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Subspace Predictive Repetitive Control with Lifted Domain Identification for Wind Turbine Individual Pitch Control ⋆

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Abstract: Individual pitch control (IPC) is gaining increasing acceptance as a method for mitigation of periodic disturbances in wind turbines. This paper aims at formulating a repetitive control (RC) methodology capable of adapting online to changing turbine dynamics. This is achieved by performing system identification in a reduced-dimensional space using basis functions and using the identified parameters to synthesise an RC law to reject periodic disturbances: this methodology is termed Subspace Predictive Repetitive Control (SPRC). The method is tested on an industrial simulation test bench and is able to identify and perform load control on the turbine without affecting its power production.

Keywords: Subspace Predictive Control, Repetitive Control, Identification, IPC, Basis Functions.

1. INTRODUCTION

While wind energy deployment has risen significantly, the associated capital costs remain an impediment to its continued growth. Active control of turbine loads is currently under research to enable lighter and more cost-effective turbine design. Individual pitch control (IPC), Bossanyi (2003), is a readily implementable method to reduce the dynamic loading in wind turbines which is periodic in nature, with the dominant load frequency equal to the rotor speed (1P). IPC is implemented via the Multi-Blade Coördinate (MBC) transformation to decouple and linearise the multivariable periodic wind turbine system. The other periodic loads at the harmonics 2P, 3P,... can be attenuated by additional control loops for each harmonic and residual periodic loading, Van Engelen (2006). The effectiveness of the method has been shown on a research turbine, Stol et al. (2006).

Despite substantial load reduction potential, this IPC method demands a drastic increase in pitch activity. Modern wind turbines enforce hard limits on pitch activity to avoid costs related to pitch system breakdown; these limits reduce the achievable load alleviation, Kanev and van Engelen (2009). Another disadvantage in this approach is the lack of control over the frequency content of the actuation signals, which also reduces pitch actuator reliability. To minimise control effort for load reduction, the periodic nature of the loading should be exploited. Further, the current approach is not multivariable and entails an increasing number of tuning parameters and decoupling complexity for additional load components.

Iterative Learning Control (ILC), Bristow et al. (2006), is a multivariable control method designed to reject periodic disturbances. Repetitive Control (RC), Longman (2000), is the extension of ILC to continuously operating systems that do not undergo initial condition resets. RC is directly applicable to turbine load reduction, and has been applied to a simulation model in Tutty et al. (2013). It is also possible to project the inputs and outputs into a basis function space, Van de Wijdeven and Bosgra (2010), so that the shape of the actuation signals can be precisely controlled. Using sinusoidal basis functions, a simulation study for IPC using RC was done in Houtzager et al. (2013), which showed promising load reductions while maintaining strict control over the frequencies in the input.

Although RC is fairly insensitive to uncertainties, the controller design requires a linear time-invariant (LTI) model that approximates the true system. An RC law can also be formulated in the lifted domain, Dijkstra and Bosgra (2002), for a system (like a wind turbine) whose model parameters vary periodically over time. Obtaining such a model for a turbine can prove difficult, since the dynamics depend on slowly changing parameters such as the wind speed, which are difficult to measure. Further, field experience shows that turbine dynamics can be strongly influenced by factors such as site location and manufacturing differences (e.g. rotor balancing effects). An adaptive RC law capable of reacting to such factors would be able to enhance load reduction potential. The Subspace Predictive Repetitive Control (SPRC) methodology, Navalkar et al. (2014), combines online subspace identification with RC implementation for wind turbine IPC.

The main contribution of this paper is a novel online closed-loop identification paradigm that estimates system parameters in the lifted domain. This is combined with RC law formulation and implementation. For the first time, basis functions are used for online identification in conjunction with repetitive control, to drastically reduce the dimensions of the problem. Compared with existing system identification methods, this strongly reduces computational complexity and ensures control over the frequency content of the actuator signals. Further, the new lifted

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domain identification algorithm extends the applicability of the SPRC technique to a larger class of systems, such as wind turbines, that admit a periodic linear time-varying model formulation. With the use of basis functions for identification, the requirement for persistency of excitation for identification, Verhaegen and Verdult (2007), is relaxed such that persistently exciting input is required only along the basis vector directions, minimising its negative effect on control performance. Also, for the first time, equality constraints on the actuator input signal are translated to constraints on the basis functions; this can be used to enforce perfect decoupling of the load alleviation controller from the nominal power production of the wind turbine.

The outline of the paper is as follows: in Section 1 the background of the problem was established. In Section 2, the plant and simulation environment are described. In Section 3, the theoretical extensions to the SPRC methodology are described. Simulation results are presented in Section 4 and conclusions are drawn in Section 5.

2. TURBINE MODEL & SIMULATION ENVIRONMENT

To test the proposed control strategy in a high-fidelity simulation environment, the software GH Bladed™ was used. This software is used by manufacturers and certification bodies for load analysis of new turbine designs. Research on load control strategies has also been done using Bladed, Navalkar et al. (2014). The turbine is modelled in Bladed with a multibody representation, with flexible blades and tower. Stationary realisations of turbulent wind fields can be generated and marched through the turbine model. An extended bladed-element-momentum (BEM) theory is used to define the interaction of the turbine with the wind. The controller is designed in Simulink and compiled into a DLL file that can be read directly by Bladed, Houtzager et al. (2013). The turbine loads over the duration of the simulation are then made available for post-processing.

The wind turbine model of the XEMC Darwind XD115 turbine was provided by the manufacturer and used for the simulations. General details of this turbine are given in Table 1.

Table 1. XD115 Wind Turbine specifications

Description	Symbol	Value
Rated power	P_{rated}	5000 kW
Rotor diameter	d_{ro}	115 m
Cut-in wind speed	v_{cutin}	4 m/s
Rated wind speed	v_{rated}	12 m/s
Cut-out wind speed	v_{cutout}	25 m/s
Rated rotational rotor speed	Ω_{ro}	18 rpm
Gearbox ratio	ν	1.0 [Direct-Drive]
Pitch-rate limit	$\dot{\theta}_{\text{limit}}$	6°/s

A baseline controller was designed for the turbine to control the generator torque and collective blade pitch. This controller also incorporates active damping of structural modes. However, the dynamic loading of the major components occurs mainly at the 1P frequency (or rotor speed) and its harmonics; this loading cannot be addressed by torque or collective pitch control. For this loading, an adaptive IPC controller will be designed using the SPRC methodology in the next section.

3. THEORETICAL FRAMEWORK

The extension of Subspace Predictive Repetitive Control (SPRC) with reduced-dimension identification is presented below.

3.1 Problem Formulation

An ideal IPC controller should be able to satisfy the following:

(A) Asymptotically reject periodic disturbances
(B) Adapt online to changes in system dynamics
(C) Produce smooth pitch control input signals
(D) Not interfere with the power production process.

The first three requirements are satisfied by SPRC, Navalkar et al. (2014), however, the present paper uses a novel lifted domain technique with basis functions to reduce the computational complexity of the algorithm. The last requirement will be satisfied by enforcing constraints on the basis functions. The SPRC approach entails online identification of the requisite system parameters and the formulation of an RC control law which produces an optimal feedforward sequence that can be repetitively applied to attenuate periodic disturbances. Each of the steps involved in SPRC is explained in the next subsections.

3.2 Step 1: Predictor Formulation

The wind turbine system can be modelled as a discrete-time system, which admits the following description in the predictor form:

$$x_{k+1} = Ax_k + Bu_k + Fd_k + Ky_k \tag{1}$$

$$y_k = Cx_k + Du_k + e_k. \tag{2}$$

Here, $x_k \in \mathbb{R}^n$ is the state, $u_k \in \mathbb{R}^r$ is the vector of the three pitch inputs, $y_k \in \mathbb{R}^\ell$ are the three blade load signals, $d_k \in \mathbb{R}^\ell$ are the periodic disturbances with period P, induced by wind loading, and $e_k \in \mathbb{R}^\ell$ is white noise sequences representing measurement noise. The system matrices A, B, C, D, F and K have the appropriate dimensions. In this predictor form, the matrix A is equal to $\tilde{A} - KC$, where $\tilde{A}$ is the state-transition matrix of the system in the innovation form given in Ljung (1999), and K is the Kalman gain. Per definition, the matrix A is stable. The system matrices are not constant, but vary with time and should be indicated as such by the subscript $_k$. For brevity, the system matrices have been shown constant, but the analysis can easily be extended to time-varying system matrices. Now, the stacked output vector is defined as:

$$Y_k = \begin{bmatrix} y_k \\ y_{k+1} \\ \vdots \\ y_{k+P-1} \end{bmatrix}. \tag{3}$$

Similarly, the stacked input and disturbance vectors are represented by U_k, D_k and E_k. Since the length of the stacked vector is the same as the period of the periodic disturbance d_k, the stacked vector D_k is constant and will be denoted by $\bar{D}$. In a manner similar to Bamieh et al. (1991), the time-domain system is “lifted” to the iteration or trial domain:

$$x_{k+P} = A^P x_k + \mathscr{K}_u U_k + \mathscr{K}_d \bar{D} + \mathscr{K}_y Y_k$$

$$Y_k = \Gamma x_k + HU_k + \mathscr{J}\bar{D} + E_k.$$

The time index k is replaced by the iteration index j, such that $(k, k+P, k+2P, \ldots) \rightarrow (j, j+1, j+2, \ldots)$:

$$x_{j+1} = A^P x_j + \mathscr{K}_u U_j + \mathscr{K}_d \bar{D} + \mathscr{K}_y Y_j \tag{4}$$

$$Y_j = \Gamma x_j + HU_j + \mathscr{J}\bar{D} + E_j. \tag{5}$$

Here the extended controllability matrix $\mathscr{K}_u$ is given by:

$$\mathscr{K}_u = \left[A^{P-1}B \; A^{P-2}B \; \dots \; B\right]. \tag{6}$$

The matrices $\mathscr{K}_d$ and $\mathscr{K}_y$ are defined in the same way, replacing B by F and K respectively. The extended observability matrix Γ and the Toeplitz matrix H are given by:

$$\Gamma = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{P-1} \end{bmatrix} \qquad H = \begin{bmatrix} D & 0 & \dots & 0 \\ CB & D & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{P-2}B & CA^{P-3}B & \dots & D \end{bmatrix}. \tag{7}$$

The Toeplitz matrix $\mathscr{J}$ is obtained by replacing B in the above equations by F, and replacing D by $0_{\ell\times\ell}$. Now, to remove the effect of the initial state, the assumption is made that $A^j \approx 0$ for $j \geq P$. For an adequately large P, this would be valid since A is stable. Hence, equation (4) reduces to:

$$x_{j+1} = \left[\mathscr{K}_u, \; \mathscr{K}_y, \; \mathscr{K}_d\bar{D}\right] \begin{bmatrix} U_j \\ Y_j \\ 1 \end{bmatrix}. \tag{8}$$

Substituting this in equation (5) to predict the output over the next period:

$$Y_j = \left[\Gamma\mathscr{K}_u, \; \Gamma\mathscr{K}_y, \; H, \; (\Gamma\mathscr{K}_d + \mathscr{J})\bar{D}\right] \begin{bmatrix} U_{j-1} \\ Y_{j-1} \\ U_j \\ 1 \end{bmatrix} + E_j. \tag{9}$$

The noise sequence E_j is uncorrelated with the input-output data of the previous iteration. Further, since an RC control law is used, the lifted control input U_j for the next iteration is determined at the end of the current iteration $(j-1)$. Hence, U_j is correlated with the lifted noise sequence E_{j-1} but it is not correlated with E_j. So, in the lifted domain, the sequence E_j forms an uncorrelated zero-mean white noise sequence. Equation (9) can be used to determine the system parameters $[\Gamma\mathscr{K}_u, \Gamma\mathscr{K}_y, H, (\Gamma\mathscr{K}_d + \mathscr{J})\bar{D}]$ if input-output data is available.

This identification problem is typically large in dimension, since the input-output data is stacked over the period P.

3.3 Step 2: Basis Function Space Projection

This step addresses the requirements (C) and (D), while it also reduces computational complexity of the controller by reducing the dimensionality of the problem. This is done by projecting the stacked input-output data into a basis function space. The identified system will then have reduced dimensions and will be able to describe the system behaviour only in the reduced basis function space. The use of input basis functions is desirable in the current application, since the shape of the input signal is required to be precisely controlled; this can be done by constraining the input signal to remain within the user-defined basis function space. Output basis functions are used to indicate that only a restricted subspace of the lifted output space is amenable to be controlled by the restricted control inputs.

Consider that θ_j is the control input projected into the input basis function space and $\bar{Y}_j$ is the output projected into the output basis function space. The projection matrices are ϕ_u and ϕ_y respectively:

$$\theta_j = \phi_u U_j, \qquad \bar{Y}_j = \phi_y Y_j. \tag{10}$$

Here, the projection matrices are composed of basis vectors:

$$\phi_u = \left[\phi_0^T, \; \phi_1^T, \; \cdots, \; \phi_b^T\right]^T,$$

where the basis vectors are $\phi_i \in \mathbb{R}^{Pr}, i = 0, 1, ..., b$. The number of basis vectors is thus b and $b \leq Pr$. Typically b is much smaller than Pr, so that the dimensions of the lifted input are drastically reduced through the projection. By defining $\phi_u = I_{Pr\times Pr}$ the original full input space can be recovered. Hence, the case of non-projected input can be considered to be a special case of projection with the projection matrix equal to identity.

To ensure that the IPC algorithm does not interfere with power production, it is necessary to constrain the summation of all input signals at any time instant to equal 0, as in Bossanyi (2003). This results in an equality constraint on the actuator input signals, which can be enforced via the input basis functions. Defining the matrices:

$$\bar{I} \in \mathbb{R}^{r\times 1}, \qquad \bar{I} = [1, 1, ..., 1]^T,$$

$$\bar{\bar{I}} \in \mathbb{R}^{Pr\times P}, \qquad \bar{\bar{I}} = \begin{bmatrix} \bar{I} & 0_{r\times 1} & \cdots & 0_{r\times 1} \\ 0_{r\times 1} & \bar{I} & \cdots & 0_{r\times 1} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{r\times 1} & 0_{r\times 1} & \cdots & \bar{I} \end{bmatrix}.$$

The constraint posed on each input basis vector is given by:

$$\phi_i \bar{\bar{I}} = 0. \tag{11}$$

In the current application, it is desired to restrict the control input to sinusoids with frequency equal to P and $2P$. This analysis can also be extended for higher harmonics in the load signal, e.g. $3P$, $4P$,... However, it is seen from simulations that under ordinary conditions, these harmonics are not dominant in the load spectra. Hence the analysis covers only the $1P$ and $2P$ components of the loads. The input basis functions are chosen as sinusoids of frequencies $1P$ and $2P$ that are $120°$ out of phase with each other, so that Equation (11) is satisfied for $r = 3$:

$$\phi_u = \begin{bmatrix} \sigma_1 & \sigma_2 & \cdots & \sigma_P \\ \zeta_1 & \zeta_2 & \cdots & \zeta_P \\ \sigma_2 & \sigma_4 & \cdots & \sigma_{2P} \\ \zeta_2 & \zeta_4 & \cdots & \zeta_{2P} \end{bmatrix},$$

where the constants σ_m and ζ_m for $m = 1, ..., 2P$ are given by:

$$\sigma_m = \left[\sin(\tfrac{2\pi m}{P}), \; \sin(\tfrac{2\pi m}{P} + \tfrac{2\pi}{3}), \; \sin(\tfrac{2\pi m}{P} + \tfrac{4\pi}{3})\right],$$
$$\zeta_m = \left[\cos(\tfrac{2\pi m}{P}), \; \cos(\tfrac{2\pi m}{P} + \tfrac{2\pi}{3}), \; \cos(\tfrac{2\pi m}{P} + \tfrac{4\pi}{3})\right].$$

A similar analysis can be done for the output basis functions ϕ_y. Since the disturbances and the control input have energy primarily concentrated at the 1P and 2P frequencies, and the system is approximately linear, the output will mainly contain energy along the same basis vectors as the control input. Hence, for this application, the input and output basis functions are taken to be identical $\phi_u = \phi_y$.

To cast equation (9) in the basis function space, it is also required to be able to reconstruct the original signals in the full-dimensional space. Since the control input is restricted to have energy only along the basis vectors, the full-dimensional input can be directly constructed as:

$$U_j = \phi_u^{\dagger}\theta_j.$$

Here $\dagger$ represents the Moore-Penrose pseudo-inverse. When projecting the output and the noise in the basis function space, it must be noted that these signals have energy in the basis function subspace as well as its null space. For the output,

$$Y_j = \phi_y^{\dagger}\bar{Y}_j + \phi_y^{\perp}Y_j^{\perp}.$$

The residual output signal, $Y_j^{\perp}$, lies outside the basis function subspace, while $\phi_y^{\perp}$ is the projection matrix that maps the output into the null space of the basis vectors. This matrix can be obtained from the singular value decomposition of ϕ_y:

$$\phi_y = U\,[\Sigma\ 0]\begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix},$$
$$\phi_y^{\perp} = V_2.$$

Since $\phi_y \phi_y^{\perp}$ is identically 0, we obtain once again $\bar{Y}_j = \phi_y Y_j$. Thus, the effect of the input on the residual output $Y_j^{\perp}$ that lies outside the basis function subspace is projected away by using ϕ_y. Since the control effort is not targetted at reducing the norm of this signal, the transfer between θ_j and $Y_j^{\perp}$ does not need to be identified and the signal will not be considered further. The same treatment can be done for the noise E_j and the effect of the periodic disturbance $(\Gamma\mathscr{K}_d + \mathscr{J})\bar{D}$.

Projecting the equation (9) on the basis function space, we have:

$$\bar{Y}_j = \left[\phi_y\Gamma\mathscr{K}_u\phi_u^{\dagger},\ \phi_y\Gamma\mathscr{K}_y\phi_y^{\dagger},\ \phi_y H\phi_u^{\dagger},\ \phi_y(\Gamma\mathscr{K}_d + \mathscr{J})\bar{D}\right]\begin{bmatrix}\theta_{j-1}\\ \bar{Y}_{j-1}\\ \theta_j \\ 1\end{bmatrix} + \phi_y E_j.$$

The objective of the identification step is to model the transfer between the projected input θ and the projected output $\bar{Y}$, so that it can be used to formulate the control law.

3.4 Step 3: Identification

Now that the dimensionality of the system description has been reduced, a novel online identification method is implemented to estimate the reduced-dimension system parameters at reduced computational complexity. This step addresses requirement (B): the controller is herewith able to identify changes in the system dynamics. This identification scheme is valid for both linear and periodic linear time-varying systems. Denoting the matrix of unknown coefficients in the equation (9) by Ξ:

$$\Xi = \left[\phi_y\Gamma\mathscr{K}_u\phi_u^{\dagger},\ \phi_y\Gamma\mathscr{K}_y\phi_y^{\dagger},\ \phi_y H\phi_u^{\dagger},\ \phi_y(\Gamma\mathscr{K}_d + \mathscr{J})\bar{D}\right]$$

If input-output data is available at iteration j, then system identification can be done to arrive at an estimate of the matrix, $\hat{\Xi}_j$ at this iteration. To minimise interference with the operation of the plant, it is desirable to perform this identification recursively, online and in a closed loop.

The identification problem can now be stated: given the relation between input-output data and system parameters,

$$\bar{Y}_j = \Xi\begin{bmatrix}\theta_{j-1}\\ \bar{Y}_{j-1}\\ \theta_j \\ 1\end{bmatrix} + \phi_y E_j. \tag{12}$$

arrive at an estimate $\hat{\Xi}_j$, recursively, for every iteration j:

$$\hat{\Xi}_j = \arg\min_{\Xi}\sum_{q=0}^{j-1}\left\|\bar{Y}_q - \Xi\begin{bmatrix}\theta_{q-1}\\ \bar{Y}_{q-1}\\ \theta_q \\ 1\end{bmatrix}\right\|_2^2. \tag{13}$$

Since e_k is a white noise sequence in the time domain, E_j is a white noise sequence in the lifted domain. The term E_j represents both periodic and non-periodic disturbances arising out of wind stochastics. As discussed in Section 3.1, it is uncorrelated with θ_j, $\bar{Y}_{j-1}$ and θ_{j-1}. The product of uncorrelated white noise with a non-zero constant matrix ϕ_y will remain uncorrelated white noise. So, equation (12) constitutes a standard least squares regression problem, per Verhaegen and Verdult (2007). The parameter estimate $\hat{\Xi}_j$ will then be asymptotically unbiased, Verhaegen and Verdult (2007) , and an increase in turbulence intensity (the variance of E_j) will directly lead to increased variance of $\hat{\Xi}_j$.

To ensure that a unique parameter estimate is obtained, the input should be persistently exciting of a sufficiently high order. This condition is applicable to the control input in the basis function space, thus θ_j has to be persistently exciting. This translates to smoother control inputs in the time domain. Further, since the persistency of excitation is required in the lifted domain, the energy of the persistently exciting control input is concentrated to a lower frequency band.

For system identification, recursive least squares using a square root algorithm, Van der Veen (2013), is implemented. At iteration j, an estimate $\hat{\Xi}_j$ then becomes available:

$$\hat{\Xi}_j = \left[\widehat{\phi_y\Gamma\mathscr{K}_u\phi_u^{\dagger}},\ \widehat{\phi_y\Gamma\mathscr{K}_y\phi_y^{\dagger}},\ \widehat{\phi_y H\phi_u^{\dagger}},\ \phi_y(\Gamma\widehat{\mathscr{K}_d + \mathscr{J}})\bar{D}\right]_j.$$

It is possible to arrive at $(\widehat{\phi_y\Gamma\mathscr{K}_u\phi_u^{\dagger}})_j$, $(\widehat{\phi_y\Gamma\mathscr{K}_y\phi_y^{\dagger}})_j$ and $(\widehat{\phi_y H\phi_u^{\dagger}})_j$ by partitioning $\hat{\Xi}_j$. These estimates can now be used to formulate an RC control law.

3.5 Step 4: Infinite Horizon Repetitive Control

With the estimated system parameters from the previous step, an RC law to asymptotically reject periodic disturbances is formulated, to address requirement (A). The main improvement here is that the use of basis functions yields reduced computational complexity of the optimisation routine to arrive at an optimum feedback law and stacked control input for the next iteration.

The generic form of the ILC law with basis functions, Van de Wijdeven and Bosgra (2010), is extended to RC:

$$\theta_{j+1} = \alpha\theta_j + \beta\begin{bmatrix}x_j\\ \varepsilon_{j-1}\end{bmatrix}.$$

The updated control input (in the basis function space) is a linear combination of the control input of the previous iteration, the new initial state x_j and the disturbance rejection error ε_{j-1}. Here, $\beta \in \mathbb{R}^{b\times(n+\ell)}$ is the learning gain matrix, while α is the "Q-filter" incorporated for robustness considerations.

Now, an RC controller is to be synthesised that minimises the output error in the basis function space (here simply equal to $\bar{Y}_j$ since the reference is 0). A Q-filter will not be used, instead, an infinite horizon cost is minimised to ensure stability for the case where the true system parameters are made available by the identification step. The output predictor (9) in terms of the identified parameters is:

$$\bar{Y}_j = \left[(\widehat{\phi_y\Gamma\mathscr{K}_u\phi_u^{\dagger}})_j,\ (\widehat{\phi_y\Gamma\mathscr{K}_y\phi_y^{\dagger}})_j,\ (\phi_y(\Gamma\widehat{\mathscr{K}_d + \mathscr{J}})\bar{D})_j\right]\begin{bmatrix}\theta_{j-1}\\ \bar{Y}_{j-1}\\ 1\end{bmatrix} + (\widehat{\phi_y H\phi_u^{\dagger}})_j U_j. \tag{14}$$

As this is a predictor, the noise sequence E_j is omitted. To eliminate the effect of the periodic disturbance, the operator "δ" is used:

$$\delta\bar{Y}_j = \bar{Y}_j - \bar{Y}_{j-1}, \qquad \delta\theta_j = \theta_j - \theta_{j-1}, \qquad \delta(1) = 0.$$

Applying this operator to equation (14), we have:

$$\bar{Y}_j - \bar{Y}_{j-1} = \left[\widehat{(\phi_y \Gamma \mathscr{K}_u \phi_u^\dagger)}_j,\ \widehat{(\phi_y \Gamma \mathscr{K}_y \phi_y^\dagger)}_j \right] \begin{bmatrix} \delta\theta_{j-1} \\ \delta\bar{Y}_{j-1} \end{bmatrix} + \widehat{(\phi_y H \phi_u^\dagger)}_j \delta U_j. \tag{15}$$

This is rewritten in a form equivalent to an LQ (linear-quadratic) problem, the solution of which would yield a stabilising controller for the case with true system parameters. For brevity, the hat notation is dropped. The LQ form is given by:

$$\underbrace{\begin{bmatrix} \bar{Y}_j \\ \delta\theta_j \\ \delta\bar{Y}_j \end{bmatrix}}_{\mathscr{X}_{j+1}} = \underbrace{\begin{bmatrix} I_b & (\phi_y(\Gamma\mathscr{K}_u)\phi_u^\dagger)_j & (\phi_y(\Gamma\mathscr{K}_y)\phi_y^\dagger)_j \\ 0_{b\times b} & 0_{b\times b} & 0_{b\times b} \\ 0_{b\times b} & (\phi_y(\Gamma\mathscr{K}_u)\phi_u^\dagger)_j & (\phi_y(\Gamma\mathscr{K}_y)\phi_y^\dagger)_j \end{bmatrix}}_{\mathscr{A}_j} \times \underbrace{\begin{bmatrix} \bar{Y}_{j-1} \\ \delta\theta_{j-1} \\ \delta\bar{Y}_{j-1} \end{bmatrix}}_{\mathscr{X}_j} + \underbrace{\begin{bmatrix} (\phi_y H \phi_u^\dagger)_j \\ I_b \\ \phi_y H \phi_u^\dagger)_j \end{bmatrix}}_{\mathscr{B}_j} \delta\theta_j. \tag{16}$$

A state feedback matrix that acts on the (fully observable) state $\mathscr{X}_j$ can now be synthesised to minimise the weighted norm of $\bar{Y}_k$ over an infinite horizon. The norm J is:

$$J = \sum_{j=0}^{\infty} ||(\mathscr{X}_{j+1})^T Q_f \mathscr{X}_{j+1} + (\delta\theta_j)^T R_f \delta\theta_j||_2^2. \tag{17}$$

Here, Q_f and R_f are user-defined weighting matrices. This formulation is similar to an LQ problem, however to be noted is that this is a trial domain formulation. Also, the norm minimisation is restricted to a reduced-order subspace. To obtain the optimal state feedback gain, a trial-domain discrete algebraic Riccati equation (DARE) can be solved. For this, an initial estimate of the DARE solution, $P_{R,j}$ is chosen, and the true solution is iterated to by using the DARE as an update law:

$$P_{R,j+1} = Q_f + \mathscr{A}_j^T (P_{R,j} - P_{R,j}\mathscr{B}_j^T (R_f + \mathscr{B}_j^T P_{R,j}\mathscr{B}_j)^{-1} \mathscr{B}_j^T P_{R,j})\mathscr{A}_j,$$

and the state feedback gain $K_{f,j}$ is then:

$$K_{f,j} = (R_f + \mathscr{B}_j^T P_{R,j}\mathscr{B}_j)^{-1} \mathscr{B}_j^T P_{R,j}\mathscr{A}_j.$$

This update law is adaptive since $\mathscr{A}_j$ and $\mathscr{B}_j$ are not constant, but recursively estimated in Step 3. The law shows good convergence in practice, although a formal stability proof is not given here. From the state feedback matrix, it is now possible to arrive at the optimal control input sequence in the basis function space that has to be implemented in the next iteration, θ_{j+1}. The lifted control input to be applied for the next iteration can be determined from the previous data in the full input space:

$$\delta\theta_{j+1} = K_{f,j} \begin{bmatrix} \bar{Y}_j \\ \delta\theta_j \\ \delta\bar{Y}_j \end{bmatrix} \tag{18}$$

$$U_{j+1} = U_j + \phi_u^\dagger K_{f,j} \begin{bmatrix} \phi_y Y_j \\ \phi_u \delta U_j \\ \phi_y \delta Y_j \end{bmatrix} \tag{19}$$

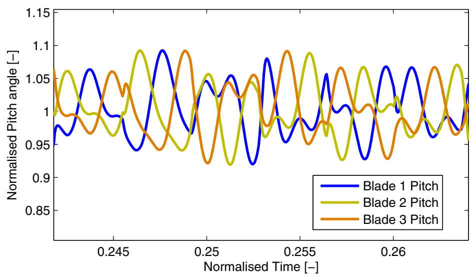

Fig. 1. Persistency of excitation in the lifted domain basis function space: wind speed 18 m/s, 0% turbulence

In the implementation, the matrix $\phi_u^\dagger$ is synthesised online based on rotor speed measurements; this reduces the sensitivity of the method to variations in the period P.

Thus, an adaptive repetitive control law has been formulated, and the results of implementation in the simulation environment will be discussed in the next section.

4. SIMULATION STUDY

The wind turbine model used for validating the above theory has been described in Section 2. The simulation model is equivalent to a high-fidelity representation of a wind turbine which can only be described fully as a time-varying state-space system. The baseline controller used with this model for simulating nominal operation of the turbine incorporates generator torque control and collective pitch control. SPRC with reduced-dimension identification was implemented for IPC specifically for periodic load alleviation.

For the simulations, an average wind speed of 18 m/s was chosen. This wind speed is relatively high such that the loading is significant, while its probability of occurrence is also high. The total time of simulation per realisation was 800 seconds. Two cases were considered:

- Zero turbulence wind field, which leads to perfect periodicity of loading; in order to understand the behaviour of the algorithm.
- Turbulence 14%; for a more realistic simulation.

The basis functions chosen are described in Section 3.3. To fulfill the condition for persistency of excitation, a white noise sequence in the lifted domain, which contains energy only in the basis function space, is superposed on the pitch signals. The persistently exciting input can be seen in Figure 1. It can be observed that it is much smoother than a white noise sequence in the time domain. Further, the mean value of the three pitch angles is exactly equal to the nominal pitch angle for this operating point, which ensures that the persistently exciting signals do not affect the nominal power production of the wind turbine.

For the case with zero turbulence, the identification converges from a cold start (initial estimate = 0) within 100 seconds, or 30 iterations. White noise excitation is switched off after 300 seconds, so the parameter estimate does not change any further. The RC law is now able to reduce the blade loading by 95% within 100 seconds, in this noise-free case.

The case with turbulence intensity 14% is considered next. The reduction in the blade loads can be seen in Figure 2. Since the basis functions chosen are sinusoids at the frequencies 1P and

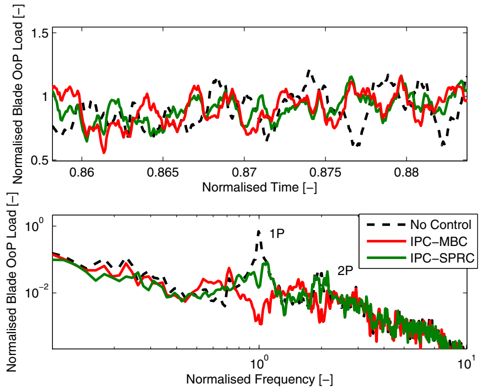

Fig. 2. SPRC achieves blade load reductions at 1P and 2P frequencies: wind speed 18 m/s and 14% turbulence

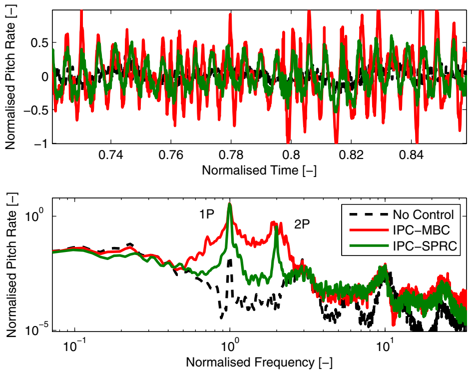

Fig. 3. Control inputs required mainly at 1P and 2P; much less control effort is required as compared to the traditional controller: wind speed 18 m/s and 14% turbulence

2P, the periodic loading at these frequencies is attentuated by the use of SPRC. For the sake of comparison, the results of a "traditional" IPC controller, designed per Bossanyi (2003), have also been shown in the figure and indicated as "IPC-MBC". The overall load reduction achieved by both controllers is similar. Load reduction of 20% is achieved with the ILC controller, while load reduction of 27% is achieved with the MBC controller. In Figure 3, the control action required for load reduction can be seen. Both controllers demand control input primarily at the 1P and 2P frequencies, however, the energy spectrum of the traditional controller input covers a much broader band. The use of SPRC reduces pitch activity in this case by 38.65%.

Thus, pitch activity is reduced substantially at the expense of a small reduction in load alleviation by using SPRC.

5. CONCLUSIONS

SPRC appears to possess several positive characteristics for wind turbine load reduction using IPC. The dynamics of the plant can be identified online, recursively. An ideal input sequence for rejecting periodic disturbances (which dominate turbine loads) can be synthesised from the identified parameters. Finally, the use of basis functions ensures that the control input shape can be constrained. With SPRC, the control effort is significantly lower than that required with a traditional IPC controller, while a similar level of load reduction is achieved.

Equality constraints are imposed on input signals by shaping the basis vectors. Using this, for wind turbine control, it is ensured that the nominal power production of the wind turbine is entirely decoupled from load alleviation control.

For the first time, the identification problem is cast into the lifted domain basis function space, which drastically reduces the dimension of the identification and RC synthesis problem. Further, SPRC is extended to be applicable to linear and linear periodically varying systems (like wind turbines). Finally, the required persistency of excitation is limited to the basis function and reduces to a large extent the high-frequency component of persistently exciting signals.

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