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Model predictive control of a sea wave energy converter: a convex approach

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Abstract: This paper investigates model predictive control (MPC) of a sea wave energy converter(WEC). A novel objective function is adopted in the MPC design, which brings obvious benefits: First, the quadratic program (QP) derived from this objective function can be easily convexified, which facilitates the employment of existing efficient optimization algorithms. Second, this novel design can trade off the energy extraction, the energy consumed by the actuator and safe operation. The effectiveness of this MPC strategy is demonstrated by numerical simulations.

1. INTRODUCTION

A sea wave energy converter (WEC) is a device used to harvest sea wave energy. Extracting the maximum possible time average power from WECs, while reducing the risk of device damage and at the same time minimizing the device cost, involves a combination of good fundamental engineering design of a device and effective control of its operation. In this paper, we investigate the control aspect of WECs. In particular, we focus on a typical type of WECs, called point absorbers, whose dimensions are small compared with the wave length of incoming waves.

Various control methods have been explored to improve energy extraction, such as impedance matching by tuning the dynamical parameters of the devices Budal et al. (1982); Nolan et al. (2005), and latching control by locking the body at some moments to keep the velocity in phase with the excitation force Eidsmoen (1996); Falnes (2002b); Korde (2002); Babarit and Clément (2006).

More recent works Cretel et al. (2011); Hals et al. (2011); Brekken (2011); Li et al. (2012); Fusco and Ringwood (2013) show that maximizing energy extraction while maintaining the safe operation of WEC is essentially a constrained optimization problem and the concept of model predictive control (MPC), can be potentially employed as the WEC control strategy. MPC is an online optimization technique, which requires a fast optimization algorithm, especially when it is applied to mechanical systems, e.g. Li et al. (2010). Conventionally, the optimization is formulated as a convex quadratic programme (QP), so that efficient optimization algorithms such as the interior point method and the active set method can be employed. However, the optimization associated with the WEC control may not be guaranteed to be convex as shown later in this paper. This problem impedes the implementation of these efficient algorithms. In this paper we show how to overcome this problem by adopting a novel cost function.

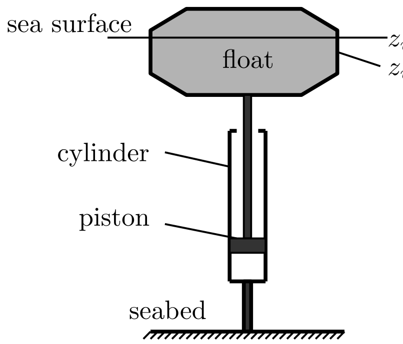

Fig. 1. Schematic diagram of the point absorber

The point absorber studied in this paper is illustrated in Fig. 1. On the sea surface is a float, below which hydraulic cylinders are vertically installed. The cylinder is attached at the bottom to the seabed. The heave motion of the float drives the piston inside the hydraulic cylinders to produce a liquid flow. The liquid drives hydraulic motors attached to a synchronous generator. From here, the power reaches the grid via back-to-back AC/DC/AC converters; see Weiss et al. (2012) for more details related to the power electronics. Here z_w is the water level, z_v is the height of the mid-point of the float. The control input is the q-axis current in the generator-side power converter, to control the electric torque of the generator. The generator torque is proportional to the force f_u acting on the pistons from the fluid in the cylinders. Since the motion of the float imposes a velocity $v = \dot{z}_v$ on the piston, the extracted power $P(t)$ at time t is expressed as $P = -f_u v$. The extracted energy over a period $[0, T]$ is therefore $-\int_0^T f_u v dt$. MPC aims to maximize the energy in its discrete time version, which amounts to minimize the cost function

$$J = \sum_{k=0}^{N} f_u(k)v(k) \tag{1}$$

where $f_u(k)$ and $v(k)$ are the discrete time values of $f_u(t)$ and $v(t)$ sampled with a sampling period T_s. For safety reasons, two constraints have to be considered. One concerns the relative motion of the float to the sea surface (it should neither sink nor raise above the water and then slam), which can be expressed as $|z_w - z_v| \leq \Phi_{\max}$. Since $z_w - z_v$ is proportional to the buoyancy force f_s, this constraint can be equivalently represented as

$$|f_s| \leq z_{\max}. \tag{2}$$

The other constraint is on the control signal set by limitations on the allowable converter current. This constraint can be expressed as

$$|f_u| \leq \gamma. \tag{3}$$

The control objective is to maximize the extracted energy subject to the constraints (2) and (3).

However, this constrained optimization problem leads to a non-convex QP, which prevents us from using efficient optimization algorithms to resolve it efficiently online. Some methods have been proposed to overcome this problem. In Bacelli et al. (2011), the WEC control is formulated as a constrained optimization problem which is approximated by a resulting concave quadratic function. In Li et al. (2012), we aim to resolve this non-convex optimization problem directly using dynamic programming (DP). However, although simulations show that the computational speed is fast enough to guarantee the real time implementation of DP for a second order model, the exponentially increased computational burden for a higher order model, namely "the curse of dimensionality of DP", can invalidate its application.

An alternative approach is to use a modified objective function to approximate the original one (1). This modified objective function takes the form of

$$J = \sum_{k=0}^{N} f_u(k)v(k+1) \tag{4}$$

which contains one sampling instant delay from input to output. In Cretel et al. (2011); Hals et al. (2011), similar approaches are used, and the QP resulting from this approximated objective function is assumed convex, which enables the application of the conventional MPC. We acknowledge the efficacy of this approximation method for many cases. However, we can show that the assumption on the QP's convexity associated with the modified cost function may not always hold.

Motivated by the existing results, the present paper aims to propose an efficient MPC control strategy to directly optimize the energy output and control signal. The MPC employs the following objective function

$$J = \sum_{k=0}^{N} \left[f_u(k)v(k) + rf_u^2(k) + qf_s^2(k)\right] \tag{5}$$

with the weights $r > 0$ and $q > 0$. Here the weighted term $rf_u^2(k)$ represents the consumed energy of the input signal, and q is used to penalize f_s. The QP can be guaranteed to be convexified when the weight r is chosen bigger than a certain value. The weight q provides an extra degree of freedom for tuning, so that the constraint on the heave motion of the buoy can be satisfied for large incoming waves. Simulation results show that the energy output loss due to the extra terms in the cost function is trivial. Based on this novel cost function, a trade-off between the safe operation (or design limit) of the WEC and the energy output under different wave conditions can be achieved.

The approach developed in this paper is based on the assumption that at each sampling instant the wave profile for a certain future period can be estimated by some wave prediction algorithms, e.g. deterministic sea wave prediction (DSWP) (Abusedra and Belmont, 2011; Naaijen et al., 2009), as presented in Li et al. (2012), and perhaps also other alternative wave prediction algorithms, e.g. Fusco and Ringwood (2012). We do not conduct robustness and performance analysis of the proposed methods regarding the wave prediction accuracy and prediction horizon, as this can be completed in a similar way as Li et al. (2012).

The structure of this paper is as follows. In Section 2, the dynamic model of the point absorber is established. In Section 3, we present the WEC optimization problem and the QPs associated with the cost function (5). Section 4 addresses the convexity problems associated with the cost functions (1) and (4), and justifies the necessity of including the extra weighted term rf_u^2 in the cost function (5); moreover, to make a direct comparison between the QP solutions related to the cost functions (1) and (4), the difference-convex optimization is introduced. Simulation results are demonstrated in Section 5. Finally, the paper is concluded in Section 6.

2. MODEL SETUP

Many existing approaches for modelling a point absorber exist in literature. For a more thorough investigation of the modeling issues of point absorbers, see Falnes (2002b); Wacher and Nielsen (2010); Price (2009).

The mathematic model for this WEC can be described by

$$m_s\ddot{z}_v = -f_s - f_r - f_f + f_u \tag{6}$$

Here the buoyancy force is $f_s := k(z_w - z_v)$, where the hydrostatic stiffness is $k = \rho g S$, with ρ as water density, g as gravitational constant, and S as the cross sectional area of the float; the mechanical force $f_f := D_f\dot{z}_v$ is due to friction and viscosity Falnes (2002a); the force applied on the piston is used as the control input f_u; the radiation force is calculated by $f_r := \int_{-\infty}^{t} h_r(\tau)[\dot{z}_v(t-\tau) - \dot{z}_w(t-\tau)]d\tau + m_a\ddot{z}_v$ with the convolution part h_r computed by boundary element methods (e.g. Newman (1977), WAMIT WAM (2006)) or approximated using analytical solutions for specific float geometry Havelock (1955); Hulme (1982). If we represent the convolution term by f_d and assume the Fourier transform of $h(t)$ is $\hat{D}(j\omega) \sim (A_r, B_r, C_r, 0)$, then f_d can be equivalently represented as a state space model

$$\dot{x}_r = A_r x_r + B_r(\dot{z}_v - \dot{z}_w) \tag{7a}$$

$$f_d = C_r x_r \tag{7b}$$

where $x_r \in \mathbb{R}^{n_r}$. Using these relations, we can express the ODE (6) by a state-space model

$$\dot{x} = A_c x + B_{uc}u + B_{wc}w \tag{8a}$$

$$y = C_c x \tag{8b}$$

$$z = C_z x \tag{8c}$$

where $w := \dot{z}_w$, $u := f_u$, $y := \dot{z}_v$, $z := f_s$ and $x := [f_s, \dot{z}_v, x_r]^T$,

$$A_c = \begin{bmatrix} 0 & -k & 0 \\ \frac{1}{m} & -\frac{D_f}{m} & -\frac{C_r}{m} \\ 0 & B_r & A_r \end{bmatrix} \quad B_{wc} = \begin{bmatrix} k \\ 0 \\ -B_r \end{bmatrix} \quad B_{uc} = \begin{bmatrix} 0 \\ \frac{1}{m} \\ 0 \end{bmatrix}$$

$$C_c = [0\ 1\ 0_{1\times n_r}] \qquad C_z = [1\ 0\ 0_{1\times n_r}]$$

with $m := m_s + m_a$. In this model, $\hat{D}(j\omega)$ can be deemed as a damping coefficient which varies with frequency. The added mass m_a is also frequency dependent, but this dependence is relatively weak, as shown in e.g. Journèe and Pinkster (2002).

If the convolution term f_d is approximated by $f_d := D(\dot{z}_v - \dot{z}_w)$, then the state space model degenerates to a second order model with state variable $x := [f_s, \dot{z}_v]^T$, and

$$A_c = \begin{bmatrix} 0 & -k \\ \frac{1}{m} & -\frac{D+D_f}{m} \end{bmatrix} \quad B_{wc} = \begin{bmatrix} k \\ \frac{D}{m} \end{bmatrix} \quad B_{uc} = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}$$

$$C_c = [0\ 1] \qquad C_z = [1\ 0]$$

Note that normally system identification methods together with some numerical tools are required to derive $\hat{D}(j\omega)$ and even the dynamics of the whole WEC from experimental data. The model we derived here is mainly for the purpose of demonstration of the MPC strategies by numerical simulation.

3. QUADRATIC PROGRAMMING FORMULATION FOR THE WEC OPTIMAL CONTROL PROBLEM

To develop MPC scheme, (8) needs to be discretized to a discrete time model

$$x(k+1) = Ax(k) + B_u u(k) + B_w w(k) \tag{9a}$$
$$y(k) = Cx(k) \tag{9b}$$
$$z(k) = C_z x(k) \tag{9c}$$

where $w := \dot{z}_w$, $u := f_u$, $y := \dot{z}_v$, $z := f_s$ and $x \in \mathbb{R}^n$. Based on (9), the constrained optimization problem is

$$\min_{U_0^N} \sum_{k=0}^{N} \left[y(k)u(k) + ru^2(k) + qz^2(k) \right] \tag{10a}$$
$$\text{s.t. } |z(k)| \le z_{\max} \text{ for } k = 0, 1, \ldots, N \tag{10b}$$
$$|u(k)| \le u_{\max} \text{ for } k = 0, 1, \ldots, N \tag{10c}$$
$$|\Delta u(k)| \le \Delta u_{\max} \text{ for } k = 0, 1, \ldots, M \tag{10d}$$

where the state constraint (10b) and input constraint (10c) correspond to (2) and (3) respectively. (10d) represents the constraint on the input slew rate.

3.1 MPC with input magnitudes as optimization variables

Define the notation $U_i^j := [u(k+i), u(k+i+1), \ldots, u(k+j)]$, with $i < j$. If the sequence of input U_0^N is used as optimization variable, then the optimization (10) without the input slew rate constraint (10d) can be converted into a QP

$$\begin{aligned} U^* = \arg\min_{U_0^k} \frac{1}{2}(U_0^N)^T \mathcal{H}_u U_0^N + \mathcal{F}_u^T U_0^N \\ \text{s.t. } A_u U_0^N \preceq b_u \end{aligned} \tag{11}$$

where

$$\mathcal{H}_u := \Phi_U + \Phi_U^T + 2R + 2\Phi_{U,z}^T Q \Phi_{U,z},$$
$$\mathcal{F}_u := (\Lambda_x + 2\Phi_{U,z}^T Q \Lambda_{x,z})x(k) + (\Phi_W + 2\Phi_{U,z}^T Q \Phi_{W,z})W_0^{N-1}$$

$$\Lambda_x = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^N \end{bmatrix}, \Phi_U = \begin{bmatrix} 0 & & & & \\ CB_u & 0 & & & \\ CAB_u & CB_u & 0 & & \\ \vdots & \vdots & \ddots & \ddots & \\ CA^{N-1}B_u & CA^{N-2}B_u & \cdots & CB_u & 0 \end{bmatrix} \tag{12}$$

$$\Phi_W = \begin{bmatrix} 0 & & \\ CB_w & & \\ CAB_w & CB_w & \\ \vdots & \vdots & \ddots \\ CA^{N-1}B_w & CA^{N-2}B_w & \cdots & CB_w \end{bmatrix} \tag{13}$$

$$A_u = \begin{bmatrix} I \\ -I \\ \Phi_{U,z} \\ -\Phi_{U,z} \end{bmatrix}, \quad b_u = \begin{bmatrix} U_{\max} \\ U_{\max} \\ Z_{\max} - \Lambda_{x,z}x(k) - \Phi_{W,z}W_0^{N-1} \\ Z_{\max} + \Lambda_{x,z}x(k) + \Phi_{W,z}W_0^{N-1} \end{bmatrix}$$

with $R = r \times I_{N+1}$, $Q = q \times I_{N+1}$, $U_{\max} = [\underbrace{1, \ldots, 1}_{N+1}]^T \times u_{\max}$ and $Z_{\max} = [\underbrace{1, \ldots, 1}_{N+1}]^T \times z_{\max}$. Here $\Lambda_{x,z}$, $\Phi_{U,z}$ and $\Phi_{W,z}$ take the same forms as (12) and (13), but with C replaced by C_z. The time derivative of the wave elevation at the current instant k is $w(k)$; we assume that at instant k, the future estimated values, $\hat{w}(k+1|k), \cdots, \hat{w}(k+N-1|k)$, are available. These future wave data can be derived by some wave prediction algorism, e.g. deterministic sea wave prediction (DSWP) algorithm.

3.2 MPC with input changing rates as optimizers

If the actuator's slew rate needs to be limited, the input slew rate has to be used as optimization variable. Assuming input slew rate horizon is M, and the prediction horizon is N with $M \le N$, we have the following relations $u(k+i) = u(k-1) + \sum_{j=0}^{i} \Delta u(k+j-1)$, with $i = 0, 1, \ldots, M$, and $u(k+M) = u(k+M+1) = \cdots = u(k+N)$. Substituting these relations into the QP (11) gives the QP with input slew rate constraint:

$$\begin{aligned} \Delta U^* = \arg \min_{\Delta U_0^{M-1}} \frac{1}{2}(\Delta U_0^{M-1})^T \mathcal{H}_{\Delta u} U_0^{M-1} + \mathcal{F}_{\Delta u}^T \Delta U_0^{M-1} \\ \text{s.t. } A_{\Delta u} \Delta U_0^{M-1} \preceq b_{\Delta u} \end{aligned} \tag{14}$$

where

$$\begin{aligned} \mathcal{H}_{\Delta u} =& T_{\Delta U}^T (\Phi_U + \Phi_U^T + 2R + 2\Phi_{U,z} Q \Phi_{U,z}) T_{\Delta U}, \\ \mathcal{F}_{\Delta u} =& T_{\Delta U}^T \left[(\Lambda_x + 2\Phi_{U,z}^T Q \Lambda_{x,z}) x(k) \right. \\ &+ (\Phi_U + \Phi_U^T + 2R + 2\Phi_{U,z}^T Q \Phi_{U,z}) T_u u(k-1) \\ &\left. + (\Phi_W + 2\Phi_{U,z}^T Q \Phi_{W,z}) W_0^{N-1} \right], \end{aligned}$$

with

$$T_u = \begin{bmatrix} I \\ I \\ \vdots \\ I \\ \vdots \\ I \end{bmatrix} \quad T_{\Delta U} = \begin{bmatrix} I & 0 & 0 & 0 \\ I & I & 0 & 0 \\ \vdots & \vdots & \ddots & \\ I & I & \cdots & I \\ \vdots & \vdots & \cdots & \vdots \\ I & I & \cdots & I \end{bmatrix} \quad A_{\Delta u} = \begin{bmatrix} I_N \\ -I_N \\ T_{\Delta U} \\ -T_{\Delta U} \\ \Phi_{U,z} T_{\Delta U} \\ -\Phi_{U,z} T_{\Delta U} \end{bmatrix},$$

$$b_{\Delta u} = \begin{bmatrix} \Delta U_{\max} \\ \Delta U_{\max} \\ U_{\max} - T_u u(k-1) \\ U_{\max} + T_u u(k-1) \\ Z_{\max} - \Lambda_{x,z} x(k) - \Phi_{U,z} T_u u(k-1) - \Phi_{W,z} W_0^{N-1} \\ Z_{\max} + \Lambda_{x,z} x(k) + \Phi_{U,z} T_u u(k-1) + \Phi_{W,z} W_0^{N-1} \end{bmatrix}$$

and $\Delta U_{\max} = \underbrace{[1, \ldots, 1]}_{M}^T \times \Delta u_{\max}$, $I \in \mathbb{R}^{n_u}$, $T_u \in \mathbb{R}^{(N+1)n_u \times n_u}$ and $T_{\Delta U} \in \mathbb{R}^{(N+1)n_u \times M n_u}$.

4. THE CONVEXITY OF THE QPS AND DIFFERENCE-CONVEX OPTIMIZATION

In this section, we discuss some issues related to the convexity of the QPs associated with the cost functions (1) and (4). Then we introduce how to use the difference-convex (DC) optimization method to approximately resolve the non-convex QP associated with (1).

4.1 The convexity of the QP formulations

We use simple examples to justify the necessity of including the weighted term $ru^2(t)$ in the cost function (10a) and demonstrate that convexity of (4) is not always guaranteed.

When $q = 0$ and $r = 0$ in the cost function (10a), the Hessian matrices $\mathcal{H}_u$ in (11) and $\mathcal{H}_{\Delta u}$ in (14) degenerate to $\mathcal{H}_u = \Phi_U + \Phi_U^T$ and $\mathcal{H}_{\Delta u} = T_{\Delta U}^T(\Phi_U + \Phi_U^T)T_{\Delta U}$ respectively. In a similar way, we can also derive the Hessian matrices of the QPs corresponding to the cost function (4). When U_1^{N+1} is used as the optimization variable, the Hessian matrix is $\mathcal{H}_a = \bar{\Phi}_U + \bar{\Phi}_U^T$; when ΔU_1^{N+1} is used as the optimization variable, the Hessian matrix is $\mathcal{H}_b = \bar{T}_{\Delta U}^T(\bar{\Phi}_U + \bar{\Phi}_U^T)\bar{T}_{\Delta U}$. Here

$$\bar{\Phi}_U = \begin{bmatrix} CB_u & & & \\ CAB_u & CB_u & & \\ \vdots & \vdots & \ddots & \\ CA^{N-1}B_u & CA^{N-2}B_u & \cdots & CB_u \end{bmatrix}$$

and $\bar{\Phi}_W$ take the same form with $\bar{\Phi}_U$ but with B_u replaced by B_w; $\bar{T}_{\Delta U}$ is derived by deleting the last row of $T_{\Delta U}$.

In Hals et al. (2011), the QPs corresponding to the cost function (4) are assumed convex, i.e. $\mathcal{H}_a > 0$ and $\mathcal{H}_b > 0$. However, we can show by simple examples that this claim does not always hold. For ease of presentation, suppose the second order state space model, with the assumption $D_f = 0$ without loss of generality, is discretized using the zero-order hold method with a sampling period of T_s; the analysis below can be extended to the higher order WEC model. The matrices associated with this discrete time model are

$$A = \begin{bmatrix} 1 & -kT_s \\ T_s/m & 1 - DT_s/m \end{bmatrix} \qquad B_u = \begin{bmatrix} 0 \\ T_s/m \end{bmatrix}$$

$$B_w = \begin{bmatrix} kT_s \\ DT_s/m \end{bmatrix} \qquad C = [0 \ 1] \tag{15}$$

Suppose prediction horizons are $N = 2$, $M = 2$, then

$$T_{\Delta U} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad \Phi_U = \begin{bmatrix} 0 & 0 & 0 \\ CB_u & 0 & 0 \\ CAB_u & CB_u & 0 \end{bmatrix} \tag{16}$$

$$\bar{T}_{\Delta U} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \qquad \bar{\Phi}_U = \begin{bmatrix} CB_u & 0 \\ CAB_u & CB_u \end{bmatrix} \tag{17}$$

In this scenario, we can investigate the convexity of the QPs corresponding to the cost function (4) by checking the positive definiteness of their associated Hessian matrices. It can be easily shown that the convexity of the QPs corresponding to the cost function (4) can only be guaranteed within a limited range of values for parameters, and the range for the case with input slew rates as optimization variables is bigger than that for the case with input magnitudes as optimization variables.

Within this scenario, it can also be shown that the Hessian matrices $\mathcal{H}_u$ and $\mathcal{H}_{\Delta u}$ of the QPs corresponding to the cost function (1) can not be positive definite for any possible parameter values. This justifies the inclusion of the extra term $ru^2(k)$ with $r > 0$ in the cost functions (5) or (10a) to *convexify* the QPs, since this is equivalent to adding positive diagonal entries to the QPs. Suppose the Hessian of the QP associated with the cost function (1), i.e. $\sum_{k=0}^{N} f_u(k)v(k)$, is $\mathcal{H}$. Then we can choose $r > -\lambda_{\min}(\mathcal{H})I$ such that $\mathcal{H}_p := \mathcal{H} + rI > 0$. Here $\mathcal{H}_p$ is the Hessian of the convex QP corresponding to the cost function $\sum_{k=0}^{N} \left[f_u(k)v(k) + rf_u^2(k)\right]$.

4.2 Difference-convex (DC) optimization

Based on the analysis in the last subsection, we can use DC as a benchmark optimization method to make a direct comparison of the QP solutions corresponding to the two cost functions (1) and (4) respectively.

The Hessian matrices of the QPs associated with the cost function (1), i.e. $\sum_{k=0}^{N} f_u(k)v(k)$ can be expressed as

$$U_k^* = \arg\min_{U_k} \left[\frac{1}{2}U_k^T \mathcal{H}_p U_k + U_k^T \mathcal{F}\right] \tag{18}$$

with $\mathcal{H}_e > -\lambda_{\min}(\mathcal{H})I$ and $\mathcal{H}_p := \mathcal{H} + \mathcal{H}_e$. Here $\lambda_{\min}(\mathcal{H})$ denotes the minimum eigenvalue of the matrix $\mathcal{H}$. Since $\mathcal{H}$ is not positive definite, we have $-\lambda_{\min}(\mathcal{H}) > 0$, so that $\mathcal{H}_e > 0$ and $\mathcal{H}_p > 0$. Define

$$f(U_k) := \frac{1}{2}U_k^T \mathcal{H}_p U_k + U_k^T \mathcal{F} \tag{19}$$

$$g(U_k) := \frac{1}{2}U_k^T \mathcal{H}_e U_k \tag{20}$$

Then both $f(U_k)$ and $g(U_k)$ are convex. Alternatively, they can be defined as the convex QPs corresponding to the cost functions $\sum_{k=0}^{N}[f_u(k)v(k) + rf_u^2(k) + qf_s^2(k)]$ and $\sum_{k=0}^{N}[rf_u^2(k) + qf_s^2(k)]$ respectively. Optimization of (18) is thus equivalent to minimizing the difference between the two convex functions, which is known as a difference-convex (DC) optimization problem Tao and An (1998). The first order Taylor series expansion of $g(U_k)$ is

$$g(U_k) = \bar{g}(U_k) - \frac{1}{2}U_{k-1}^T \mathcal{H}_e U_{k-1} + O(U_k - U_{k-1})$$

with $\bar{g}(U_k) := U_k^T \mathcal{H}_e U_{k-1}$. We approximate (18) by replacing $g(U_k)$ in (18) by $\bar{g}(U_k)$

$$U_k^* = \arg\min_{U_k} \left[\frac{1}{2}U_k^T \mathcal{H}_p U_k + U_k^T (\mathcal{F} - \mathcal{H}_e U_{k-1})\right] \tag{21}$$

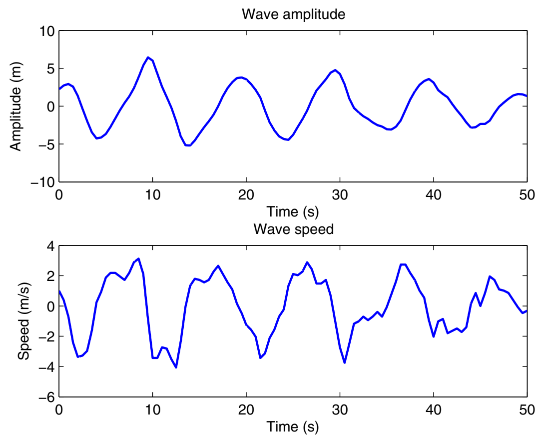

Fig. 2. The wave amplitude and its derivative data used in simulations

Then the suboptimal solution can be found by resolving the convex QP (21) iteratively. The algorithm is summarized in Algorithm 1.

Algorithm 1 Difference of convex optimization

given U_0, $k = 1$, $\epsilon > 0$
do Compute
$U_k^* = \arg\min_{U_k} \left[\frac{1}{2}U_k^T(\mathcal{H} + \mathcal{H}_e)U_k + U_k^T(\mathcal{F} - \mathcal{H}_e U_{k-1})\right]$.
s.t. constraints.
while $\|f(U_k^*) - g(U_k^*) - f(U_{k-1}) + g(U_{k-1})\|^2 > \epsilon$,
or $k >$ maximum iterations.

5. NUMERICAL SIMULATION

In this section, we present the simulation results using the MPC control strategies based on the cost functions (1), (4) and (5) respectively. For ease of presentation, we refer to the MPC based on (1) as *exact MPC* (i.e. the MPC with the cost function *exactly* reflecting the extracted energy), the MPC based on (4) as *approximated MPC*, while the MPC based on (5) as the *novel* MPC. The model adopted contains a frequency dependent added damping term $\hat{D}(j\omega)$. Apart from damping, this model has similar dynamics to the 2nd order model in Li et al. (2012). The stiffness is $k = 6.39 \times 10^5$ N/m. The mass of the float is $m_s = 1 \times 10^4$kg. The frequency independent added mass is $m_a = 7 \times 10^4$kg. Then the total mass is $m = 8 \times 10^4$kg. The input magnitude constraint is $u_{\max} = 3 \times 10^5$ N and the slew rate constraint is $\Delta u_{\max} = 0.4 \times 10^5$ N. The heave motion limit of the buoy is $\Phi_{\max} = 1.2$ m. The frequency dependent added damping is

$$\hat{D}(j\omega) = \frac{1.5 \times 10^4 \times (j\omega + 0.01)(j\omega + 0.02)}{(j\omega + 0.1)(j\omega + 0.2)^2} \qquad (22)$$

This transfer function is estimated from real experimental data provided by OPT Inc. for the PB150 device. The resulting 5th order continuous-time model rules out the implementation of the DP algorithm on computational ground. The WEC model is discretized with a sampling rate $T_s = 0.02$ sec. Real sea wave data gathered off the coast of Cornwall, UK is used.

The wave heave magnitude and its derivative for a period of 50 seconds used for simulations are shown in Fig. 2. The sequence of input slew rates is used as the optimization variable and the input slew rate constraint is incorporated into QP formulation. Prediction horizons are $N = 50$ and $M = 30$. For the novel MPC, the weights in the cost function (5) are chosen as $r = 3 \times 10^{-7}$ and $q = 2 \times 10^{-7}$, which guarantees the positive definiteness of the Hessian matrix. It is noted that when the constraint on the relative heave motion is $[-1.2, 1.2]$ m, the approximated MPC cannot always yield feasible solutions during simulations. For this reason, this limit is relaxed to $[-1.4, 1.4]$ m for the case of the approximated MPC simulations.

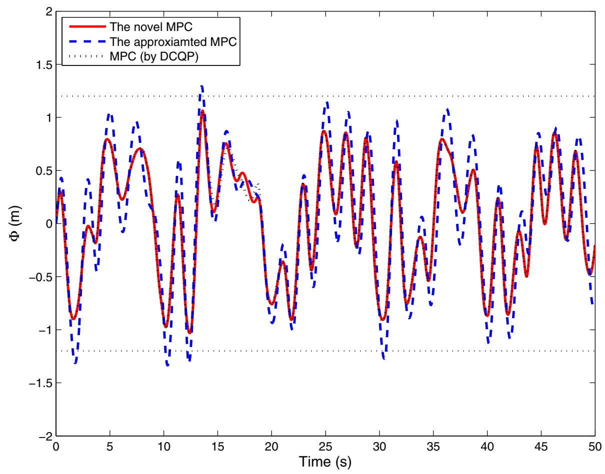

Fig. 3. The vertical displacement difference between the water level and the mid-point of the float. Constraint violations occur for approximate MPC.

In the Figs. 3-4, the solid lines and dashed lines correspond to the simulation results from the novel MPC and the approximated MPC respectively.

Fig. 3 shows the heave motion trajectories: solid line is for the novel MPC and dashed line is for the approximated MPC. When the WEC is controlled by the proposed novel MPC, the relative heave motion constraint is satisfied for the whole period. But when the WEC is controlled by the approximated MPC, the constraint on relative heave motion is violated around 1.6, 10.3, 12.3, 13.5 and 30.5 seconds. These constraint violations can potentially damage the WEC.

Fig. 4 shows the energy generated by the three MPC WEC controllers respectively. The energy generated by the novel MPC and exact MPC are still indistinguishable and slightly less than that of the approximated MPC. This indicates two aspects: First, it should be noted that this relative smaller energy output is traded off by the smaller energy consumption by the actuator, and most importantly, the relative motion constraint is always satisfied for the whole simulation period compared with the constrained violation by the approximated MPC controller. In reality, this comparison is not even necessary because the approximate MPC violates the state constraints and so could not even be used under the real conditions. Second, the extra terms involved in the novel MPC for penalizing the input variable and constrained state variable are nearly negligible compared with the case when only the output energy is used as the objective function.

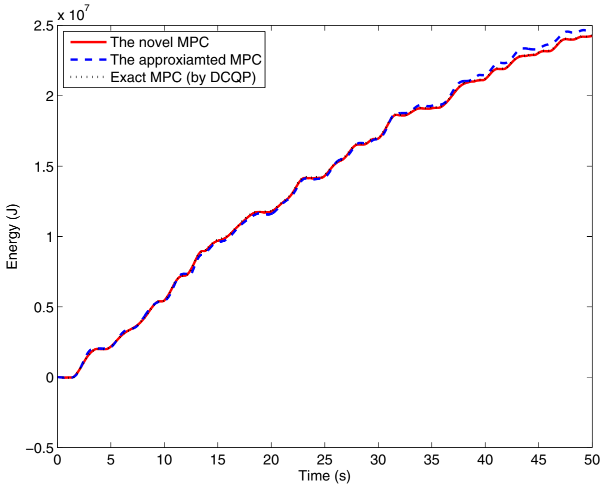

Fig. 4. Extracted energy over time. The energy generated by the novel MPC is indistinguishable from that by DCQP control.

6. CONCLUSION

We have proposed a novel MPC strategy for WEC control. This novel MPC can trade off the amount of energy output against the input energy consumption requirements of the actuator. It also explicitly penalizes the relative heave motion of the WEC, which guarantees feasible optimal solution and safe operation. The quadratic programme associated with this novel MPC can be tuned to be convex, which facilitates efficient online implementation. A typical type of point absorber is used as a study case. The simulation results confirm the efficacy of the proposed novel MPC strategy for WEC control.

REFERENCES

Wamit. WAMIT, User manual, htt://wamit.com, 2006.

L. Abusedra and M. R. Belmont. Prediction diagrams for deterministic sea wave prediction and the introduction of the data extension prediction method. *Int. J. Sipbuilding Prog.*, 48(1):59–81, 2011.

A. Babarit and A. H. Clément. Optimal latching control of a wave energy device in regular and irregular waves. *Applied Ocean Research*, 28:79–91, 2006.

Giorgio Bacelli, John V. Ringwood, and Jean-Christophe Gilloteaux. A control system for a self-reacting point absorber wave energy converter subject to constraints. In *the 18th IFAC World Congress*, Milano, Italy, 2011.

Ted K. A. Brekken. On model predictive control for a point absorber wave energy converter. In *PowerTech, 2011 IEEE Trondheim*, pages 1–8, June 2011.

K. Budal, J. Falnes, L. C. Iversen, P.M. Lillebekken, G. Oltedal, T. Hals, T. Onshus, and A.S. Høy. The Norwegian wave-power buoy project. pages 323–344, Tapir, Trondheim, 1982.

J. A. M. Cretel, G. Lightbody, G. P. Thomas, and A. W. Lewis. Maximisation of energy capture by a wave-energy point absorber using model predictive control. In *Proc. 18th IFAC World Congress*, Milano, 2011.

H. Eidsmoen. *On theory and simulation of heaving buoy wave energy converters with control.* PhD thesis, Norwegian University of Science and Technology, Trondheim, 1996.

J. Falnes. Optimum control of oscillation of wave-energy converters. *International Journal of Offshore and Polar Engineering*, 12(2):147–155, 2002a.

Johannes Falnes. *Ocean Waves and Oscillating Systems.* Cambridge University Press, 2002b.

F. Fusco and J.V. Ringwood. A study of the prediction requirements in real-time control of wave energy converters. *Sustainable Energy, IEEE Transactions on*, 3 (1):176–184, 2012.

F. Fusco and J.V. Ringwood. A simple and effective real-time controller for wave energy converters. *Sustainable Energy, IEEE Transactions on*, 4(1):21–30, 2013.

Jorgen Hals, Johannes Falnes, and Torgeir Moan. Constrained optimal control of a heaving buoy wave-energy converter. *J. of Offshore Mechanics and Arctic Engineering*, 133(1):011401, 2011.

Thomas Havelock. Waves due to a floating sphere making periodic heaving oscillations. volume 231, London, 1955. Proceedings of Royal Society, A.

A Hulme. The wave forces acting on a floating hemisphere undergoing forced periodic oscillations. *Journal Fluid Mech.*, 121, 1982.

J.M.J. Journèe and Jakob Pinkster. *Introduction in ship hydromechanics, Lecture MT519.* Delft University of Technology, draft edition, April 2002.

U. A. Korde. Latching control of deep water wave energy devices using an active reference. 29(11):1343–1355, 2002.

G. Li, D. P. Stoten, and J. Tu. Model predictive control of dynamically substructured systems with application to a servohydraulically actuated mechanical plant. *IET Control Theory and Applications*, 4(2):253–264, 2010.

G. Li, G. Weiss, M. Mueller, S. Townley, and M. B. Belmont. Wave energy converter control by wave prediction and dynamic programming. *Renewable Energy*, 48:392–403, 2012.

P. Naaijen, R. R. T. van Dijk, R. H. M. Huijsmans, and A. A. El-Mouhandiz. Real time estimation of ship motions in short crested seas. pages 243–255. ASME, 2009.

J. N. Newman. *Marine Hydrodynamics.* The MIT Press, Cambridge, MA, 1977.

G. A. Nolan, J. V. Ringwood, W. E. Leithead, and S. Butler. Optimal damping profiles for a heaving buoy wave energy converter. In *The Fifteenth Int. Offshore and Polar Eng. Conf.*, pages 477–484, Seoul, South Korea, June 2005.

Alexandra Anne Elizabeth Price. *New Perspectives on Wave Energy Converter Control.* PhD thesis, The University of Edinburgh, Edingburgh, UK, 2009.

P D Tao and L T H An. A dc optimization algorithm for solving the trust-region subproblem. *SIAM Journal on Optimization*, 8(2):476–505, 1998.

A. Wacher and K. Nielsen. Mathematical and numerical modeling of the aquabuoy wave energy converter. *Mathematics-in-Industry Case Studies*, 2:16–33, 2010.

G. Weiss, G. Li, M. Mueller, S. Townley, , and M. R. Belmont. Optimal control of wave energy converters using deterministic sea wave prediction. In *The Energy & Materials Research Conference*, Torremolinos, Malaga, Spain, 2012.