

H_∞ networked fuzzy control for vehicle lateral dynamic with limited communication

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Abstract: This paper concerns the problem of networked control for vehicle lateral dynamic stability. Based on the Takagi-Sugeno (T-S) fuzzy representation of the vehicle, the state feedback and observer-based control designs are addressed. These controllers should guarantee the global stability of the resulting closed-loop fuzzy system with a prescribed H_∞ disturbance attenuation level. Digital communication network conditions, such as network-induced delays, data packet dropouts and limited communication capacity due to signal quantization are taken into consideration. Using fuzzy Lyapunov-Krasovski functional, we derive a less conservative delay-dependent criterion for stability analysis and control synthesis of networked control systems with a quantizer. Simulation results illustrate the effectiveness of the proposed approach.

Keywords: Vehicle lateral dynamic stability, H_∞ networked fuzzy control, state feedback and observer-based control, signal quantization and fuzzy Lyapunov-Krasovski functional.

1. INTRODUCTION

In order to cope with some critical driving situations, the general tendency in automotive vehicle field is to improve passengers safety by integrating various active control systems (such as ABS, ESP, TCS, ASR, DYC ...) Madauand et al. (1993). These systems are generally based on the distributed assistant systems which work through the communication networks such as Controller Area Network (CAN) Froberg et al. (2004). The main characteristic of a Networked control system (NCS) is that its components (sensors, controller, and actuators) are connected to a feedback controller via a shared communication network. These embedded distributed control systems, used in the vehicle electronic control systems, permit an economic, flexible, remote monitoring and adjustment of (NCS) components. However, the limited bandwidth and the complex network resources due to the introduction of communication network into the closed-loop control can caused some unavoidable problems such as Network-induced delays, random, time delay, data packet loss and quantized measurements; that may lead to instability and performance degradation. It is pointed out that the communication delay, which has time-varying characteristics, is one of the important issues to be considered in NCS analysis and synthesis Peng et al. (2008); Jiang and Han (2008). Also, the quantization effect is an important issue to be addressed. In fact, due to the bandwidth limitation, the real communication networks are not able to send data with high level precision. Therefore, effective quantization

of sensor and actuator signals can help to reduce the sizes of data packets and the effect of signal quantization, must be considered into control design problem Kchaou and Toumi (2014). In another hand, it is noticed that most existing yaw moment control strategies rely on the measurement of both sideslip angle and yaw rate. However, if the yaw rate can be directly measurable by a yaw rate sensor (gyroscope), the sideslip angle will have to be estimated using an observer because the current available sensors for sideslip angle measurement are all too expensive to be acceptable by customers.

Our main objective is to develop an observer-based fuzzy control scheme that uses a communication network to exchange the sensor and actuator data transmission in order to improve stability and performances of vehicle lateral dynamics. In the analysis and design, the vehicle lateral will be represented by a Takagi-Sugeno (T-S) fuzzy model Takagi and Sugeno (1985). It is usually referred to as the bicycle model Dahmani et al. (2013). The structure of this paper is organized as follows : the second section describes the (T-S) fuzzy model of nonlinear lateral dynamics. In section 3, the proposed lateral dynamic networked control structure based on the estimation on sideslip angle is introduced and its design method is described. In section 4, simulation results are carried out to demonstrate the effectiveness of the proposed networked active safety system in terms of improving the vehicle stability. Finally, concluding remarks are made in section 5.

Notations: $W + W^T$ is denoted as $Sym(W)$.

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Symbol (*) within a matrix represents the symmetric entries. l_2 is the space of square integrable functions over $[0, \infty)$, and $\|\cdot\|_2$ denotes the l_2 -norm.

2. VEHICLE MODEL DESCRIPTION

The two-dimensional model with nonlinear tire characteristics of the vehicle behavior can be described by differential equations. Catino et al. (2003); ElHajjaji et al. (2005, 2006) :

$$\begin{cases} \dot{\beta}(t) = \frac{2F_f + 2F_r}{mU} - r(t) \\ \dot{r}(t) = \frac{2a_f F_f - 2a_r F_r + M_Z(t)}{I_z} \end{cases} \quad (1)$$

Where $\beta(t)$ denotes the sideslip angle, $r(t)$ is the yaw rate, F_f is the cornering force of the two front tires, F_r is the cornering force of the two rear tires. U is the vehicle velocity, I_z is the yaw moment of inertia, m is the vehicle mass and $M_Z(t)$ is the control input. (cf. Fig. 1). The parameters of the vehicle are given in the following table :

Table 1. Vehicle parameters

Parameters	$I_z(Kg^2m)$	$m(Kg)$	$a_f(m)$	$a_r(m)$	$U(m/s)$
Values	3000	1500	1.3	1.2	20

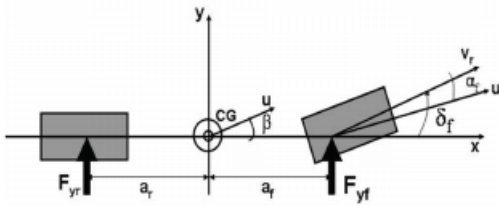


Fig. 1. Bicycle model

Based on the (T-S) fuzzy model representation and using the some idea in Chadli and ElHajjaji (2006); Dahmani et al. (2013), the front and rear lateral forces can be modeled as follows

$$\begin{cases} F_f = h_1(|\alpha_f|)C_{f1}\alpha_f + h_2(|\alpha_f|)C_{f2}\alpha_f \\ F_r = h_1(|\alpha_f|)C_{r1}\alpha_r + h_2(|\alpha_f|)C_{r2}\alpha_r \end{cases} \quad (2)$$

Where α_f and α_r represent tyre slip-angles at the front and rear of the vehicle respectively, C_{fi} and C_{ri} are the stiffness coefficients, and $h_j(j = 1, 2)$ is the j^{th} bell curve membership function of fuzzy set M_j . The membership function parameters and consequence parameters of fuzzy rules are obtained using an identification method that combines RLS (Recursive Least Squares) and based on the LM (Levenberg-Marquardt) algorithm Lee et al. (2003), defined as

$$\begin{cases} h_1(|\alpha_f|) = \frac{w_1(|\alpha_f|)}{w_1(|\alpha_f|) + w_2(|\alpha_f|)}, \\ h_2(|\alpha_f|) = 1 - h_1(|\alpha_f|) \end{cases} \quad (3)$$

With $w_1(|\alpha_f|) = (1 + |\frac{|\alpha_f| - c_1}{a_1}|)^{-2b_1}$, $w_2(|\alpha_f|) = (1 + |\frac{|\alpha_f| - c_2}{a_2}|)^{-2b_2}$ and $a_1 = 0.0908$, $b_1 = 23.3421$, $c_1 = 0.7237$, $a_2 = 204.0533$, $b_2 = 0.0415$, $c_2 = 23.4094$.

According to (1)-(2) and considering that:

$$\begin{aligned} \alpha_f &\cong \beta(t) + \frac{a_f r(t)}{U} - \delta_f(t) \\ \alpha_r &\cong \beta(t) + \frac{a_r r(t)}{U} \end{aligned} \quad (4)$$

we obtain the following TS fuzzy model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^2 h_i(|\alpha_f|)[A_i x(t) + B_{fi} \delta_f(t) + B M_Z(t)], \\ z(t) = C_1 x(t), \\ y(t) = C_2 x(t) \end{cases} \quad (5)$$

Where

$$\begin{aligned} A_i &= \begin{bmatrix} \frac{2C_{fi} + 2C_{ri}}{mU} & -\frac{2a_f C_{fi} - 2a_r C_{ri}}{mU^2} - 1 \\ \frac{2a_f C_{fi} - 2a_r C_{ri}}{I_z} & -\frac{2a_f^2 C_{fi} + 2a_r^2 C_{ri}}{I_z U} \end{bmatrix}, \\ B_{fi} &= \begin{bmatrix} \frac{2C_{fi}}{mU} \\ \frac{2a_f C_{fi}}{I_z} \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \\ I_z \end{bmatrix}, \text{ and } C_1 = C_2 = [0 \ 1]. \end{aligned}$$

With $x(t) = [\beta^T(t) \ r^T(t)]^T$ is the state vector, $\delta_f(t)$ is the external disturbance input, $y(t)$ is the measured output and $z(t)$ is the controlled output.

3. MAIN RESULTS

In this section, networked fuzzy state feedback control design for vehicle lateral dynamics is developed.

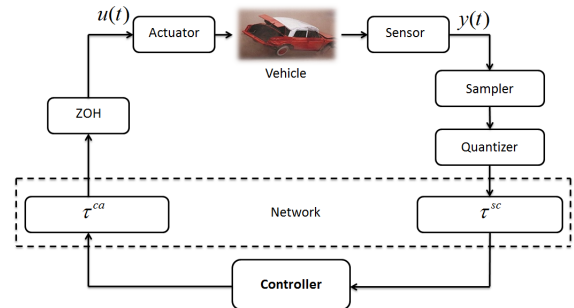


Fig. 2. Framework of networked control system

A typical NCS model with network-induced delays is shown in Fig. 2, where τ_{sc} is sensor-to-controller delay and τ_{ca} is the controller-to-actuator delay. It is assumed that the controller computational delay can be absorbed into either τ_{sc} or τ_{ca} .

The following assumptions, which are common for NCSs research in the open literature, are also made in this work:

- (1) The sensors are clock driven, the controller and actuators are event driven.
- (2) Data, either from measurement or for control, are transmitted with a single packet.
- (3) The real input $M_Z(t)$, realized through a zero-order hold, is a piecewise constant function.

The measurement output signals will be quantized before they are transmitted to next nodes. The logarithmic quan-

tizer is considered here. It is called logarithmic if the set of quantized levels is characterized by

$$u = \{u_i, u_i = \rho^i u_0, i = 0 \pm 1 \pm 2, \dots\} \cup \{0\}, \quad u_0 > 0$$

where the parameter $0 < \rho < 1$ is called the quantization density, and the logarithmic quantizer $q(\nu)$ is

$$q(\nu) = \begin{cases} u_i & \text{if } \frac{1}{1+\delta}\rho^i u_0 < \nu \leq \frac{1}{1-\delta}\rho^i u_0 \\ 0 & \text{if } \nu = 0 \\ -q(-\nu) & \text{if } \nu < 0 \end{cases} \quad (6)$$

where $\delta = \frac{1-\rho}{1+\rho}$.

By the sector bound method, $q(\nu)$ can be expressed as in Mahmoud and Al-Rayyah (2011) by

$$q(\nu) = (I + \Delta(t))\nu$$

where

$$\Delta(t) = \text{diag}\{\Delta_1(t), \Delta_2(t), \dots, \Delta_n(t)\}, \quad \|\Delta_j(t)\| \leq \delta, \quad j = 1, 2, \dots, n \quad (7)$$

3.1 Networked fuzzy state feedback control design

Suppose that state of system (5) is measurable and will be quantized before it will be transmitted to the controller through a communication network. To determine the state feedback control for system (5) subject to quantization, following Parallel Distributed Compensation scheme (PDC) is :

$$M_Z(t_k) = \sum_{i=1}^2 h_i(|\alpha_f(t_k)|) \bar{K}_i x(t_k - \tau_k)$$

Where $\bar{K}_i = K_i(I + \Delta(t))$ and t_k are sampling instants. From the ZOH, the input signal is

$$M_Z(t) = \sum_{i=1}^2 h_i(|\alpha_f(t_k)|) \bar{K}_i x(t - \tau_k), \quad t_k \leq t \leq t_{k+1} \quad (8)$$

Then, the closed-loop control networked system can be written for $t_k \leq t \leq t_{k+1}$ as

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^2 h_i(|\alpha_f(t_k)|) (A_i x(t) + B \bar{K}_i x(t - \tau_k) + B_{f_i} \delta_f(t)) \\ z(t) = C_1 x(t) \\ y(t) = C_2 x(t) \end{cases}$$

- Measurement quantization : It is assumed that the sampler and quantizer are clock driven, while the zero-order hold (ZOH) is event driven. It is assumed also that, the sampled measurements of $y(t)$ are first quantized via a quantizer, and then transmitted with a single packet.

- Network-induced delay (τ_k): Network-induced delays always exist when the data transmits through a network, and obviously, it has both lower and upper bounds. A natural assumption on τ_k can be made as

$$0 < \tau_m \leq \tau_k \leq \tau_M \quad (9)$$

- Packet dropouts : The effect of one packet dropout in the transmission is just a case that one sampling period delay is induced in the updating interval of ZOH. Rahmani and Markazi (2013)

$$t_{k+1} - t_k = (\sigma_{k+1} + 1)T_e + \tau_{k+1} - \tau_k$$

where T_e denotes the sampling period and σ_{k+1} is the number of accumulated packet dropouts in this period.

Let consider $\eta(t) = t - t_k + \tau_k$, $t_k \leq t \leq t_{k+1}$, then

$$\tau_m \leq \tau_k \leq \eta(t) \leq (\bar{\sigma} + 1)T_e + \tau_{k+1} \quad (10)$$

where $\bar{\sigma}$ denotes the maximum number of packet dropouts in updating periods, $\eta_1 = \tau_m$ and $\eta_2 = (\bar{\sigma} + 1)T_e + \tau_M$. Thus, we get

$$\eta_1 \leq \eta(t) \leq \eta_2 \text{ and } \dot{\eta}(t) \leq \eta_d \quad (11)$$

Where η_d is constant parameter. Since $\sum_{k=0}^{\infty} [t_k, t_{k+1}) = [0, \infty)$, we have

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^2 h_i (A_i x(t) + B \bar{K}_i x(t - \eta(t)) + B_{f_i} \delta_f(t)) \\ z(t) = C_1 x(t) \\ y(t) = C_2 x(t) \\ x(t) = \phi(t), t \in [t_0 - \eta_2, t_0] \end{cases} \quad (12)$$

Where $\phi(t)$ can be viewed as initial condition sequence.

Defining

$$A(t) = \sum_{i=1}^2 h_i A_i, \quad B_f(t) = \sum_{i=1}^2 h_i B_{f_i}, \quad (13)$$

$$H(t) = \sum_{i=1}^2 h_i H_i, \quad H_i = B K_i (I + \Delta(t))$$

then closed-loop system (12) can be described by

$$\begin{cases} \dot{x}(t) = A(t)x(t) + H(t)x(t - \eta(t)) + B_f(t)\delta_f(t) \\ z(t) = C_1 x(t) \\ y(t) = C_2 x(t) \\ x(t) = \phi(t), t \in [t_0 - \eta_2, t_0] \end{cases} \quad (14)$$

The objective now is to determine controller (8) such that the feedback closed-loop system is asymptotically stable with H_∞ performance. In order to obtain the main results in this paper, the following lemmas are needed:

Lemma 3.1. Yu et al. (2012) For any scalars $M > 0$, $N > 0$, $h(t)$ is a continuous function and satisfies $h_m < h(t) < h_M$, then

$$\begin{aligned} & -\frac{h_M - h_m}{h(t) - h_m} M - \frac{h_M - h_m}{h_M - h(t)} N \\ & \leq \max(-(M + 3N), -(3M + N)) \end{aligned} \quad (15)$$

Lemma 3.2. Wang et al. (1992) Given matrices $D, E, F(t)$ with compatible dimensions and $F(t)$ satisfying $F(t)^T F(t) \leq I$. Then, the following inequality holds for any $\epsilon > 0$: $DF(t)E + E^T F(t)^T D^T \leq \epsilon DD^T + \epsilon^{-1} E^T E$

From Theorem 1 in Kchaou and Toumi (2014), the following result summarizes how the fuzzy state feedback controller can be designed by a solution of matrix inequalities given as follows:

Theorem 3.1. For given scalars $\eta_1 > 0$, $\eta_2 > 0$, $\mu_1, \mu_2, \mu_3, \lambda, \epsilon > 0$ and quantization density $\rho > 0$, closed-loop system (14) is asymptotically stable with H_∞ norm bounded γ , if there exist positive matrices $\bar{P}, \bar{Q}_1, \bar{Q}_2, \bar{Q}_3, \bar{Z}_1, \bar{Z}_2$ and matrices \bar{G} and Y_i with appropriate dimensions, such that the following conditions hold

$$\begin{bmatrix} \bar{\Phi}_i + \Phi_1(\bar{Z}_2) & \bar{\Gamma}_i & \epsilon \bar{G}^T \bar{\Delta}^T \\ * & \epsilon \bar{\Xi}_1 & 0 \\ * & * & -\epsilon I \end{bmatrix} < 0 \quad (16)$$

$$\begin{bmatrix} \bar{\Phi}_i + \Phi_2(\bar{Z}_2) & \bar{\Gamma}_i & \epsilon \bar{G}^T \bar{\Delta}^T \\ * & \epsilon \bar{\Xi}_1 & 0 \\ * & * & -\epsilon I \end{bmatrix} < 0 \quad (17)$$

where

$$\bar{\Phi}_i = \begin{bmatrix} \bar{\Phi}_{11i} & \bar{\Phi}_{12i} & \bar{Z}_1 & 0 & \bar{\Phi}_{15i} & \mu_1 B_{fi} & \bar{\Phi}_{17} \\ * & \bar{\Phi}_{22i} & 0 & 0 & \bar{\Phi}_{25i} & \mu_2 B_{fi} & 0 \\ * & * & -\bar{Q}_2 - \bar{Z}_1 & 0 & 0 & 0 & 0 \\ * & * & * & -\bar{Q}_3 & 0 & 0 & 0 \\ * & * & * & * & \bar{\Phi}_{55} & \mu_3 B_{fi} & 0 \\ * & * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} \quad (18)$$

$$\Phi_1(\bar{Z}_2) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & -4\bar{Z}_2 & 3\bar{Z}_2 & \bar{Z}_2 & 0 & 0 & 0 \\ * & * & -3\bar{Z}_2 & 0 & 0 & 0 & 0 \\ * & * & * & -\bar{Z}_2 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & 0 \end{bmatrix} \quad (19)$$

$$\Phi_2(\bar{Z}_2) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & -4\bar{Z}_2 & \bar{Z}_2 & 3\bar{Z}_2 & 0 & 0 & 0 \\ * & * & -\bar{Z}_2 & 0 & 0 & 0 & 0 \\ * & * & * & -3\bar{Z}_2 & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & 0 \end{bmatrix} \quad (20)$$

$$\begin{aligned} \bar{\Phi}_{11i} &= \bar{Q}_1 + \bar{Q}_2 + \bar{Q}_3 + \mu_1 \text{sym}(A_i \bar{G}) - \bar{Z}_1, \\ \bar{\Phi}_{12i} &= \mu_2 \bar{G}^T A_i^T + \mu_1 B Y_i, \\ \bar{\Phi}_{22i} &= \mu_2 \text{sym}(B Y_i) - (1 - \eta_d) \bar{Q}_1, \\ \bar{\Phi}_{15i} &= \bar{P} - \mu_1 \bar{G} + \mu_3 \bar{G}^T A_i^T, \\ \bar{\Phi}_{25i} &= -\mu_2 \bar{G} + \mu_3 Y_i^T B^T, \\ \bar{\Phi}_{55} &= \eta_1^2 \bar{Z}_1 + \eta_r^2 \bar{Z}_2 - \mu_3 \text{sym}(\bar{G}), \\ \bar{\Phi}_{17} &= \bar{G}^T C_1^T, \quad d_r = \eta_2 - \eta_1, \\ \bar{\Gamma}_i &= [\bar{\Phi}_{18i}^T \quad \bar{\Phi}_{28i}^T \quad 0 \quad 0 \quad \bar{\Phi}_{58i}^T \quad 0 \quad 0]^T, \\ \bar{\Xi}_1 &= \lambda^2 I - \lambda \text{Sym}(\bar{G}), \quad \bar{\Delta} = [0 \quad \delta I \quad 0 \quad 0 \quad 0 \quad 0 \quad 0], \\ \bar{\Phi}_{18i} &= \mu_1 B Y_i, \quad \bar{\Phi}_{28i} = \mu_2 B Y_i, \quad \bar{\Phi}_{58i} = \mu_3 B Y_i \end{aligned}$$

Where $Y_i = K_i \bar{G}$

3.2 Observer-based networked fuzzy state feedback design

Currently, the sideslip angle is unavailable for measurement. This problem can be overcome by introducing the fuzzy observer theory. The overall fuzzy observer is represented as follows:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^2 h_i(|\alpha_f(t)|) [A_i \hat{x}(t) + B_{fi} \delta_f(t) + B M_Z(t) \\ \quad + L_i (y(t) - \hat{y}(t))] \\ \hat{y}(t) = C_2 \hat{x}(t) \end{cases} \quad (21)$$

Where $\hat{x}^T(t) = [\beta_e^T(t) \quad r_e^T(t)]$ is the estimated state, and $\hat{y}(t)$ is the estimated measured output, L_1 and L_2 are the constant observer gains to be determined.

The overall networked PDC fuzzy controller is represented as follows:

$$M_Z(t) = \sum_{i=1}^2 h_i(|\alpha_f(t_k)|) K_i (I + \Delta(t)) \hat{x}(t - \tau_k) \quad (22)$$

Where K_1 and K_2 are the constant feedback gains to be determined.

Define

$$e(t) = x(t) - \hat{x}(t) \quad (23)$$

From systems (5), (21), (22), and (23), the augmented system can be expressed as :

$$\begin{cases} \dot{\hat{x}}(t) = \tilde{A}(t) \hat{x}(t) + \tilde{H}(t) (I + \tilde{\Delta}(t)) \hat{x}(t - \eta(t)) + \tilde{B}_f(t) \delta_f(t), \\ z(t) = \tilde{C}_1 \hat{x}(t) \end{cases} \quad (24)$$

Where

$$\tilde{A}(t) = \sum_{i=1}^2 h_i \tilde{A}_i, \quad \tilde{B}_f(t) = \sum_{i=1}^2 h_i \tilde{B}_{fi}, \quad \tilde{H}(t) = \sum_{i=1}^2 h_i \tilde{H}_i. \quad (25)$$

$$\tilde{A}_i = \begin{bmatrix} A_i & 0 \\ 0 & A_i - L_i C_2 \end{bmatrix}, \quad \tilde{H}_i = \begin{bmatrix} B K_i & -B K_i \\ 0 & 0 \end{bmatrix}, \quad \tilde{B}_{fi} = \begin{bmatrix} B_{fi} \\ 0 \end{bmatrix}, \quad \tilde{C}_1 = [C_1 \quad 0], \quad \tilde{\Delta}(t) = \begin{bmatrix} \Delta(t) & 0 \\ 0 & \Delta(t) \end{bmatrix} \text{ and } \tilde{x}(t) = [x^T(t) \quad e^T(t)]^T.$$

Next, we focus our attention on the fuzzy observer-based control for system (5), which is summarized in the following theorem.

Theorem 3.2. For given scalars $\eta_1 > 0, \eta_2 > 0, \mu_1, \mu_2, \mu_3, \epsilon > 0, \lambda_1$, and quantization density $\rho > 0$, the closed-loop networked system (24) is asymptotically stable with H_∞ norm bounded γ , if there exist positive matrices $\tilde{P}, \tilde{Q}_1, \tilde{Q}_2, \tilde{Q}_3, \tilde{Z}_1, \tilde{Z}_2$ and matrices \tilde{G}, g, Y_i and F_i with appropriate dimensions, such that the following conditions hold

$$\begin{bmatrix} \tilde{\Phi}_i + \Phi_1(\tilde{Z}_2) & \tilde{\Gamma}_i & \epsilon \tilde{G}^T \tilde{\Delta}^T \\ * & -\epsilon \tilde{\Xi}_1 & 0 \\ * & * & -\epsilon I \end{bmatrix} < 0 \quad (26)$$

$$\begin{bmatrix} \tilde{\Phi}_i + \Phi_2(\tilde{Z}_2) & \tilde{\Gamma}_i & \epsilon \tilde{G}^T \tilde{\Delta}^T \\ * & -\epsilon \tilde{\Xi}_1 & 0 \\ * & * & -\epsilon I \end{bmatrix} < 0 \quad (27)$$

where

$$\tilde{\Phi}_i = \begin{bmatrix} \tilde{\Phi}_{11i} & \tilde{\Phi}_{12i} & \tilde{Z}_1 & 0 & \tilde{\Phi}_{15i} & \tilde{\Phi}_{16i} & \tilde{\Phi}_{17} \\ * & \tilde{\Phi}_{22i} & 0 & 0 & \tilde{\Phi}_{25i} & \tilde{\Phi}_{26i} & 0 \\ * & * & -\tilde{Q}_2 - \tilde{Z}_1 & 0 & 0 & 0 & 0 \\ * & * & * & -\tilde{Q}_3 & 0 & 0 & 0 \\ * & * & * & * & \tilde{\Phi}_{55} & \tilde{\Phi}_{56i} & 0 \\ * & * & * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} \quad (28)$$

$$\begin{aligned}
 \tilde{\Phi}_{11i} &= \tilde{Q}_1 + \tilde{Q}_2 + \tilde{Q}_3 + \text{sym}(\mathbf{A}_i) - \tilde{Z}_1, \\
 \tilde{\Phi}_{12i} &= \mathbf{A}_i + \mathbf{A}_{\eta i}^T, \\
 \tilde{\Phi}_{22i} &= \text{sym}(\mathbf{A}_{\eta i}) - (1 - \eta_d)\tilde{Q}_1, \\
 \tilde{\Phi}_{15i} &= \tilde{P} - \mathbf{G} + \mathbf{A}_i, \\
 \tilde{\Phi}_{16i} &= \mu_1 \mathbf{B}_{\text{fn}}, \quad \tilde{\Phi}_{26i} = \mu_2 \mathbf{B}_{\text{fn}}, \\
 \tilde{\Phi}_{56i} &= \mu_3 \mathbf{B}_{\text{fn}}, \quad \tilde{\Phi}_{25i} = -\mathbf{G}^T + \mathbf{A}_{\eta i}, \\
 \tilde{\Phi}_{55} &= \eta_2^2 \tilde{Z}_1 + \eta_r^2 \tilde{Z}_2 - \text{sym}(\mathbf{G}), \quad \tilde{\Phi}_{17} = [C_1 g \ C_1]^T, \\
 \tilde{\Gamma}_i &= [\tilde{\Phi}_{18i}^T \ \tilde{\Phi}_{28i}^T \ 0 \ 0 \ \tilde{\Phi}_{58i}^T \ 0 \ 0]^T, \\
 \tilde{\Phi}_{18i} &= \mu_1 \mathbf{A}_{\eta i}, \quad \tilde{\Phi}_{28i} = \mu_2 \mathbf{A}_{\eta i}, \quad \tilde{\Phi}_{58i} = \mu_3 \mathbf{A}_{\eta i}, \\
 \tilde{\Xi}_1 &= \lambda_1^2 I - \lambda_1 \text{Sym}(\mathcal{G}), \\
 \mathbf{A}_i &= \begin{bmatrix} A_i g & A_i \\ 0 & \hat{G}^T A_i - F_i C_2 \end{bmatrix}, \quad \mathbf{A}_{\eta i} = \begin{bmatrix} B Y_i & 0 \\ 0 & 0 \end{bmatrix}, \\
 \tilde{G} &= \begin{bmatrix} g & G \\ 0 & G \end{bmatrix}, \quad \mathbf{B}_{\text{fn}} = \begin{bmatrix} B_{fi} \\ 0 \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} g & I \\ 0 & \hat{G} \end{bmatrix}, \\
 \mathcal{G} &= \begin{bmatrix} g & I \\ 0 & I \end{bmatrix}.
 \end{aligned}$$

Where $Y_i = K_i g$, $F_i = \hat{G} L_i$

Proof 3.1. Under the conditions of the Theorem 3.2, a feasible solution satisfies the condition $\tilde{\Phi}_{55} < 0$ which implies that \tilde{G} is nonsingular. Thus, g and G are also nonsingular. Define $\hat{G} = G^{-1}$ and $\dot{G} = \begin{bmatrix} I & 0 \\ 0 & \hat{G} \end{bmatrix}$.

Note that $\mathbf{A}_i = \dot{G}^T \tilde{A}_i \tilde{G} \dot{G}$, $\mathbf{A}_{\eta i} = \dot{G}^T \tilde{A}_{\eta i} \tilde{G} \dot{G}$, $\mathbf{G} = \dot{G}^T \tilde{G} \dot{G}$ and $\mathcal{G} = \tilde{G} \dot{G}$.

Notting $\tilde{P} = \mathcal{G}^T P \mathcal{G}$, $\tilde{Q}_1 = \mathcal{G}^T Q_1 \mathcal{G}$, $\tilde{Q}_2 = \mathcal{G}^T Q_2 \mathcal{G}$, $\tilde{Q}_3 = \mathcal{G}^T Q_3 \mathcal{G}$, $\tilde{Z}_1 = \mathcal{G}^T Z_1 \mathcal{G}$, and $\tilde{Z}_2 = \mathcal{G}^T Z_2 \mathcal{G}$.

Following the similar lines in the proof of Theorem 3.1, by checking congruence transformations to (26)-(27) by

$\text{diag}(\dot{G}^{-1}, \dot{G}^{-1}, \dot{G}^{-1}, \dot{G}^{-1}, \dot{G}^{-1}, I, I, \dot{G}^{-1}, I)$ and then by

$\text{diag}(\tilde{G}^{-1}, \tilde{G}^{-1}, \tilde{G}^{-1}, \tilde{G}^{-1}, \tilde{G}^{-1}, I, I, \tilde{G}^{-1}, I)$, we conclude that the fuzzy observer (21) exists and guaranties for closed-loop system (24) to be asymptotically stable.

4. SIMULATION RESULTS

To show the effectiveness of the proposed networked active safety system, we have carried the following simulation. In the design, considered stiffness coefficients are: ElHajjaji et al. (2006)

stiffness coefficients	Cf1	Cf2	Cr1	Cr2
Values	60712	4812	60088	3455

The network-related parameters are assumed: $T_e = 5ms$, minimum delay $\eta_1 = 6ms$, maximum delay $\eta_2 = 28ms$ and maximum number of packet dropouts $\bar{\sigma} = 3$. With quantizer parameter $\rho = 0.85$, $\eta_d = 0.1$, $\mu_1 = 1$, $\mu_2 = 0.1$, $\mu_3 = 0.2$, $\lambda_1 = 1.05$, we find a minimum allowable γ is 15.563 and

$$K_1 = 10^3 [-6.4610 \ -1.7955], \quad (29)$$

$$K_2 = 10^3 [4.5565 \ -5.7045],$$

$$L_1 = \begin{bmatrix} -1.5833 \\ -0.3077 \end{bmatrix}, \quad L_2 = \begin{bmatrix} -2.5108 \\ 3.4384 \end{bmatrix}$$

$$\hat{G} = \begin{bmatrix} -4.5944 & -1.2072 \\ -0.6470 & -3.9235 \end{bmatrix}, \quad g = \begin{bmatrix} -2.2324 & -0.2287 \\ -1.1440 & -2.2044 \end{bmatrix}$$

The controller and observer gains for the case without considering delay and packet dropout in design given by Theorem in ElHajjaji et al. (2006) are given as follows

$$K_3 = [-541.9428 \ 575.1236], \quad (30)$$

$$K_4 = 10^3 [4.3076 \ -3.5320],$$

$$L_3 = \begin{bmatrix} -0.4522 \\ 2.8041 \end{bmatrix}, \quad L_4 = \begin{bmatrix} 4.4232 \\ -1.5365 \end{bmatrix}$$

We will tested the vehicle behavior by considering the steering angle shown in Fig 3. In the first, we considered the case when we don't consider the communication network in control design (30), we remark that the behavior of the vehicle is controllable but the sideslip and yaw rate are very important as it will be shown in Fig 4 and Fig 5.

To overcome this problem, we have tested the proposed method with considering communication network induced time varying delay and packet dropouts in the control design with (29), Fig 4 and Fig 5 show state variable evolutions and Fig 6 shows the input control in the two cases. We remark that our approach is efficient and improve the stability and the performances of the vehicle through communication.

For simulation, the initial condition is assumed to be $x_0 = [0.1, 1]^T$. The state responses of the NCS with control input are depicted in Fig 4 and 5 which we can see that the estimated variables converge towards the measured state variables. The simulation results are in accordance with the analysis and support the effectiveness of the developed design strategy.

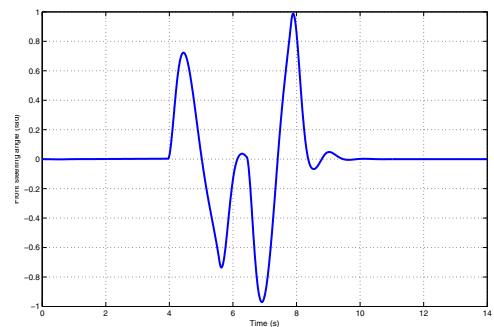


Fig. 3. Front steering angle

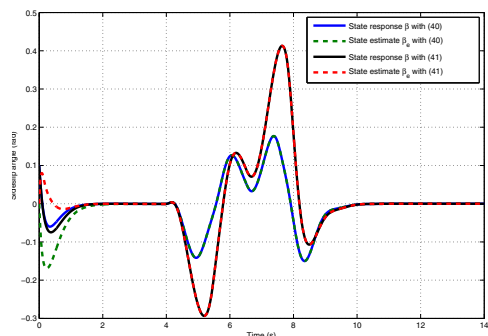


Fig. 4. Response and estimate of the sideslip angle .

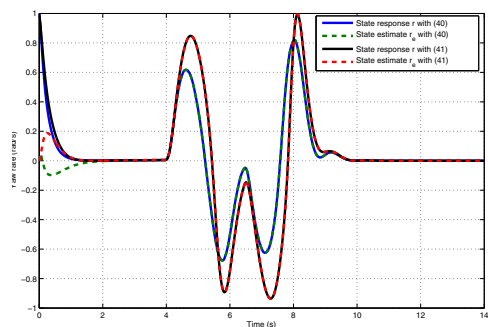


Fig. 5. Response and estimate of the yaw rate .

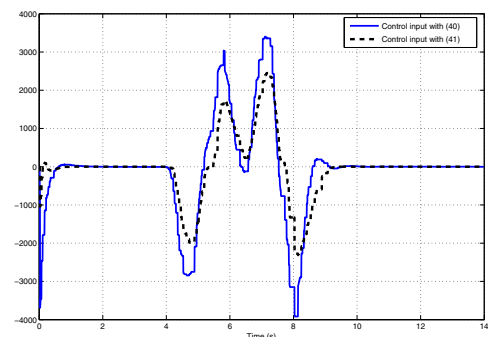


Fig. 6. Curves of moment M_Z .

5. CONCLUSION

In this paper, we have presented new stabilization conditions for networked controlled vehicle lateral dynamics with unmeasurable sideslip angle. By considering network induced delay, data packet dropout and quantized measurements constraints as well as the unavailability of the sideslip measurement, observer-based networked fuzzy state feedback design for vehicle active safety is developed. The optimal allowable delay bound and the controller and the observer gains have been derived by solving a set of LMIs based on the Lyapunov-Krasovski functional. The obtained simulations show the improvements of the vehicle stability with this proposed networked control scheme.

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