

## An analysis of risks and vulnerabilities in supply networks

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**Abstract:** Today's supply networks have become more and more efficient but also complex and prone to strong disturbances that may generate blocking and starving effects. This study provides a risk analysis of supply networks based on a stochastic model of product flows subject to strong disturbances and threshold constraints. The main objective is to construct vulnerability indices from simulation of the multistage system, both in nominal conditions when an ARIMA model describes the system dynamics and in disturbed running conditions when one or several state variables are subject to saturation effects.

**Keywords:** Manufacturing systems, supply chains, time series analysis, risk, performance analysis, performance indices

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### 1. INTRODUCTION

Globalization and new technologies increase the complexity of supply chains and as a consequence, expose them to risks. Owing to the fact that every component in a supply chain is prone to accidents, undesirable events may lead to important consequences or damages. The recent interest in supply chain risk management focuses on coordination between various members to protect the companies and the chain as a whole. C.Chopra and S.Sodhi (2004) consider that "failure anywhere can cause failure everywhere". In fact, "individual risks are often interconnected. As a result, actions that mitigate one risk can end up exacerbating another". They also show the interest of supply chain risk management by comparing the consequences of the fire at Royal Philips electronics plant, in the year 2000 on the Scandinavian mobile phone manufacturer Nokia Corp. and Telefon AB L.M Ericson. Under these circumstances, Nokia's production suffered less than Ericson's, which was disrupted for months. In a similar situation during the 1990s, the Yoplait Company increased its market share for fresh and ultra-fresh products in a spectacular way thanks to its ability to manage transportation crises caused by road strikes.

The literature on supply chains distinguishes various types of risks and vulnerabilities and proposes different classifications. Demand with random and non-stationary evolution creates dangerous disturbances at upstream stages of supply chains. This phenomenon is known as the "bullwhip effect" (Lee et al., 1997, Towill, 2005). Conversely, disturbances on prices for raw material generate price variations from suppliers to producers and then from producers to customers. Amplification of disturbances in the downstream stages of a supply chain is known as "the reverse bullwhip effect" (Özelkan and Çakanyıldırım, 2009).

This paper seeks to evaluate the vulnerability of supply chains both to internal and external disturbances. Supply chain dynamics and disturbance amplification or attenuation

strongly depend on the ordering policy (Hennet, 2009, Choi 2013). The ordering policy considered in this study is the "order up to" policy with respect to the inventory position. This policy has been shown to be optimal with respect to a model predictive approach (Hennet 2003). Then, in agreements with the findings of (Gilbert, 2005), we propose to use a time series representation of the supply chain in the form of an ARIMA (Auto Regressive Integrated Moving Average) model that propagates along the supply chain and makes possible to represent the bullwhip effect. The limits of validity of this model correspond to the hitting of positivity and capacity constraints as the result of strong disturbances on product flows. It is then possible to simulate the constrained system evolution and compute some vulnerability indices related to the frequency of constraints saturation.

To develop this study, section II describes the concept of risks and tries to identify risks and vulnerabilities in supply chains. Then, section III presents the ARIMA time series representation and its behaviour in cases of strong disturbances. Some vulnerability indices are then proposed to be computed from simulation of the constrained system trajectories. An illustrative example with one manufacturer and one retailer is described in section IV, and some concluding remarks are given in section V.

### II. SUPPLY CHAINS RISKS AND VULNERABILITIES

A supply chain can be viewed as a "virtual systems subject to dynamic reconfigurations, through arrival or departure of partner enterprises." (Hennet et al, 2008). However, in spite of their resilience resulting from natural flexibility, supply chains are prone to risks related to political, social, natural, technological, organizational or economic aspects. They can downgrade the system performance or lead to material and immaterial damages.

#### 2.1 On the definitions of risks

Risk is a very complex concept due to the complexity of the world in which we live, and due to the fact that it is a social construction grounded on a cultural dimension. Risk is inevitable, and even crucial, in many sectors of today's society. Several researchers from a wide range of disciplines have looked at risk in a number of different contexts and proposed different definitions of risk. This study follows a generic framework based on norm ISO/CE73 and contained in official documents such as (Circ, 2005). Risk is defined as the combination of the probability of an undesired event (accident) and its consequences (damages). Accident is defined as an unwanted event resulting from uncontrolled evolutions during the exploitation of equipment. It is the realization of a hazardous phenomenon combined with the presence of vulnerable targets exposed to the effects of these phenomena. This undesired event implies consequences for the people, the assets, the environment, or the system as a whole. Consequence or damage is the combination of the intensity of effects and vulnerability of targets (stakes) which are located in areas exposed to these effects. Ultimately, vulnerability of a target to an effect (or sensitivity) represents a factor of proportionality between the effects on a vulnerable element (or target) and the damages that it suffers (see Fig. 1).

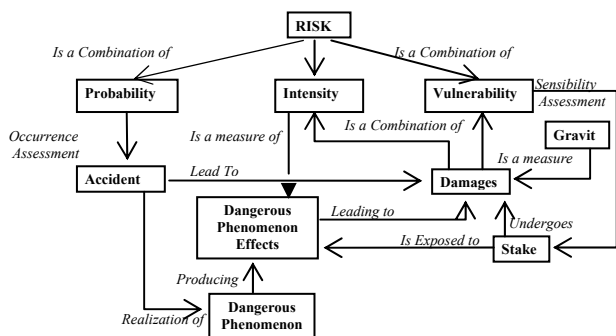


Fig. 1 Some concepts and relations to define risks.

## 2.2 Risks in supply networks

Considering the supply chain as a virtual entity, risks seem to be in the nodes (supplier, manufacturers, distributor, retailer, transporter, etc) rather than in the network. In particular, Finch (2004) shows that while becoming a member of a supply chain, SMEs considerably increase their exposure to risk. However, Stevens (1989) considers that supply chain is "concerned with two distinct flows (material and information) through the organization". This underlines the importance of flows in supply chain. In reality, we distinguish three types of flow connecting the various entities according to their directions of circulation: (1) flow of products (goods or services) from the suppliers towards the customers, (2) financial flow in the opposite direction and (3) the flow of information, in both directions. So, risks can exceed the limits of the organizations and can touch the network itself. Furthermore, information flows have a direct impact on the inventory control, production plans and delivery scheduling. We can give the example of "love bug" computer virus which caused billions of dollars estimated damages by infecting and shutting down e-mail at Pentagon, NASA, Ford and others (S.Chopra, and S.Sodhi, 2004).

## 2.3 What are the main sources of risks in supply networks?

In order to apprehend and analyze supply chain risks, it is necessary to first identify and classify them. Due to the complexity and diversity of global supply chains, we distinguish variety of risks such as: long lead times, demand and supply uncertainty, seasonality, product variety, short life cycles of products, inventory disruption, poor quality or low yield at supply sources, machine failure, problems of information systems and data base, exchange rate risk, natural disaster, etc. many classifications and categories of risks coexist in the SCRM literature. Chopra and Sodhi (2004) suggest nine categories of risks and show its different sources and also indicate how to mitigate them. These categories are: disruptions, delays, systems, forecast, intellectual property, procurement, receivables, inventory and capacity. Mason-Jones and Towill (1998) identify risk sources related to supply chain. The five categories proposed by the authors are presented on Fig.2: environment, demand, supply, process and control. Similarly Jüttner (2005) classified risks based on supply chain basic constructs: environmental-risk, organizational risk and network risks.

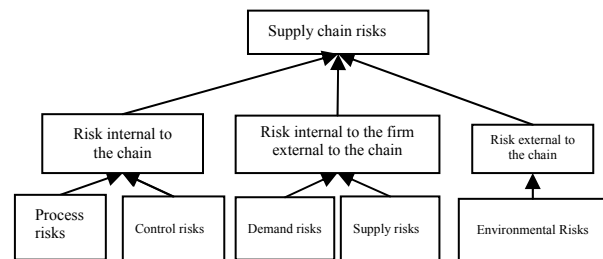


Fig.2 A typology of risks in supply networks

## 4. Risk indicators

For managing supply chain risks, many researchers have developed different strategies and models to mitigate supply chain disruptions (Tang, 2006). In the literature we distinguish a variety of methods to model the supply chain. There are the semi-formal models such as SCOR, and multi-agent models, on the one hand and the analytical models based on mathematical expressions such as time series, on the other hand. The SCOR model generates complex data from various sources and allows modelling, diagnose and assess supply chains. It revolves around five principle processes: plan, source, manufacture, deliver, and return. Multi-agent models are mainly used to organize information flows in supply chains and improve the performance. In the recent years, several risk indicators have been generated by these approaches. However, the focus has been put on good practices, risk avoidance and risk management rather than on risk measurement.

## 4. Vulnerability indicators

Conceptually, vulnerability can be defined as a risk increasing factor. Mason-Jones and Towill (1998) define it as "an exposure to serious disturbance arising from supply chain risks and affecting the supply chain's ability to effectively serve the end customer market". Jüttner et al

(2003) suggest the following definition: “the propensity of risk sources and risk drivers to outweigh risk mitigating strategies, thus causing adverse supply chain consequences”.

The random and non-stationary nature of demand can cause disturbances on order quantities and even dangerous fluctuations in the upstream stages of a supply chain, known as Bullwhip effect. Bullwhip effect is a phenomenon in which the variation of demand produces larger variations in upstream orders and inventory (Gilbert, 2005). Importance of the Bullwhip effect can be considered as a symptom of vulnerability for a supply chain.

Risk assessment requires building indicators of vulnerability related to the identified risks. These indicators can be measured in a quantitative manner (frequency, percentage, ratio, compared variation, etc.), or qualitative way (measurements based on judgment or perception).

In the literature, very few indicators of vulnerability in the supply chains were proposed. Based on the organic decomposition of the supply chain into its basic constructs we can distinguish the internal indicators for a firm such as costs, added value, flexibility, quality and lead times, and indicators external to the firm and internal to the chain related to supply and demand lead times, flow variability, network complexity, organization and contracts. Most vulnerability indicators considered in this study have been constructed from performance indicators. The main assumption is that supply chain vulnerability becomes high when one or several performance indicators reach their critical level.

### III. HOW TO MODEL A SUPPLY NETWORK

In order to study the behaviour of a supply network, it is necessary to first select a modelling paradigm and then to specialize it to the studied system and phenomenon, before identification and validation on real data. In the case of supply networks, modelling is the key to system resilience. An accurate model is required to characterize and quantify the system dynamics, understand and predict its evolution under some critical conditions, to drive the system into safe and efficient running conditions.

#### 3.1 Time series

Time series represent analytically the discrete-time evolution of variables. They allow describing and analyzing a system, to represent its past evolution and predict its future behaviour by using mathematical expressions. Classically, a time series representation of a process starts by identifying its stationary part, which can be represented by an ARMA (Auto Regressive Moving Average) model (Box and Jenkins, 1976) with generic form (1):

$$\phi(B)z_t = \theta(B)a_t \quad (1)$$

where  $B$  is the backward shift operator applied to input series  $\{a_t\}$  and output series  $\{z_t\}$ . In the SISO (Single Input Single Output) version of model (1),  $\phi(B)$  and  $\theta(B)$  are polynomials in  $B$ , while in the MIMO (Multi Input Multi Output) version, they are polynomial matrices. In this study, the considered models are of the SISO type. By definition, an ARMA( $p,q$ ) model is a model given by formula (1), in

which the autoregressive polynomial in  $B$ ,  $\phi(B)$ , has order  $p$  and the moving average polynomial  $\theta(B)$ , has order  $q$ . Stationarity is characterized by the fact that the roots in  $B$  of  $\phi(B) = 0$  lie strictly outside the unit circle of the complex plane. In the nonstationary case, the unity ( $1+0i$ ) is a solution of  $\phi(B) = 0$  with multiplicity  $r$ . Then, by defining the integrated series:  $\tilde{z}_t = (1-B)^r z_t$ , equation (1) can be re-written in the stationary ARIMA form:

$$\tilde{\phi}(B)\tilde{z}_t = \theta(B)a_t \quad (2)$$

with  $\tilde{\phi}(B) = (1-B)^r \phi(B)$ , also written  $\tilde{\phi}(B) = \nabla^r \phi(B)$  with by definition,  $\nabla = 1 - B$ .

#### 3.2 Supply chain modelling with ARIMA

Consider a supply chain dedicated to production and sale of products. This is a multi-stage system in which each intermediate manufacturing stage is located in a particular firm, requires input products from one or several firms and delivers manufactured products to other firms. All the firms involved in the different manufacturing and logistics stages constitute a network with flow and information arcs connecting the nodes.

One of the basic requirements for a supply chain model is its ability to represent random fluctuations of demand and their propagation upward the supply chain.

##### 3.2.1 The Demand Process

Consider a discrete time representation of a demand process defined by a series of values over elementary time intervals with unit time duration. Assuming that demand at periods  $k \in \mathbb{N}$  forms a stationary time series with mean value  $d$ , it can be represented by the following ARMA model:

$$\phi(B)(d_k - d) = \theta(B)w_k \quad (3)$$

where  $w_k$  is a white noise characterized by the following

$$\text{moments: } \begin{cases} E(w_k) = 0, E(w_k^2) = \sigma_w^2 \quad \forall k \in \mathbb{N} \\ E(w_k w_l) = 0 \quad \forall (k, l) \in \mathbb{N}^2, k \neq l \end{cases}$$

Model (3) can also be written as the output of a linear filter:

$$d_k = d + \Psi(B)w_k \text{ with } \Psi(B) = \frac{\theta(B)}{\phi(B)} = 1 + \sum_{i=1}^{\infty} \psi_i B^i \quad (4)$$

##### 3.2.2 The downward stage of the chain

Consider the last stage of the chain. It involves the customers, through the current value of demand,  $d_k$ , given by (3) or (4), the retailer, through his inventory level at successive periods  $k-1$  and  $k$  and past orders to his supplier:  $O_{k-L}, \dots, O_{k-1}$ . The value of lead time  $L$ , assumed to be constant, depends on the supply and transportation of final products.

The inventory balance equation describing the last stage of the supply chain is written:

$$I_k = I_{k-1} + O_{k-L} - d_k \quad (5)$$

where  $I_k$  is the inventory at the end of period  $k$  and  $O_{k-L}$  the order placed at the end of period  $k-L$  and expected in period

$k$ . The last stage model can then be completely determined by the choice of the retailer ordering policy.

The ordering policy considered in this study is the “order up to” policy with respect to the inventory position. This policy has been shown to be optimal with respect to a model predictive approach (Hennet 2003):

$$O_k = S + \hat{d}_{k+1|k} + \dots + \hat{d}_{k+L|k} - I_k - O_{k-1} - \dots - O_{k-L+1} \quad (6)$$

In equation (6), the order level  $O_k$  is computed using conditional expectations of demand  $d_{k+j}$  computed at period  $k$ , denoted  $\hat{d}_{k+j|k}$  for  $j=1, \dots, L$ .

Equation (3) can be used to compute the sequence of predictions  $(\hat{d}_{k+1|k}, \dots, \hat{d}_{k+L|k})$ , using the division of polynomials in the form (see e.g. Kučera, 1979):

$$\theta(B) = \phi(B)Q_p(B) + B^p R_p(B) \quad (7)$$

with  $\deg(Q_p(B)) = p-1$ , to obtain from (3):

$$\theta(B)(\hat{d}_{k+p|k} - d) = R_p(B)(d_k - d) \text{ for } p=1, \dots, L \quad (8)$$

Gilbert (2005) has shown that under the assumptions of an ARMA(p,q) model of demand (3), constant lead-times for all the products, and order up-to policies (6), the sequence of inventories  $\{I_k\}$  and the sequence of orders,  $\{O_k\}$  can be represented by ARIMA models. (9) and (10).

$$I_k = S + \varepsilon_k^{(I)} + (1 + \psi_1)\varepsilon_{k-1}^{(I)} + \dots + (1 + \psi_1 + \dots + \psi_{L-1})\varepsilon_{k-L+1}^{(I)} \quad (9)$$

$$\Phi(B)\nabla^\delta(O_k - d) = \Theta^{(O)}(B)w_k^{(O)} \quad (10)$$

where  $q^{(O)} = \max(p + \delta, q - l)$  and a noise series  $(w_k^{(O)})$  multiple of the noise series  $(w_k)$  of the demand process.

### 3.2.3 A dynamic model for each stage of the chain

If we neglect saturation effects, the time series ARIMA representation property is valid at all the stages of the supply chain. A key result for multistage supply chains under the “order up to” policy is then that the bullwhip effect at any stage  $s$  of a supply chain depends on the total of lead times from stage  $s$  down to final stage 1 and not on the number of stages (Gilbert 2005). An indicator of the supply chain bullwhip effect can thus be obtained from the sum of lead times and the parameters of the ARIMA models.

## 3.3 Vulnerability indicators

Due to the ability of the ARMA to represent the fluctuations of flows and inventories in supply networks, it seems interesting to construct indicators for assessing the vulnerabilities of the system, such as the bullwhip effect, the low-demand indicator, the starving indicator and the over-cost indicator.

### 3.3.1 Bullwhip effect indicators

Random variations in supply chains have a cumulative effect known as “Bullwhip Effect”. Random variations on lead times (production time or supply time) in the MRP method and uncertainties on predictions of orders, generate the bullwhip effect and have a significant impact on the system performance.

Lee et al (1997) have identified four causes for the bullwhip effect: demand signal processing, rationing game, order batching and price variation.

Under the ARMA model (9), (10), of a supply chain driven by the order up to policy, the ratios of standard deviations corresponding to the “bullwhip” effect are given by the following expressions (Gilbert,2005):

- Bullwhip effect related to the Inventory:

$$K_I = [1 + (1 + \psi_1)^2 + (1 + \psi_1 + \psi_2)^2 + \dots + 1 + \psi_1 + \psi_2 + \dots + \psi_L - 1212] \quad (11)$$

- Bullwhip effect related to the Order:

$$K_O = 1 + \phi + \phi^2 + \phi^3 + \dots + \phi^L = \frac{1 - \phi^{L+1}}{1 - \phi} \quad (12)$$

These expressions clearly show that the bullwhip effect depends on the total lead time  $L$  and not on the number of stages in a supply chain.

### 3.3.2 Order index

When issuing a command, a company may face three possible cases, two of which are sources of risk. The first one is the case when the command computed by the unconstrained model is negative, meaning that the retailer would like to sell back some products that he has previously ordered. The second case is when the retailer wants to order a quantity greater than what the supplier possesses or can deliver. Assuming that  $\Omega_k$  is the actual order and  $\bar{O}$  is the available capacity at the supplier’s, the order and order index  $O I_k$  are linked as follows:

$$\begin{cases} \Omega_k = 0 \text{ and } O I_k = -1 \text{ if } O_k < 0 \\ \Omega_k = O_k \text{ and } O I_k = 0 \text{ if } 0 \leq O_k \leq \bar{O} \\ \Omega_k = \bar{O}_k \text{ and } O I_k = 1 \text{ if } O_k > \bar{O} \end{cases} \quad (13)$$

### 3.3.3 Inventory level index

Classically demand  $d_k$  is supposed known before order  $\Omega_k$  is placed. Then, demand  $d_k$  is delivered from the stock. The inventory level may characterize two critical cases. The first one is when it is negative, representing the case when demand  $d_k$  is greater than the quantity available in stock,  $I_k$ . In this case, the excess demand is backordered. The second critical case is relative to the storage capacity that cannot be exceeded. Assuming that  $J_k$  represents the current stock and that  $\bar{I}$  is the storage capacity, the current stock  $J_k$  and stock indicator  $I I_k$  are determined as follows:

$$\begin{cases} J_k = 0 \text{ and } I I_k = -1 \text{ if } I_k < 0 \\ J_k = I_k \text{ and } I I_k = 0 \text{ if } 0 \leq I_k \leq \bar{I} \\ J_k = \bar{I}_k \text{ and } I I_k = 1 \text{ if } I_k > \bar{I} \end{cases} \quad (14)$$

Inventory level and order indices reveal the system vulnerability to strong external disturbances. They will now be used to construct several vulnerability indicators, namely the delay indicator, the over-cost indicator and the nervousness indicator.

### 3.3.4 Low-demand indicator

The Low-demand indicator can be calculated as follows:

$$\begin{cases} LI(D) = \frac{1}{D} \sum_{k=1}^D \max(-O I_k, 0) \\ LI(k) = LI(k-1) + \frac{1}{D} (\max(-O I_k, 0) - \max(-O I_{k-D}, 0)) \text{ for } k > D. \end{cases} \quad (15)$$

The case when the index  $OI_k=-1$  means that, for a given stage, the buyer (retailer) would like to sell back some products. It can be explained by the insufficiency in the demand (consumer) side.

### 3.3.5 Starving indicator

The value  $OI_k=1$  represents the case when the retailer would like to order more than the available capacity per time interval at the retailer's. To describe this situation from the retailer's viewpoint, this index is called a starving indicator. In a dual manner, it represents an insufficient capacity for the supplier.

$$\begin{cases} SI(D) = \frac{1}{D} \sum_{k=1}^D \max(OI_k, 0) \\ SI(k) = SI(k-1) + \frac{1}{D} (\max(OI_k, 0) - \max(OI_{k-D}, 0)) \\ \text{for } k > D. \end{cases} \quad (16)$$

### 3.3.6 Delay indicator

A possible indicator of the delay for the considered stage of the supply chain is the experimental probability of backorder. It can be constructed from the sequence of inventory level indices over a time duration of D times the elementary time interval:

$$\begin{cases} DI(D) = \frac{1}{D} \sum_{k=1}^D \max(-II_k, 0) \\ DI(k) = DI(k-1) + \frac{1}{D} (\max(-II_k, 0) - \max(-II_{k-D}, 0)) \\ \text{for } k > D. \end{cases} \quad (17)$$

$DI(k)$  represents the experimental probability to have backorders during the time interval  $[k+1, k+D]$ . If the indicator value is near to 1, it means that the quality of service is low. This is also a vulnerability indicator relatively to next stage of the supply chain, which may suffer from starving conditions and look for other suppliers.

### 3.3.7 Over-cost Indicator

When demand is satisfied, either immediately or after some delay, over-costs may be caused by the violation of positivity and capacity constraints on stock and order.

The inventory capacity saturation indicator may be computed by formula (18):

$$\begin{cases} IC(D) = \frac{1}{D} \sum_{k=1}^D \max(II_k, 0) \\ IC(k) = IC(k-1) + \frac{1}{D} (\max(II_k, 0) - \max(II_{k-D}, 0)); \\ \text{for } k > D \end{cases} \quad (18)$$

The over-cost indicator can then be computed by summation of the all the cost indicators:  $CI(k) = LI(k) + SI(k) + IC(k) + DI(k)$ . Noting that  $|II_k| = \max(II_k, 0) + \max(-II_k, 0)$  and  $|OI_k| = \max(OI_k, 0) + \max(-OI_k, 0)$ , this indicator is recursively computed by:

$$\begin{cases} CI(D) = \frac{1}{D} \sum_{k=1}^D (|II_k| + |OI_k|) \\ CI(k) = CI(k-1) + \frac{1}{D} (|II_k| - |II_{k-D}|) \\ + \frac{1}{D} (|OI_k| - |OI_{k-D}|); \text{ for } k > D \end{cases} \quad (19)$$

If the over-cost indicator is frequently greater than a certain threshold, it means that the supply chain is not efficient and that its production and/or inventory capacity must be revised.

## IV EXAMPLE AND CASE STUDY

In this section, we model a simple supply chain with one supplier and one retailer. This system incorporates a non-stationary demand with a random walk. It is generated by Monte-Carlo generation with the following model:

$$d_k = V(k) + w_k \quad (21)$$

and  $\{w_k\}$  is the white noise ( $E(w_k)=0$  and  $E(w_k^2)=\sigma$ ).

The average demand profile in (21) is as follows:

$$V(1:300) = 20, V(301:700) = 30, V(701:1000) = 10.$$

The non-stationary average value,  $V(k)$  in (21) is not supposed to be known by the retailer. He needs to predict it from its past and current values.

The demand model estimated by the retailer is supposed to take the form:

$$\hat{d}_k = 0.5d_{k-1} + 0.5d_{k-2} + \varepsilon_k \quad (22)$$

and  $\{\varepsilon_k\}$  is a Gaussian white noise ( $E(\varepsilon_k)=0$  and  $E(\varepsilon_k^2)=\eta$ ).

The delivery time is assumed fixed and known:  $L=3$ . From model (20), it is easy to derive predictions  $\hat{d}_{k+j|k}$  for  $j=1, 2, 3$ :

$$\begin{cases} \hat{d}_{k+1|k} = 0.5d_k + 0.5d_{k-1} \\ \hat{d}_{k+2|k} = 0.75d_k + 0.25d_{k-1} \\ \hat{d}_{k+3|k} = 0.625d_k + 0.375d_{k-1} \end{cases} \quad (23)$$

The ordering policy is the order up to policy with authorized backorders. It is defined by:

$$O_k = S + \hat{d}_{k+1|k} + \hat{d}_{k+2|k} + \hat{d}_{k+3|k} - I_k - O_{k-1} - O_{k-2}. \quad (24)$$

The order  $O_k$  is selected to lift up the inventory level to  $S$ , while covering the expected demand over  $L$  periods, taking into account the orders at times  $k-1$  and  $k-2$ . In this example, we chose the value  $S = 20$ .

The system will now be simulated in two stages. In the first stage, the system dynamics are simulated without taking into account the constraints of positivity and capacity on inventory and order levels. These constraints are then introduced in the second stage, and vulnerabilities indicators are computed from the constrained model, to study the risk of reaching critical levels for the stock or the order. The dynamics of the unconstrained system are defined by relations (5), (22), (24) and shown on Fig.3

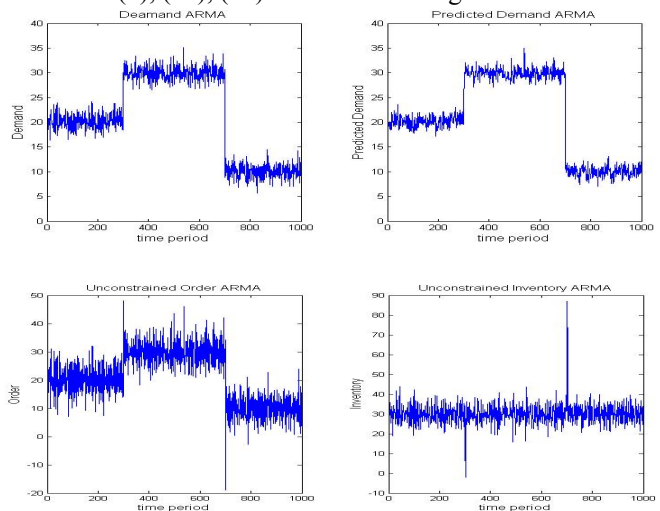


Fig. 3 Simulation of the unconstrained system

It can be observed that in the unconstrained case, the inventory level fluctuates around the level of reference. To represent reality more accurately, constraints and saturation functions on inventory and order levels are then integrated into the ARIMA models. At each step of the simulation, the values of  $O_k$  and  $I_k$  are respectively replaced by their actual values  $\Omega_k$  and  $J_k$  and the dynamic equations are run step by step. The constrained evolution of the inventory level is represented on Fig. 4

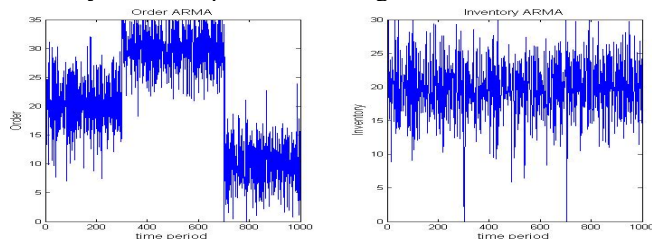


Fig. 4 Simulation of the constrained system

Simulation of the constrained system indicates that the system frequently hits the constraints. The corresponding vulnerability indicators are displayed on Fig. 5.

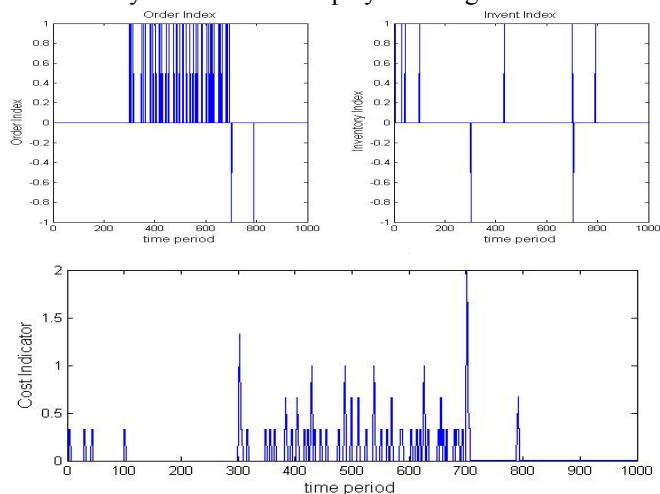


Fig. 5 Simulation of vulnerability indicators

The evolution of vulnerability indicators and more specifically the cost indicator in the situations of average, high and low demand, indicate that the system is more vulnerable in high demand conditions. In fact, its inventory and ordering capacities are better adapted to an average demand per period between 10 and 20 per period.

## V CONCLUSIONS

Supply networks are facing the important challenge of adjusting the size of their manufacturing capacity and product storage, especially when demand and supply are subject to strong and unpredicted variations. In such conditions, how to mitigate supply chain risks without eroding profits? Risk analysis has become a key issue to manage supply chains and supply networks. Firstly, it is necessary to identify the risks in the system, then to model their possible impacts on the system, taking into account the constraints on production and storage for each firm and their effects on the products and value chains. The recursive nature of ARMA allows to model

supply chains at each manufacturing stage as an elementary node and iterating the process to represent the whole network. In this paper, it has been proposed to add positivity and capacity constraints to this model, to simulate the system under strongly disturbed conditions, and to measure vulnerabilities indicators to assess the internal effects of strong external disturbances. Application of this methodology on a simple example has shown the relevance of the approach and its possible use in managerial decision support systems.

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