

Design of stabilizing strategies for dual switching stochastic-deterministic linear systems

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Abstract: Dual switching linear systems are systems with piecewise linear dynamics governed by two distinct external switching signals. In this paper one switching signal is stochastic and modeled as a time-homogeneous Markov chain, and the second one is assumed to be a control signal taking values in a finite set. This situation arises, for instance, in multi-loop networked control systems (NCS) with limited transmission capacity and lossy communication channels, where the control variable is the scheduling signal and the stochastic switching accounts for packet dropout. The aim of this paper is to derive suitable switching strategies for dual switching linear systems, ensuring mean-square stability and attaining guaranteed bounds on \mathcal{H}_2 and \mathcal{H}_∞ performance indices. The relevant sufficient conditions are expressed in terms of feasibility of sets of coupled matrix inequalities. The application to a scheduling design problem in NCS's is also discussed.

Keywords: Markov Jump Linear Systems, Dual switching, Stability, \mathcal{H}_2 performance, \mathcal{H}_∞ performance, Networked control.

1. INTRODUCTION

Switched linear systems are piecewise linear systems evolving according to a finite number of operating modes, subject to an external switching signal that selects, at any time instant, which mode is currently active. These models are very useful to describe systems affected by sudden faults, configuration changes or switching control laws. The switching signal can be modeled as either a deterministic or a random signal taking values in a finite set. The former case has been widely studied and an extensive literature is now available on stabilization and control of deterministic switched linear systems, see e.g. the books Liberzon (2003), Sun and Ge (2011), the survey papers Colaneri (2009), Lin and Antsaklis (2009) and the references therein. On the other hand, the models where the switching signal is described as a stochastic Markov chain are referred to as Markov Jump Linear Systems (MJLS) and their properties have been deeply investigated, as well (see again Colaneri (2009) and the recent book Costa et al. (2013)).

In various applications (such as networked control, fault tolerant systems, communication networks) problems are encountered where the system is jointly affected by two independent external sources, for instance a manipulated switching signal and stochastic jumps. These systems are referred to in the literature as *dual switching* systems, Bolzern et al. (2010, 2013); Bolzern and Colaneri (2013). They exhibit a complex dynamic behavior due to the interplay between the two switching signals. A typical instance of dual switching systems might be a networked control system (NCS) with deterministically switching control laws and stochastic jumps between levels of network congestion, see e.g. Hespanha et al. (2007).

Some results on the stochastic stability of switching MJLS,

i.e. systems with random Markovian jumps and a deterministic switching rule with dwell-time constraints, were discussed in Bolzern et al. (2010) and Bolzern et al. (2013). In Bolzern and Colaneri (2013), dual switching discrete-time systems with a deterministic periodic signal and uncontrolled switches were studied in terms of stability and performance. In all these papers the deterministic switching signal was not considered as a decision variable. The problem addressed in the present paper, on the other hand, concerns the design of the deterministic switching signal for switched MJLS. The design requirements are stochastic stability of the dual (deterministic/stochastic) switching system and guaranteed bounds on its \mathcal{H}_2 and \mathcal{H}_∞ performances. The relevant sufficient conditions are expressed in terms of feasibility of coupled matrix inequalities. An application of the theory to a scheduling design problem in NCS's with multiple plants, packet dropout and capacity limitation in the communication channel between the regulator and the actuators, see e.g. Hristu-Varsakelis (2005), is also discussed.

The paper is organized as follows. After introducing a general setting for dual switching systems in Section 2, a specific motivating problem of scheduling design in networked control systems is presented in Section 3. The main results of the paper are derived in Section 4, where the design of switching strategies ensuring stability and guaranteed \mathcal{H}_2 and \mathcal{H}_∞ performances is worked out. In Section 5, these design tools are applied to the networked control example. The paper ends with some concluding remarks reported in Section 6.

The notation adopted in the paper is fairly standard. In particular, the set of all signals with finite 2-norm is denoted by \mathcal{L}_2 and $\delta(t)$ is the unitary Dirac delta function. Moreover, \mathcal{T}_N is the set of zero row-sum Metzler matrices of size N , i.e. square $N \times N$

real matrices with nonnegative off-diagonal entries and whose diagonal terms are such that each row sums up to zero. For a stochastic variable x , the notation $E[x]$ represents its expected value. The symbol $\mathcal{L}V(x, s)$ is used to denote the infinitesimal generator of the stochastic function $V(x, s)$, see e.g. Kushner (1967). For symmetric matrices, the symbol \star stands for each of its symmetric blocks.

2. PROBLEM FORMULATION

Consider the class of dual switching linear systems described by

$$\dot{x}(t) = A_{\sigma(t)}^{\gamma(t)} x(t) + B_{\sigma(t)}^{\gamma(t)} w(t), \quad x(0) = x_0 \quad (1)$$

$$z(t) = C_{\sigma(t)}^{\gamma(t)} x(t) \quad (2)$$

where $x(t) \in \mathcal{R}^n$ is the state, $w(t) \in \mathcal{R}^m$ is a deterministic disturbance with $w(\cdot) \in \mathcal{L}_2$, $z(t) \in \mathcal{R}^p$ is the performance output, $\gamma(t)$ is a deterministic right-continuous switching signal taking values in the finite set $\mathcal{M} = \{1, 2, \dots, M\}$, and $\sigma(t)$ is a right-continuous time homogeneous Markov process taking values in the set $\mathcal{N} = \{1, 2, \dots, N\}$, with transition rate matrix Λ . More precisely, the off-diagonal entry $\lambda_{ij} \geq 0, i \neq j$ of Λ represents the probabilistic transition rate from mode i at time t to mode j at time $t + h$, namely

$$\Pr\{\sigma(t+h) = j | \sigma(t) = i\} = \lambda_{ij}h + o(h) \quad , \quad h > 0$$

Moreover

$$\lambda_{ii} = - \sum_{j=1, j \neq i}^N \lambda_{ij}$$

so that Λ is a zero row-sum Metzler matrix, i.e. $\Lambda \in \mathcal{T}_N$. In the sequel, we assume that Λ is irreducible, so that the Markov process admits a unique stationary (strictly positive) distribution π satisfying $\pi' \Lambda = 0$, see e.g. Bremaud (1998).

In summary, the system is subject to both stochastic jumps governed by the form process $\sigma(t)$ and deterministic switches dictated by the control signal $\gamma(t)$. Therefore, the state dynamics of the overall system is characterized by NM triplets $(A_i^r, B_i^r, C_i^r), i \in \mathcal{N}, r \in \mathcal{M}$.

In accordance with standard notions of stochastic stability, for a given deterministic switching signal $\gamma(t)$, system (1) is mean-square stable (MS-stable) if, for $w(t) = 0$, it follows that

$$\lim_{t \rightarrow \infty} E[\|x(t)\|^2] = 0$$

for any initial condition x_0 and any initial probability distribution of $\sigma(0)$. Here and afterwards, the symbol $E[\cdot]$ will denote the expectation with respect to the stationary distribution π .

We will consider two performance indices inspired by the standard \mathcal{H}_2 and \mathcal{H}_∞ indices of deterministic LTI systems. Precisely, let $x_0 = 0$, $\gamma(t)$ be given, and define $z^{(k)}(t)$ as the impulse response generated by $w(t) = \delta(t)e_k$, where e_k is the k -th column of the $m \times m$ identity matrix. The \mathcal{H}_2 performance is defined as the following expected quadratic cost

$$J_2(\gamma) = E \left[\sum_{k=1}^m \int_0^\infty (z^{(k)}(t))' z^{(k)}(t) dt \right] \quad (3)$$

As for \mathcal{H}_∞ -type performance, we consider $x_0 = 0$ and the worst-case index

$$J_\infty(\gamma) = \sup_{w \in \mathcal{L}_2} \left(E \left[\int_0^\infty z(t)' z(t) dt \right] - \rho^2 \int_0^\infty w(t)' w(t) dt \right) \quad (4)$$

where $\rho > 0$ is the prescribed level of disturbance attenuation. When $J_\infty(\gamma) \leq 0$ the \mathcal{H}_∞ requirement is guaranteed.

The design of an optimal switching signal $\gamma(t)$ yielding the minimum of (3) is a formidable task which would require the use of the Maximum Principle for optimal stochastic control. In the following, we will consider the simpler problem of designing a suboptimal feedback control $\gamma(t)$ so that the overall system is mean-square stable and an upper bound \bar{J}_2 of the optimal cost is guaranteed. As for the \mathcal{H}_∞ -type performance, we will work out a switching design procedure ensuring $J_\infty(\gamma) \leq 0$. In both cases, sufficient conditions will be provided. Note that, in our derivation, the switching signal $\gamma(t)$ is allowed to switch at infinitely high frequency, possibly giving rise to sliding mode state trajectories.

Remark 1. In a full information context, the switching signal $\gamma(t)$ may exploit the knowledge of both $x(t)$ and $\sigma(t)$, namely $\gamma(t) = g(x(t), \sigma(t))$. This is the case that will be considered in the paper. For what concerns the case of partial information, two situations are possible, depending whether just $\sigma(t)$ or $x(t)$ is accessible.

In the former case, the system is described by

$$\dot{x}(t) = A_{\sigma(t)}^{g(\sigma(t))} x(t) + B_{\sigma(t)}^{g(\sigma(t))} w(t) \quad (5)$$

$$z(t) = C_{\sigma(t)}^{g(\sigma(t))} x(t) \quad (6)$$

and a possible control design strategy consists in constructing the static decision map $g : \mathcal{N} \rightarrow \mathcal{M}$. Note that, for a given map g , system (5), (6) is a standard MJLS, for which stability analysis and performance assessment can be carried out by means of well-established tools, Costa et al. (2013). In order to find the optimal map, an exhaustive combinatorial search can be rather easily implemented.

When just $x(t)$ is available for feedback, a possible strategy would consist in using present and past values of $x(t)$ to reconstruct the current value of $\sigma(t)$, see e.g. Doucet et al. (2001); Tugnait (1982). Then, relying on a kind of certainty equivalence principle, one might select $\gamma(t)$ as a function of $x(t)$ and the estimate $\hat{\sigma}(t)$ along with the techniques developed later in this paper. Proving stability and performance properties of this heuristic approach is an interesting open issue.

3. SCHEDULING DESIGN IN NETWORKED CONTROL SYSTEMS

To illustrate a potential application of the dual switching framework, consider a scheduling problem for a multi-loop networked control system subject to packet dropout. To be precise, assume that M linear (possibly unstable) plants have to be controlled by a single regulator exchanging input-output data through a shared network, as depicted in Figure 1. The regulator is allowed to attend only one plant at a time according to the scheduling signal $\gamma(t)$ taking values in the set \mathcal{M} . Transmission of sensor/actuator data over the network is subject to random failures modeled by the Markov process $\sigma(t)$, taking values in the set \mathcal{N} . For simplicity, we assume that full state information is transmitted by each sensor without failures, so that the regulator has full access to the state information of all plants. As for the regulator-actuator channel, let $\sigma(t) = 1$ stand for the no-fault mode, when all packets are correctly transmitted, and $\sigma(t) = 2$ stand for packet dropout mode, when no packet is delivered. A Markov chain model with transition rate matrix Λ

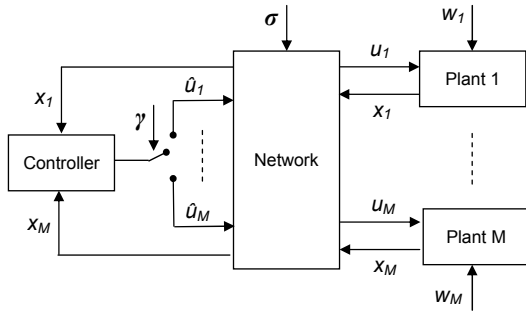


Fig. 1. The NCS considered in the application.

is assumed to describe the jumps between these two modes. Of course, more complex models could fit within the given framework, by increasing the number of logical states (e.g. allowing for packet loss also in the sensor-regulator channel), augmenting the state to account for time delay effects, and introducing suitable state observers in the regulator when full-state information is not available. Assume that the regulator is equipped with M control laws tailored to the individual plants and only the scheduling signal has to be designed so as to satisfy stability and performance requirements. If the regulator has access to the value of $\sigma(t)$, the scheduling design problem can be cast in the formulation of Section 2. Consider, for instance, a NCS with two plants ($M = 2$) described by

$$\dot{x}_i(t) = F_i x_i(t) + G_i u_i(t) + L_i w_i(t), \quad i = 1, 2 \quad (7)$$

The control law issued by the regulator is modeled as

$$\hat{u}_i(t) = \begin{cases} K_i x_i(t), & \text{if } \gamma(t) = i \\ 0, & \text{if } \gamma(t) \neq i \end{cases}$$

and the true actuator signals, affected by random packet loss, are given by

$$u_i(t) = \begin{cases} \hat{u}_i(t), & \text{if } \sigma(t) = 1 \\ 0, & \text{if } \sigma(t) = 2 \end{cases}$$

Letting $x(t) = [x_1(t)' \quad x_2(t)']'$, the overall system can be written as in (1), (2) with

$$A_1^1 = \begin{bmatrix} F_1 + G_1 K_1 & 0 \\ 0 & F_2 \end{bmatrix}, \quad A_1^2 = \begin{bmatrix} F_1 & 0 \\ 0 & F_2 + G_2 K_2 \end{bmatrix}$$

$$A_2^1 = A_2^2 = \begin{bmatrix} F_1 & 0 \\ 0 & F_2 \end{bmatrix}, \quad B_i^r = \begin{bmatrix} L_1 & 0 \\ 0 & L_2 \end{bmatrix}, \quad i, r = 1, 2$$

Finally, it might be reasonable to take $z(t) = Cx(t)$, independently of $\sigma(t)$ and $\gamma(t)$, to represent the performance output. The objective of the scheduling design is to guarantee simultaneous MS-stabilization of all plants, along with the fulfillment of some global \mathcal{H}_2 or \mathcal{H}_∞ performance specifications.

4. SWITCHING STRATEGIES DESIGN

The first result of this section deals with the design of a state-feedback switching strategy ensuring MS-stability of system (1) when the disturbance $w(t)$ is absent.

Theorem 2. Consider system (1) with $w(t) = 0$. Assume that there exist positive definite matrices P_i^r , $i \in \mathcal{N}$, $r \in \mathcal{M}$ and a Metzler matrix $\Phi = [\varphi_{rs}] \in \mathcal{T}_M$ satisfying, $\forall i, r$, the matrix inequalities

$$(A_i^r)' P_i^r + P_i^r A_i^r + \sum_{j=1}^N \lambda_{ij} P_j^r + \sum_{k=1}^M \varphi_{rk} P_i^k < 0 \quad (8)$$

Then, the feedback switching law

$$\gamma^* = g(x, \sigma) = \operatorname{argmin}_r x' P_r^r x$$

makes the closed-loop system MS-stable.

Proof. Consider the stochastic Lyapunov function $V(x, s) = \min_r x' P_s^r x$ and compute the corresponding infinitesimal generator at time t with the positions $x(t) = x$, $\sigma(t) = s$ and $g = \operatorname{argmin}_r x' P_s^r x$. For brevity, the event $\sigma(t) = s$ and the joint event $(x(t), \sigma(t)) = (x, s)$ will be indicated by \mathcal{E}_s and $\mathcal{E}_{x,s}$, respectively. It results that:

$$\begin{aligned} \mathcal{L}V(x, s) &= \lim_{h \rightarrow 0} \frac{1}{h} (E[V(x(t+h), \sigma(t+h)) | \mathcal{E}_{x,s}] - V(x, s)) \\ &= \lim_{h \rightarrow 0} \frac{1}{h} (E[\min_r x(t+h)' P_r^r x(t+h) | \mathcal{E}_{x,s}] - \min_r x' P_s^r x) \\ &= \lim_{h \rightarrow 0} \frac{1}{h} (E[\min_r x'(I + A_s^g h)' P_r^r \sigma(t+h) (I + A_s^g h) x | \mathcal{E}_s] - x' P_s^g x) \end{aligned}$$

Notice now that the expected value of the minimum of a function is not greater than the minimum of the expectation. Moreover,

$$E[P_r^r \sigma(t+h) | \mathcal{E}_s] = h \sum_{j \neq s} \lambda_{sj} P_j^r + (1 + \lambda_{ss} h) P_s^r + o(h)$$

Therefore, by neglecting the terms of order $o(h)$, it follows

$$\mathcal{L}V(x, s) \leq \lim_{h \rightarrow 0} \frac{1}{h} (\min_r x' F_r x - x' P_s^g x)$$

where

$$F_r = h \left(\sum_j \lambda_{sj} P_j^r + (A_s^g)' P_s^r + P_s^r A_s^g \right) + P_s^r$$

Now, since the function $x' F_r x$ evaluated in $r = g$ is not less than its minimum, i.e. $x' F_g x \geq \min_r x' F_r x$, we obtain

$$\mathcal{L}V(x, s) \leq x' \left((A_s^g)' P_s^g + P_s^g A_s^g + \sum_j \lambda_{sj} P_j^g \right) x$$

Then, thanks to (8), it results

$$\mathcal{L}V(x, s) < -x' \left(\sum_{k=1}^M \varphi_{gk} P_s^k \right) x$$

Since $x' P_s^r x \geq x' P_s^g x, \forall k$ and $\sum_{k=1}^M \varphi_{gk} = 0$, it holds that

$$\begin{aligned} \mathcal{L}V(x, s) &< -x' \left(\sum_{k \neq g} \varphi_{gk} P_s^k \right) x - x' \varphi_{gg} P_s^g x \leq \\ &\leq -x' \left(\sum_{k \neq g} \varphi_{gk} P_s^k \right) x - x' \varphi_{gg} P_s^g x = \\ &= -\sum_{k=1}^M \varphi_{gk} x' P_s^k x = 0 \end{aligned}$$

Since $\mathcal{L}V(x, s) < 0$, MS-stability follows from standard results on stochastic Lyapunov functions, see e.g. Kushner (1967). \square

By slightly strengthening the conditions of Theorem 2, it is possible to design a stabilizing switching strategy which yields a guaranteed \mathcal{H}_2 performance.

Theorem 3. Consider system (1), (2) with $x_0 = 0$, $B_\sigma^r = B_\sigma, \forall r \in \mathcal{M}$, and the performance index (3). Assume that there exist positive definite matrices $P_i^r, i \in \mathcal{N}, r \in \mathcal{M}$ and a Metzler matrix $\Phi = [\varphi_{rs}] \in \mathcal{T}_M$ satisfying $\forall i, r$ the matrix inequalities

$$(A_i^r)'P_i^r + P_i^r A_i^r + \sum_{j=1}^N \lambda_{ij} P_j^r + \sum_{s=1}^M \varphi_{rs} P_i^s + (C_i^r)'C_i^r < 0 \quad (9)$$

Then, the feedback switching law

$$\gamma^* = g(x, \sigma) = \operatorname{argmin}_r x' P_\sigma^r x \quad (10)$$

makes the closed-loop system MS-stable and guarantees that $J_2(\gamma^*) < \bar{J}_2 = \min_r E[\operatorname{trace}((B_\sigma)^r P_\sigma^r B_\sigma)]$.

Proof. First observe that feasibility of inequalities (9) implies feasibility of inequalities (8), so that the system is MS-stable.

Using again the stochastic Lyapunov function

$$V(x, s) = \min_r x' P_s^r x$$

and applying the same arguments as in the proof of Theorem 2, it can be shown that

$$\mathcal{L}V(x, s) < -x' \left((C_s^g)'C_s^g + \sum_{k=1}^M \varphi_{gk} P_s^g \right) x = -x' (C_s^g)'C_s^g x$$

Consider now the trajectories of system (1), (2) when $w(t) = \delta(t)e_k$ and $x_0 = 0$. In view of the well-known Dynkin's Formula, one obtains (recall that the expectation is taken with respect to the stationary distribution of $\sigma(t)$)

$$E[V(x(\infty), \sigma)] - E[V(x(0^+), \sigma)] < -E \left[\int_0^\infty (z^{(k)}(t))' z^{(k)}(t) dt \right]$$

Thanks to stability and noticing that

$$E[V(x(0^+), \sigma)] = E[\min_r e_k' B_\sigma^r P_\sigma^r B_\sigma e_k]$$

one can conclude that

$$\begin{aligned} J_2(\gamma^*) &= E \left[\sum_{k=1}^m \int_0^\infty (z^{(k)}(t))' z^{(k)}(t) dt \right] \\ &< E \left[\sum_{k=1}^m \min_r e_k' B_\sigma^r P_\sigma^r B_\sigma e_k \right] \\ &< E \left[\sum_{k=1}^m e_k' B_\sigma^{\bar{r}} P_\sigma^{\bar{r}} B_\sigma e_k \right] \end{aligned}$$

for any $\bar{r} \in \mathcal{M}$. Hence

$$J_2(\gamma^*) < \min_r E[\operatorname{trace}((B_\sigma)^r P_\sigma^r B_\sigma)] \quad (11)$$

so that the result follows. \square

In the theorem above we have assumed that the matrix B_σ^γ does not depend on the controlled switching signal $\gamma(t)$. This was done for simplicity. Indeed, when dealing with impulse responses, the value of the input matrix is relevant only at time 0. If the input matrix did depend on γ , the value $\gamma(0)$ would be an additional degree of freedom in minimizing the cost. To be precise, the feedback switching law (10) would be valid for $t > 0$ and (11) would become

$$J_2(\gamma^*) < \min_r E[\operatorname{trace}((B_\sigma^{\gamma(0)})' P_\sigma^r B_\sigma^{\gamma(0)})]$$

so that a minimization with respect to $\gamma(0)$ could be further performed.

Observe that the performance bound (11) depends both on $\Phi \in \mathcal{T}_M$ and the matrices P_i^r satisfying the bilinear matrix inequalities (9). In order to strengthen this bound, an optimization procedure can be worked out, e.g. by gridding the free parameters of Φ in a finite set and solving, for each selected Φ , a convex optimization problem.

Finally, consider the \mathcal{H}_∞ performance associated with the index (4). We can prove the following result.

Theorem 4. Consider system (1), (2) with $x_0 = 0$ and the performance index (4) with a given value of $\rho > 0$. Assume that there exist positive definite matrices P_i^r , $i \in \mathcal{N}$, $r \in \mathcal{M}$ and a Metzler matrix $\Phi = [\varphi_{rs}] \in \mathcal{T}_M$ satisfying $\forall i, r$ the matrix inequalities

$$\begin{bmatrix} (A_i^r)'P_i^r + P_i^r A_i^r + \sum_{j=1}^N \lambda_{ij} P_j^r + \sum_{s=1}^M \varphi_{rs} P_i^s + (C_i^r)'C_i^r & P_i^r B_i^r \\ \star & -\rho^2 I \end{bmatrix} < 0 \quad (12)$$

Then, the feedback switching law

$$\gamma^* = g(x, \sigma) = \operatorname{argmin}_r x' P_\sigma^r x$$

makes the closed-loop system MS-stable and guarantees that $J_\infty(\gamma) \leq 0$.

Proof. First of all, feasibility of (12) implies feasibility of (8), so that the switching law γ^* guarantees MS-stability.

Notice also that, by the Schur complement Lemma, inequalities (12) can be equivalently rewritten as

$$\begin{aligned} (A_i^r)'P_i^r + P_i^r A_i^r + \sum_{j=1}^N \lambda_{ij} P_j^r + \sum_{s=1}^M \varphi_{rs} P_i^s \\ + (C_i^r)'C_i^r + \rho^{-2} P_i^r B_i^r (B_i^r)' P_i^r < 0 \end{aligned}$$

Now, consider again the stochastic Lyapunov function $V(x, s) = \min_r x' P_s^r x$, and compute the infinitesimal generator $\mathcal{L}V(x, s)$ along the systems trajectories. It is easily obtained that

$$\begin{aligned} \mathcal{L}V(x, s) &< -x' (C_s^g)'C_s^g x + \rho^2 w' w - \rho^2 \|w - \rho^{-2} (B_s^g)' P_s^g x\|^2 \\ &< -x' (C_s^g)'C_s^g x + \rho^2 w' w \end{aligned}$$

Using again the Dynkin's formula, and recalling that $x_0 = 0$, it results that, for all $w \in \mathcal{L}_2$,

$$0 < -E \left[\int_0^\infty z(t)' z(t) dt \right] + \rho^2 \int_0^\infty w(t)' w(t) dt$$

so that the thesis follows. \square

5. APPLICATION TO THE SCHEDULING DESIGN PROBLEM

The design techniques of Section 4 can be applied to the two-plant NCS scheduling problem of Section 3. Precisely, assume that both open-loop plants (7) are unstable first-order systems, with $F_1 = 1$, $F_2 = 2$, $G_1 = G_2 = 1$, $L_1 = L_2 = 1$, and the controller gains are $K_1 = K_2 = -10$. Moreover, consider the performance output $z(t) = x(t)$, so that $C = I$, irrespective of $\sigma(t)$ and $\gamma(t)$. Finally, let the transition rate matrix Λ of the Markov process $\sigma(t)$ be given by

$$\Lambda = \begin{bmatrix} -5 & 5 \\ 10 & -10 \end{bmatrix}$$

Note that, for this Markov chain, the stationary probability distribution is $\pi = [2/3 \ 1/3]'$, meaning that the stationary probability of the packet dropout mode is $1/3$. Since both open-loop plants are unstable and only a single plant can be attended at any time-instant, the scheduling signal $\gamma(t)$ must be suitably calibrated so as to balance the control action between the two loops, taking also into account the possible effect of packet dropout.

Consider the design of a switching strategy aimed at minimizing the bound on the \mathcal{H}_2 performance. With the given data, the input matrix B_i^r coincides with the identity matrix for all i and r . Due to the diagonal structure of all the matrices, the problem of minimizing the expected quadratic cost $J_2(\gamma)$ of (3) is trivial. As a matter of fact, when the impulse is applied to the first input channel, the second plant remains indefinitely in the null state and the optimal scheduling strategy consists in keeping the loop 1 under control, and viceversa. Therefore, in order to make the problem more significant, by enforcing coupling between the control loops, it is necessary to consider a nondiagonal input matrix, for instance

$$B_i^r = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad i, r = 1, 2$$

In order to design a suboptimal switching strategy, the condition (9) of Theorem 3 has to be satisfied with a given choice of the Metzler parameter Φ . Once a feasible solution is obtained, the guaranteed \mathcal{H}_2 cost is computed as

$$\bar{J}_2 = \min_r E[\text{trace}((B_\sigma)' P_\sigma' B_\sigma)] = \min_r \sum_{i=1}^2 \pi_i \text{trace}((B_i)' P_i' B_i)$$

By gridding the parameters φ_{12} and φ_{21} in the box $[0, 2000] \times [0, 2000]$, it is found that the best choice that minimizes such an upper bound is $\varphi_{12} = 1800$, $\varphi_{21} = 1100$, resulting in $\bar{J}_2 = 1.9008$.

In order to compare the guaranteed performance bound with the actual cost, 100 simulations were run with different signals $\sigma(t)$, randomly generated according to the assumed stochastic model. By averaging the results, a sample estimate \hat{J}_2 of the expected cost J_2 of eq. (3) was obtained. It turned out that $\hat{J}_2 = 1.8090$, which is very close to the guaranteed bound \bar{J}_2 .

6. CONCLUDING REMARKS

The problem of designing a state-feedback switching law for dual switching linear systems subject to Markov jumps has been solved. Design requirements include mean-square stability and the achievement of guaranteed \mathcal{H}_2 and \mathcal{H}_∞ costs. These results may prove useful in several contexts, such as scheduling problems for NCS's with capacity limitations and random faults. Further research will address the issue of mitigation of chattering along possible sliding modes generated by the control strategies. Moreover, NCS's with more complex structure and/or dynamical behaviour will be considered.

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