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Frequency Domain Tuning Method for Unconstrained Linear Output Feedback Model Predictive Control

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Abstract:
This paper provides a novel solution to the problem of tuning linear output feedback model predictive control (MPC). A systematic tuning method that allows to obtain all the parameters of an unconstrained output feedback MPC based on disturbance model and observer is presented. It is shown that such tuning can be translated to a frequency domain control design problem, which can be solved using existing techniques. Experimental results on a quadrotor reference tracking problem show the effectiveness of the proposed MPC tuning method.

Keywords: MPC, Output Feedback, Observers, Disturbance Model

1. INTRODUCTION

Although model based predictive control theory promises the solution to many industrial control challenges such as control of multivariable systems and constraint handling, the most recent model predictive control (MPC) formulations tend to have a very complex structure with several parameters to be tuned, see [3] for a survey. One of the main limitations of MPC is that it requires full knowledge of the plant states, and one of the most common approaches is to use an observer to estimate them [9]. With this approach two new problems arise: first, the complexity and number of tuning parameters of MPC, such as cost function weights and prediction horizons, are increased by the observer parameters; second, it has been shown that in the presence of perturbations, the closed-loop of observer based state feedback controllers can show arbitrary stability margins [10], thus affecting the robustness of the closed-loop system.

The problem of robustness of output feedback MPC is still an open research area where several approaches have been investigated. Since the early implementations of MPC, robustness against uncertainties has been achieved by adding disturbance models to the nominal model of the system to be controlled (see [12] for a survey). For example, to remove an offset, the nominal model of the system to be controlled can be augmented with an integrator. This kind of approach has shown to improve MPC robustness by removing offsets from the predictions, which are used to compute the optimal control inputs in MPC [15]. If there is some knowledge of the frequency characteristics of the disturbances, more complex disturbance models can be used, and complete tuning methods have been developed based on this idea [14]. The augmented model, i.e. the nominal model with disturbance model, is then used both in the observer and MPC formulation, and Loop Transfer Recovery techniques have been used to obtain robustness with respect to system/model mismatch [13]. One of the disadvantages already mentioned is that the MPC tuning effort is increased by the choice of disturbance model and observer tuning. Furthermore, there are some authors that have pointed out that this approach based on the 'certainty equivalence principle' only takes uncertainty into account indirectly [11].

In more recent approaches, set theory and modified versions of MPC are used to account for uncertainty directly, see [16] and [11]. The estimation error is considered as an unknown uncertainty but bounded by a pre-computed invariant set, and a 'tube-based' robust MPC algorithm is designed to explicitly deal with this bounded uncertainty. Although results of these type of approaches are very promising, the structure and tuning complexity of such MPC formulations can be considerably large. This contrasts with the most recent extensions of classical control theory to multivariable robust control, e.g. H_∞ control, where the tuning of the controller is more intuitive for practitioners based on frequency domain weights or loop-shaping and modeled uncertainty can be explicitly taken into account [8]. Nevertheless, multivariable modern robust control approaches do not offer the flexibility and advantages of MPC such as feedforward, cost selection and constraint handling [9].

The linear output feedback MPC tuning problem is to find an MPC and observer match, such that desired closed-loop performance and robustness margins can be obtained up to a useful degree. The main contribution of this paper is a systematic method for solving the unconstrained linear output feedback MPC tuning problem using frequency domain design is proposed. The main characteristic of this method is the ability to provide most of the MPC parameters out of a frequency domain design problem, in this way the complexity of the MPC tuning problem is considerably reduced. As a result, the tuned disturbance model and observer result in improved predictions that make the proposed method specially suitable for predictive control. The analysis of the inclusion of constraints with the proposed method is left for future work. A quadrotor reference tracking problem is used to show experimental results obtained by implementing a tracking MPC formulation with the proposed tuning method.

An unconstrained linear output feedback MPC formulation based on disturbance model and observer will be used, which is described in Section 2. Based on this MPC formulation the tuning problem is defined in Section 3. The principal results of this paper are also presented in Section 3, where a frequency domain design problem is introduced based on a continuous time model of the system to be controlled. Discretization steps are discussed and it is shown how to obtain the unconstrained linear output feedback MPC formulation parameters out of the frequency domain design, including a disturbance model and observer that are used to improve predictions in MPC. In Section 4 experimental results are shown, obtained by applying the method to a quadrotor reference tracking problem.

2. CONTROLLER STRUCTURE

In this Section, the main ingredients of the unconstrained linear output feedback MPC formulation used in this paper are presented. It is composed by an internal model, a cost function and an observer. Since the goal of this paper is to describe a continuous-time tuning method for a discrete time algorithm, both continuous and discrete time models will be used. Each of the MPC formulation ingredients are described in detail below.

2.1 Internal Model

It will be assumed that a continuous time nominal model is available that captures the main dynamics of the input/output behavior of the system to be controlled. Furthermore, this nominal model has to be a strictly proper state space minimal realization of the form

$$\begin{aligned} \dot{x}_n &= A_n x_n + B_n u \\ y &= C_n x_n \end{aligned} \tag{1}$$

where $x_n \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^l$ are state, input and output vectors respectively with $n \geq m$ and $n \geq l$. The matrices A_n, B_n and C_n are the state transition, input and output matrices respectively. For this model to be used in the proposed MPC formulation, it needs to be transformed to a discrete time form. By using the zero-order hold (ZOH) discretization method, the discrete time version of the system to be controlled becomes

$$\begin{aligned} x_n(k+1) &= A_{nk} x_n(k) + B_{nk} u(k) \\ y(k) &= C_{nk} x_n(k) \end{aligned} \tag{2}$$

where k denotes the discrete time variable. The matrices A_{nk}, B_{nk} and C_{nk} are the matrices of the nominal discrete time state space model.

In order to account for disturbances, a standard disturbance model will be used [9], in which the continuous time nominal model dynamics are augmented with the disturbance model leading to an augmented model of the form

$$\begin{aligned} \begin{bmatrix} \dot{x}_n \\ \dot{x}_d \end{bmatrix} &= \begin{bmatrix} A_n & X \\ 0 & A_d \end{bmatrix} \begin{bmatrix} x_n \\ x_d \end{bmatrix} + \begin{bmatrix} B_n \\ 0 \end{bmatrix} u \\ y &= \begin{bmatrix} C_n & 0 \end{bmatrix} \begin{bmatrix} x_n \\ x_d \end{bmatrix} \end{aligned} \tag{3}$$

where $x_d \in \mathbb{R}^d$ is the disturbance state vector. The matrix A_d is the disturbance model transition matrix and X is the matrix that maps the effects of the disturbance states to the nominal states. The discrete time augmented model obtained by ZOH discretization has the form

$$\begin{aligned} \begin{bmatrix} x_n(k+1) \\ x_d(k+1) \end{bmatrix} &= \begin{bmatrix} A_{nk} & X_k \\ 0 & A_{dk} \end{bmatrix} \begin{bmatrix} x_n(k) \\ x_d(k) \end{bmatrix} \\ &\quad + \begin{bmatrix} B_{nk} \\ 0 \end{bmatrix} u(k) \\ y(k) &= \begin{bmatrix} C_{nk} & 0 \end{bmatrix} \begin{bmatrix} x_n(k) \\ x_d(k) \end{bmatrix} \end{aligned} \tag{4}$$

where the matrices A_{dk} and X_k are the matrices of the discrete time disturbance model. It is assumed that the disturbance model matrices, either in the continuous or discrete time form, are not known in advance and are part of the MPC formulation tuning parameters, which will be obtained with the proposed tuning method.

2.2 Cost Function

The cost function of the unconstrained linear MPC is defined as

$$\begin{aligned} V(x_n, U) = \sum_{i=0}^{N-1} \Big(x_n(i)^T Q x_n(i) + u(i)^T R u(i) \Big) \\ + x_n(N)^T P x_n(N) \end{aligned} \tag{5}$$

where $N > 0$ is the prediction horizon, $U = (u(0), u(1), \cdots, u(N-1))$ is the vector of stacked inputs and $R = R^T \succeq 0$, $Q = Q^T \succeq 0$, $P = P^T \succeq 0$ are the input cost, stage cost and final stage cost matrices respectively. The corresponding optimization problem for the MPC formulation is defined below.

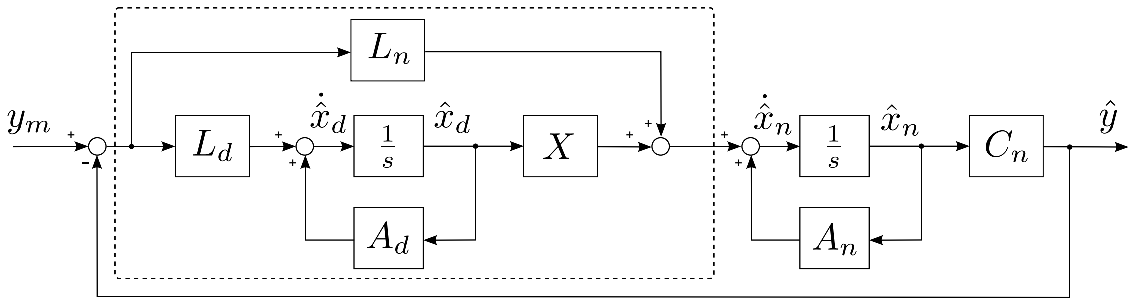

Fig. 1. Continuous time observer gain and disturbance model design as a standard tracking control problem.

$$\begin{aligned} &\min_{U} V(x_n, U) \\ &subject\ to: \\ &x_n(k+1) = A_{nk}x_n(k) + X_k x_d(k) + B_{nk}u(k) \\ &x_d(k+1) = A_{dk}x_d(k) \end{aligned} \tag{6}$$

For this MPC formulation the final cost P is set to the solution to the Ricatti equation corresponding to the equivalent infinite horizon control problem for some given Q and R, since it is well known that in the absence of constraints, an MPC formulation with a final stage cost equal to the solution of the infinite horizon control problem, is equivalent to the well known Linear Quadratic Regulator (LQR) for any prediction horizon $N \geq 1$ [1]. The solution to the unconstrained MPC optimization problem from Equation (6) can be obtained by solving a least squares problem, see [9] for more details.

It is assumed that the stage cost Q and input cost R are unknown and form part of the MPC formulation tuning parameters and will be obtained by the proposed method.

2.3 Observer

A standard observer will be used in this MPC formulation to estimate the states of the augmented model. This observer is defined by the continuous time equations

$$\begin{aligned} \begin{bmatrix} \dot{\hat{x}}_n \\ \dot{\hat{x}}_d \end{bmatrix} &= \begin{bmatrix} A_n & X \\ 0 & A_d \end{bmatrix} \begin{bmatrix} \hat{x}_n \\ \hat{x}_d \end{bmatrix} + \begin{bmatrix} B_n \\ 0 \end{bmatrix} u \\ &+ \begin{bmatrix} L_n(y_m - C_n\hat{x}_n) \\ L_d(y_m - C_n\hat{x}_n) \end{bmatrix} \end{aligned} \tag{7}$$

where $\hat{x}_n$ and $\hat{x}_d$ are the estimated nominal model states and estimated disturbance model states respectively, y_m is the output measurement which might contain the effects of noise and disturbances, L_n and L_d are the observer gain matrices for the nominal and disturbance models respectively. The reason why the observer has been introduced in its continuous time form will be more clear in Section 3. For the moment it is assumed that the discrete time counter part of the observer has the form

$$\begin{aligned} \begin{bmatrix} \hat{x}_n(k+1) \\ \hat{x}_d(k+1) \end{bmatrix} &= \begin{bmatrix} A_{nk} & X_k \\ 0 & A_{dk} \end{bmatrix} \begin{bmatrix} \hat{x}_n(k) \\ \hat{x}_d(k) \end{bmatrix} + \begin{bmatrix} B_{nk} \\ 0 \end{bmatrix} u(k) \\ &+ \begin{bmatrix} L_{nk}(y_m(k) - C_{nk}\hat{x}_n(k)) \\ L_{dk}(y_m(k) - C_{nk}\hat{x}_n(k)) \end{bmatrix} \end{aligned} \tag{8}$$

where L_{nk} and L_{dk} are the discrete time observer gain matrices, which are used for the actual observer implementation. The observer gain matrices, either in the continuous or discrete time form, are not known in advance and are part of the MPC formulation tuning parameters which will be obtained by the proposed tuning method.

3. TUNING METHOD

In the previous section, the unconstrained linear output feedback formulation has been introduced. For this formulation it is known that for a prediction horizon $N \geq 1$ the MPC is equivalent to a discrete LQR controller and therefore is stabilizing given that the system is controllable, observable, $R = R^T \succeq 0$, $Q = Q^T \succeq 0$ and $P = P^T \succeq 0$ [1]. Therefore, in this Section the main focus will be in obtaining the tuning parameters given by the discrete time disturbance model matrices A_{dk}, X_k, observer gains L_{nk}, L_{dk}, stage cost Q and input cost R.

3.1 Problem Definition

Consider the unconstrained output feedback MPC formulation from Equations (5), (6) and (8), and the system to be controlled from Equation (4). In closed loop they form what will be defined as the Actual Feedback Loop transfer function $\Phi_{AFL}(z)$.

The output feedback MPC tuning problem is to find appropriate disturbance model matrices A_{dk}, X_k, observer gains L_{nk}, L_{dk}, stage cost Q and input cost R such that the the Actual Feedback Loop (AFL) discrete transfer function $\Phi_{AFL}(z)$ approximates a (stable) Desired Feedback Loop (DFL) discrete transfer function $\Phi_{DFL}(z)$.

3.2 Observer Gains and Disturbance Model

To solve the unconstrained output feedback MPC tuning problem, we start with the continuous time observer design. The main result of this paper is to show that the design of the observer gains and disturbance model matrices simplifies to a standard tracking control problem, and that the resulting disturbance model and observer improve the MPC predictions. The simplification can be shown easily by noticing that the block diagram of the observer transfer between y_m and $C_n\hat{x}_n$ defined in Equation 7 can be rearranged as a standard tracking control problem as shown in Figure 1, where the effect of u has been omitted in the tracking control problem from Figure 1, since u can be regarded for this observer tracking control problem as a measured bounded known disturbance given by the state feedback controller block, i.e. the MPC. The effect of u does not compromise the stability of the observer in closed loop with the MPC due to the separation principle.

As it can be observed from Figure 1, the observer gains, disturbance model matrices and states altogether form a complete dynamic compensator block

$$J(s) = X\left(sI_d - A_d\right)^{-1} L_d + L_n \tag{9}$$

where I_d is an identity matrix of suitable dimension.

Define $\Psi(s) = C_n\left(sI_n - A_n\right)^{-1}$, the observer gains and disturbance model design problem in the continuous time domain can be defined as finding a dynamic compensator $J(s)$ such that the closed loop

$$\Phi_{DFL}(s) = \frac{\hat{y}(s)}{y_m(s)} = \left[I + \Psi(s)J(s)\right]^{-1}\Psi(s)J(s) \tag{10}$$

meets the desired stability-robustness and performance specifications. The Desired Feedback Loop (DFL) $\Phi_{DFL}(s)$ can then be defined by shaping the transfer between $y_m(s)$ and $\hat{y}(s)$. In the next Subsection it will be shown how to recover the DFL stability-robustness and performance properties in the overall Actual Feedback Loop (AFL) $\Phi_{AFL}(z)$.

Note that since it was assumed that the system in Equation (1) is a minimal realization, i.e. $[A_n, C_n]$ detectable, it implies that there exists a $J(s)$ which will make the continuous time Desired Feedback Loop $\Phi_{DFL}(s)$ stable. Furthermore, the dynamic compensator $J(s)$ can be synthesized using any of the available multivariable robust control techniques such as weighted sensitivity or loop-shaping [8], and modeled uncertainty can also be included in the design.

Once a suitable dynamic compensator $J(s)$ has been found, it can be discretized to obtain $J(z)$ and with it the discrete disturbance model matrices A_{dk}, X_k and discrete observer gains L_{nk}, L_{dk}. The resulting disturbance model and observer improve the predictions made with the augmented model, as it is shown in Section 4. Finally, the discrete time observer from Equation (8) can be built using $J(z)$ and Equation (2). Discretization of $J(s)$ is not restricted to the ZOH method, since $J(z)$ does not have to be strictly proper.

3.3 Cost Function Weights

Due to the equivalence of the unconstrained linear MPC formulation of this paper with the LQR controller, it is possible to leverage all the theory and methods developed for the LQR controller, such as the Loop Transfer Recovery (LTR) theory which was developed in the continuous time by [6] and [7], and in the discrete time by [5], in which it is shown that if the system in Equation (2) is square, minimum phase, $det(C_{nk}, B_{nk}) \neq 0$, the stage cost $Q = C_{nk}^T C_{nk}$ and the input cost $R \to 0$, then for the unconstrained linear output feedback MPC formulation

$$\Phi_{AFL}(z) \to \Phi_{DFL}(z). \tag{11}$$

The LTR theory shows that the convergance of the Actual Feedback Loop to the Desired Feedback Loop gets closer as R is reduced towards zero. In the discrete time case and under the given assumptions, the recovery of the Desired Feedback Loop can be made perfect if $R = 0$ [5]. In practice the choice of $R = 0$ is relaxed to $R = \rho I_m$, where ρ is a small value.

The LTR convergance theorem assumptions mentioned before might seem very restrictive, but many authors from the LTR theory have shown that the technique can still be used with non-square systems [17]. They have also shown that even if the system is non-minimum phase, convergance is still obtained at least for the frequencies below the effects of the non-minimum phase zeros. In general the LTR theory indicate that even if some of the assumptions are not met, recovery is often obtained over a useful degree for $Q = C_{nk}^T C_{nk}$ and $R = \rho I_m$ [9].

4. EXPERIMENTAL RESULTS

A quadrotor XY reference tracking problem is used to test the MPC tuning method presented in this paper. The setup consists of a AR Drone v1.0 quadrotor platform. The quadrotor is set to fly at a fixed altitude following a desired trajectory in the XY plane, where there are internal control loops that allow to control the pitch θ and the roll ψ angles of the quadrotor. The input vector is then $u = [u_\theta, u_\psi]^T$, both are normalized to lie in the range $[-1, 1]$. u_θ and u_ψ actuate the pitch and roll of the quadrotor, causing movements in y and x axis respectively. The output vector $z_{meas} = [x, \dot{x}, y, \dot{y}]^T$ corresponds to measurements of the position and velocity in the $x - axis$, and position and velocity in the $y - axis$. These measurements are obtained from the inertial sensors and are provided by the AR Drone platform (see [18] for details). The positions are measured in meters $[m]$ and the velocities in meter per second $[m/s]$. Note that for this example it was decided to change the notation of the output vector to z rather than y in order to avoid confusion with the position in the $y - axis$, the same for the state vector which is now defined as w instead of x to avoid confusion with the position in the $x - axis$.

4.1 Simplified Quadrotor Model

To obtained a fairly simple model of the quadrotor, a series of step functions were applied to it on each of its inputs, u_θ and u_ψ. It was observed that dynamics from u_θ to $\dot{y}$ and those of u_ψ to $\dot{x}$ are close to those of second order models, with the form

$$G(s, u, \xi, \omega) = \frac{K_u(u)}{\left(\frac{s}{\omega}\right)^2 + 2\xi\left(\frac{s}{\omega}\right) + 1} \tag{12}$$

in which the gain of the system varies with the magnitude of the input. Such non-linearity was modeled as static

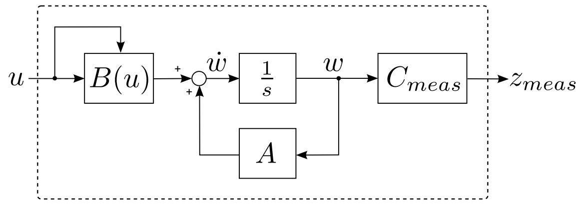

Fig. 2. Quadrotor control-oriented model.

nonlinear gains in the input channels. Additionally, the quadrotor presented some coupling between u_θ and $\dot{x}$, and between u_ψ and $\dot{y}$, however their magnitude was very small and such couplings were treated as disturbances. Therefore, we consider the models

$$\frac{y}{u_\theta} = \frac{1}{s}G(s, u_\theta, \xi_\theta, \omega_\theta), \; \frac{x}{u_\psi} = \frac{1}{s}G(s, u_\psi, \xi_\psi, \omega_\psi) \tag{13}$$

Finally we can use the model structure shown in Figure 2, were w denotes the state vector and $B(u)$ the input dependent non-linear gain.

4.2 MPC formulation

The resulting control-oriented model is non-linear as shown in Figure 2. The corresponding observer design problem remains linear because its design does not depend on the (scheduled) input matrix $B(u)$ and the compensator $J(s)$ can still be synthesized using linear techniques. For this application, the observer design problem is solved using H_∞ loop-shaping techniques. The open-loop observer transfer $\Upsilon(s) = C_{meas}(sI_n - A)^{-1} \cdot J(s)$ was shaped using the procedure described by McFarlane and Glover (see [8] for details), aiming to achieve an ideal open-loop transfer ω_B/s for all the singular values of $\Upsilon(s)$, where ω_B is the desired observer open-loop bandwidth.

The MPC formulation for this application is a reference tracking formulation designed to leverage the feedforward capabilities of MPC. The cost function is modified to penalize deviations of the x and y positions around their desired references and to penalize the change of the inputs Δu with respect to the previous inputs. The cost function used in this MPC formulation is

$$V(e_z, U) = \sum_{i=0}^{N-1} \left(e_z(i)^T Q_z e_z(i) + \Delta u(i)^T R \Delta u(i) \right) + e_z(N)^T P_z e_z(N) \tag{14}$$

where $e_z(i) = z_{control}(i) - z_{ref}(i)$ is the output tracking error defined as the difference between the controlled output vector $z_{control} = C_{control} \cdot w = [x, y]^T$ and the desired reference vector z_{ref}. In this case the stage cost $Q_z = I_z$ is an identity matrix, since the cost already penalizes the output. Note that P_z has little influence on the solution since from equation (14) it is expected that for large N, $e_z(N) \to 0$. Thus, the final cost is chosen $P_z = I_z$, this also avoids the computation of a Ricatti equation on each sample time due to the variation of $B(u)$.

The corresponding optimization problem is subject to the nominal model dynamics plus the disturbance model dynamics resulting from the observer design problem, as

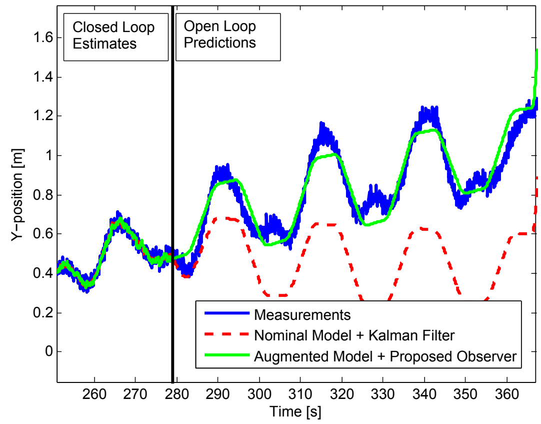

Fig. 3. Comparison between Kalman Filter/Nominal Model and Proposed Observer/Augmented Model.

described in Section 3. Note that the output matrix used in the observer C_{meas} differs from the one used in the MPC optimization problem $C_{control}$. The resulting control is implemented in a receding horizon form (see [9]), such that the optimization problem is solved at every sampling period T_S, returning an optimized vector ΔU_{opt}. Then the optimal input applied at time k is

$$\Delta u(k)_{opt} = [I_m, 0_m, ..., 0_m] \, \Delta U_{opt} \tag{15}$$

$$u(k) = u(k-1) + \Delta u(k)_{opt}. \tag{16}$$

At each sampling period, the input matrix is scheduled with respect to the previous input $B(u(k-1))$ to account for the static non-linearity in both the observer and the MPC formulation. $B(u)$ is assume not to change in the receding horizon to compute the MPC output on each sampling time, otherwise the computation time would increase considerably. This simplification was used due to the time restrictions of the sampling period, which for this application is $T_S = 60ms$ and the control algorithm takes around $5ms$ to compute.

4.3 Tuning

The tuning procedure was done using the measurement data obtained from the identification process. The observer open-loop bandwidth ω_B was then tuned iteratively, we

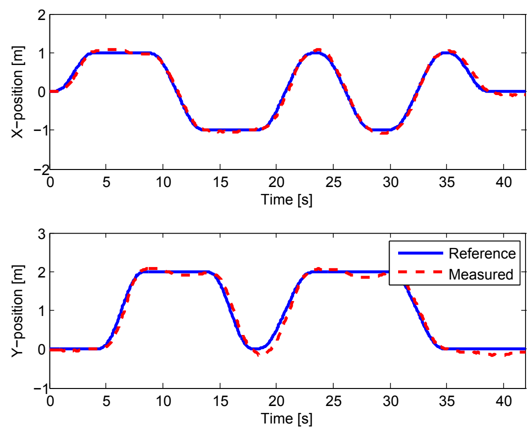

Fig. 4. Output responses to third order set-points.

designed the observer starting with a low value, then it was simulated using the measured data and ω_B was increased gradually until the output of the observer $\hat{z}$ tracked the measurements z_{meas} with keeping a balance in the trade-off between performance and noise rejection. The final bandwidth value used was $\omega_B = 0.9\,[rad/s]$.

A comparison of predictions obtained using the nominal model and a standard Kalman Filter, versus the proposed augmented model and observer is shown in Figure 3. The black line in Figure 3 marks the time at which the observer loops are open and predictions are obtained using both approaches. It can be observed that the usage of the tuned disturbance model and observer gain $(J(s))$, lead to more accurate predictions, effectively compensating mismatches between the real plant and the nominal model.

Finally, to recover approximately this performance of the observer in the overall MPC implementation, an input cost of $R = \rho I_m$ with $\rho = 0.001$ was chosen. The obtained parameters were tested initially with simulations performed in closed-loop with the nominal model.

4.4 Results

The closed loop tests consisted on following a set of points in the XY plane one after the other. From one point to the next one, third order set-point motion profiles were designed. The MPC algorithm was fed with the third order set-point profile 50 samples in advance (prediction horizon $N = 50$), in both the $x - axis$ and $y - axis$. The results of one of the closed-loop tests are shown in Figure 4. From Figure 4 one can see that the system can follow the desired trajectories, more satisfactorily in the $x - axis$. The quadcopter is able to follow the desired trajectory, however it can be noticed that tracking in the XY plane shows some deviations at certain points. This comes from the fact that the proposed simplified quadrotor model accounts for non-linearities only at the input channels. Even though the disturbance model compensates for some of the mismatches, the real system presents much more complex non-linear behavior. The focus was on simplifying the tuning procedure to obtain suitable observer and disturbance model parameters, and despite uncertainty in the modeling was not explicitly taken into account, yet the MPC showed robust behavior during all the experiments.

5. CONCLUSION

A complete output feedback MPC tuning method was introduced that considerably reduces the tuning effort of an unconstrained linear output feedback MPC formulation by translating the MPC tuning problem into a frequency domain tracking problem. It was shown that a desired feedback loop can be designed in the observer such that it reflects stability against disturbances. It was also shown that the proposed disturbance model and observer improve the predictions used in MPC. The desired feedback loop can be obtained by making use of H_∞ control techniques and under certain conditions the desired feedback loop can be perfectly recovered.

The main goal of the experiments was to show that the tuning effort of the output feedback MPC for such a complex system can be reduced to a frequency domain design problem. Further theoretical analysis of the loop recovery in presence of constraints and other MPC formulations, remains as future work for research.

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