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Network design for distributed consensus estimation over heterogeneous sensor networks

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Abstract: This paper considers distributed estimation over heterogeneous sensor networks. We propose a distributed estimation strategy based on PageRank algorithm, where the link weight depends on the edge estimation covariances. We prove that the proposed estimator obtains better estimates than one typical estimator under identical initial conditions. Motivated by the advantage of the sensors with high accuracy locating at important positions, we propose an optimal sensor deployment strategy to arrange locations for sensors in a given physical network. Numerical examples are provided to demonstrate the effectiveness of the proposed estimator, showing a better performance under the optimal sensor deployment.

1. INTRODUCTION

Fueled by applications in a variety of fields including battlefield surveillance, intelligent transportation, environment monitoring, health care, etc., there has been a recent surge of interest in distributed state estimation using a wireless sensor network (WSN) which is composed of a large number of geographically distributed sensor nodes, which are capable of measuring certain parameters of interest such as temperature, humidity, position and velocity of a vehicle. In the last decade, many works on consensus-based distributed estimation have been reported since it can drastically reduce the communication resource requirements, where each sensor can observe the target state and exchange the estimates with its neighbors. By doing that, some advantages are obtained such as no requirement on network topology, lower energy cost, more flexible for ad-hoc deployment when compared with centralized and decentralized estimations, see Anderson et al. [1979]-Iftar [1993].

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The design of efficient consensus-based distributed estimation algorithm is a current focus of active research in the literature. Under standard assumptions, distributed estimators combined with a consensus term give conditions to ensure that the estimate of each node approaches the state of the target asymptotically, see Spanos et al. [2005]-Shen et al. [2010]. The existing distributed estimation algorithms can be classified into two categories: one is adding a consensus term to the update step, see Olfati-Saber et al. [2005]-Olfati-Saber [2009] and the other is driving consensus on the priori estimate in the prediction step, see Stanković et al. [2009], Cattivelli et al. [2010]. More precisely, they can be referred to as distributed weighted average consensus algorithm in (Spanos et al. [2005], Spanos et al. [2005]), distributed Kalman filtering in (Olfati-Saber et al. [2005]-Olfati-Saber [2009]), decentralized state estimation with intermittent observations and communication faults in (Stanković et al. [2009]), diffusion strategies for distributed Kalman filtering and smoothing in (Cattivelli et al. [2010]), distributed estimation of deterministic signals with noisy links in (Schizas [2008]), distributed parameter estimation over a WSN with bit rate constraint in (Li et al. [2007]), adaptive consensus filter in (Demetriou [2010], Xi et al. [2010]), distributed consensus filtering algorithm with pinning observers in (Yu et al. [2009]), distributed H_∞-consensus filtering over a finite-horizon for sensor networks with multiple missing measurements in (Shen et al. [2010]).

From a practical point of view, however, the network estimation accuracy given identical credit to every node could be improved in a heterogeneous sensor network. In last decade, the PageRank algorithm employed at Google, which assigns a measure of importance to each webpage for rankings in search results, has gained more and more attraction. In the algorithm, those webpages who are more related to the keywords obtain higher rankings. As a result, they can be searched with higher probability. An intuitive conclusion is, giving higher credits to those sensors with higher estimation accuracy will lead to better estimates compared with identical credit to all the sensors. Similar to the PageRank algorithm, we can design edge weight between each pair of neighboring nodes to assign credit to each node. Currently, some related works also consider weight design but only provide numerical simulations. Stanković et al. [2009] designed relative weights which are proportional to the diagonal elements of the inverse of the covariance matrices of the local estimators to the communicated estimates. Demetriou [2010] proposed adaptive observers to the estimates including a coupling term which penalizes the disagreement of the estimates. In this paper, a distributed estimation algorithm based on the PageRank algorithm is proposed. The weight of the link from sensor i to sensor j depends on the diagonal elements of the inverse of the edge covariance matrices between sensor i and sensor j over the sum of the edge covariance matrices between sensor i and its neighboring sensors. Under standard assumptions, sensors with the proposed estimator approach the target state asymptotically. Given identical initial conditions, the proposed estimator obtains better estimates when compared with a typical consensus-based distributed estimator at each time step.

In parallel with weight design problem, sensor deployment under a given physical network topology influences network estimation accuracy in a heterogeneous sensor network as well. Intuitively, a sensor with high accuracy locating at an important location obtains better estimates when compared with a sensor with low accuracy. In this paper, an optimal sensor deployment strategy is designed by solving an optimization problem. To implement sensor deployment efficiently in practice, some indices characterizing node importance in complex network are utilized to order the positions in physical sensor networks.

The remainder of the paper is organized as follows. In Section 2, we describe the system model and introduce some notations. In Section 3, we design link weight for a consensus-based distributed estimation algorithm. A sufficient condition is given to guarantee the convergence of the proposed estimation algorithm. Differences between the improved weighted estimator and the constant weighted estimator are analyzed. In Section 4, an optimal sensor deployment strategy are proposed. In Section 5, we verify the results derived in Section 3 by a numerical example. Finally, some concluding remarks and future work are given in Section 6.

Notation. We use $\mathrm{tr}(\cdot)$ to denote the trace of a matrix. $\mathrm{vec}(A)$ is the vector formed by "stacking" the columns of A in the natural order. $\mathrm{diag}(A)$ denotes a diagonal matrix with the elements of main diagonal of A. The cardinality of a set A is denoted $|A|$. $A \otimes B$ is the Kronecker product of matrices A and B. We write $A \geq 0$ if A is positive semi-definite, and $A \geq B$ if $A - B \geq 0$. Moreover, $A > 0$ if A is positive-definite, and $A > B$ if $A - B > 0$. We use $\mathbf{1}$ to denote a vector of arbitrary dimension with each component equal to one. Let I denotes the identity matrix. For an $n \times n$ symmetric matrix A, $\lambda(A) \in \mathbb{R}^n$ denotes the vector of eigenvalues of A. The minimal product of two vectors x and y in $\mathbb{R}^n$ is denoted $< x, y >_-$, and is defined by $< x, y >_- = \min_\pi \prod_{i=1}^n x_i y_{\pi(i)}$, where $\pi(\cdot)$ is a permutation of $1, 2, \ldots, n$.

2. PROBLEM SETUP

Consider a given target system

$$x(k+1) = Ax(k) + w(k), \tag{1}$$

where $x(k) \in \mathbb{R}^m$ is the target system state at time step k, $w(k) \in \mathbb{R}^m$ is the system noise with zero mean and covariances $\mathbb{E}\{w(k)w(s)^T\} = Q\delta_{k,s}$, where $\delta_{k,s}$ is the Kronecker Delta function and $Q > 0$. The initial state $x(0)$ is also zero-mean Gaussian with covariance π_0, and is independent of $w(k)$ for all k. We consider $m = 2$ as all the results can be extended to $m > 2$ using Kronecker product.

Assume that a wireless sensor network consisting of n sensors, where each sensor can observe the target state. The measurement equation of the i-th sensor is given by,

$$y_i(k) = H_i x(k) + v_i(k), \tag{2}$$

where $y_i(k) \in \mathbb{R}^m$ is the measurement of i-th sensor at time step k, $v_i(k)$ is the measurement noise with zero mean and $\mathbb{E}\{v_i(k)v_i^T(k)\} = R_i\delta_{k,s}$, $R_i \geq 0$. Assume that $\mathbb{E}\{v_i(k)v_j^T(s)\} = 0$ for all $i \neq j$ and all s, k.

The wireless sensor network is modeled as an undirected graph $G = (V, E)$ with the nodes $V = \{1, 2, \ldots, n\}$ being sensors, and the edges $E \subset V \times V$ representing the available communication links. The existence of edge (i, j) means sensor i can exchange information with sensor j. Define the neighboring sensors of sensor i by $N_i = \{j : (i, j) \in E\}$. Let $d_i = |N_i|$ be the number of neighboring sensors of the i-th sensor. Define the Laplacian of G as $L = [l_{ij}]$, where $l_{ij} = -1$ if $(i, j) \in E, i \neq j$ and $l_{ii} = -\sum_{j=1}^n l_{ij}$.

Each sensor can receive the prior estimates of its neighbors. The estimator of i-th sensor is designed as follows:

$$\begin{aligned}
\hat{x}_i(k|k) &= \hat{x}_i(k|k-1) + K_p^i(k)[y_i(k) - H_i\hat{x}_i(k|k-1)], \quad (3)\\
\bar{x}_i(k|k) &= \hat{x}_i(k|k) + \epsilon m_{ij}(k) \sum_{j \in N_i} (\hat{x}_j(k|k) - \hat{x}_i(k|k)),\\
\hat{x}_i(k+1|k) &= A\bar{x}_i(k|k),
\end{aligned}$$

where

$$K_p^i(k) = \tilde{P}^i(k) H^{iT} (H_i \tilde{P}^i(k) H_i^T + R_i)^{-1},$$

$$\begin{aligned}
\tilde{P}^i(k+1) = \tilde{P}^i(k) &+ Q\\
&- \tilde{P}^i(k) H_i^T [H_i \tilde{P}^i(k) H_i^T + R_i]^{-1} H_i \tilde{P}^i(k),
\end{aligned}$$

and $0 < \epsilon < 1$ is the consensus gain, $m_{ij}(k)$ is the link weight. Different from the existing works, we design the link weight based on the PageRank algorithm, which represents the credit of sensor j to sensor i. The sensor with higher estimation accuracy is assigned with higher credit. Let the initial state $\tilde{P}^i(0) > 0$ for all i.

3. WEIGHT DESIGN FOR DISTRIBUTED CONSENSUS FILTER

In this section, we design link weight for sensors to improve the estimation accuracy, and compare the estimation performance of the proposed estimator with that of a constant weighted estimator.

Define the estimation error as $\hat{e}_i(k|k) = \hat{x}_i(k|k) - x(k)$, $\bar{e}_i(k|k) = \bar{x}_i(k|k) - x(k)$ and $\hat{e}_i(k|k-1) = \hat{x}_i(k|k-1) - x(k)$, respectively. Then one has

$$\hat{e}_i(k|k) = (I_m - K_p^i(k)H_i)\hat{e}_i(k|k-1) + K_p^i(k)v_i(k), \quad (4)$$
$$\bar{e}_i(k|k) = \hat{e}_i(k|k) + \epsilon \sum_{j \in N_i} m_{ij}(k)(\hat{e}_j(k|k) - \hat{e}_i(k|k)),$$
$$\hat{e}_i(k+1|k) = A\bar{e}_i(k|k) - w(k).$$

Further define their corresponding estimation error covariances as

$$\hat{P}_i(k|k) = \mathbb{E}[\hat{e}_i(k|k)\hat{e}_i^T(k|k)],$$
$$\bar{P}_i(k|k) = \mathbb{E}[\bar{e}_i(k|k)\bar{e}_i^T(k|k)],$$
$$\hat{P}_i(k+1|k) = \mathbb{E}[\hat{e}_i(k+1|k)\hat{e}_i^T(k+1|k)].$$

Then

$$\hat{P}_i(k|k) = (I_m - K_p^i(k)H_i)\hat{P}_i(k|k-1)(I_m - K_p^i(k)H_i)^T + K_p^i(k)R_iK_p^{iT}(k), \quad (5)$$

$$\bar{P}_i(k|k) = (1-\epsilon)^2\hat{P}_i(k|k) + \epsilon^2 \sum_{j \in N_i} m_{ij}(k)\hat{P}_j(k|k)m_{ij}(k)^T + (1-\epsilon)\epsilon \sum_{j \in N_i} \hat{P}_{ij}(k|k)M_{ij}^T(k) + (1-\epsilon)\epsilon \sum_{j \in N_i} m_{ij}(k)\hat{P}_{ij}(k|k), \quad (6)$$

$$\hat{P}_i(k+1|k) = A\bar{P}_i(k|k)A^T + Q, \quad (7)$$

and

$$\hat{P}_{ij}(k|k) = (I_m - K_p^i(k)H_i)\hat{P}_{ij}(k|k-1)(I_m - K_p^j(k)H_j)^T,$$
$$\bar{P}_{ij}(k|k) = \hat{P}_{ij}(k|k) + \epsilon \sum_{r \in N_i} m_{ir}(k)(\hat{P}_{rj}(k|k) - \hat{P}_{ij}(k|k)) + \epsilon \sum_{s \in N_j} m_{js}(k)(\hat{P}_{is}(k|k) - \hat{P}_{ij}(k|k)) + \epsilon^2 \sum_{r \in N_i}\sum_{s \in N_j} m_{ir}(k)\cdot m_{js}(k)(\hat{P}_{rs}(k|k) - \hat{P}_{rj}(k|k) - \hat{P}_{is}(k|k) + \hat{P}_{ij}(k|k)),$$
$$\hat{P}_{ij}(k+1|k) = A\bar{P}_{ij}(k|k)A^T + Q,$$

where

$$m_{ij}(k) = \mathbf{diag}\left(\hat{P}_{ij}^{-1}(k|k)\left[\sum_{j \in N_i}\hat{P}_{ij}^{-1}(k|k)\right]^{-1}\right), j \in N_i. \quad (8)$$

First, we investigate the convergence properties of the presented estimator (3). Let

$$\hat{e}(k+1|k) = [\hat{e}_1(k+1|k)^T, \hat{e}_2(k+1|k)^T, \ldots, \hat{e}_n(k+1|k)^T]^T,$$
$$v(k) = [v_1(k), v_2(k), \ldots, v_n(k)]^T.$$

Define the block matrix $\tilde{M}(k) = \tilde{m}_{ij}(k)$ as,

$$\tilde{m}_{ij}(k) = \begin{cases} -m_{ij}(k), & \text{if } (i,j) \in E, i \neq j, \\ \sum_{j \in N_i} m_{ij}(k), & \text{if } i = j, \\ \mathbf{0}, & \text{otherwise.} \end{cases}$$

Remark 1. If $m_{ij}(k) = I_m \otimes (1/d_i), j \in N_i$, then the second equation in the proposed estimator (3) utilizes the constant weighted consensus protocol. For comparison in subsequent discussion, we call it **constant weighted estimator**, and denote its estimation error covariance for each step by $\hat{P}_i^0(k|k)$, $\bar{P}_i^0(k|k)$ and $\hat{P}_i^0(k+1|k)$, respectivly.

The entire estimation error is

$$\hat{e}(k+1|k) = \Gamma(k)\hat{e}(k|k-1) + W(k), \quad (9)$$

where

$$\Gamma(k) = (I_n \otimes I_m - \epsilon\tilde{M}(k))(I_n \otimes A)diag(I_m - K_p^i(k)H_i),$$

$$W(k) = (I_n \otimes I_m - \epsilon\tilde{M}(k))(I_n \otimes A)diag(K_p^i(k))v(k) - \mathbf{1}_n \otimes w(k).$$

Define $\hat{P}(k|k-1) = \mathbb{E}[\hat{e}(k|k-1)\hat{e}(k|k-1)^T]$ as the associated estimation error covariance. Then one obtains

$$\hat{P}(k+1|k) = \Gamma(k)\hat{P}(k|k-1)\Gamma(k)^T + \mathbb{E}(W(k)W(k)^T). \quad (10)$$

Assumption 2. The pair (A, H_i) is detectable for all i, and $(A, Q^{1/2})$ is stabilizable.

First, we analyze the stability of the proposed estimator (3). By using similar methods in (Stanković et al. [2009], Cattivelli et al. [2010]), we can obtain the following results.

Lemma 3. Under Assumption 2, if $0 < \epsilon < 1$, then

$$\lim_{k\to\infty} \mathbb{E}[\hat{e}(k|k-1)] = 0$$

and $\hat{P}(k+1|k)$ converges to a constant positive semi-definite matrix $\hat{P}$ as $k \to \infty$.

Proof: Assumption 2 guarantees that $\lim_{k\to\infty}\tilde{P}^i(k) = \tilde{P}^i$ for any initial condition $\prod_0 \geq 0$. Thus, $\tilde{M}(k)$ and $K_p^i(k)$ also converge to $\tilde{M}$ and $\bar{K}_p^i$ for all i. Let $\mathcal{K} = (I_n \otimes A)\text{diag}(I_m - \bar{K}_p^iH_i)$. By Lemma 1 in Cattivelli et al. [2010], it is easy to prove that the matrix $\mathcal{K}$ is stable. Furthermore, since $0 < \epsilon < 1$, the matrix $I_n \otimes I_m - \epsilon\tilde{M}$ has non-negative entries with the sum of each row equals to one (i.e., $(I_n \otimes I_m - \epsilon\tilde{M})\mathbf{1_n} = \mathbf{1_n}$. By Lemma 2 in (Cattivelli et al. [2010]), the matrix $\Gamma(k)$ converges to a stable matrix $\bar{\Gamma}$. Therefore

$$\lim_{k\to\infty} \mathbb{E}[\hat{e}(k|k-1)] = 0$$

and steady-state estimation covariance is bounded as k goes infinity.

Remark 4. Here, $\hat{P}$ satisfies

$$\bar{\Gamma}\hat{P}\bar{\Gamma}^T - \hat{P} + \bar{Q} = 0,$$

where

$$\bar{\Gamma} = (I_n \otimes I_m - \epsilon\tilde{M})(I_n \otimes A)diag(I_m - \bar{K}_p^iH_i),$$
$$\bar{Q} = \text{diag}(A\bar{K}_p^i)[(I_n - \epsilon\tilde{M}) \otimes I_m]\text{diag}(R_i) \cdot [(I_n - \epsilon\tilde{M}) \otimes I_m]^T\text{diag}(A\bar{K}_p^i)^T + \mathbf{1}\mathbf{1}^T \otimes Q.$$

Lemma 5. Let

$$C_1 = l_1 P_1 + l_2 P_2 + \cdots + l_n P_n,$$
$$C_2 = \frac{1}{n} P_1 + \frac{1}{n} P_2 + \cdots + \frac{1}{n} P_n.$$

If

$$i)\ l_1 + l_2 + \cdots + l_n = 1,$$
$$ii)\ 0 < l_1 \le l_2 \le \cdots \le l_n,$$
$$iii)\ P_1 \ge P_2 \ge \cdots \ge P_n > 0,$$

then $C_1 \le C_2$.

Proof: According to the condition i) and ii), there exists a l_i such that $l_i \le \frac{1}{n}$ and $l_{i+1} > \frac{1}{n}$. Then

$$\begin{aligned} C_1 - C_2 &= (l_1 - \frac{1}{n})P_1 + \cdots + (l_i - \frac{1}{n})P_i + (l_{i+1} - \frac{1}{n})P_{i+1} \\ &\quad + \cdots + (l_n - \frac{1}{n})P_n \\ &\le (l_1 + l_2 + \cdots + l_i - \frac{1}{n}i)P_i \\ &\quad + (l_{i+1} + \cdots l_n - \frac{1}{n}(n-i))P_{i+1} \\ &= (l_1 + l_2 + \cdots + l_i - \frac{1}{n}i)(P_i - P_{i+1}) \\ &\le 0 \end{aligned}$$

The last inequality follows from condition iii).

Theorem 6. Assume that the proposed estimator (3) with link weight (8) and constant weighted estimator are initialized identically. Then $\hat{P}_i(k+1|k) \le \hat{P}_i^0(k+1|k)$.

Proof: Since $\hat{P}_i(1|0) = \hat{P}_i^0(1|0)$, we have that $\hat{P}_i(1|1) = \hat{P}_i^0(1|1)$ for all i. Since $\hat{P}_j(k|k)$ is bounded for all j, we can always find a sufficiently small ϵ such that $\epsilon^2 \sum_{j\in N_i} m_{ij}(k)\hat{P}_j(k|k)m_{ij}(k)^T$ is close to zero. Note that

$$m_{ij}(k)\hat{P}_{ij}(k|k) = \sum_{l=1}^{m} V_l \cdot m_{ij}(k)_{l,l} \cdot (\mathbf{1}_m \otimes \hat{P}_{ij}(k|k)(:,l)),$$

where V_l denotes a matrix in which the entries outside the l-th element of main diagonal are all zero, $\hat{P}_{ij}(k|k)(:,l)$ denotes the l-th row of matrix $\hat{P}_{ij}(k|k)$. Setting $m_{ij}(k)$ as equation (8), the $m_{ij}(k)_{l,l}$ is arranged in decreasing order by j if the $\hat{P}_{ij}(k|k)$ is arranged in increasing order by j. Since $\hat{P}_{ij}(1|1) = \hat{P}_{ij}^0(k|k)$, by Lemma 5, then

$$\sum_{j\in N_i} V_l \cdot m_{ij}(1)_{l,l} \cdot (\mathbf{1}_m \otimes \hat{P}_{ij}(1|1)(:,l)) \le \sum_{j\in N_i} V_l \cdot \frac{1}{d_i} \cdot (\mathbf{1}_m \otimes \hat{P}_{ij}^0(1|1)(:,l))$$

for all l. Thus,

$$\sum_{j\in N_i} m_{ij}(1)\hat{P}_{ij}(1|1) \le \sum_{j\in N_i} m_{ij}(1)\hat{P}_{ij}^0(1|1).$$

Then one has $\hat{P}_i(2|1) \le \hat{P}_i^0(2|1)$ for all i. Note that $\hat{P}_{rj}(k|k) - \hat{P}_{ij}(k|k) = cov(\hat{e}_r(k|k) - \hat{e}_i(k|k), \hat{e}_j(k|k))$. Similarly, one has $\hat{P}_{ij}(2|1) \le \hat{P}_{ij}^0(2|1)$. Furthermore, by induction, it is easy to show that $\hat{P}_i(k+1|k) \le \hat{P}_i^0(k+1|k)$. Therefore, the proof is completed.

4. SENSOR DEPLOYMENT FOR DISTRIBUTED CONSENSUS FILTER

In Section 3, when the position of each sensor is given, we have designed link weight to improve the estimation performance. In this section, we consider a related problem: if given fixed physical network topology, how to deploy sensors at optimal positions to improve the estimation accuracy? In heterogeneous sensor network, putting sensor with higher accuracy at important position leads to better estimates. In contrast, the estimation error increases if sensors with lower accuracy located at important positions. We label the sensors as $s_1, s_2, \ldots, s_n$ and the node positions on network as $n_1, n_2, \ldots, n_n$. Consider a random initial sensor positions and let the Laplacian of sensor network topology be L_0. The problem is transformed to find optimal sensor deployment $n_k, k \in \{1, \ldots, n\}$ for sensor s_i. We formulate the problem as follows

$$\begin{aligned} (P_1):\ \min_K\ & \text{tr } \hat{P} \\ \text{s.t. } & L = K L_0 K^T \\ & K \in \Pi, \end{aligned} \tag{11}$$

where Π is the set of $n \times n$ permutation matrices.

Since problem P_1 is an quadratic assignment problem (QAP), which belongs to the class of combinatorial optimization problems, there is no known algorithm for solving it in polynomial time. In (Anstreicher et al. [2001]), Anstreicher et. al. proposed a parallel B&B algorithm to solve the QAP based on the serial algorithm in (Brixius et al. [1998]).

The quadratic programming bound for P_1 is of the form

$$\begin{aligned} (P_2):\ \min\ & \text{tr } \hat{P} + \tilde{\lambda}(\text{vec}(K)^T \tilde{Q} \text{vec}(K) + \gamma) \\ \text{s.t. } & K\mathbf{1} = K^T\mathbf{1} = \mathbf{1} \\ & \tilde{\lambda} > 0, K \ge 0, \end{aligned} \tag{12}$$

where $\tilde{Q} = (L_0 \otimes I) - (I \otimes S) - (T \otimes I)$. Let V be an $n \times (n-1)$ matrix whose columns are an orthonormal basis for the nullspace of $\mathbf{1}^T$. The matrices S and T are obtained from the spectral decompositions of $V^T V$ and $V^T L_0 V$, and $\gamma =< \lambda(V^T V), \lambda(V^T L_0 V) >_-$.

In (Brixius et al. [1998]), a Frank-Wolfe algorithm is introduced to solve the problem P_2 for a small-scale network. Then we can deploy the sensors to the optimal positions from K. Here, K gives an order of position importance in a sensor network. When those sensors with high estimation accuracy are located at important positions, the entire network estimation error decreases. The above method is, however, not effective for a large-scale network. Moreover, to solve P_2, we need all the information of sensors including network topology and sensor estimation accuracy. It is difficult to implement in practice. Instead, we try to find a suboptimal method which is easy to implement. A natural idea is to arrange the node positions in a network by their importance in decreasing order using some indices, by doing that, we know which position in sensor network is important.

In the area of complex network, some indices have been found to characterize node importance in network, such as degree centrality (DC), betweenness centrality (BC), eigenvector centrality (EC) and PageRank (PR), see Newman [2010].

Degree centrality: is defined as the number of links incident upon a node.

Betweenness centrality: is an index of a node's centrality in a network equal to the number of shortest paths from all vertices to all others that pass through that node.

Eigenvector centrality: is an index of the influence of a node in a network. It assigns relative scores to all nodes in the network based on the concept that connections to high-scoring nodes contribute more to the score of the node in question than equal connections to low-scoring nodes.

PageRank: is a link analysis algorithm that assigns a numerical weighting to each element of a hyperlinked set of documents with the purpose of "measuring" its relative importance within the set. The PR is also an index of the influence of a webpage in World Wide Web.

In real applications, if the physical network topology is given, it is easy to calculate those indices for nodes according to the Laplacian matrix. Thus, the importance of the positions in the network could be arranged in order. In Section 5, we compare the estimation performance of sensor deployment strategy under five different indices.

5. SIMULATION RESULTS

In this section, we illustrate the results derived in Section 3 and 4 by numerical simulations. Moreover, the estimation performance of the improved weighted estimator and the constant weighted estimator are compared.

Consider a wireless sensor network with $n = 30$ sensors. The discrete-time system and sensor parameters are given as follows:

$$A = \begin{pmatrix} 1.01 & 0 \\ 0 & 1.01 \end{pmatrix}, \ Q = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix},$$

$$H^i = \begin{pmatrix} 2\delta_i & 0 \\ 0 & 2\delta_i \end{pmatrix}, \ R_i = \begin{pmatrix} 2\nu_i & 0 \\ 0 & 2\nu_i \end{pmatrix},$$

where $\delta_i \in (0, 1.5]$, $\nu_i \in (0, 5]$ for all i. We choose an undirected network topology G with its second eigenvalue $\lambda_2(L) = 1.4536$ and maximal degree $\Delta = 18$. We also choose $\epsilon = 0.01$. As Fig. 1 shows, all the sensors track the unstable object system of (1) effectively. Define the mean-squared estimation error as $\sum_{i=1}^{n} e_k^{iT} e_k^i / n$. Fig. 2 shows that mean squared estimation error decreases to a bounded region. Similarly to Olfati-Saber [2005], the disagreement of the estimates is measured by $\|\delta(k)\| = (\sum_{i=1}^{n} (\delta^i(k))^2)^{1/2}$ with $\delta^i(k) = \hat{x}^i(k|k-1) - m(k)$, where $m(k) = \frac{1}{n} \sum_i \hat{x}^i(k|k-1)$. From Fig. 3, the improved weighted estimator has cohesive estimates. The simulation results in Fig. 4 demonstrate that $\hat{P}_i(k+1|k)$ converges to a $\hat{P}_i$ for all i, and $\hat{P}_{i,j}(k+1|k)$ also converges for all i, j. Moreover, it is easy to find that the improved weighted estimator has lower trace of estimation covariance than the constant weighted estimator, which verifies the results derived in Section 3.

Fig. 5 shows that the average trace of estimation covariance of the proposed estimator under five sensor deployment strategies. The performance of the PR strategy is better than that of the other four strategies, and the performance of the DC strategy is close to that of the PR strategy. The intuitive reason is that degree centrality has similar statistical performance with PageRank value for undirected network.

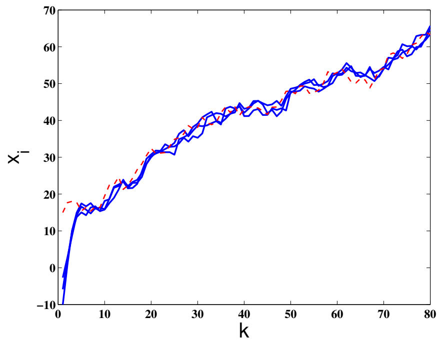

Fig. 1. Tracking performance of the proposed estimator

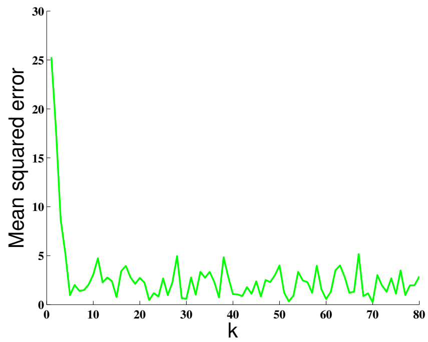

Fig. 2. Mean squared estimation error

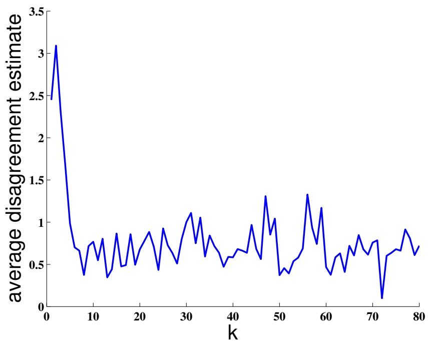

Fig. 3. Average disagreement estimate

6. CONCLUSIONS

In this paper, we considered distributed estimation over heterogeneous sensor network. First, we have designed link weight for sensors by the diagonal elements of the inverse of the edge covariance matrices between linked sensors based on the PageRank algorithm. We proved that the improved weighted estimator obtained lower estimation error than the constant weighted estimator at each time step. Second, we proposed an optimal sensor deployment strategy to arrange optimal positions for the sensors. We also have

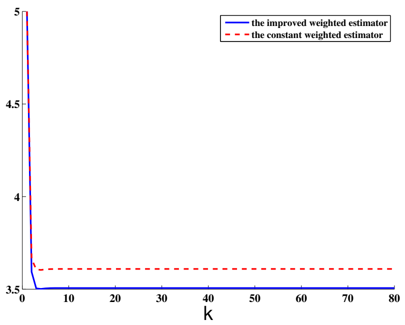

Fig. 4. Trace of covariance of the proposed estimator: node 1

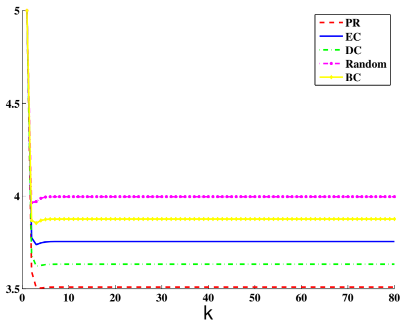

Fig. 5. Average trace of covariance of the proposed estimator

verified that the position of the sensors sorted by some indices obtains better estimates.

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