with the addressed period, influence performance. One may need to make a compromise.

Evaluating the Needed Notch Width: Ideally, one has data that indicates not only the nominal period, but the nominal amplitudes of each harmonic, so that one knows how important it is to address each harmonic. Note that all notches are widened by the same amount. Given an expected fluctuation range for the disturbance period we can widen the notch for the fundamental frequency by the appropriate amount. But when the fundamental period varies by $\Delta\omega$, the first harmonic varies by $2\Delta\omega$, etc. Hence, the actual desired notch widening could be determined by harmonics instead of the fundamental. As a result, a chosen amount of widening can attenuate errors for the fundamental and lower harmonics, but higher harmonics can be amplified. One can make appropriate choices, and then use the cutoff filter $F(z)$ to prevent amplification of higher harmonics.

Cutoff Filter and Sample Rate: Consider the effects of doubling the sample rate, this doubles the value of p, cuts the value of T in half, doubles Nyquist frequency, and allows one to see roughly twice as many harmonics. Increasing the sample rate gives better fidelity representation of the continuous time signals, but probably introduced more harmonics that should be cut out of the RC learning process. The cutoff frequency needs to be determined based on: (1) The considerations above related to period fluctuation and harmonics. (2) The limit imposed by inaccurate knowledge of the system dynamics at high frequencies, a limit that can only be determined by hardware tests. (3) Possible need to limit the learning frequency range to avoid requiring too large actuator outputs trying to correct errors far above the control system bandwidth.

Evaluation of Stability Range: The RC system is likely to be unstable for particularly small values of overall gain ϕ. When the pole on the unit circle is a triple pole, the three departure angles will be spaced 60 degrees apart, and two of the roots in the root locus plot will depart outward from the unit circle. It is not a particularly important issue, but a fourth standard plot is the minimum gain ϕ needed for stability, as a function of value of *c*. One can also record the maximum gain for stability, which can become of interest when we generalize the class of systems.

Generalization of the Design Process: The above discussion considered that all zeros of $G(z)$ were inside the unit circle so that all zeros could be cancelled by poles. When the pole excess of $G(s)$ is three or more, and the sample rate is not unusually slow, there is one or more zeros introduced outside the unit circle when converting to $G(z)$. There are at least two main approaches to dealing with these zeros. One is the compensator design optimization in the frequency domain due to Panomruttanarug and Longman discussed in Longman (2010). And the second is due to Tomizuka, also discussed in the same work. Here we consider the latter approach because it lends itself to making design tool plots analogous to those given above. One designs $\Phi(z)$ to contain the gain ϕ as before, and to cancel all poles and zeros of $G(z)$ that are inside the unit circle. Then for any zero outside the unit circle on the negative real axis, one introduced a zero at the reciprocal location inside the unit circle, and also introduces a pole at the origin. Then one can normalize so that the DC gain without considering ϕ is unity.

Generally, there is one zero outside the unit circle for pole excess of 3 or 4 in $G(s)$, two outside for excess of 5 or 6, etc. And the locations of these zeros approach asymptotic values as *T* approaches zero (given by Astrom and also discussed in Longman (2010)), and does so reasonably quickly with sample rate. Given any pole excess and the associated zero location(s), one can again produce standard plots to use in the design process, analogous to those discussed above. A new aspect is that the notches no longer need to be identical, and become a function of *p*. One can use the magnitude frequency response of the sensitivity transfer function and the root locus plot as a function of ϕ, to see its effect. Again one can produce the same standard plots to assist in creating the design tradeoffs, the notch width plot, the height of the peaks inside the notches, the height of the peaks between the notches, and the stability range for ϕ.

5. SIMULATION RESULTS

There are many methods to design the compensator Φ(z), in order to focus our method, we assume the $G(z)$ is minimal phase and stable, then create a compensator that is equal to $\phi / G(z)$, where ϕ is a scalar gain parameter. From the subsection 3.2, we have the third order RC as follows

$$R_3(z) = \frac{\Phi(z)\left[\alpha_1 z^{2p} + \alpha_2 z^p + \alpha_3\right]}{z^{3p} - \left[\alpha_1 z^{2p} + \alpha_2 z^p + \alpha_3\right]} = \frac{\Phi(z)\left[\alpha_1 z^{2p} + \alpha_2 z^p + \alpha_3\right]}{(z^p - 1)(z^p - q_1)(z^p - q_2)}$$

We choose the sampling rate f=100 [Hz], and the reference signal is periodic with period *p*=10 time steps. So the fundament frequency is 10 [Hz], Nyquist frequency is 50 [Hz], and

$$\Delta\theta = \frac{2\pi}{p} = \frac{2\pi}{10} = 36[\deg] = 0.628[rad], \quad \Delta f = \frac{f}{p} = 10[\text{Hz}].$$

The root locus plot for the 3rd order RC system is defined by

$$\frac{\phi\left[\alpha_1 z^{2p} + \alpha_2 z^p + \alpha_3\right]}{(z^p - 1)(z^p - q_1)(z^p - q_2)} = -1$$

when ϕ changed from 0 to ∞. The sensitivity transfer function is defined by $S(z) = [1 + R_3(z)G(z)]^{-1}$ We use Matlab for simulation. For *c*=0, 0.1, 0.2, we plot out the loot locus and the sensitivity transfer functions $(\phi = 1)$.

Fig.2 is the case of subsection 3.1, which makes each of the p original poles into triple poles. It does widen notches but the effect is limited. Fig.3 is the case for *c*=0. 1. Compared with Fig.2, the triple pole in same place split a litter. The interval frequency between these poles is 10% of the fundamental frequency, that is $10 \times 0.1 = 1[\text{Hz}]$. That means the notch is extended from 9 [Hz] to 11 [Hz] around the fundamental frequency (Fig.3 (b)). Fig.4 is the case for *c*=0. 2, compare with the Fig.3, the triple pole in same place split more wide. The interval frequency between these poles is 20% of the fundamental frequency, that is $10 \times 0.2 = 2[\text{Hz}]$. That means the notch is extended from 8 [Hz] to 12 [Hz] around the fundamental frequency (Fig.4 (b)). These simulations show using the parameter *c* as a freedom, we can obtain the desired

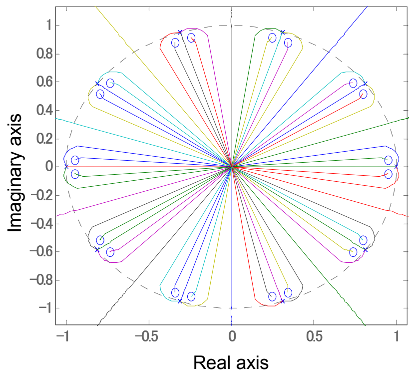

Fig.2. (a) Rout locus for *c*=0.

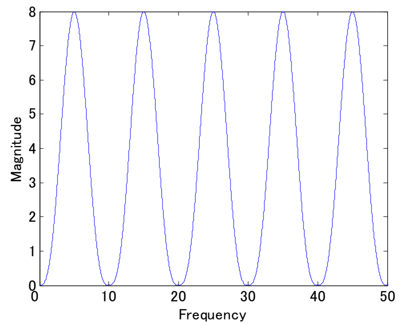

Fig.2 (b) Sensitivity transfer function for *c*=0.

range of the notch. It is very useful in practical applications.

6. CONCLUSIONS

In feedback control, normally one can only attenuate the effects of disturbances on the output. However, repetitive control is capable of completely eliminating the influence of a periodic disturbance, but it uses knowledge of the disturbance period to do so. The usual first order RC has a frequency response plot from disturbance to error with a cusp that goes to zero at all frequencies of the given period. But slight changes in the period move the response up the sides of the cusp very quickly, making RC very sensitive to accurate knowledge of the disturbance period being addressed. In some applications, the disturbance period fluctuates with time, and to address such applications one needs to have RC that has wider notches in place of the cusps, making the RC robust to period fluctuations. Often one uses a sensor to keep the RC period tuned to the actual period of the disturbance. However, Ahn et al. (to appear) cites situations in which the discretization produced by picking an integer value p for the number of time steps in a period, is enough to see a need for widening notches. This need also appears when one is monitoring a drifting period, one that has non negligible change within a period. This paper develops a systematic design process to produce RC that is robust to period fluctuations or period uncertainties, and which can also be used when the period drifts. Design aids or design tools are presented to help the designer address the issues and tradeoffs. These plots show what benefits are obtained by a choice of design variables, and at what expense in terms of tradeoffs with other performance characteristics.

REFERENCES

Ahn ES, Longman RW, and Kim JJ (to appear). Comparison of multiple-period and higher order repetitive control used to produce robustness to period fluctions. *Advances in the Astronautical Sciences*.

Ahn ES, Longman RW, Kim JJ, and Agrawal BN (2013) Improving laser communication between formation flying satellites using repetitive control jitter mitigation. 7th International Workshop on Satellite Constellations and Formation Flying, 13-15 March, Lisbon, Portugal.

Lo C-P, Longman RW. (2005). Root locus analysis of higher order repetitive control. *Advances in the Astronautical Sciences,* 120, 2021–2040

Lo C-P, Longman RW. (2006). Frequency response analysis of higher order repetitive control. *Advances in the Astronautical Sciences,* 123, 1183–1202

Longman, R.W. (2010). On the theory and design of linear repetitive control systems. *European Journal of Control,* 5, 447-496

Steinbuch, M. (2002). Repetitive control for systems with uncertain period. *Automatica*, 38(12), 2103-2109

Steinbuch, M., Weiland, S., Eerenbeemt, J.V.d., and Singh, T. (2004). On noise– and period-time sensitivity in high order repetitive control. *43rd IEEE Conference on Decision and Control*, 1295-1300.

Tsao TC, Qian XY, Nemani M. (2000). Repetitive control for asymptotic tracking of periodic signals with an uncertain period. *J Dyn Syst Meas Control,* 122, 364-368.

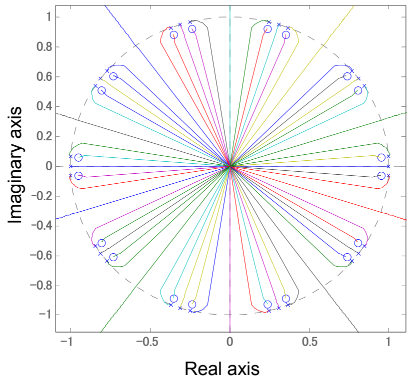

Fig.3 (a) Rout locus for c=0.1.

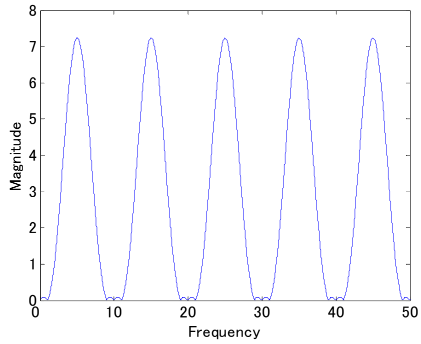

Fig.3 (b) Sensitivity transfer function for c=0.1

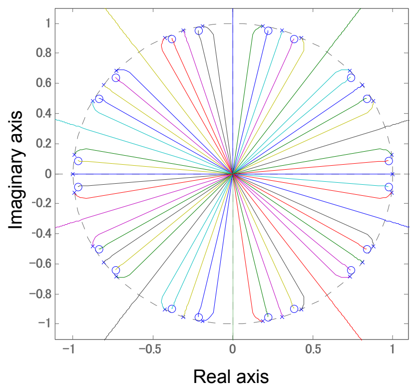

Fig.4 (a) Rout locus for c=0.2.

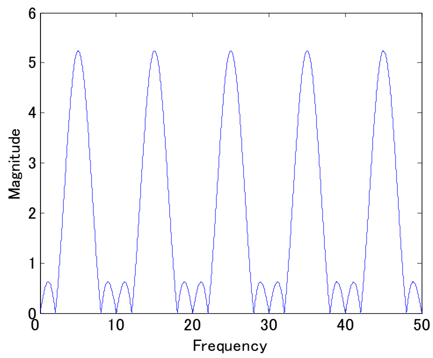

Fig.4 (b) Sensitivity transfer function for c=0.2.