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Fault Tolerant Control Strategy for an Overactuated Autonomous Vehicle Path Tracking

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Abstract: The paper presents a fault tolerant control (FTC) strategy for maintaining the lateral stability of a 4WS4WD autonomous vehicle in presence of an unknown component fault. It is designed using the flatness theory and the backstepping technique, and consists in actively controlling the vehicle's rear-wheel steering system. In a normal situation, only the front-wheel steering actuator is used since it is able to ensure the desired tracking performances on its own. However, for different faulty scenarios, it is needed to use actively the vehicle rear-wheel steering system in order to preserve the system's lateral stability. We prove that, by using this strategy, faults on the traction and the steering systems can be tolerated. The main advantage of our fault tolerant scheme is that these faults can be compensated by online computing new references for the control loops without changing the controller, providing the necessary time for a diagnosis system to precisely isolate the faulty component. This method is tested and validated on a realistic vehicle dynamic model co-simulated using CarSim and Matlab-Simulink softwares.

Keywords: Automated guided vehicles, autonomous control, trajectory planning, fault-tolerant systems, active control, four-wheel steering, dynamic reference generator.

INTRODUCTION

The control of standard and overactuated vehicles has been subject of intensive investigations in recent years (Kiencke U. et al. 2006, Li, X. et al. 2010). It is proved that overactuated vehicles are superior over traditional ones in different scenarios (Song J. and al 2009). These types of vehicles give the possibility of combining front-wheel steering control with rear-wheel steering control, as well as active differential control.

The four-wheel steering four-wheel driving (4WS4WD) vehicle safety and stability were recently studied in the literature (Casavola, A. et al 2008, Yang, H., et al. 2010, Wang, R et al. 2011). It is demonstrated that, in specific faulty scenarios, the vehicle stability cannot be ensured by controlling only the front-wheel steering system (Haddad A. et al., 2012, Zheng B. et al., 2009). In this paper, we present a fault tolerant strategy to preserve the lateral stability of a 4WS4WD autonomous vehicle in presence of an unknown component fault. In normal situation, the path tracking is maintained by using only the front wheels steering system. As soon as an abnormal behaviour is detected, i.e. a lateral deviation of the vehicle trajectory, the rear-wheel steering system is activated. In the considered control strategy two loops are designed: an outer loop and an inner loop. In the outer loop, a dynamic reference generator for rear-wheel steering is designed based on the flatness theory (Fliess, M. et al., 1997). This generator computes desired rear-wheel steering references for the inner loop. In the inner loop, the real input for the rear-wheel steering system is computed using the backstepping technique (Kirstić et al., 1995) for a nonlinear vehicle model.

The active use of the rear-wheel steering system reduces energy consumption (Chen, Y. et al., 2011, McCoy, G. A., et al., 1996) compared to passive fault tolerant control that would use both front and rear wheels steering actuators in normal and faulty situations. We show that the elaborated strategy is efficient for maintaining the lateral stability of the vehicle in presence of faults such as sensor fault on the traction system, fault on the wheels braking system, actuators drop of efficiency, front-wheel steering blocking...

The paper is organized as follows: Section 2 describes the kinematic model of a 4WS4WD vehicle. Section 3 describes the strategy used for controlling the rear-wheel steering system. Section 4 presents simulation results. Conclusions and future work are presented in section 5.

2. VEHICLE NONLINEAR MODEL

A nonlinear model is presented in order to model the overactuated 4WS4WD autonomous vehicle as in (Gillespie, T. D., 1992, Sotelo, M.A., 2003). This model is valid under the assumption of planar motion, rigid body, non-slipping tires, and considering that the two front wheels (resp. two rear wheels) turn at the same angle. These assumptions give the possibility of determining the position of the rotation center using kinematic rules.

The state-space model of the nonlinear vehicle dynamics, in the frame OXYZ fixed to the ground, can be written as follows:

$$\dot{x} = f(x) + g(u) \tag{1}$$

$$x = \begin{bmatrix} X & Y & \psi & \delta_{wf} & \delta_{wr} & \dot{\delta}_{wf} & \dot{\delta}_{wr} & \dot{\theta}_1 & \dot{\theta}_2 & \dot{\theta}_3 & \dot{\theta}_4 \end{bmatrix}^T \tag{2}$$

$$u = \begin{bmatrix} U_f & U_r & u_1 & u_2 & u_3 & u_4 \end{bmatrix}^T \tag{3}$$

X and Y are respectively the vehicle longitudinal and lateral positions, ψ is the vehicle orientation, δ_{wf} and δ_{wr} are respectively the front and rear wheels steering angles, $\dot{\theta}_i$ is the angular velocity of the wheel i with $i \in \{1,2,3,4\}$, U_i is the traction torque applied on the wheel i, and U_f and U_r are respectively the torques applied respectively on the front and rear steering actuators.

$f(x)$ and $g(u)$ are expressed as follows:

$$f(x) = \left(V_G\cos(\psi) \quad V_G\sin(\psi) \quad \frac{V_G(Tan(\delta_{wf}) + Tan(\delta_{wr}))}{L} \quad \dot{\delta}_{wf} \quad \dot{\delta}_{wr} \right.$$
$$\frac{-B_f\dot{\delta}_{wf} + M_{Tf}}{J_f} \quad \frac{-B_r\dot{\delta}_{wr} + M_{Tr}}{J_r} \quad \frac{-f_1\dot{\theta}_1 + F_{x1}R\cos(\delta_{wf}) + F_{y1}R\sin(\delta_{wf})}{J_1}$$
$$\frac{-f_2\dot{\theta}_2 + F_{x2}R\cos(\delta_{wf}) + F_{y2}R\sin(\delta_{wf})}{J_2} \quad \frac{-f_1\dot{\theta}_1 + F_{x3}R\cos(\delta_{wr}) + F_{y3}R\sin(\delta_{wr})}{J_3}$$
$$\left. \frac{-f_1\dot{\theta}_1 + F_{x4}R\cos(\delta_{wr}) + F_{y4}R\sin(\delta_{wr})}{J_4} \right)^T \tag{4}$$

$$g(u) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & \frac{U_f}{J_f} & \frac{U_r}{J_r} & \frac{U_1}{J_1} & \frac{U_2}{J_2} & \frac{U_3}{J_3} & \frac{U_4}{J_4} \end{pmatrix}^T \tag{5}$$

In these equations, V_G is the vehicle velocity at its center of gravity, L is the wheelbase, B_f and B_r are respectively the front and rear tires rolling resistances, M_{Tf} and M_{Tr} are respectively the total moments applied on front and rear steering system, J_f and J_r are respectively the front and rear wheels steering inertia, R is the wheel radius, J_i and f_i are respectively the inertia and the friction of the wheel i, and F_{xi} and F_{yi} are respectively the longitudinal and lateral forces applied on the wheel i.

The vehicle lateral position Y is monitored in order to detect undesirable deviations. A residual is generated based on the difference between the reference trajectory and the vehicle lateral position measurement. This residual is then compared to a threshold. When the considered residual exceeds this threshold, the control allocation algorithm is activated. This algorithm will generate dynamic references for the rear-wheel steering system online in order to maintain the vehicle's lateral stability.

3. FAULT TOLERANT CONTROL STRATEGY

Our objective is to preserve the vehicle's stability by using the rear-wheel steering system, as soon as a lateral deviation is detected. Since the studied system is overactuated, the control allocation problem needs to be managed.

Existing strategies dealing with the control allocation problem are either based on centralized control or on modular control (Levine, W.S., 2010). When applying a centralized control strategy, the system is viewed as a whole: a single algorithm is used to control all system inputs. The main disadvantage of this strategy is the necessity of redesigning the entire controller if a single component is modified. As for the modular control, the system is divided hierarchically into two control loops: outer loop and inner loop (Vermillion, C. et al, 2007). In the outer loop, a virtual control input is computed regardless of the dynamics of the actuators. It is selected depending on the desired optimal system behaviour. In the inner loop, the control input is computed in order to track exactly the virtual input obtained from the outer loop. This strategy, used essentially in aerospace applications (Härkegård, O. et al., 2005, Luo, Y. et al. 2004) is applied to the studied vehicle system.

Let's first express the equations describing the vehicle lateral behavior (derived from equation (4)) as follows:

$$\dot{x}_1 = f_1(x_1, \delta_{wf}, \delta_{wr}) \tag{6}$$

$$\dot{x}_2 = f_2(x_2, U_f) \tag{7}$$

$$\dot{x}_3 = f_3(x_3, U_r) \tag{8}$$

with $x_1 = (Y \quad \dot{Y})^T$, $x_2 = (\delta_{wf} \quad \dot{\delta}_{wf})^T$, and $x_3 = (\delta_{wr} \quad \dot{\delta}_{wr})^T$

In order to obtain a global steering control strategy, we will design two controllers as shown in figure 1: the first one for the outer loop and the one second for the inner loop. In the outer loop, the controller is designed using the flatness theory based on the model described in equation (6). As for the inner loop, the controller is designed using the model described by equations (6) and (8) in order to obtain a global lateral stability. We consider that a nominal control strategy is previously designed for the front-wheel steering system.

3.1 *Outer loop controller design*

The desired rear-wheel steering reference δ_{wrdes} is generated in the outer loop of the allocation control strategy. It is elaborated using the flatness property of the studied system. We recall that a system is flat if we can find a set of outputs (equal in number to the number of inputs) such that all states and inputs can be determined from these outputs without integration (Chamseddine et al., 2012, Fliess, M. et al., 1997).

A flat nonlinear system may be transformed into a linear one by using a special type of feedback, namely endogenous feedback, and controlled using linear systems control methods. The drawback of transforming a nonlinear system to a linear one could be the zero dynamics resulting from the choice of the linear submodel.

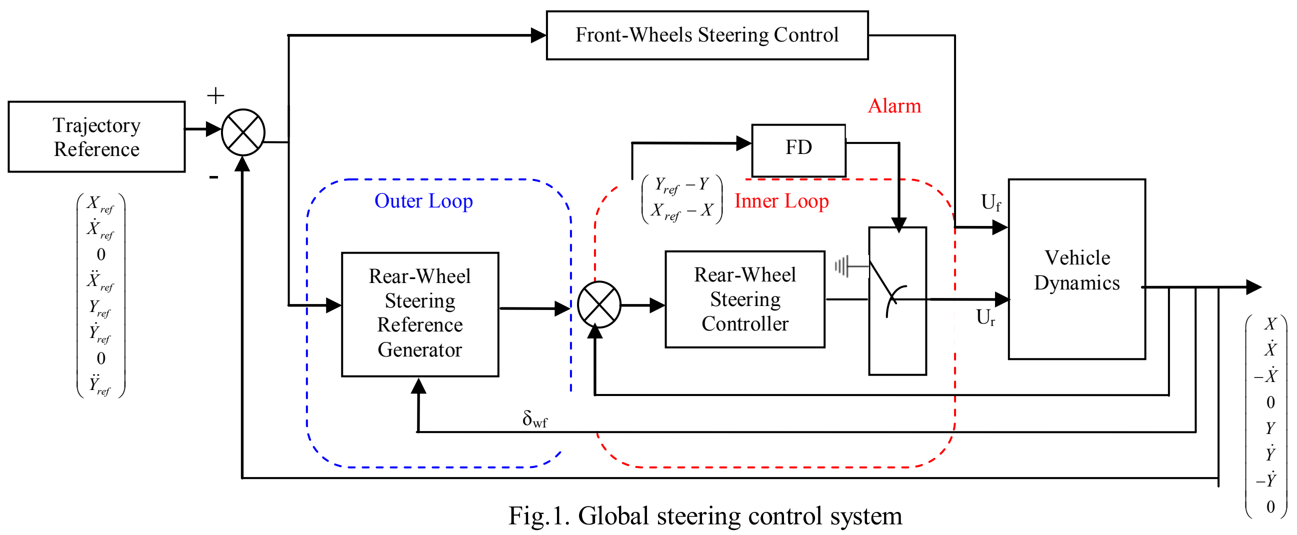

Fig.1. Global steering control system

The zero dynamics represents the dynamics of the submodel of maximal dimension that can be made unobservable by feedback. We will show that this is not the case in the proposed strategy since the relative degree of the output is equal to the number of states of the considered system.
We will use the model presented in equation (6) for generating δ_{wrdes}. In this model, the measured output Y of the considered system is the vehicle lateral position. We will prove that the state variables and the system inputs expressed in (6) can be presented using only the output Y of the system and its time-derivatives verifying the flatness property. Once verified, we will control this output and its derivatives in order to control all system variables expressed in (6) and track a desired nominal trajectory.

Let's first demonstrate that the system presented in (6) is flat. From the model expressed in (4), the output Y of the system can be written as follows:

$$Y = \int_0^t V_G \sin(\psi) dt = \beta_1(t,\psi) \tag{9}$$

Deriving equation (9) with respect to time leads to:

$$\dot{Y} = V_G \sin(\psi) \tag{10}$$

We consider that the vehicle's velocity V_G is constant. Yaw angle ψ can be written as follows:

$$\psi = Arc\sin(\frac{\dot{Y}}{V_G}) = \beta_2(\dot{Y}) \tag{11}$$

And (24) can be rewritten as follows:

$$\beta_1(t,\psi) = \beta_1(t,\beta_2(\dot{Y})) = Y \tag{12}$$

Deriving equation (10) with respect to time leads to:

$$\ddot{Y} = V_G \cos(\psi)\dot{\psi} \tag{13}$$

For:

$$u_1 = Tan(\delta_{wf}) + Tan(\delta_{wrdes}) \tag{14}$$

and $\dot{\psi}$ written as in equation (4), equation (13) can be rewritten as follows:

$$\ddot{Y} = \frac{V_G^{\ 2}\cos(\psi)u_1}{L} \tag{15}$$

When substituting ψ in (15) with the expression obtained in (11) we can write:

$$\ddot{Y} = \frac{V_G^{\ 2}\cos(Arc\sin(\frac{\dot{Y}}{V_G}))u_1}{L} \tag{16}$$

and u_1 can then be expressed as follows:

$$u_1 = \frac{L\ddot{Y}}{V_G^{\ 2}\cos(Arc\sin(\frac{\dot{Y}}{V_G}))} = \beta_3(\dot{Y},\ddot{Y}) \tag{17}$$

Since the system variables and input (t,ψ,u_1) can be expressed using $(Y,\dot{Y},\ddot{Y})$, the studied nonlinear system is flat and therefore controllable. We can also conclude that the system does not have zero dynamics since the relative degree of the output is equal to the number of states.

Our objective is to compute in the outer loop a desired rear-wheel steering reference which, if it is exactly followed using the inner loop, will ensure the lateral stability of the vehicle as well as desired nominal performances for the system.

Let us first define the following errors:

$$e_1 = Y_{ref} - Y,\ \dot{e}_1 = \dot{Y}_{ref} - V_G\sin(\psi),\ \ddot{e}_1 = \ddot{Y}_{ref} - V_G\cos(\psi)\dot{\psi} \tag{18}$$

The lateral stability of the vehicle is obtained if $(e_1,\dot{e}_1)$ point converges to $(0,0)$, meaning that $(Y, V_G\sin(\psi))$ converges to $(Y_{ref},\dot{Y}_{ref})$. For this purpose, we will elaborate a Lypunov function that verifies the following conditions:

C$_1$: $V_1(e_1,\dot{e}_1) > 0$ for $(e_1,\dot{e}_1) \neq 0$, and $V_1(0,0) = 0$.

C_2: $\frac{dV_1(e_1,\dot{e}_1)}{dt} < 0$ for $(e_1,\dot{e}_1) \neq 0$.

If obtained, $(e_1,\dot{e}_1) = (0,0)$ is then an asymptotically stable equilibrium point for the system.

Consider the following continuous differential function verifying C_1:

$$V_1(e_1,\dot{e}_1) = \frac{K_0(Y_{ref}-Y)^2 + (\dot{Y}_{ref} - V_G\sin(\psi))^2}{2} > 0 \tag{19}$$

for $e_1 \neq 0$, $\dot{e}_1 \neq 0$ and $K_0 > 0$, and with $V_1(0,0) = 0$.

If $\frac{dV_1(e_1,\dot{e}_1)}{dt} < 0$ we verify C_2, and $V_1(e_1,\dot{e}_1)$ is then a Lyapunov function with $(e_1,\dot{e}_1) = (0,0)$ an asymptotically stable equilibrium point for the system.

Deriving $V_1(e_1,\dot{e}_1)$ with respect to time leads to:

$$\begin{aligned}\frac{dV_1(e_1,\dot{e}_1)}{dt} &= K_0(Y_{ref}-Y)(\dot{Y}_{ref} - V_G\sin(\psi)) + \\ &(\dot{Y}_{ref} - V_G\sin(\psi))(\ddot{Y}_{ref} - \frac{V_G^{\,2}\cos(\psi)}{L}(Tan(\delta_{wf}) + Tan(\delta_{wr})))\end{aligned} \tag{20}$$

Our objective is to ensure that $\dot{V}_1(e_1,\dot{e}_1) < 0$ verify C_2. For this purpose, we will calculate desired rear-wheel steering position value δ_{wr} for equation (20) satisfying this objective.

A solution for obtaining $\dot{V}_1(e_1,\dot{e}_1) < 0$ is to have:

$$\begin{aligned}&\ddot{Y}_{ref} - \frac{V_G^{\,2}\cos(\psi)}{L}(Tan(\delta_{wf}) + Tan(\delta_{wr})) \\ &= -K_0(Y_{ref}-Y) - K_1(\dot{Y}_{ref} - V_G\sin(\psi))\end{aligned} \tag{21}$$

with $K_1 > 0$.

From equation (21), $\dot{V}(e_1,\dot{e}_1)$ in equation (20) can be written as follows:

$$\begin{aligned}\dot{V}_1(e_1,\dot{e}_1) &= K_0(Y_{ref}-Y)(\dot{Y}_{ref} - V_G\sin(\psi)) + \\ &(\dot{Y}_{ref} - V_G\sin(\psi))(-K_0(Y_{ref}-Y) - K_1(\dot{Y}_{ref} - V_G\sin(\psi)))\end{aligned} \tag{22}$$

From equation (22) we obtain:

$$\dot{V}_1(e_1,\dot{e}_1) = -K_1(\dot{Y}_{ref} - V_G\sin(\psi))^2 < 0 \tag{23}$$

$V_1(e_1,\dot{e}_1)$ is then a Lyapunov function and the system is asymptotically stable for e_1 and $\dot{e}_1$ equal to zero.

From equation (21) we obtain:

$$\frac{V_G^{\,2}\cos(\psi)(Tan(\delta_{wf}) + Tan(\delta_{wr}))}{L} = -K_0 e_1 - K_1\dot{e}_1 + \ddot{Y}_{ref} \tag{24}$$

From the vehicle model expressed in (4), we can express the vehicle speed as follows:

$$V_G = \sqrt{\dot{X}^2 + \dot{Y}^2} \tag{25}$$

and the vehicle orientation angle as follows:

$$\psi = ArcTan(\frac{\dot{Y}}{\dot{X}}) \tag{26}$$

From (24), (25), and (26), we can compute the desired rear-wheel steering position δ_{wrdes} as follows:

$$\begin{aligned}\delta_{wrdes} &= ArcTan(\frac{L}{(\dot{X}^2+\dot{Y}^2)\cos(ArcTan(\frac{\dot{Y}}{\dot{X}}))}(-K_0 e_1 \\ &-K_1\dot{e}_1 + \ddot{Y}_{ref}) - Tan(\delta_{wf}))\end{aligned} \tag{27}$$

The gains K_0 and K_1 in equation (27) can later be tuned in order to obtain desired performances.

Remark 1: Singularities exist in equation (27) for $ArcTan(\frac{\dot{Y}}{\dot{X}}) = \pm\frac{\pi}{2}$. These singularities can be avoided by changing the basis in the frame OXYZ, when vehicle rotation angle does not verify one of the following conditions (Rajamani et al., 2003):

C_3: $-\frac{\pi}{4} <= ArcTan(\frac{\dot{Y}}{\dot{X}}) <= \frac{\pi}{4}$

C_4: $\frac{3\pi}{4} <= ArcTan(\frac{\dot{Y}}{\dot{X}}) <= \frac{5\pi}{4}$.

The transition matrix can be expressed as follows:

$$T = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{28}$$

By applying this transformation, controlling the vehicle in the frame OXYZ yields controlling it in the frame OYXZ. In that case, rear-wheel steering reference can be rewritten as follows:

$$\begin{aligned}\delta_{wrdes} &= ArcTan(-\frac{L}{(\dot{X}^2+\dot{Y}^2)\sin(ArcTan(\frac{\dot{Y}}{\dot{X}}))}(K_0(X_{ref}-X) \\ &+K_1(\dot{X}_{ref}-\dot{X}) + \ddot{X}_{ref}) - Tan(\delta_{wf}))\end{aligned} \tag{29}$$

Remark 2: It is clear that singularities exist also for $ArcTan(\frac{\dot{Y}}{\dot{X}}) = 0$ and $ArcTan(\frac{\dot{Y}}{\dot{X}}) = \pi$. A transformation is also applied when vehicle rotation angle does not verify C_5 : $\frac{\pi}{4} < ArcTan(\frac{\dot{Y}}{\dot{X}}) < \frac{3\pi}{4}$ or C_6 : $\frac{5\pi}{4} < ArcTan(\frac{\dot{Y}}{\dot{X}}) < \frac{7\pi}{4}$. In this case, the transition matrix can be expressed as follows:

$$T^{-1} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{30}$$

In other words, when conditions C_5 or C_6 are not verified, the vehicle is controlled in the frame OXYZ .

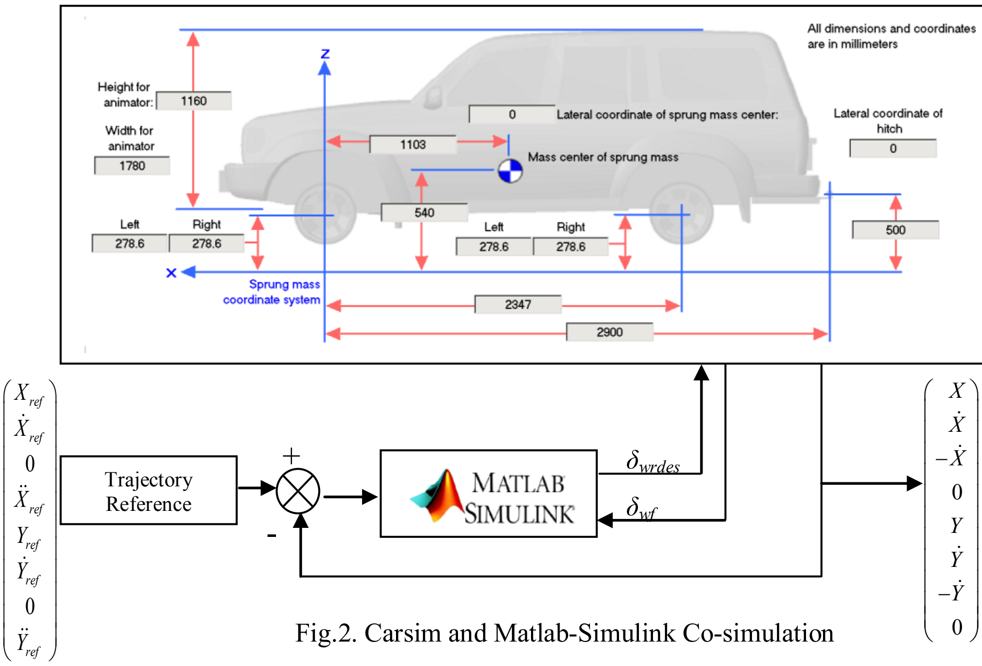

Fig.2. Carsim and Matlab-Simulink Co-simulation

3.1 Inner Loop controller design

After generating the desired rear-wheel steering position in the outer loop, we will compute the control input U_r to track this reference using the backstepping technique. This control law will ensure vehicle lateral stability as well as desired performances.

In equations (27) and (29) δ_{wrdes} represents the desired rear-wheel steering position, which is not necessarily equal to the measured value δ_{wr} when taking into consideration the rear-wheels steering actuator dynamics. It is then necessary to include these dynamics when computing U_r.

The relationship between the measured rear-wheel steering position and the desired one can be expressed as follows:

$$\delta_{wr} = \delta_{wrdes} + \Delta\delta_{wr} \tag{31}$$

where $\Delta\delta_{wr}$ represents the difference between these two values.

After substituting δ_{wr} value presented in equation (6) with the one obtained in equation (29) we can write:

$$\dot{x}_1 = f_1(x_1, \delta_{wf}, \delta_{wrdes} + \Delta\delta_{wr}) \tag{32}$$

with $\Delta\delta_{wr} = \delta_{wr} - \delta_{wrdes}$

Equation (32) can be rewritten as follows:

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, \delta_{wf}, \delta_{wrdes} + \Delta\delta_{wr}) = f_1(x_1, \delta_{wf}, \delta_{wrdes}) \\ &+ \lambda_1(x_1, \delta_{wf}, \Delta\delta_{wr})\Delta\dot{\delta}_{wr} \end{aligned} \tag{33}$$

with:

$$\begin{aligned} f_1(x_1, \delta_{wf}, \delta_{wrdes} + \Delta\delta_{wr}) &= \begin{pmatrix} f_{11}(x_1, \delta_{wf}, \delta_{wrdes} + \Delta\delta_{wr}) \\ f_{12}(x_1, \delta_{wf}, \delta_{wrdes} + \Delta\delta_{wr}) \end{pmatrix} \\ &= \begin{pmatrix} V_G \sin(\psi) \\ \dfrac{V_G{}^2 \cos(\psi)(Tan(\delta_{wf}) + Tan(\delta_{wrdes} + \Delta\delta_{wr}))}{L} \end{pmatrix} \end{aligned} \tag{34}$$

and $\lambda_1(x_1, \delta_{wf}, \Delta\delta_{wr})$ expressed as follows:

$$\lambda_1(x_1, \delta_{wf}, \Delta\delta_{wr}) = \frac{f_1(x_1, \delta_{wf}, \delta_{wr}) - f_1(x_1, \delta_{wf}, \delta_{wrdes})}{\Delta\dot{\delta}_{wr}} \tag{35}$$

with:

$$\begin{aligned} \lambda_1(x_1, \delta_{wf}, \Delta\delta_{wr}) &= \begin{pmatrix} \lambda_{11}(x_1, \delta_{wf}, \Delta\delta_{wr}) \\ \lambda_{12}(x_1, \delta_{wf}, \Delta\delta_{wr}) \end{pmatrix} \\ &= \begin{pmatrix} \dfrac{f_{11}(x_1, \delta_{wf}, \delta_{wr}) - f_{11}(x_1, \delta_{wf}, \delta_{wrdes})}{\Delta\dot{\delta}_{wr}} \\ \dfrac{f_{12}(x_1, \delta_{wf}, \delta_{wr}) - f_{12}(x_1, \delta_{wf}, \delta_{wrdes})}{\Delta\dot{\delta}_{wr}} \end{pmatrix} \end{aligned} \tag{36}$$

Let us define a continuous differential function as follows:

$$\begin{aligned} V_2(e_1, \dot{e}_1, \Delta\delta_{wr}, \Delta\dot{\delta}_{wr}) &= V_1(e_1, \dot{e}_1) + \frac{K_2(\delta_{wr} - \delta_{wrdes})^2}{2} \\ &+ \frac{(\dot{\delta}_{wr} - \dot{\delta}_{wrdes})^2}{2} \end{aligned} \tag{37}$$

with $\Delta\delta_{wr} = \delta_{wr} - \delta_{wrdes}$, $\Delta\dot{\delta}_{wr} = \dot{\delta}_{wr} - \dot{\delta}_{wrdes}$, and $K_2 > 0$.

If $V_2(e_1, \dot{e}_1, \Delta\delta_{wr}, \Delta\dot{\delta}_{wr})$ verifies condition C_1 and C_2, it is then a Lyapunov function and $(e_1, \dot{e}_1, \Delta\delta_{wr}, \Delta\dot{\delta}_{wr}) = (0,0,0,0)$ is then an asymptotically stable equilibrium point.

The function defined in (37) verifies condition C_1 since $V_2(e_1, \dot{e}_1, \Delta\delta_{wr}, \Delta\dot{\delta}_{wr}) > 0$ for: $e_1 \neq 0$, $\dot{e}_1 \neq 0$, $\Delta\delta_{wr} \neq 0$, and $\Delta\dot{\delta}_{wr} \neq 0$, and $V_2(0,0,0,0) = 0$. Our objective is to calculate U_r satisfying condition C_2 for $V_2(e_1, \dot{e}_1, \Delta\delta_{wr}, \Delta\dot{\delta}_{wr})$ function. If it is successfully obtained, then the function expressed in equation (37) is a Lyapunov function and the system converges asymptotically to the point $(Y, \dot{Y}, \delta_{wr}, \dot{\delta}_{wr}) = (Y_{ref}, \dot{Y}_{ref}, \delta_{wrdes}, \dot{\delta}_{wrdes})$.

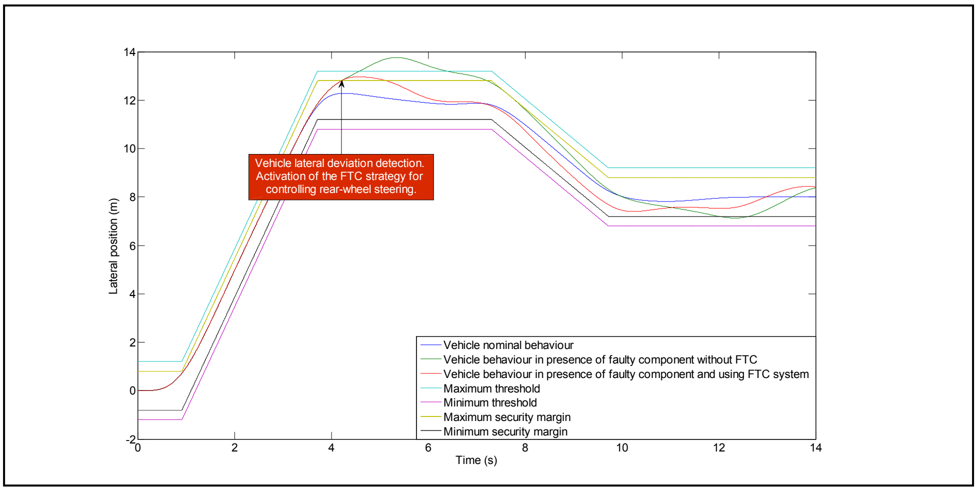

Fig. 3 Vehicle lateral behaviour when performing a double lane-change maneuver

The derivative of the function $V_2(e_1,\dot{e}_1,\Delta\delta_{wr},\Delta\dot{\delta}_{wr})$ with respect to time is expressed as follows:

$$\dot{V}_2(e_1,\dot{e}_1,\Delta\delta_{wr},\Delta\dot{\delta}_{wr})=\dot{V}_1(e_1,\dot{e}_1)+K_2(\delta_{wr}-\delta_{wrdes})(\dot{\delta}_{wr}-\dot{\delta}_{wrdes})+(\dot{\delta}_{wr}-\dot{\delta}_{wrdes})(\ddot{\delta}_{wr}-\ddot{\delta}_{wrdes}) \quad (38)$$

In equations (4) and (8), $\ddot{\delta}_{wr}$ is expressed as follows:

$$\ddot{\delta}_{wr}=\frac{-B_r\dot{\delta}_{wr}+M_{Tr}+U_r}{J_r} \quad (39)$$

In our study we consider that the wheels sideslip angles are small. We can then neglect M_{Tr} moment in equation (39) (Gillespie, T. D., 1992). We then introduce $\ddot{\delta}_{wr}$ to equation (38). We obtain:

$$\dot{V}_2(e_1,\dot{e}_1,\Delta\delta_{wr},\Delta\dot{\delta}_{wr})=\dot{V}_1(e_1,\dot{e}_1)+K_2(\delta_{wr}-\delta_{wrdes})(\dot{\delta}_{wr}-\dot{\delta}_{wrdes})+(\dot{\delta}_{wr}-\dot{\delta}_{wrdes})(\frac{-B_r\dot{\delta}_{wr}+U_r}{J_r}-\ddot{\delta}_{wrdes}) \quad (40)$$

Using equations (20) and (33), $\dot{V}_1(e_1,\dot{e}_1)$ can be rewritten as follows:

$$\dot{V}_1(e_1,\dot{e}_1)=K_0e_1\dot{e}_1+\dot{e}_1(\ddot{Y}_{ref}-f_{12}(Y,\dot{Y},\delta_{wrdes})+\lambda_{12}(Y,\dot{Y},\Delta\delta_{wr})\Delta\dot{\delta}_{wr})=-K_1\dot{e}_1^{\,2}+\lambda_{12}(Y,\dot{Y},\Delta\delta_{wr})\Delta\dot{\delta}_{wr}\dot{e}_1 \quad (41)$$

We can verify that if $\Delta\delta_{wr}=0$, meaning that $\delta_{wrdes}=\delta_{wr}$, equation (41) becomes equal to equation (23).
Using the result obtained from equation (41), equation (40) can now be expressed as follows:

$$\dot{V}_2(e_1,\dot{e}_1,\Delta\delta_{wr},\Delta\dot{\delta}_{wr})=-K_1\dot{e}_1^{\,2}+\lambda_{12}(Y,\dot{Y},\Delta\delta_{wr})\Delta\dot{\delta}_{wr}\dot{e}_1+K_2(\delta_{wr}-\delta_{wrdes})(\dot{\delta}_{wr}-\dot{\delta}_{wrdes})+(\dot{\delta}_{wr}-\dot{\delta}_{wrdes})(\frac{-B_r\dot{\delta}_{wr}+U_r}{J_r}-\ddot{\delta}_{wrdes}) \quad (42)$$

A sufficient condition for verifying C2 is to have U_r expressed as follows:

$$U_r=+B_r\dot{\delta}_{wr}+J_r(\ddot{\delta}_{wrdes}+\lambda_{12}(Y,\dot{Y},\Delta\delta_{wr})\dot{e}_1+K_2(\delta_{wrdes}-\delta_{wr})+K_3(\dot{\delta}_{wrdes}-\dot{\delta}_{wr}))=+B_r\dot{\delta}_{wr}+J_r(\ddot{\delta}_{wrdes}+\lambda_{12}(Y,\dot{Y},\Delta\delta_{wr})\dot{e}_1+K_2\Delta\delta_{wr}+K_3\Delta\dot{\delta}_{wr}) \quad (43)$$

With $K_3>0$.

We rewrite equation (42) after substituting U_r value with the one presented in equation (43). We obtain:

$$\dot{V}_2(e_1,\dot{e}_1,\Delta\delta_{wr},\Delta\dot{\delta}_{wr})=-K_1\dot{e}_1^{\,2}+\lambda_{12}(Y,\dot{Y},\Delta\delta_{wr})\Delta\dot{\delta}_{wr}\dot{e}_1+K_2\Delta\delta_{wr}\Delta\dot{\delta}_{wr}+\Delta\dot{\delta}_{wr}(-K_2\Delta\delta_{wr}-K_3\Delta\dot{\delta}_{wr}-\lambda_{12}(Y,\dot{Y},\Delta\delta_{wr})\dot{e}_1) \quad (44)$$

From equation (44) we obtain:

$$\dot{V}_2(e_1,\dot{e}_1,\Delta\delta_{wr},\Delta\dot{\delta}_{wr})=-K_1\dot{e}_1^{\,2}-K_3\Delta\dot{\delta}_{wr}^{\,2} \quad (45)$$

C2 is then verified as shown in equation (45). We can then conclude that $V_2(e_1,\dot{e}_1,\Delta\delta_{wr},\Delta\dot{\delta}_{wr})$ is a Lyapunov function and that $(Y,\dot{Y},\delta_{wr},\dot{\delta}_{wr})=(Y_{ref},\dot{Y}_{ref},\delta_{wrdes},\dot{\delta}_{wrdes})$ is an asymptotically stable equilibrium point.

The gains K_2 and K_3 expressed in equation (43) can later be tuned in order to obtain desired performances.

4. SIMULATION RESULTS

The elaborated strategy is tested using a co-simulation between CarSim, a professional simulator used by automobile manufacturers, and Matlab-Simulink software (as shown in figure 2). In this test, an overactuated autonomous vehicle is circulating with an initial constant speed of 60 km/h, and performing a double lane-change maneuver on a dry asphalt road (friction coefficient $\mu_{\max}$ = 1.2). At t=3.6s, a drop of efficiency is created at the front-wheel steering actuator. Two scenarios are then considered.

In the first scenario, the vehicle is controlled using its front-wheel steering system only. It can be seen in figure 3 that the front-wheel steering controller is not able to ensure alone the lateral stability of the vehicle. The vehicle exceeds the limits of the road at t=4.58s.

In the second scenario, fault detection (FD) system is used for monitoring the lateral deviation. When this deviation violates the acceptable security margins (determined based on the width of the road) at t=4.16s, the FD system activates the rear-wheel steering controller elaborated in section 3. The rear-wheel steering system is then able to maintain the lateral stability of the global system in presence of the component fault.

5. CONCLUSIONS

In this paper, a fault tolerant control strategy is developed for a 4WS4WD autonomous vehicle. We demonstrate that this strategy can ensure the vehicle lateral stability in presence of an unknown component fault. This strategy is developed using the flatness theory and the backstepping technique, and consists on dividing the control strategy into two loops: outer loop and inner loop. In the outer loop, a desired steering position is computed for obtaining nominal vehicle performances. In the inner loop, the steering actuator input is computed in order to follow the desired reference calculated in the outer loop. When a vehicle lateral deviation is detected, the elaborated algorithm is activated in order to maintain the global system's lateral stability. The efficiency of this strategy is then illustrated using a co-simulation between Carsim and Matlab-Simulink softwares.

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