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Multi-Sensor State Estimation Using SDRE Information Filters

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Abstract: This paper presents a derivative free state estimation algorithm for nonlinear systems called the state dependent Riccati equation information filter (SDREIF). The SDREIF is developed by fusing an SDRE filter with an information filter. Similar to the extended Kalman filter (EKF) or the extended information filter (EIF), the proposed SDREIF consists of two stages, prediction and update. However, unlike the EKF or EIF, the SDREIF does not use Jacobians and hence the filter is suitable for highly nonlinear systems. Furthermore, the framework is extended to handle state estimation for multi-sensor measurements. The efficacy of the proposed SDREIF is shown by a simulation example of a permanent magnet synchronous motor.

Keywords: SDRE filters, information filters, state estimation.

1. INTRODUCTION

Nonlinear state estimation is an active research area which plays an important role in real-life applications ranging from engineering to finance. Nonlinear state estimation can broadly be classified as derivative and derivative-free filters. Derivative filters use Jacobians, which are the truncated first order Taylor series of the nonlinear functions; whereas derivative-free filters explicitly avoid the use of Jacobians. The most famous Jacobian based derivative filter for nonlinear systems is the extended Kalman filter (EKF). The EKF has been extensively used in industry and academia over the last six decades. Another important class of derivative filter for nonlinear systems is the extended H_∞ filter; which is based on robust H_∞ theory (Simon, 2006). In many cases, the derivative filters are not preferred because of the errors introduced by first order linearisation and inconvenience of the involvement of Jacobians. Recently, a few researchers have proposed derivative free filters like particle filters (Arulampalam et al., 2002), unscented Kalman filters (Julier, 1996; Julier and Uhlmann, 2000; Wan and Van Der Merwe, 2000), Gaussian filters (Ito and Xiong, 2000), etc. These may be called sigma-point filters. Another class of derivative-free filters is the state dependent Riccati equation (SDRE) filter (Cloutier, 1997; Mracek et al., 1996; Çimen, 2008; Nemra and Aouf, 2010), etc. In sigma-point filters, a few sigma-points are selected and processed to capture the mean and covariance. However, in SDRE filters, the nonlinear functions are parameterised in a state dependent coefficient (SDC) form which is then followed by an estimation procedure. When it comes to multi-sensor state estimation, the state and covariance are propagated in the information space and the state estimators derived in the information domain are called information filters and for nonlinear systems, they are called extended information filters (EIFs) (Mutambara, 1998).

In this paper, we present a new nonlinear state estimator called the state dependent Riccati equation information filter (SDREIF). The SDREIF is based on the SDRE filter and the information filter and has the advantages of both. The SDREIF is a derivative free filter like the SDRE filter and can easily handle multi-sensor measurements like the information filter. The remainder of this paper is organised as follows. Section 2 describes the basic formulation of the SDRE and the information filters. The details of the proposed SDREIF are given in Section 3; Section 4 describes the numerical simulations for the permanent magnet synchronous motor and Section 5 concludes the paper.

2. SDRE AND EXTENDED INFORMATION FILTERS

A brief introduction to the SDRE filter and the EIF is presented in this section. For a detailed formulation and derivation of these methods see for example (Cloutier, 1997) for the SDRE filter and (Mutambara, 1998) for the EIF.

2.1 SDRE Filter

Consider the discrete-time process and measurement models

$$\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}) + \mathbf{w}_{k-1} \tag{1}$$

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{v}_k, \tag{2}$$

where, k is a current time index, $\mathbf{x}_k \in \mathbb{R}^n$ represents a state vector, $\mathbf{u}_k \in \mathbb{R}^q$ is a control input vector, $\mathbf{z}_k \in \mathbb{R}^p$ is a measurement vector, and $\mathbf{w}_{k-1}$ and $\mathbf{v}_k$ are the process and measurement noises. In this paper, the process and measurement noises are assumed to be Gaussian with zero means and covariances of $\mathbf{Q}_{k-1}$ and $\mathbf{R}_k$,

$$\mathbf{w}_{k-1} = \mathcal{N}(\mathbf{0}, \mathbf{Q}_{k-1})$$

$$\mathbf{v}_k = \mathcal{N}(\mathbf{0}, \mathbf{R}_k),$$

where, $\mathcal{N}$ represents the Gaussian or normal probability distribution.

The nonlinear models in Eqs. (1) and (2) can be represented in the state dependent coefficient (SDC) (Cloutier, 1997) form as:

$$\mathbf{x}_k = \mathbf{F}(\mathbf{x}_{k-1})\mathbf{x}_{k-1} + \mathbf{G}(\mathbf{x}_{k-1})\mathbf{u}_{k-1} + \mathbf{w}_{k-1} \tag{3}$$

$$\mathbf{z}_k = \mathbf{H}(\mathbf{x}_k)\mathbf{x}_k + \mathbf{v}_k. \tag{4}$$

Note that the SDC matrices, $\mathbf{F}$, $\mathbf{G}$ and $\mathbf{H}$, for nonlinear systems are non-unique.

Consider a simple example, where the nonlinear plant dynamics is given by:

$$\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}) = \begin{bmatrix} x_{2,k-1} \\ x_{1,k-1}^2 \end{bmatrix}. \tag{5}$$

The SDC form of Eq. (5) can be written as: $\mathbf{f}_{k-1} = \mathbf{F}_1(\mathbf{x}_{k-1})\mathbf{x}_{k-1} = \mathbf{F}_2(\mathbf{x}_{k-1})\mathbf{x}_{k-1} = \mathbf{F}_3(\mathbf{x}_{k-1})\mathbf{x}_{k-1} = \ldots$, where the SDC matrices are:

$$\mathbf{F}_1(\mathbf{x}_{k-1}) = \begin{bmatrix} 0 & 1 \\ x_{1,k-1} & 0 \end{bmatrix},$$

$$\mathbf{F}_2(\mathbf{x}_{k-1}) = \begin{bmatrix} \frac{x_{2,k-1}}{x_{1,k-1}} & 0 \\ x_1 & 0 \end{bmatrix},$$

$$\mathbf{F}_3(\mathbf{x}_{k-1}) = \lambda \mathbf{F}_1(\mathbf{x}_{k-1})\mathbf{x}_{k-1} + (1-\lambda)\mathbf{F}_2(\mathbf{x}_{k-1}).$$

A similar example is considered in (Banks et al., 2007). Different parameterisations can be obtained for different 'λ' in the interval $[0,1]$, which satisfies the convexity of the chosen parameterisation. Similarly, different combinations of the SDC measurement matrices, $\mathbf{H}_i$, and the state vector, $\mathbf{x}_k$, can yield the same $\mathbf{h}(\mathbf{x}_k)$, like $\mathbf{H}_1(\mathbf{x}_k)\mathbf{x}_k = \mathbf{H}_2(\mathbf{x}_k)\mathbf{x}_k = \mathbf{H}_3(\mathbf{x}_k)\mathbf{x}_k = \mathbf{h}(\mathbf{x}_k)$. As the SDC matrices are non-unique, before designing the SDRE controller (or the SDRE observer), one should make sure that the state-dependent controllability matrix (or the state-dependent observability matrix) has full rank (Banks et al., 2007).

Similar to the Kalman filter, the SDRE filter can be written in prediction and update stages (Jaganath et al., 2005). The predicted state, $\hat{\mathbf{x}}_{k|k-1}$, and positive definite matrix, $\mathbf{P}_{k|k-1}$, can be written as:

$$\widehat{\mathbf{x}}_{k|k-1} = \mathbf{f}(\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{u}_{k-1}) \tag{6}$$

$$\mathbf{P}_{k|k-1} = \mathbf{F}(\hat{\mathbf{x}}_{k-1|k-1})\mathbf{P}_{k-1|k-1}\mathbf{F}(\hat{\mathbf{x}}_{k-1|k-1})^T + \mathbf{Q}_{k-1}. \tag{7}$$

The updated state, $\hat{\mathbf{x}}_{k|k}$, and positive definite matrix, $\mathbf{P}_{k|k}$, are:

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k[\mathbf{z}_k - \mathbf{H}(\hat{\mathbf{x}}_{k|k-1})] \tag{8}$$

$$\mathbf{P}_{k|k} = (\mathbf{I}_n - \mathbf{K}_k\mathbf{H}(\hat{\mathbf{x}}_{k|k-1}))\mathbf{P}_{k|k-1}, \tag{9}$$

where, $\mathbf{I}_n$ denotes the identity matrix of dimension $n \times n$ and the SDRE filter gain is:

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{H}(\hat{\mathbf{x}}_{k|k-1})^T \left[\mathbf{H}(\hat{\mathbf{x}}_{k|k-1})\mathbf{P}_{k|k-1}\mathbf{H}(\hat{\mathbf{x}}_{k|k-1})^T + \mathbf{R}_k\right]^{-1}. \tag{10}$$

It can be noted that the matrices $\mathbf{P}_{k|k-1}$ and $\mathbf{P}_{k|k}$ satisfy the Riccati recursion and are state dependent. At first glance, the above formulation looks similar to that of the EKF; but it is not. The main difference between the SDRE filter and the EKF is the way in which the matrices $\mathbf{F}(\hat{\mathbf{x}}_{k-1|k-1}))$ and $\mathbf{H}(\hat{\mathbf{x}}_{k|k-1})$ are defined. In the SDRE filter they are the parameterised SDC matrices and for the EKF they are the Jacobians of the process and measurement models. The SDRE filter avoids the evaluation of Jacobians and hence avoids errors introduced by truncation of Taylor series. It is interesting to note that different parameterisations in SDRE filter may yield different results (Liang and Lin, 2013).

2.2 *Extended Information Filter*

For nonlinear state estimation with multi-sensor measurements, the extended version of the information filter (the EIF), is preferred over the EKF (Mutambara, 1998). The EIF is an algebraic equivalent of the EKF, in which the parameters of interest are the information vector and the information matrix rather than the states and covariance. The EIF is summarised below.

Consider the discrete nonlinear process and measurement models given in Eqs. (1) and (2). The information filter deals with the information vector, $\mathbf{y}$, and the corresponding matrix, $\mathbf{Y}$. The EIF can also be expressed in prediction and update stages. The predicted information vector, $\hat{\mathbf{y}}_{k|k-1}$, and the predicted information matrix, $\mathbf{Y}_{k|k-1}$, are given as:

$$\hat{\mathbf{y}}_{k|k-1} = \mathbf{Y}_{k|k-1}\hat{\mathbf{x}}_{k|k-1} \tag{11}$$

$$\mathbf{Y}_{k|k-1} = \mathbf{P}_{k|k-1}^{-1} = \left[\nabla\mathbf{f}_x \mathbf{Y}_{k-1|k-1}^{-1} \nabla\mathbf{f}_x^T + \mathbf{Q}_{k-1}\right]^{-1}, \tag{12}$$

where, $\mathbf{P}_{k|k-1}$ is the predicted covariance matrix and

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{f}(\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{u}_{k-1}). \tag{13}$$

The updated information vector, $\hat{\mathbf{y}}_{k|k}$, and the updated information matrix, $\mathbf{Y}_{k|k}$, are

$$\hat{\mathbf{y}}_{k|k} = \hat{\mathbf{y}}_{k|k-1} + \mathbf{i}_k \tag{14}$$

$$\mathbf{Y}_{k|k} = \mathbf{Y}_{k|k-1} + \mathbf{I}_k. \tag{15}$$

The information vector contribution, $\mathbf{i}_k$, and its associated matrix, $\mathbf{I}_k$, are

$$\mathbf{i}_k = \nabla\mathbf{h}_x^T \mathbf{R}_k^{-1} \left[\nu_k + \nabla\mathbf{h}_x \hat{\mathbf{x}}_{k|k-1}\right] \tag{16}$$

$$\mathbf{I}_k = \nabla\mathbf{h}_x^T \mathbf{R}_k^{-1} \nabla\mathbf{h}_x, \tag{17}$$

where, the measurement residual, ν_k, is

$$\nu_k = \mathbf{z}_k - \mathbf{h}(\hat{\mathbf{x}}_{k|k-1}, \mathbf{u}_k) \tag{18}$$

and $\nabla\mathbf{f}_x$, and $\nabla\mathbf{h}_x$ are the Jacobians of $\mathbf{f}$ and $\mathbf{h}$ evaluated at the latest available state (Jacobians for prediction and update equations are evaluated at $\hat{\mathbf{x}}_{k-1|k-1}$ and $\hat{\mathbf{x}}_{k|k-1}$, respectively).

For multi-sensor data fusion, the update stage of the EIF is computationally simpler and has several other advantages over the EKF. But some of the drawbacks inherent in the EKF still affect the EIF; these include the nontrivial nature of the derivations of the Jacobian matrices and linearisation instability (Mutambara, 1998).

3. SDRE INFORMATION FILTER

This section presents the formulation for the SDRE information filter (SDREIF). The SDRE filter and the EIF are fused to form the SDREIF, to preserve the benefits of both these filters. The SDREIF is a derivative free filter like SDRE filter, and like the EIF, it is handy to deal with multi-sensor state estimation. Similar to the information filter, the SDREIF is developed in the information space, where the information vector and its associated matrix are propagated.

Consider the discrete time process and measurement models given in Eqs. (1) and (2) and their SDC form in Eqs. (3) and (4). The predicted information vector, $\hat{\mathbf{y}}_{k|k-1}$, and the inverse of the positive definite matrix, $\mathbf{Y}_{k|k-1} = \mathbf{P}^{-1}_{k|k-1}$, for the SDREIF are:

$$\hat{\mathbf{y}}_{k|k-1} = \mathbf{Y}_{k|k-1}\hat{\mathbf{x}}_{k|k-1} \tag{19}$$

$$\mathbf{Y}_{k|k-1} = \left[\mathbf{F}(\hat{\mathbf{x}}_{k-1|k-1})\mathbf{Y}^{-1}_{k-1|k-1}\mathbf{F}(\hat{\mathbf{x}}_{k-1|k-1})^T + \mathbf{Q}_{k-1}\right]^{-1}, \tag{20}$$

where,

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{f}(\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{u}_{k-1}). \tag{21}$$

It can be noted that the predicted information vector in Eq. (19) is the same as that of the EIF, but the expression for $\mathbf{Y}_{k|k-1}$ is different. In the EIF, the updated information matrix is the function of the Jacobian, $\nabla \mathbf{f}_x$, whereas in the SDREIF the $\nabla \mathbf{f}_x$ is replaced by the SDC $\mathbf{F}(\mathbf{x}_{k-1})$.

The updated information vector, $\hat{\mathbf{y}}_{k|k}$, and $\mathbf{Y}_{k|k}$ are:

$$\hat{\mathbf{y}}_{k|k} = \hat{\mathbf{y}}_{k|k-1} + \mathbf{i}_k \tag{22}$$

$$\mathbf{Y}_{k|k} = \mathbf{Y}_{k|k-1} + \mathbf{I}_k. \tag{23}$$

The information vector contribution, $\mathbf{i}_k$, and its associated information matrix, $\mathbf{I}_k$, are:

$$\mathbf{i}_k = \mathbf{H}(\hat{\mathbf{x}}_{k|k-1})^T \mathbf{R}_k^{-1}\left[\nu_k + \mathbf{H}(\hat{\mathbf{x}}_{k|k-1})\hat{\mathbf{x}}_{k|k-1}\right] \tag{24}$$

$$\mathbf{I}_k = \mathbf{H}(\hat{\mathbf{x}}_{k|k-1})^T \mathbf{R}_k^{-1}\mathbf{H}(\hat{\mathbf{x}}_{k|k-1}), \tag{25}$$

where, the measurement residual, ν_k, is:

$$\nu_k = \mathbf{z}_k - \mathbf{H}(\hat{\mathbf{x}}_{k|k-1}), \tag{26}$$

and $\mathbf{H}(\hat{\mathbf{x}}_{k|k-1})$ is a parameterised SDC matrix of the measurement model.

3.1 SDREIF in Multi-Sensor State Estimation

State estimation performed in the information domain has the ability to easily handle multi-sensor state estimation (Mutambara, 1998; Bar-Shalom et al., 2004). In the update stage, the information from different sensors can be easily fused (Mutambara, 1998; Bar-Shalom et al., 2004). The prediction step for the SDREIF multi-sensor state estimation is the same as that of the SDREIF (with single sensor). In the update stage, the measurements from different sensors are fused for an efficient and reliable estimation (Raol and Girija, 2002).

Let the different nonlinear sensors used for the state estimation be:

$$\mathbf{z}_{j,k} = \mathbf{h}_{j,k}(\mathbf{x}_{j,k}) + \mathbf{v}_{j,k}; \qquad j = 1, 2, ...D, \tag{27}$$

where, 'D' is the number of sensors. The parameterised SDC form for the multi-sensor models can be written as:

$$\mathbf{z}_{j,k} = \mathbf{H}(\mathbf{x}_{j,k})\mathbf{x}_{j,k} + \mathbf{v}_{j,k}; \qquad j = 1, 2, ...D. \tag{28}$$

The prediction step of multi-sensor SDREIF is the same as that of SDEIF. The updated information vector and the corresponding matrix for multi-sensor SDREIF are:

$$\hat{\mathbf{y}}_{k|k} = \hat{\mathbf{y}}_{k|k-1} + \sum_{j=1}^{D} \mathbf{i}_{j,k} \tag{29}$$

$$\mathbf{Y}_{k|k} = \mathbf{Y}_{k|k-1} + \sum_{j=1}^{D} \mathbf{I}_{j,k}, \tag{30}$$

where,

$$\mathbf{i}_{j,k} = \mathbf{H}(\hat{\mathbf{x}}_{k|k-1})^T \mathbf{R}_{j,k}^{-1}\left[\nu_k + \mathbf{H}(\hat{\mathbf{x}}_{k|k-1})\hat{\mathbf{x}}_{k|k-1}\right] \tag{31}$$

$$\mathbf{I}_{j,k} = \mathbf{H}(\hat{\mathbf{x}}_{k|k-1})^T \mathbf{R}_{j,k}^{-1}\mathbf{H}(\hat{\mathbf{x}}_{k|k-1}). \tag{32}$$

The SDREIF is summarised in Algorithm 1.

4. SIMULATIONS

Simulations are performed on a two phase permanent magnet synchronous motor (PMSM) model. The state vector of the PMSM is $[i_a, i_b, \omega, \theta]^T$. It is assumed that the first two states, i_a and i_b, are available and the remaining two states, ω and θ, are to be estimated. The inputs to the PMSM are the voltages, $u_{1,k}$ and $u_{2,k}$.

The discrete-time nonlinear model of PMSM is (Simon, 2006; Chandra et al., 2013)

$$\begin{bmatrix} i_{a,k+1} \\ i_{b,k+1} \\ \omega_{k+1} \\ \theta_{k+1} \end{bmatrix} = \begin{bmatrix} i_{a,k} + T_s\left(-\frac{R}{L}i_{a,k} + \frac{\omega\lambda}{L}\sin\theta_k + \frac{1}{L}u_{1,k}\right) \\ i_{b,k} + T_s\left(-\frac{R}{L}i_{b,k} - \frac{\omega\lambda}{L}\cos\theta_k + \frac{1}{L}u_{2,k}\right) \\ \theta_k + T_s\left(-\frac{3\lambda}{2J}i_{a,k}\sin\theta_k + \frac{3\lambda}{2J}i_{b,k}\cos\theta_k - \frac{F\omega_k}{J}\right) \\ \theta_k + T_s\omega_k \end{bmatrix}$$

the outputs and inputs are

$$\begin{bmatrix} y_{1,k} \\ y_{2,k} \end{bmatrix} = \begin{bmatrix} i_{a,k} \\ i_{b,k} \end{bmatrix}, \quad \begin{bmatrix} u_{1,k} \\ u_{2,k} \end{bmatrix} = \begin{bmatrix} \sin(0.002\pi k) \\ \cos(0.002\pi k) \end{bmatrix}.$$

The following parameters are used for the simulations (Simon, 2006; Chandra et al., 2013): R = 1.9Ω, λ=0.1, L = 0.003H, J = 0.00018, F = 0.001 and T_s = 0.001 s. It is assumed that the plant and measurement models are excited with additive Gaussian noises. In this paper, emphasis is given to the state estimation rather than the control design. The control strategy is based on an open-loop mechanism and hence reference speed tracking is not considered.

The first step in implementing the SDREIF is to parameterise the nonlinear process and measurement equations in the SDC form. In this paper the following SDC form has been used:

$$\mathbf{x}_k = \mathbf{F}(\mathbf{x}_{k-1})\mathbf{x}_{k-1} + \mathbf{G}(\mathbf{x}_{k-1})\mathbf{u}_{k-1} + \mathbf{w}_{k-1} \tag{33}$$

$$\mathbf{z}_k = \mathbf{H}(\mathbf{x}_k)\mathbf{x}_{k-1} + \mathbf{v}_k, \tag{34}$$

where,

$$\mathbf{F}(\mathbf{x}_{k-1}) = \mathbf{I}_4 + T_s \begin{bmatrix} -\frac{R}{L} & 0 & \frac{\lambda}{L}\sin\theta_k & 0 \\ 0 & -\frac{R}{L} & \frac{\lambda}{L}\cos\theta_k & 0 \\ -\frac{3}{2}\frac{\lambda}{J}\sin\theta_k & \frac{3}{2}\frac{\lambda}{J}\cos\theta_k & \frac{-F}{J} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{G}(\mathbf{x}_{k-1}) = \begin{bmatrix} \frac{1}{L} & 0 & 0 & 0 \\ 0 & \frac{1}{L} & 0 & 0 \end{bmatrix}^T$$

$$\mathbf{H}(\mathbf{x}_k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

Two sets of simulations are performed on the PMSM to analyse the proposed SDREIF. The first set is simulated with a single group of sensors and the second set of simulations are performed with two groups of sensors. In the first set of simulations, the covariance of the process and measurement noises are:

$$\mathbf{Q} = \begin{bmatrix} 11.1111 & 0 & 0 & 0 \\ 0 & 11.1111 & 0 & 0 \\ 0 & 0 & 0.0025 & 0 \\ 0 & 0 & 0 & 1\times10^{-6} \end{bmatrix}, \quad \mathbf{R} = 1\times10^{-4}\mathbf{I}_2.$$

The initial information vector is selected from

$$\mathcal{N}\left([1\ 1\ 1\ 1]^T, \mathbf{I}_4\right).$$

It was further assumed that the first sensor (i_a measurement) and the second sensor (i_b measurement) are faulty from 0.5 s to 0.8 s and 2 s to 2.3 s, respectively. In this paper, the term 'faulty' refer to sensors with zero measurements from 0.5 s to 0.8 s and from 2 s to 2.3 s. Fig. 1 shows the plots of actual and measured data; where the sensors are faulty. In the first set of simulations, a single group of faulty sensors are used to measure the currents, i_a and i_b; and ω and θ are estimated using the SDREIF given in Section 3. For the second set of simulations, two groups of sensors are used and the state estimation procedure given in the Section 3.1 is used. For the multi-sensor state estimation, the first group of sensors are the same as that of the single-sensor state estimation; the noise covariance of the second group of sensors is: $\mathbf{R}_2 = 4\times10^{-6}\mathbf{I}_2$. The corresponding results are depicted in Fig. 2. It can be seen that the SDREIF using single group of sensors are not capable to handle the faulty sensors; whereas the multi-sensor SDREIF can handle it efficiently. It is interesting to note that, during the faulty measurements, the multi-sensor SDREIF is able to keep the estimation error within an acceptable range. The error plots using the single- and multi-group of sensors are shown in Fig. 3. Several spikes in the rotor position error are due to the sudden changes in the trapezoidal rotor position at 2π Radians.

Remark: Similar to the SDRE filter, the selection of an appropriate SDC form plays a crucial role in the SDREIF. For the state estimation of the PMSM, if the SDC matrix $\mathbf{F}(\mathbf{x}_{k-1})$ given in Eq. (35) is selected and assuming all the remaining parameters are the same as used in the Section 4, then the estimation errors are found to be quite large. One of the main reasons for this is the $(3,3)$ and $(3,4)$ elements of the matrix in Eq. (35).

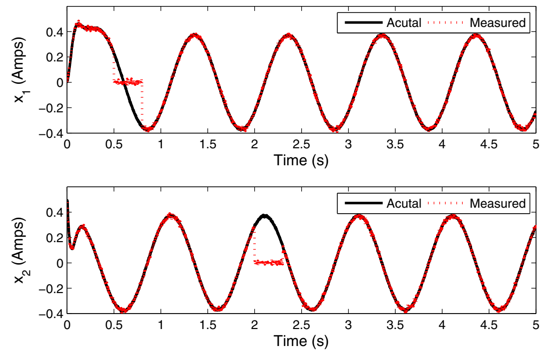

Fig. 1. Actual and measured states.

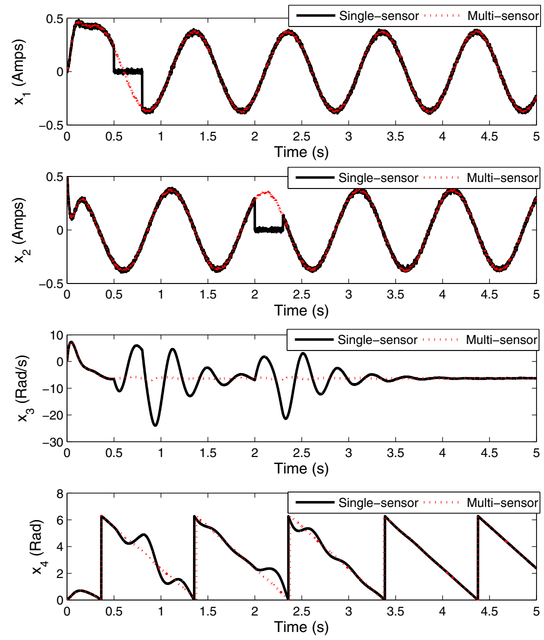

Fig. 2. State estimation using single- and multi-sensor SDREIF.

$$\mathbf{F}(\mathbf{x}_{k-1}) = \mathbf{I}_4 + T_s \begin{bmatrix} -\frac{R}{L} & 0 & \frac{\lambda}{L}\sin\theta_k & 0 \\ 0 & -\frac{R}{L} & \frac{\lambda}{L}\cos\theta_k & 0 \\ -\frac{3}{2}\frac{\lambda}{J}\sin\theta_k & \frac{3}{2}\frac{\lambda}{J}\cos\theta_k & -\frac{F}{J}(1-\theta_k) & -\frac{F}{J}\omega_k \\ 0 & 0 & 1 & 0 \end{bmatrix}. \quad (35)$$

In the PMSM model, ω and θ are sensitive states and hence the SDC matrices formed with a combination of these states lead to an erroneous state estimation. The selection of an appropriate SDC form for the SDRE filters is an active area of research as reported in (Liang and Lin, 2013).

5. CONCLUSIONS

In this paper, we have presented the state dependent Riccati equation information filters (SDREIFs). The proposed filter is derived from the SDRE filter and an extended information filter. Further it has been extended to deal with a multi-sensor platform. The SDREIF has the following advantages:

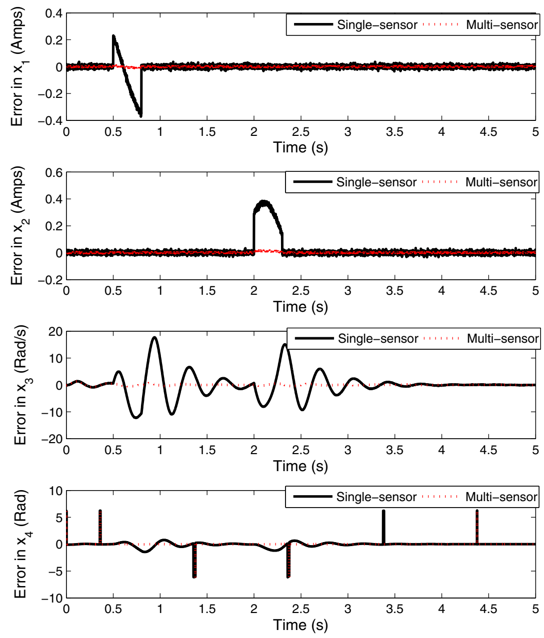

Fig. 3. Error plots using single- and multi-sensor SDREIF.

(1) It is a derivative free filter and has the ability to deal with highly nonlinear systems.
(2) The multi-sensor SDREIF has a simpler update stage.
(3) The multi-sensor SDREIF can handle multiple sensors and faulty measurements.

The efficacy of the multi-sensor SDREIF has been demonstrated on a permanent magnet synchronous motor example and the results are promising. Further areas of study include:

(1) A square-root version of the SDREIF to improve the numerical stability.
(2) Different parameterisations and their effects on the state estimation error.
(3) Stability of the SDREIF.

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Algorithm 1 State Dependent Riccati Equation Information Filter (SDREIF)

State Dependent Coefficient Form:

1: Use paramerisation to bring the nonlinear process and measurement models in to the below SDC form:

$$\mathbf{x}_k = \mathbf{F}(\mathbf{x}_{k-1})\mathbf{x}_{k-1} + \mathbf{G}(\mathbf{x}_{k-1})\mathbf{u}_{k-1} + \mathbf{w}_{k-1}$$
$$\mathbf{z}_k = \mathbf{H}(\mathbf{x}_k)\mathbf{x}_{k-1} + \mathbf{v}_k.$$

Prediction:

Initialise the information vector, $\hat{\mathbf{y}}_{k-1|k-1}$, and the associated matrix, $\mathbf{Y}_{k-1|k-1}$; by setting $k = 1$.

1: The predicted information vector and the associate matrix are:

$$\hat{\mathbf{y}}_{k|k-1} = \mathbf{Y}_{k|k-1}\hat{\mathbf{x}}_{k|k-1}$$
$$\mathbf{Y}_{k|k-1} = \left[\mathbf{F}(\hat{\mathbf{x}}_{k-1|k-1})\mathbf{Y}^{-1}_{k-1|k-1}\mathbf{F}(\hat{\mathbf{x}}_{k-1|k-1})^T + \mathbf{Q}_{k-1}\right]^{-1},$$

where,

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{f}(\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{u}_{k-1}).$$

Measurement Update:

1: Evaluate the information vector contribution and its associated information matrix

$$\mathbf{i}_k = \mathbf{H}(\hat{\mathbf{x}}_{k|k-1})^T\mathbf{R}_k^{-1}\left[\nu_k + \mathbf{H}(\hat{\mathbf{x}}_{k|k-1})\hat{\mathbf{x}}_{k|k-1}\right]$$
$$\mathbf{I}_k = \mathbf{H}(\hat{\mathbf{x}}_{k|k-1})^T\mathbf{R}_k^{-1}\mathbf{H}(\hat{\mathbf{x}}_{k|k-1})$$

where, the measurement residual, ν_k, is

$$\nu_k = \mathbf{z}_k - \mathbf{H}(\hat{\mathbf{x}}_{k|k-1}).$$

2: The updated information vector and the corresponding matrix are

$$\hat{\mathbf{y}}_{k|k} = \hat{\mathbf{y}}_{k|k-1} + \mathbf{i}_k$$
$$\mathbf{Y}_{k|k} = \mathbf{Y}_{k|k-1} + \mathbf{I}_k.$$

For multi-sensor state estimation, the updated information vector and the corresponding matrix are:

$$\hat{\mathbf{y}}_{k|k} = \hat{\mathbf{y}}_{k|k-1} + \sum_{j=1}^{D}\mathbf{i}_{j,k}$$
$$\mathbf{Y}_{k|k} = \mathbf{Y}_{k|k-1} + \sum_{j=1}^{D}\mathbf{I}_{j,k},$$

where,

$$\mathbf{i}_{j,k} = \mathbf{H}(\hat{\mathbf{x}}_{j,k|k-1})^T\mathbf{R}_{j,k}^{-1}\left[\nu_k + \mathbf{H}(\hat{\mathbf{x}}_{j,k|k-1})\hat{\mathbf{x}}_{j,k|k-1}\right]$$
$$\mathbf{I}_{j,k} = \mathbf{H}(\hat{\mathbf{x}}_{j,k|k-1})^T\mathbf{R}_{j,k}^{-1}\mathbf{H}(\hat{\mathbf{x}}_{j,k|k-1})$$

Recovery of State and the Corresponding Matrix:

1: The state and the corresponding positive definite matrix can be recovered as:

$$\hat{\mathbf{x}}_{k|k} = \mathbf{Y}_{k|k}\backslash\hat{\mathbf{y}}_{k|k}$$
$$\mathbf{P}_{k|k} = \mathbf{Y}_{k|k}\backslash\mathbf{I}_n$$

where, $\mathbf{I}_n$ is the state vector sized identity matrix and '\' is left divide operator.

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