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# A Novel Design of Unknown Input Observer for Fault Diagnosis in Non-minimum Phase Systems

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**Abstract**: Fault diagnosis has become a significant issue due to rising demands for reliability as well as higher system performance and product quality. There are various approaches for the goal of fault detection and isolation among which we focus on the unknown input observer since observer-based schemes have been successfully adopted in a variety of application fields. This paper presents a novel design of Unknown Input Observer (UIO) for fault detection and isolation which can be applied to non-minimum phase systems. Unlike many previous UIO methods developed in the literature, the proposed scheme can carry out real-time reconstruction of unknown input in the presence of unstable invariant zero (with respect to the relation between the output and the unknown input). We extend the system by augmenting low pass filter(s) in the input(s)-output(s) path to tackle the non-minimum phase problem and generate robust residuals. Finally, the effectiveness of the proposed design method is confirmed through demonstrating a simulation-base example.

*Keywords:* Fault diagnosis, unknown input observer, non-minimum phase, augmentation.

## 1. INTRODUCTION

Fault diagnosis has become a crucial subject due to rising demands for reliability on one side and higher system performance and product quality on another. (Termehchy, Afshar and Javidsharifi, 2013). There are various approaches for the goal of fault detection and isolation (FDI) among which unknown input observer is focused on in this research since the observer-based schemes have been successfully adopted in a variety of applications (Zarei and Poshtan, 2011). UIO is a Luenberger type observer that delivers a state estimation independent of unknown input (Ding, 2008, p. 142). The design of the unknown input observer and its application to the Fault Detection and Isolation (FDI) is an important subject and has been put forward by several authors (Guan and Saif 1991, Hou and Muller 1992, Fernando and Trinh 2007, Aldeen and Sharma 2008, Jo, Shim and Son 2010, Ting, Chang and Chen 2011, Jia, Zhan, Chen and Shen 2012, Capisani, Ferrara, Ferreira de Loza and Fridman 2012, Mrugalski and Witczak 2013, Fernando, Dougall, Sreeram and Trinh 2013). The basic idea behind the development of UIO-based fault diagnosis technique is to make greater robustness against model uncertainties and unknown input disturbances. It can be demonstrated that the existence of a stable UIO has two necessary and sufficient conditions which comprise the transfer function matrix between the unknown input and the system output must be minimum phase and with relative degree one. If one of the conditions mentioned above is not satisfied, how to design a stable UIO will be a difficult research problem (Ting, Chang and Chen, 2011). Xiong and Saif (2003) have proposed two reduced-order input estimators, one of which an extension of a state/input estimator and the other based on the adaptive observer design technique. The proposed adaptive observer can be designed for certain non-minimum phase systems. Marro and Zattoni (2010), have shown that the unknown-state, unknown-input reconstruction problem can be solved for non-minimum phase systems, provided that a reconstruction delay is allowed, whose length is related to the dynamics of the unstable invariant zeros. The same problem is considered in (Kirtikar, Palanthandalam-Madapusi, Zattoni and Bernstein 2011,Ting, Chang and Chen 2011). Although algebraic methods devised in (Kirtikar, Palanthandalam-Madapusi, Zattoni and Bernstein, 2011) do not consider any reconstruction delay related to the unstable zero dynamics, it is concluded that real-time reconstruction of the unknown input is not possible in the presence of unstable invariant zeroes. Similar conclusion is presented in Ting, Chang and Chen (2011) which proposed a reduced-order UIO design method to effectively estimate the system state and avoid the peaking phenomenon. This paper deals with UIO design problem for actuator fault detection and isolation for systems having unstable invariant zeroes (with respect to the relation between the output and the unknown input). We extend the system by augmenting low pass filter(s) in the input(s)-output(s) path to tackle non-minimum phase problem and reconstruct the unknown input as real-time. The organization of this paper is as follows: Section 2 presents the FDI technique using UIO. Section 3 describes the problem definition and the proposed approach. Section 4 applies the proposed approach to a simplified version of the Tennessee Eastman Control and shows simulation results which demonstrate the effectiveness and capabilities of the suggested method.

## 2. FAULT DETECTION AND ISOLATION TECHNIQUE USING UIO

### *2.1Unknown Input Observer Design*

 

Consider a system with the unknown input described by the following equations:

$$\dot{x}(t) = Ax(t) + Bu(t) + Ed(t),$$

$$y(t) = Cx(t) \qquad (1)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^p$ is the input vector, $d(t) \in R^q$ is the unknown input vector and $y(t) \in R^m$ is the output vector. A, B, E and C are constant matrices of appropriate dimensions. Despite the fact that the input $d(t)$ is unknown, UIO's goal is to estimate the system state $x(t)$ accurately. Busawon and Kabore (2001) demonstrated that the conventional Luenberger observer is not suitable to overcome unknown inputs. For the system (1), the UIO is as follows (Darouach, 2009):

$$\dot{z}(t) = Fz(t) + TB\,u(t) + Ky(t),$$

$$\hat{x}(t) = z(t) + Hy(t),\ \ \hat{x} \in R^n, z \in R^n \qquad (2)$$

A UIO is a Luenberger type observer that delivers a state estimation $\hat{x}(t)$ independent of the unknown input $d(t)$ in the sense that (Ding, 2008):

$$\lim_{t\to\infty}(x(t) - \hat{x}(t)) = 0 \ \text{ for all } u(t), d(t), x_0 \qquad (3)$$

The state estimation error and the residual are defined by:

$$e(t) = x(t) - \hat{x}(t) \qquad (4)$$

$$r(t) = y(t) - C\,\hat{x}(t) \qquad (5)$$

If the following equations hold:

$$K = K_1 + K_2, (HC - I)E = 0, T = I - HC,\ F = A - HCA - K_1,\ K_2 = FH \qquad (6)$$

Then $\dot{e}(t)$ will become:

$$\dot{e}(t) = Fe(t) \qquad (7)$$

So, as can be seen, if the eigenvalues of F are stable, regardless of what $d(t)$ is, the state estimation error will reach zero asymptotically. This means that the observer is insensitive to the unknown input.

**Theorem1**(Ding, 2008). The necessary and sufficient conditions for the existence of a UIO for the system defined by (1) are:

- Condition I: $\mathrm{rank}(CE) = \mathrm{rank}(E)$
- Condition II: (TA, C)is detectable, where

$$TA = A_1 = A - E[(CE)^T(CE)]^{-1}(CE)^T CA = A - HCA$$

**Definition1.** Assume $s = z_0$ is not a pole of system (1), then $z_0$ is a transmission zero of the system if and only if $\mathrm{rank}(G(z_0)) < \min(m, p)$; where $G(s)$ is the transfer function of the system. In other words, there exists a vector $u_0$ and a scalar $z_0$ such that $u(t) = u_0 e^{z_0 t}$ result in the output $y(t)$ not containing $e^{z_0 t}$ terms.

Theorem 1 can be reformulated as:

**Corollary1**(Ding, 2008). Given system (1), there exists a UIO in the sense of (3) if

- $\mathrm{rank}(CE) = \mathrm{rank}(E)$
- (A, E, C) has no unstable transmission zero.

*2.2 Fault Detection and Isolation*

Fault detection and isolation can be obtained by generating structured residuals through different methods; one more applicable method is to use a bank of UIO's.

Assuming that all sensors are fault-free, the system (1) subject to actuator fault can be represented as:

$$\dot{x}(t) = Ax(t) + B\big(u(t) + f_a(t)\big) + Ed(t),$$

$$y(t) = Cx(t) \qquad (8)$$

where $f_a(t) \in R^p$ is an immeasurable vector considered as an additive bias arising from actuator fault. The residual is checked in terms of the likelihood of fault through a simple comparison between an adaptive or constant threshold (T(t)) and the Euclidean norm of the residual ($\|r(t)\|$):

$\|r(t) = y(t) - C\,\hat{x}(t)\| > T(t)$ : Faulty case

$\|r(t) = y(t) - C\,\hat{x}(t)\| \le T(t)$ : Fault-free case

In order to isolate the fault, the banks of observers are designed such that they are sensitive to certain faults provided they are insensitive to other faults of the system. To this end, the following table is helpful. In this table, "1" in the i-th column and the j-th row indicates that residual $r_j$ is a function of $f_{u_i}$ and "0" means that $r_j$ is decoupled from $f_{u_i}$.

**Table 1.Structured residuals**

| | $f_{u1}$ | $f_{u2}$ | $\cdots$ | $f_{ui}$ |
|---|---|---|---|---|
| $r_1$ | 0 | 1 | 1 | 1 |
| $r_2$ | 1 | 0 | 1 | 1 |
| $\vdots$ | | | | |
| $r_j$ | 1 | 1 | $\cdots$ | 0 |

With the purpose of generating $r_j$:

$$\dot{z}^j(t) = F^j z^j(t) + T^j B^j u^j(t) + K^j y(t),$$

$$r_j = \big(I - CH^j\big)y(t) + Cz^j(t) \qquad (9)$$

where the j-th column of B ($b_j$) is eliminated and added to E thereby it is formed $B^j \in R^{n*(p-1)}$, $E^j \in R^{n*(q+1)}$.

$H^jCE^j = E^j$, $T^j = I - H^jC$, $F^j = T^jA - K_1{}^jC$, $K_2{}^j = F^jH^j$, $K^j = K_1{}^j + K_2{}^j$ (Note that, $F^j$must be Hurwitz.)

The necessary and sufficient conditions for the each observer existence are as given in theorem1:

- Condition I: $\mathrm{rank}(CE^j) = \mathrm{rank}(E^j)$
- Condition II: (A, $E^j$, C) has no any unstable transmission zero

## 3. UIO DESIGN FOR NON- MINIMUM PHASE SYSTEM USING AUGMENTATION APPROACH

### *3.1 Problem Definition*

Consider the MIMO linear system as (1) and designing a UIO to generate $r_{j=s}$:

$$\dot{x}(t) = Ax(t) + B^s u^s(t) + E^s \begin{bmatrix} d \\ u_s \end{bmatrix},$$

$$y(t) = Cx(t) \tag{10}$$

$$x(t) \in R^n, u^s(t) \in R^{p-1}, y(t) \in R^m, d(t) \in R^q, A \in R^{n*n}, B^s \in R^{n*(p-1)}, E^s \in R^{n*(q+1)}.$$

with the following conditions:

- $\mathrm{rank}(CE^s) = \mathrm{rank}(E^s)$
- ($T^sA$, C) is not detectable, in other words (A,$E^s$,C) has unstable transmission zero(es).
- $q + 1 <= m$ => min(m, (q+1)) = q+1

According to theorem 1 and references (Ting, Chang and Chen, 2011, Kirtikar, Palanthandalam-Madapusi, Zattoni and Bernstein, 2011), it is impossible to design a real time UIO to generate $r_s$ because of the unstable transmission zero.

### *3.2 Augmentation Approach To Solve Non-minimum Phase Problem*

Consider the augmented system below:

$$\begin{bmatrix} \dot{x} \\ \dot{\bar{x}} \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & \bar{A} \end{bmatrix} \begin{bmatrix} x \\ \bar{x} \end{bmatrix} + \begin{bmatrix} B^s \\ \overline{B^s} \end{bmatrix} u^s(t) + \begin{bmatrix} E^s \\ \overline{E^s} \end{bmatrix} \begin{bmatrix} d \\ u_s \end{bmatrix},$$

$$\begin{bmatrix} y \\ \bar{y} \end{bmatrix} = \begin{bmatrix} C & 0 \\ 0 & \bar{C} \end{bmatrix} \begin{bmatrix} x \\ \bar{x} \end{bmatrix} \tag{11}$$

$$x(t) \in R^n, u^s(t) \in R^{p-1}, y(t) \in R^m, d(t) \in R^q, A \in R^{n*n},$$

$$B^s \in R^{n*(p-1)}, E^s \in R^{n*(q+1)}$$

$$\bar{x} \in R^l, \bar{y} \in R^k, \bar{A} \in R^{l*l}, \overline{B^s} \in R^{l*(p-1)}, \overline{E^s} \in R^{l*(q+1)}, \bar{C} \in R^{k*l}$$

$$A' = \begin{bmatrix} A & 0 \\ 0 & \bar{A} \end{bmatrix}, B' = \begin{bmatrix} B^s \\ \overline{B^s} \end{bmatrix}, E' = \begin{bmatrix} E^s \\ \overline{E^s} \end{bmatrix}, C' = \begin{bmatrix} C & 0 \\ 0 & \bar{C} \end{bmatrix} \tag{12}$$

where $x(t)$,$u(t)$, $d(t)$, $y(t)$ are the state, input, unknown input and output vector of system (10) respectively and A, B, E and C are constant matrices of system (10). $\bar{x}(t) \in R^l$ is the augmented state vector and $\bar{y}(t) \in R^k$ is the augmented output vector. $\bar{A}$,$\bar{B}$,$\bar{E}$ and $\bar{C}$ are constant matrices of appropriate dimensions. Comparing systems (10) and (11), it can be understood that the main system's state variable $x(t)$ and output $y(t)$ aren't affected by augmented state variables $\bar{x}(t)$ and output $\bar{y}(t)$. Inputs of both systems are also the same.

**Theorem2.** Given system (10) with unstable transmission zeroes(with respect to the relation between the output and the unknown input) at least one augmented system can always be found, whose matrices are constructed as (11), and the augmented system does not have any unstable transmission zeros.

**Proof:**

The transfer function matrix of system (A, $E^s$, C) is:

$$G(s) = C(sI - A)^{-1}E^s \tag{13}$$

The transfer function matrix of system $(A', E', C')$ is:

$$G'(s) = \begin{bmatrix} C & 0 \\ 0 & \bar{C} \end{bmatrix} \begin{bmatrix} (sI - A) & 0 \\ 0 & (sI - \bar{A}) \end{bmatrix}^{-1} \begin{bmatrix} E^s \\ \overline{E^s} \end{bmatrix}$$

$$= \begin{bmatrix} C & 0 \\ 0 & \bar{C} \end{bmatrix} \begin{bmatrix} (sI - A)^{-1} & 0 \\ 0 & (sI - \bar{A})^{-1} \end{bmatrix} \begin{bmatrix} E^s \\ \overline{E^s} \end{bmatrix}$$

$$G'(s) = \begin{bmatrix} C(sI - A)^{-1}E^s \\ \bar{C}(sI - \bar{A})^{-1}\overline{E^s} \end{bmatrix} = \begin{bmatrix} G(s) \\ \bar{G}(s) \end{bmatrix} \tag{14}$$

$$\begin{cases} \bar{A} \in R^{l*l}, \overline{E^s} \in R^{l*(q+1)} \text{ and } \bar{C} \in R^{k*l} \;=>\; \bar{G}(s) \in R^{k*(q+1)} \\ A \in R^{n*n}, E^s \in R^{n*(q+1)} \text{ and } C \in R^{m*n} \;=>\; G(s) \in R^{m*(q+1)} \end{cases}$$

$$=> G'(s) \in R^{(m+k)*(q+1)}$$

Assume that there is a $z_0 \in C^+$ which is not pole of the system (A, $E^s$, C) and $\mathrm{rank}(G(z_0)) = q$. Then the $G(z_0)$ matrix either has a zero column or includes one column which is a multiple of another. In other words:

$$\text{a)} \begin{bmatrix} g_{1i}(z_0) \\ \vdots \\ g_{mi}(z_0) \end{bmatrix} = 0, \quad 1 \le i \le q+1 \quad \text{or}$$

$$\text{b)} \begin{bmatrix} g_{1i}(z_0) \\ \vdots \\ g_{mi}(z_0) \end{bmatrix} = K \begin{bmatrix} g_{1j}(z_0) \\ \vdots \\ g_{mj}(z_0) \end{bmatrix}, 1 \le i, j \le q+1 \text{ and } K \in Z, K \neq 0$$

In both cases at least one row ($\bar{G}(s)$) can be found as follows such that by augmenting it to G(s), a $G'(s)$ is produced such that $\mathrm{rank}(G'(z_0)) = q+1$:

In the case of (a):

$$\bar{G}(s) = [\bar{g}_{m+11}(s), \dots, \bar{g}_{m+1q+1}(s)] \text{ such that } \bar{g}_{(m+1z\neq i)}(s) = 0,$$
$$\bar{g}_{m+1i}(s) = \frac{c * d}{s + a} \text{ and } \bar{g}_{m+1i}(z_0) \neq 0$$

$$\bar{A} = [-a], \overline{B^s} = 0, \bar{C} = [c], \overline{E^s} = [e_1, \dots, e_{q+1}] \text{ such that } e_{(v\neq i)} = 0 \text{ and } e_i = d, \quad c, d \in Z, a \in Z^+ \text{ and } a, c, d \neq 0$$

In the case of (b):

$\bar{G}(s) = [\bar{g}_{m+11}(s), \dots, \bar{g}_{m+1q+1}(s)]$ such that $\bar{g}_{m+1z(z\neq i,j)}(s) = 0$, $\bar{g}_{m+1i}(s) = \frac{c*d_1}{s+a}$, $\bar{g}_{m+1j}(s) = \frac{c*d_2}{s+a}$ and $\bar{g}_{m+1i}(z_0) \neq K\,\bar{g}_{m+1j}(z_0)$

$\overline{A} = [-a], \overline{B^s} = 0, \overline{C} = [c], \overline{E^s} = [e_1, \dots, e_{q+1}]$ such that
$e_v = 0(v \neq i, j)$ and $e_i = d_1, e_j = d_2$,
$c, d_1, d_2 \in Z, a \in Z^+, a, c, d_1, d_2 \neq 0$ and $d_1 \neq kd_2$

Note that if in $G(z_0)$ more than one column become equal to zero, then the number of non-zero elements in $\overline{E^s}$ should be the same as the number of these zero columns; the same is true for the condition of more than two multiple columns. In other words, if $\mathrm{rank}(G(z_0)) = (q+1) - p$, one will have:

(a):

$$\overline{G}(s) = [\overline{g}_{m+11}(s), \dots, \overline{g}_{m+1q+1}(s)] \quad \text{such that}$$
$$\overline{g}_{(m+1z\neq i_1,\dots,i_p)}(s) = 0, \overline{g}_{m+1i_1}(s) = \frac{c * d_1}{s+a}, \dots, \overline{g}_{m+1i_p}(s) = \frac{c * d_p}{s+a} \text{ and } \overline{g}_{(m+1z=i_1,\dots,i_p)}(z_0) \neq 0.$$

$\overline{A} = [-a], \overline{B^s} = 0, \overline{C} = [c], \overline{E^s} = [e_1, \dots, e_q]$

$e_{(v\neq i_1,\dots,i_p)} = 0$ and $e_{i_1} = d_1, \dots, e_{i_p} = d_p$. $c, d_1, \dots, d_p \in Z, a \in Z^+$

and $a, c, d_1, \dots, d_p \neq 0$( $d_1, \dots, d_p$ are not multiple of each other)

(b):

$$\overline{G}(s) = [\overline{g}_1(s), \dots, \overline{g}_{q+1}(s)] \quad \text{such that} \quad \overline{g}_z(s) = 0(z \neq i_1, \dots, i_p),$$
$$\overline{g}_{i_1}(s) = \frac{c * d_1}{s+a}, \dots, \overline{g}_{i_p}(s) = \frac{c * d_p}{s+a} \text{ and } \overline{g}_{i_i}(z_0) \neq K_j \overline{g}_{i_j}(z_0)$$

$\overline{A} = [-a], \overline{B^s} = 0, \overline{C} = [c], \overline{E^s} = [e_1, \dots, e_{q+1}]$:

$e_v = 0(v \neq i_1, \dots, i_p)$ and $e_{i_1} = d_1, \dots, e_{i_p} = d_p$

$c, d_1, \dots, d_p \in Z, a \in Z^+, a, c, d_1, \dots, d_p \neq 0$ and $d_i \neq k_j d_j$

Consequently, a $G'(s)$ can be always produced by augmenting appropriate row(s) to transfer function matrix $G(s)$ which is full rank at $z_0$. In other words, at least one set of $\overline{A}$, $\overline{B^s}$, $\overline{E^s}$ and $\overline{C}$ matrices can be always found such that the system $(A', E', C')$ doesn't have any unstable transmission zero.

## 4. TENNESSEE EASTMAN PROCESS CONTROL SYSTEM AND SIMULATION RESULTS

### *4.1 System Model*

The Tennessee-Eastman challenge process is a realistic simulation of chemical system that has been widely used in process control studies (Juricek, Seborg and Larimore, 2001). A simplified model of the system was proposed by Ricker (1993) and Chabukswar, Mo and Sinopoli (2011) have given the model below which is based on Ricker model:

$$\begin{bmatrix} F_4 \\ P \\ y_{A3} \\ V_L \end{bmatrix} = \begin{bmatrix} g_{11} & 0 & 0 & g_{14} \\ g_{21} & 0 & g_{23} & 0 \\ 0 & g_{32} & 0 & 0 \\ 0 & 0 & 0 & g_{44} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \tag{15}$$

The individual transfer functions are given in following equations:

$$g_{11} = \frac{1.7}{0.75s+1}, g_{21} = \frac{45(5.667s+1)}{2.5s^2+10.25s+1}$$
$$g_{23} = \frac{-15s-11.25}{2.5s^2+10.25s+1}, g_{32} = \frac{1.5}{10s+1} e^{-0.1s}$$
$$g_{14} = \frac{-3.4s}{0.1s^2+1.1s+1}, g_{44} = \frac{1}{s+1}$$

Nominal values for steady state operation of the eight state, four manipulated and four output variables of the system are given in (Termehchy, Afshar and Javidsharifi, 2013)

### *4.2 Problem Definition:*

Consider that the fault may occur because of the actuator failure of the system, so the state space representation of the TE-PCS can be written as follows:

$$\dot{x}(t) = Ax(t) + B(u(t) + f_a(t)) + Ed(t),$$
$$y(t) = Cx(t) \tag{16}$$

$$\mathbf{A} = \begin{bmatrix} -1.333 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -11 & -2.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & -4.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -20.1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 0 \\ 4.5 & 0 & -1.125 & 0 \\ 12.75 & 0 & -0.75 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1.333 & -4.25 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.075 & 1.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$
$$E = [1,1,1,1,1,1,1,1]^T$$

Generating $r_2$, we obtained that $(T^2A, C)$ is not detectable. To solve this problem, we employ our augmentation approach as given in next section. Other residuals generating procedure does not bring any insight in to the application of the proposed method and the conventional UIO methods can be implemented therefore they are not discussed here and we only show their simulation results.

### *4.3 Augmenting The System For Solving The Non minimum Phase Problem*

To generate $r_2$ , we write system (16) as:

$$\dot{x}(t) = Ax(t) + B^2(u^2(t) + f_a(t)) + E^2 \begin{bmatrix} d(t) \\ u_2 \end{bmatrix}$$
$$y(t) = Cx(t) \tag{17}$$

$$B^2 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 8 \\ 0 & 0 & 0 \\ 4.5 & -1.125 & 0 \\ 12.75 & -0.75 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E^2 = [E\, b_2]$$

The transfer function between the $[d\ u_2]^T$ and the system output ( $[F_4\ P\ y_{A3}\ V_L]^T$) will be:

$$\begin{bmatrix} F_4 \\ P \\ y_{A3} \\ V_L \end{bmatrix} = \begin{bmatrix} \frac{-3.117s^2+17.42s+25.49}{s^3+12.33s^2+24.66s+13.33} & 0 \\ \frac{8s+4}{s^2+4.1s+0.4} & 0 \\ \frac{-1.425s+31.8}{s^2+20.1s+2} & \frac{-0.15s+3}{s^2+20.1s+2} \\ \frac{1}{s+1} & 0 \end{bmatrix} \begin{bmatrix} d \\ u_2 \end{bmatrix} \quad (18)$$

The unstable transmission zero of the system between the output and the $[d, u_2]^T$ is 20 and so $(T^2A, C)$ is not detectable. Because of the unstable invariant zero, conventional UIO methods cannot be applied and to solve the problem, the augmentation approach is used. We augment the system with the state and output as follows:

$$\dot{x}_9 = -x_9 + u_2,$$

$$y_5 = x_9 \quad (19)$$

The augmented system can be considered as:

$$\dot{x}(t) = A'x(t) + B'u^2(t) + E'\begin{bmatrix} d(t) \\ u_2(t) \end{bmatrix},$$

$$y(t) = C'x(t) \quad (20)$$

where:

$$A' = \begin{bmatrix} A_{8*8} & 0_{8*1} \\ 0_{1*8} & -1_{1*1} \end{bmatrix}, B' = \begin{bmatrix} B^s \\ \overline{B^s} \end{bmatrix}, E' = \begin{bmatrix} E^s \\ \overline{E^s} \end{bmatrix}$$

$$C' = \begin{bmatrix} C_{4*8} & 0_{4*1} \\ 0_{1*8} & 1_{1*1} \end{bmatrix}, \overline{B^s} = [0\ 0\ 0],\ \overline{E} = [0 \quad 1]$$

In order to generate $r_2$:

$$H'^2C'E'^2 = E'^2, T'^2 = I - H'^2C', F'^2 = T'^2A' - K_1'^2C',$$

$$K_2'^2 = F'^2H'^2, K'^2 = K_1'^2 + K_2'^2$$

The transfer function between the $[d\ u_2]^T$ and the system output ( $[F_4\ P\ y_{A3}\ V_L\ y_5]^T$) will be:

$$\begin{bmatrix} F_4 \\ P \\ y_{A3} \\ V_L \\ y_5 \end{bmatrix} = \begin{bmatrix} \frac{-3.117s^2+17.42s+25.49}{s^3+12.33s^2+24.66s+13.33} & 0 \\ \frac{8s+4}{s^2+4.1s+0.4} & 0 \\ \frac{-1.425s+31.8}{s^2+20.1s+2} & \frac{-0.15s+3}{s^2+20.1s+2} \\ \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+1} \end{bmatrix} \begin{bmatrix} d \\ u_2 \end{bmatrix} \quad (21)$$

As for the second column of the matrix above, it is seen that each vector $u_0$ while $u(t) = u_0e^{20t}$, results in the output $y(t)$ containing $e^{20t}$ terms. By repeating UIO procedure for the augmented system to generate $r_2(t)$, it is observed that $(T^{2'}A', C')$ is detectable and the system of $(A', E', C')$ does not have any unstable zeros, figure 2 and 3 in the next section confirms this issue. Augmentation procedure is presented in figure 1.

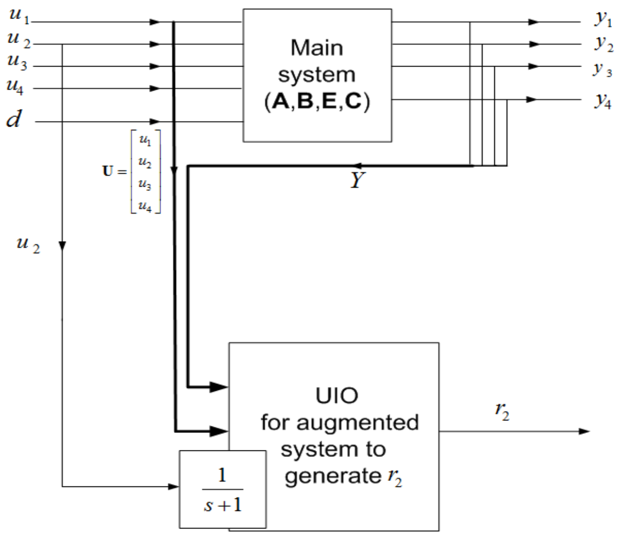


Fig.1. Augmentation Procedure to generate $r_2$ .

*5.4 Simulation Results*

Results of two actuator failures are presented here.

- $u_1$ failure at t=15s:

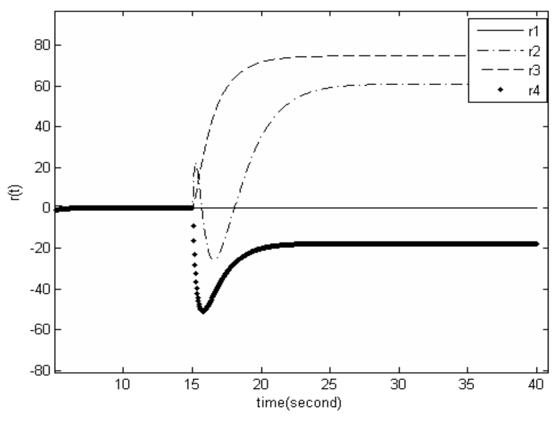


Fig.2. Residuals for $u_1$ failure at t=15s.

- $u_2$ failure at t=15s:

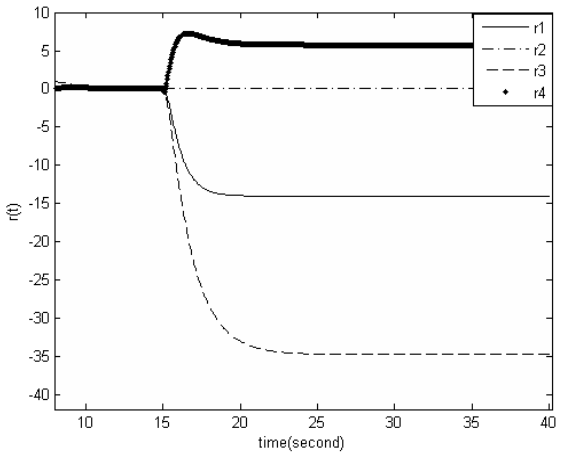


Fig.3. Residuals for $u_2$ failure at t=15s.

At t=15, a fault has occurred in $u_{i=1}$ and $u_{i=2}$. Fig. 2 and 3 show that $r_j$ $(j \neq i)$ have a change after t=15 but $r_i$ is unaffected. This means that a fault has occurred in $u_i$. Fig. 3 has demonstrated the effectiveness and capabilities of the suggested method.

According to Theorem 1, It is impossible to generate $r_2$ for system (15) because of the unstable transmission zero at s=20. Fig.2 and Fig. 3 demonstrate the effectiveness and capabilities of the suggested method to solve this problem. As is clear, the residuals obtained by running the augmentation approach do not have any reconstruction delay.

## 5. CONCLUSIONS

In this paper the augmentation approach is proposed to design UIO for fault detection and isolation in non-minimum phase systems. We extend the system by augmenting a very simple structure low pass filter(s) in the input(s)-output(s) path. The advantages of proposed approach are real time reconstruction and simple implementation.

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