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Electric Vehicle Charging: A Noncooperative Game Using Local Measurements ⋆

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Abstract: A distributed control algorithm is proposed to manage the electrical power demand for the purpose of charging electric vehicles so that (a) the overall power demand remains within the limitations of the distribution network and (b) each vehicle obtains a sufficiently charged battery at the end of the charging cycle. The charging algorithm utilises only local measurements to determine the charge current. The control problem is modelled as a non-cooperative game with weakly coupled cost functions for each vehicle. The cost function for each vehicle consist of an individual cost term and a group cost term. The group cost term expresses the aggregated demand of all vehicles and serves the purpose of ensuring that the infrastructure capacity constraints are respected. It is shown that this term can be estimated from local voltage measurements. The individual cost term reflects the need to achieve a desired charge level in the battery. Sufficient conditions for the overall system to admit a unique Nash Equilibrium are identified. Convergence and stability properties for a particular greedy algorithm implementation are described. To illustrate the efficacy of the proposed charging methodology, the algorithm is simulated in the context of an Australian suburban low voltage electricity distribution network.

Keywords: electric vehicles, distributed charging, optimization, game theory, power systems

1. INTRODUCTION

It is likely that in the future a significant amount of personal transport will make use of electric vehicles (EVs) and plug-in hybrid vehicles (PHEV). The additional electricity demand required to charge the EV (and PHEV) batteries is substantial. Indeed, in Australia, using typical household commute requirements, the corresponding EV's energy demand is roughly equal to the present average per household energy demand. Hence, if all personal transport would become EV based, domestic electrical energy demand would double [Mareels et al., 2010]. Under normal operating conditions, this demand increase could require a substantial augmentation of the current grid infrastructure. Alternatively, allowing uncontrolled charging of EVs on the present grid will lead to unacceptable frequency and voltage variations, even when only a small proportion of households use an EV [Lopes et al., 2011, Kelly et al., 2009]. Nevertheless, as the present grid is constructed to meet peak demand, its capacity is hardly ever fully utilised. In Victoria, Australia, half the total energy capacity of the grid is unused [Mareels et al., 2010]. This indicates that there is an opportunity to allow many households to use an EV provided this demand could be shifted as to use the underutilised capacity of the grid. In this paper a particular approach to such EV demand management is presented.

EV demand management can be approached from a centralized and decentralized perspective. In a centralized approach, a central controller communicates with all batteries across the network, collating all relevant information (the state of the grid and each EV), and computes an optimal charging profile for each battery, which is then communicated to the batteries [Richardson et al., 2012, Sojoudi and Low, 2011]. This approach requires all vehicles to participate in the decision making process. Although ideal from an information point of view, this will require significant communication and computation infrastructure. Moreover, given the size of the network scaling problems may be expected. A distributed decentralised approach would allow each EV to calculate its own charging profile based on information obtained locally [Gan et al., 2011, Li et al., 2011, Ahn et al., 2011, Studli et al., 2012, Ardakanian et al., 2013]. This will necessarily be suboptimal compared to the global centralised approach, but is far more realistic from an implementation point of view. Here we propose a decentralised demand management strategy, without addressing the question of how suboptimal the solution will be.

Our recent work [Xia et al., 2014] formulates EV charging as a distributed optimization problem. The algorithm al-

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Table 1. List of Symbols

m	electric vehicle m
M	number of electric vehicles
$\overline{D}$	desired total demand level on relevant phase
D	actual total demand on relevant phase
$\underline{D}$	minimum total demand on relevant phase
p_m	charging power of electric vehicle m
p_{-m}	vector of charging power of all EVs except m
$\overline{p}_{-m}$	sum of charging power of all EVs except m
$\overline{V}_m$	local voltage at house m corresponding to $\underline{D}$
V_m	local voltage at house m (or EV m)
$\underline{V}_m$	local voltage at house m corresponding to $\overline{D}$
J_m	cost function of electric vehicle m
$\overline{E}$	desired aggregated EV demand on relevant phase
α_m, β_m	constants related to the user preferences of EV m
γ_m	scheduled charging power of electric vehicle m

lows EVs to increase their charging power asynchronously until the aggregated demand reaches a desired level. The performance of the algorithm is tested through simulations on a realistic Melbourne suburban network. In this paper, a theoretical analysis is presented using (non-cooperative) game theory. EV charging problems have already been formulated using game theory when control involves a certain level of communication [Ma et al., 2010]. Also game theory can explain communication network flow control where communication for the purpose of control is minimal or even absent [Altman and Basar, 1998]. As the resource allocation problem in electricity grids is similar to the network flow control in communication networks it may be expected that non-cooperative game theory may equally elucidate the former's behaviour.

In our game formulation, the players are the EV batteries, these are non-uniform and coupled in the game only through their local objective function. Each EV's objective function consist of a term penalising total demand deviating from desired value and an individual, local term to ensure that the battery gets charged. The EVs optimise their objective functions being influenced by the aggregated charging action of the collective. We argue and show that the collective action, even though not measurable, can be approximated by a local voltage measurement. In this paper, the charging algorithm manages EV charging in a distributed way using only local information, and within the capacity of the grid. No explicit communication is required. The performance of the algorithm is illustrated via simulations against realistic Australian suburban network data.

The rest of the paper is organized in the following way. Section 2 explains the relationship between local voltage and total phase demand in an electricity network which will form the basis of our algorithm. Section 3 introduces the non-cooperative game model and proves the existence and uniqueness of the Nash Equilibrium; Section 4 proposes an iterative update scheme and shows the convergence to the Nash Equilibrium under this update scheme. The algorithm is verified via practical simulations in Section 5 using the model of a real Australian network in eastern Melbourne; the last section concludes this paper and points directions for our future work.

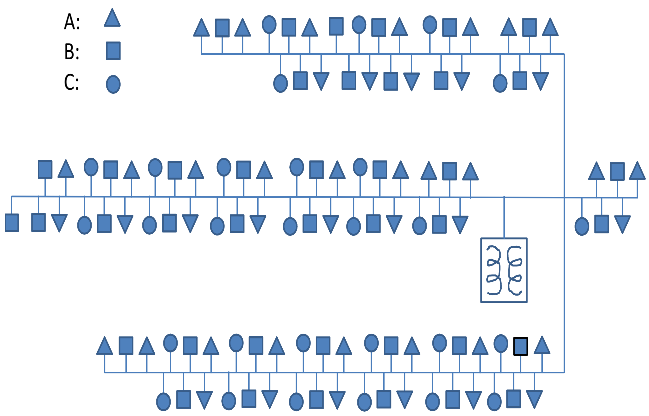

Fig. 1. Diagram of an Victorian suburban distribution network of 114 houses feed in by a transformer. Triangles denote houses on phase A; rectangles denote houses on phase B and circles denote houses on phase C.

2. LOW VOLTAGE NETWORK MODEL

This paper focus primarily on low voltage networks having radial configurations. As an example, Figure 1 shows a typical Melbourne suburban network with 114 houses. In such networks, there exist an approximately linear relationship between available capacity on phases, D, and local voltage levels, V, such that

$$\frac{\overline{D}-D}{\overline{D}-\underline{D}} = \frac{V^m - \underline{V}^m}{\overline{V}^m - \underline{V}^m} \quad (1)$$

holds. The symbols in (1) are defined in Table 1. As discussed in [Ganu et al., 2013] and [Xia et al., 2014], heavy load in a distribution network increases the line voltage drops which in turn reduces household voltage levels.

The network in Figure 1 is simulated using real daily demand data to validate (1). For each house in the network, the plot of local voltage verses total demand on the corresponding phase suggests an approximate linear relationship which can be easily obtained using linear regression. We pick a random house W in the network and show this relationship (Figure 2). The left y axis shows the 24 hour total demand curve for the house W on a typical winter day. The evening peak as well as the overnight and mid-day demand valleys can be clearly seen. The right y axis plots the local voltage level for the house W on the same day where the voltage drop in the evening is observed. We next correlate the two figures to plot the relationship between total phase demand and the local voltage of house W in Figure 3. The scatter plot clearly suggests a linear relationship. After linear regression based on least squares, a line can be easily fit. For different houses, the line has a different slope and intercept but the approximate linear relationship as per (1) holds for all houses.

3. STRATEGIC GAME

We formulate the EV charging problem as an M-player strategic (non-cooperative) game $\mathcal{G} = \langle \mathcal{M}, \mathcal{X}, \mathcal{J} \rangle$. The player set $\mathcal{M} := \{1, ..., M\}$ includes all houses with an EV on a low voltage network. The set $X_m \subset \mathcal{X}$ denotes the actions or strategies of a player m from strategy set $\mathcal{X}$ which

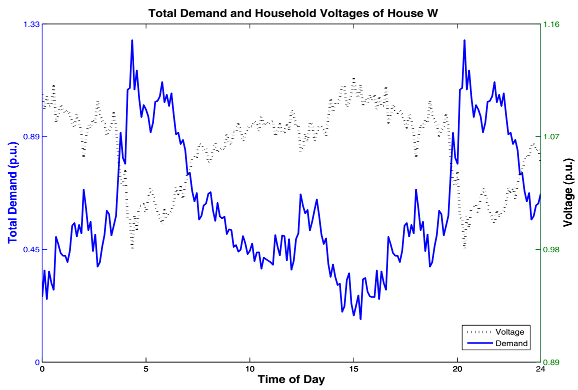

Fig. 2. Voltage profile of house W with dotted line on right axis and corresponding total phase demand with solid line on left axis.

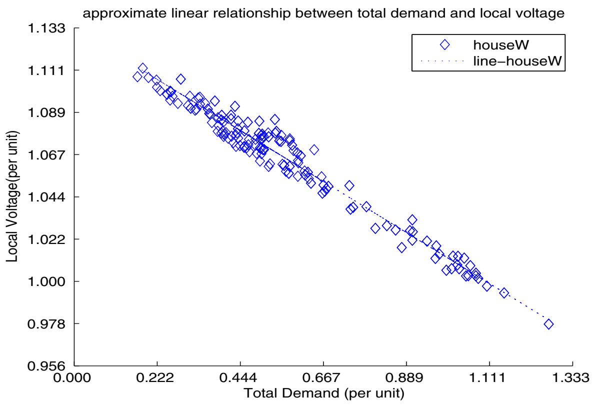

Fig. 3. Scatter plot and least square line fitting showing the relationship between local voltage at house W and the corresponding phase total demand.

in our case is the charging power. The set $\mathcal{J} := \{J_1, ..., J_m\}$ is the set of cost functions to be minimized for all players. The cost functions are chosen to be quadratic and consist of two terms: group cost and individual cost. Group cost is characterized as the square of distance between total demand level and the desired demand level; we want each agent to charge greedily such that the total demand is around the desired level $\overline{D}$. Since we are using local voltage to approximate total demand, according to (1), our goal is equivalent to local voltage of agent m being as close to $\underline{V}^m$ as possible. Through total demand or local voltage, all agents are weakly coupled. The other cost we want to minimize is the cost for individuals deviating from their charging plans. When EV m is plugged in, the user will set the total time T_m available for recharging the EV. The ratio $\gamma = R_m/T_m$ gives a an average charging power for the EV to be charged on time; where R_m is the total energy needed to fully charge the battery of EV m. This average is a schedule that we want to keep track of. If there is no such cost term, it is possible that some EVs are given much more charging power than others but the total demand is still minimized. Therefore, we guarantee a certain level of fairness by introducing this term.

It is assumed that total demand on a given phase does not fluctuate wildly over a given discrete interval of time, allowing the charging power of EVs to converge to the equilibrium value. Hence, the actual demand is $D = H + \sum_{m=1}^{M} p_m$. $\overline{E}$ thus represent network capacity that is available to be used for charging. Let $k_m = 1/(\overline{D} - \underline{D})$. Then, the cost function for EV m is defined as follows:

$$J_m(p_m, p_{-m}) = (k_m(\overline{E} - p_m - \overline{p}_{-m}))^2 + \beta_m \left(\frac{p_m - \gamma_m}{\gamma_m}\right)^2 \tag{2}$$

where γ_m is the average charging power from user setting and where β_m is a positive constant that can be used to tune the system behaviour. In practice, we do not have access to $\overline{p}_{-m}$, but we can estimate this value using the local voltage V_m as per (1). The cost function (2) is therefore approximately equivalent to the following cost function:

$$\begin{aligned} J_m &= J_m(p_m, V_m) \\ &= \left(\frac{V_m(p_m, p_{-m}) - \underline{V}^m}{\overline{V}^m - \underline{V}^m}\right)^2 + \beta_m \left(\frac{p_m - \gamma_m}{\gamma_m}\right)^2 \end{aligned} \tag{3}$$

The players minimize this cost function by adjusting their charging power p_m. Because of the convexity of (2) with respect to p_m and assuming an inner solution, we take its partial derivative with respect to p_m and let it be zero:

$$\frac{\partial J_m}{\partial p_m} = 2k_m^2(p_m + \overline{p}_{-m} - \overline{E}) + \frac{2\beta_m}{\gamma_m^2}(p_m - \gamma_m) = 0 \tag{4}$$

For simplicity, we assume here that $\overline{E} \geq \overline{p}_{-m}$. The solution to (4) is therefore given as follows:

$$\begin{aligned} p_m &= \Phi_m(\overline{p}_{-m}, \beta_m, k_m, \gamma_m) \\ &= \frac{k_m^2\gamma_m^2\overline{E} - k_m^2\gamma_m^2\overline{p}_{-m} + \beta_m\gamma_m}{k_m^2\gamma_m^2 + \beta_m} \end{aligned} \tag{5}$$

This is also the best response reaction function of player or EV m and it is in a linear form. Let $\alpha_m = (k_m^2\gamma_m^2\overline{E} + \beta_m\gamma_m)/(k_m^2\gamma_m^2 + \beta_m)$. Then, we can write the reaction functions of all player in a matrix form:

$$p^{i+1} = Ap^i + \alpha$$

$$\begin{pmatrix} p_1^{i+1} \\ p_2^{i+1} \\ \vdots \\ p_m^{i+1} \end{pmatrix} = - \begin{pmatrix} 0 & k_2^2\gamma_2^2 & \cdots & k_m^2\gamma_m^2 \\ k_1^2\gamma_1^2 & 0 & \cdots & k_m^2\gamma_m^2 \\ \vdots & \vdots & \ddots & \vdots \\ k_1^2\gamma_1^2 & k_2^2\gamma_2^2 & \cdots & 0 \end{pmatrix} \begin{pmatrix} p_1^i \\ p_2^i \\ \vdots \\ p_m^i \end{pmatrix} + \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_m \end{pmatrix} \tag{6}$$

Proposition 1. Matrix A is invertible (can be proved using contradiction), therefore, the Nash Equilibrium can be computed from (6) as:

$$p^* = (I - A)^{-1}\alpha$$

It can be shown that matrix $D := (I - A)^{-1}$ is non-singular under non-conservative assumptions using proof by contradiction again (assume there is a non-zero vector X such that $DX = \mathbf{0}$, then only under some very rare condition can D be singular). Therefore we conclude that generically, there exists a unique Nash Equilibrium in the M-player noncooperative EV charging game $\mathcal{G}$.

Corollary 2. For homogenous agents, charging power at equilibrium can be calculated as follows: if all EVs are having same setting of β, γ and k, from (5), for any EV m, we have the following equation at equilibrium:

$$p_m = \frac{k^2\gamma^2\overline{E} - k^2\gamma^2(M-1)p_m + \beta\gamma}{k^2\gamma^2 + \beta} \tag{7}$$

The solution to equation (7) is

$$p_m = \frac{k^2\gamma^2\overline{E} + \beta\gamma}{k^2\gamma^2 M + \beta} \tag{8}$$

Remark 3. The reaction function defined in (5) represents the optimal response of EV m to the charging rate of all other users. And this charging rate of all other users is only determined through the sum. Again, in practice, we do not have access to the value of the aggregated charging rate from each house's point of view. However, we can write (5) equivalently as follows with respect to (3):

$$p_m = \frac{k_m\gamma_m^2(V_m - \underline{V}^m)/(\overline{V}^m - \underline{V}^m) + \beta_m\gamma_m}{k_m^2\gamma_m^2 + \beta_m} \tag{9}$$

Remark 4. The value γ_m is considered a constant for each EV in this paper. In order for our algorithm to achieve optimality, we assume that there is a price incentive such that all customers behave rationally and do not cheat. However, in reality, γ_m can be used as a pricing parameter and associated with a tally. The users can choose to have their vehicles charged faster by paying more and γ can be adjusted to higher value to meet the requirement. Customers who do not need to use their vehicle in the next day can choose a very small γ and pay much less than normal electricity price (making benefit in some sense). We will carry out more research to study the effect of γ and how to set the value based on different user requirement.

Remark 5. In terms of hardware implementation, we can choose parameters for actual system based on calculations and simulations of a homogenous system. We first of all write (8) as follows:

$$p_m = \frac{\overline{E}/M + \beta\gamma/(k^2\gamma^2 M)}{1 + \beta/(k^2\gamma^2 M)} \tag{10}$$

There are two terms in the numerator; the first term indicates the average spare capacity that each household could have and the second term shows the desired charging speed from users. If we want the spare capacity in the grid to play a much more important role in the charging algorithm, we can set $\beta \ll 1$ and vice versa. If we want the two terms to be equally weighted, we can simply set $\beta = k^2\gamma^2 M$.

4. UPDATE SCHEME AND STABILITY ANALYSIS

The Nash Equilibrium solution derived in the previous section cannot be computed centrally if there is no communication infrastructure and no centralised decision maker. However, it can be computed in a decentralised manner where each agent executes the best-response algorithm using only local information, in this case voltage measurements. There are a variety of distributed update schemes such as asynchronous, parallel, round robin, etc. In this section, we investigate the stability property of our system under an asynchronous random update algorithm. We assume that the time is discretized into intervals of several seconds at a time. At the beginning of interval $i+1$, the player m updates with a nonzero probability $\pi_m(i+1)$ based on the residual information from last interval as shown below.

$$p_m^{(i+1)} = \begin{cases} \Phi_m(\overline{p}_{-m}^{(i)}), & \text{with probability } \pi_m(i+1) \\ p_m^{(i)}, & \text{with probability } 1-\pi_m(i+1) \end{cases}. \tag{11}$$

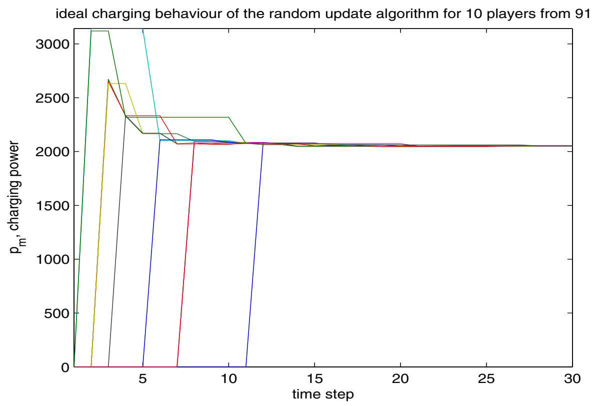

Fig. 4. Numerical demonstration of our algorithm for 10 homogenuous players among 91 from cold start.

In reality, changes in charging decisions have an instantaneous effect on the voltages measured by other users/players. Therefore, we consider an asynchronous update schemes the one most true to reality. Also note that the update scheme in (11) is very general due to update probability being dependant on time. Random round robin or parallel update are special cases under our update scheme with different probability functions.

Figure 4 shows numerically the asynchronous behaviour and the convergence of our update algorithm as per (11) for 10 homogenous players among 91. We do not consider any constraints in this numerical example and players update their charging power with a probability of 10% at each interval. We observe that it only takes a very short time for the game to converge in this ideal case. We next establish the stability properties of the algorithm in Theorem 6.

Theorem 6. Under our random update algorithm, the system asymptotically converges in the mean to the unique NE from any starting point if k_m, β_m and γ_m are chosen such that the following condition is satisfied:

$$M \leq 2 + \frac{\beta_m}{k_m^2\gamma_m^2}, \quad \forall m. \tag{12}$$

Note that this sufficient condition is easy to satisfy since $k_m^2\gamma_m^2$ is a very small unit free value in most cases.

Proof. The proof here is based on Bertsekas and Tsitsiklis [1989] and the proof of Theorem 4.1 in Alpcan et al. [2002]. At the unique Nash Equilibrium, we have:

$$p_m^* = \frac{k_m^2\gamma_m^2\overline{E} - k_m^2\gamma_m^2\overline{p}_{-m}^* + \beta_m\gamma_m}{k_m^2\gamma_m^2 + \beta_m} \tag{13}$$

Let the difference between the m^{th} user's charging power and equilibrium power level at interval i be $\Delta p_m^{(i)} = p_m^{(i)} - p_m^*$ where p_m^* is always positive. We will show that the update function (11) generates a contraction mapping. From (11), no matter what the equilibrium value p_m^* for user m is, the following holds:

$$E|\Delta p_m^{(i+1)}| = E|\Delta p_m^{(i+1)}|\pi_m + E|\Delta p_m^{(i)}|(1-\pi_m)$$
$$= \frac{k_m^2\gamma_m^2\pi_m}{k_m^2\gamma_m^2+\beta_m}\sum_{n\neq m} E|\Delta p_n^{(i)}|$$
$$+ E|\Delta p_m^{(i)}|(1-\pi_m), \quad (14)$$

where E denotes expected value.

Now let the infinity norm of the vector $(\Delta p_1, \Delta p_2 ... \Delta p_M)^T$ be $\|\Delta p\|_\infty$ which is the maximum entry in the vector, we have

$$\max_m E\|\Delta p_m^{(i+1)}\| \leq \max_m\Big(\frac{k_m^2\gamma_m^2\pi_m(M-1)}{k_m^2\gamma_m^2+\beta_m} + (1-\pi_m)\Big)\|\Delta p^{(i)}\|_\infty \quad (15)$$

Therefore, it is sufficient for the right hand side of (14) to be a contraction mapping if the condition in Theorem 6 holds.

It can also be shown that our update scheme ensures almost sure convergence. We will provide detailed proof and arguments in our future works as well as a comparison among different update schemes.

5. SIMULATIONS

In order to verify the actual performance of our algorithm, we ran simulations using a validated model of a real Victoria suburban three-phase distribution network with 114 households as shown in Figure 1 on a regular winter day. We use two case studies: 50% and 80% EV penetration. Household demand is based on data collected in this network and vehicle demand is estimated using real vehicle travel profiles obtained in the area that this network is located in (Victorian Integrated Survey of Travel and Activity, 2009). We assume that there is no distributed generation such as solar panels in the network. We set the desired demand level to be similar with the existing peak demand value, which is very low considering the additional demand from EVs, to test the performance of our algorithm under extreme conditions. The software packages used were MATLAB SimPower toolbox for load flow calculation and POSSIM Simulator which provides an interface to MATLAB such that the strategies can be calculated in POSSIM and exported to MATLAB.

Table 2. Performance comparisons under no EV, uncontrolled charging and distributed charging.

Algorithm (50% EV)	no EV	uncontrolled	distributed
voltage outliers%	0	2	0
average charging rate	n/a	3.45kW	0.89kW
peak vs valley demand	3.63	5.68	3.67
cost for charging	n/a	$15.86	$14.13
unbalance time%	0.05	1	0
adequately charged%	n/a	100	97.55
Algorithm (80% EV)	**no EV**	**uncontrolled**	**distributed**
voltage outliers%	0	2	0
average charging rate	n/a	3.45kW	0.61kW
peak vs valley demand	3.63	6.19	3.47
cost for charging	n/a	$25.34	$21.84
unbalance time%	0.05	1	0
adequately charged%	n/a	100	95.16

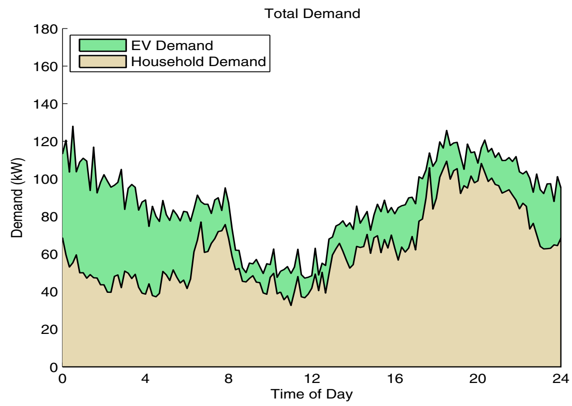

Fig. 5. Demand profile of the Melbourne network using our distributed charging method based on game theory under 80% EV penetration.

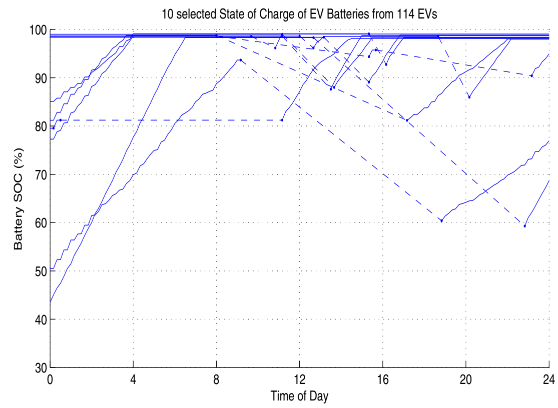

Fig. 6. POSSIM generated battery State of charge (SOC) level changes of 10 EVs from 91 (114 houses with 80% EV penetration). Each curve represents a vehicle. Dashed lines indicate vehicles being away, as opposite to solid lines. And gradients of curves denote the charging rate.

Figure 5 shows the performance of our distributed algorithm in the network on a typical working day under 80% EV penetration. It is clear from the figure that demand peak in the evening decreased and additional demand is distributed to the overnight demand valley. From 6, it is apparent that the charge rate (solid lines) are steeper during overnight period than in the evening peak. In the morning, all cars are sufficiently charged for the day and most of them are fully charged. In reality, the desired demand level can be set to higher values such that EVs can be charged more and faster as we have shown the worst scenarios in the simulations. One fact to note about the EV travelling profiles is that most vehicles are out for work during the middle of the day. Therefore charging is not feasible for most households as we only consider residential charging at this stage. Table 2 presents some key performance parameters of the system in 24 hours averaged over several typical winter working days under three conditions: no EVs, uncontrolled charging and our distributed charging. The percentage of voltage outliers indicates the percentage of time that the system is violating the voltage requirements according to distribution code. The cost of charging calculates the generation price of all electricity used within that 24 hours subject to a typical spot price. The unbalance time shows the duration that the system is experiencing 3% or more phase unbalance per house over 24 hours. An EV is called adequately charged if it is charged above 80% before 8*am* in the morning. Note that 80% is often much more than what a car needs for the following day. We can see that without any control, all EVs will start to charge as soon as they arrive at home and this increases the peak demand significantly. Furthermore, the system underperforms in terms of phase unbalance and voltage quality. Using our distributed charging control, the peak demand is decreased significantly without violating any constraints of the network and the electricity price paid for charging is significantly lower. More importantly, our algorithm requires no communication infrastructure, therefore, no update of the existing facilities and it is ready to be used.

6. CONCLUSION

In this paper, we have established a noncooperative game framework for the EV charging problem based on only local information. We have shown empirically that locally measured voltage can approximate the phase demand. We have proved the existence and uniqueness of a Nash Equilibrium as well as the stability of an asynchronous update scheme under a mild sufficient condition. Simulations conducted using realistic data show that even with a high (80%) EV penetration rate, the distributed algorithm successfully mitigates demand peaks, ensures satisfactory battery levels and certain amount of fairness without violating any grid constraints. An important aspect of our algorithm is that it requires no additional infrastructure and works only with local information.

There are still more work to be done. Some the important aspects are parameter learning (for the linear relationship mentioned above), parameter tuning (for the reaction function) and performance guarantee study. We are also hoping to generalized the algorithm so that not only it can be applied on EVs but also some other residential or commercial appliances like air-conditioning system, water heating system, etc.

7. ACKNOWLEDGEMENT

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