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Control-based strategy for effective wind speed estimation in wind turbines

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Abstract: This paper deals with a control-based observer structure used for effective wind speed estimation in a wind turbine operating in partial load. The central idea is that this input estimation can be achieved based on system output tracking using a high-gain control loop. One of the consequences is that the system entire state estimation can also be performed. The resulted observer is simple and easy to tune. Numerical simulations show that effective wind speed is reconstructed with a good dynamic, thus being able to replace anemometer information for diagnosis and output power assessment purposes. The basic idea of this paper is quite general, as it can be applied to a large class of systems having unknown inputs.

Keywords: Input estimation, windmills, observers, tracking, singular perturbations.

1. INTRODUCTION

Mass wind power generation combines the advantage of free resource with wasteless operation and supposes use of large (typically multi-MW-sized) wind turbines within wind farms counting hundreds of them. Constant increase of wind turbines size in the last years has induced new challenges on single wind turbine monitoring (*e.g.* fault detection/prediction) and its output power scheduling as part of a dispatchable power plant.

A multi-MW turbine rotor sweeps an important surface within which wind speed has a spatial distribution – it actually represents a wind field. It is well known that single-point speed measured by nacelle anemometer placed downwind the rotor (hence subjected to disturbances/supplementary turbulences due to rotor movement and nacelle proximity) does not provide a wind speed value usable on its own neither in turbine diagnosis nor output power assessment. Anyway, the wind speed field incident to the rotor plane determines lift forces that concentrate into the rotational shaft due to the aerodynamic effect induced by blade movement. The wind shaft integrates these distributed forces into a single variable, the wind torque, that produces wind turbine movement and hence mechanical power.

Therefore, it makes sense to consider the equivalent lumped wind speed that acts on the wind turbine and produces the same output power. In that spirit, one defines the concept of *equivalent wind speed* as a single-point wind speed, whose application on a lumped-parameter wind turbine model (*e.g.* represented by its power and thrust coefficients) produces the same output power as the real wind turbine fed by a wind field (van der Hooft and Engelen (2004)). It is clear that this variable is unmeasurable.

The concerned literature provides different algorithms for effective wind speed estimation, most of them being listed in (Soltani et al. (2013)). They are based either on Kalman filter estimators (Østergaard et al. (2007)), unknown input observer (Odgaard et al. (2011)), immersion and invariance estimator (Ortega et al. (2013)), power balance estimator (van der Hooft and Engelen (2004)), disturbance accommodating control (Wrigth (2004)), or frequency-domain data fusion (Xu et al. (2012)).

This paper proposes a novel effective wind speed estimator for partial-load operation, based upon a control-like observer structure. The primary purposes of such an estimation are monitoring, diagnosis and output power assessment.

The paper is organized as follows. Section 2 first recalls the wind speed estimation issue and wind turbine dynamical modeling. Section 3 then highlights the proposed control-based estimation strategy, which is subsequently illustrated in Section 4 with the effective wind speed reconstruction in a wind turbine. Section 5 validates the proposed method by numerical simulation, while Section 6 concludes the paper.

2. MODELING AND PROBLEM FORMULATION

The plant is a generic high-power variable-speed controlled wind turbine working in partial load, *i.e.*, at wind speeds lower than the rated one (Burton et al. (2001)). Its three-mass model, corresponding to interactions within the aerodynamic and drive train structures is classically represented by equation (1) (see, for example Bianchi et al. (2006)). In this equation state variables Ω_l and Ω_h are rotational speed at low-speed and high-speed shafts respectively and δ is the shaft torsion. d_A is gear box damping, d_l is low-speed shaft damping, d_h is high-speed shaft damping, k_A is gear box stiffness, J_l is low-speed shaft inertia coefficient, J_h is high-speed shaft inertia coefficient and i is drive train multiplication ratio.

$$\begin{cases} J_l\dot{\Omega}_l = -(d_A + d_l)\cdot\Omega_l + \dfrac{d_A}{i}\cdot\Omega_h - k_A\cdot\delta + T_w \\ J_h\dot{\Omega}_h = \dfrac{d_A}{i}\cdot\Omega_l - \left(\dfrac{d_A}{i^2} + d_h\right)\cdot\Omega_h - \dfrac{k_A}{i}\cdot\delta + T_G\ , \\ \dot{\delta} = \Omega_l - \dfrac{\Omega_h}{i} \end{cases} \tag{1}$$

In addition, T_w is the wind torque which drives the turbine shaft, and which is given by the nonlinear equation (2)

$$T_w = 0.5\rho\pi R_b^3 \cdot v_w^2 \cdot c_T(\lambda)\,, \tag{2}$$

where R_b is blade length, ρ is air density, v_w is wind speed, $\lambda = \dfrac{\Omega_l R_b}{v_w}$ is tip speed ratio and c_T is torque coefficient.

Finally, T_G is the generator torque that controls the wind turbine in partial load, at variable speed. Its control structure implemented as a feedback of rotational speed (Bossanyi (2003); Munteanu et al. (2008)) is out of interest in this paper. It is anyway worthy to indicate that T_G is the control input variable, and thus considered to be known.

Based upon those known plant model and parameters together with the control input and some output variable, it is required to estimate the unknown input which is the wind speed. If the output variable is the turbine power, the estimated wind speed is in fact the so-called effective wind speed. In practice, this estimation can be based on measuring the high-speed shaft rotational speed as an output variable, as it will be detailed in Section 4.

A linear model can be obtained for the turbine by linearizing the wind torque around a typical operating point:

$$\Delta T_w = k_1 \cdot \Delta\Omega_l + k_2 \cdot \Delta v_w,$$

(k_1 and k_2 being linearization coefficients) and using it into the first equation of (1).

By considering indeed the variations of $[\Omega_l\ \Omega_h\ \delta]^T$ around this point as a state vector $\mathbf{x}$, the variations of generator torque T_G as the control input u, the wind speed variations as the unknown input variable v, and that the variations of Ω_h provide the measurement y, a state space representation of the turbine dynamics results as:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}u + \mathbf{D}v \\ y &= \mathbf{C}\mathbf{x} \end{aligned} \tag{3}$$

where:

$$\mathbf{A} = \begin{bmatrix} -\dfrac{d_A + d_l - k_1}{J_l} & \dfrac{d_A}{iJ_l} & -\dfrac{k_A}{J_l} \\ \dfrac{d_A}{iJ_h} & -\left(\dfrac{d_A}{i^2} + d_h\right) & \dfrac{k_A}{iJ_h} \\ 1 & -\dfrac{1}{i} & 0 \end{bmatrix}, \tag{4}$$

$$\mathbf{B} = \left[0\ -\frac{1}{J_h}\ 0\right]^T,\ \mathbf{D} = \left[\frac{k_2}{J_l}\ 0\ 0\right]^T,\ \text{and } \mathbf{C} = [0\ 1\ 0]\,. \tag{5}$$

Notice that the linearization is considered for the purpose of simplifying the presentation, but the proposed method could be applied to estimate T_w, and from it recover v.

3. CONTROL-BASED ESTIMATION STRATEGY

As a result of the former modelling for the problem under consideration, we are brought to the issue of *estimating the unknown input* v ($\in I\!R$) for a system which can be described by (3) where, in general, $\mathbf{x} \in I\!R^n$ is the state, $\mathbf{u} \in I\!R^m$ the control input variable, and $y \in I\!R$ the measured output. It is known that some solution can be looked for by using filtering or observer techniques (cf Schrijver and van Dijk (2002) for instance). But our point here is to emphasize how this problem can alternatively be solved by a *control* approach, allowing to rely on classical tools for control design.

The main idea in short is that under 'enough observability', if for a copy of system (3), one can find a control $\hat{v}$ such that the corresponding output $\hat{y}$ copies y, then $\hat{v}$ should approach v.

It appears that this idea can be formally stated under the following assumptions:

(a1) The pair $(\mathbf{A}, \mathbf{C})$ is observable;

(a2) The extended system (with control v):

$$\begin{aligned} \dot{x}_0 &= \mathbf{C}\mathbf{x} \\ \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{D}v \end{aligned} \tag{6}$$

is controllable;

(a3) $\mathbf{x}^{(k)}(t)$ is uniformly bounded for $k = 0$ to n and $v^{(k)}(t)$ for $k = 0$ to $2n - 1$, *i.e.*,

$$\exists X, V\ :\ \forall t \geq 0, \|\mathbf{x}^{(k)}(t)\| \leq X,\ k = 0, ..n;$$
$$|v^{(k)}(t)| \leq V,\ k = 0, ..2n - 1.$$

With this indeed, we can state the following:

Proposition *3.1.* Under assumptions (a1)-(a3), for any $\varepsilon > 0$ and any $t_1 > 0$, one can find an estimate $\hat{v}$ as:

$$\begin{aligned} \hat{v} &= -\mathbf{F}\hat{\mathbf{x}} - f_0\hat{x}_0 \\ \dot{\hat{x}}_0 &= \mathbf{C}\hat{\mathbf{x}} - y \\ \dot{\hat{\mathbf{x}}} &= \mathbf{A}\hat{\mathbf{x}} + \mathbf{D}\hat{v} \end{aligned} \tag{7}$$

(where $\mathbf{F}, f_0$ are chosen as a stabilizing feedback gain for system (6), ensuring a convergence rate 'large enough'), such that:

$$\|\hat{v}(t) - v(t)\| \leq \varepsilon + \eta V,\ \forall t \geq t_1. \tag{8}$$

for some $\eta > 0$ depending on the zeros of the transfer function between input v and output y.

If in addition all those zeros have strictly negative real parts, then for any $\varepsilon > 0$ one can find $\tilde{v}$ such that:

$$\|\tilde{v}(t) - v(t)\| \leq \varepsilon,\ \forall t \geq t_2$$

for some $t_2 > 0$. ■

In short, (8) means that the unknown input v can be accurately reconstructed when varying *slowly enough* (*i.e.*, V small), by any control 'fast enough' for observer (7). Similarly, the reconstruction can also be accurate if parameter η (depending on the system) is small enough.

In addition, if the mentioned zeros are stable, the reconstruction can be arbitrarily accurate (in particular this is true if there is no zero).

This result can be established by considering the dynamics of error $\hat{x} - x$ in some canonical form, subject to some *high gain* feedback control, and using the so-called Tikhonov's theorem for singularly perturbed systems (Khalil (2002); Tikhonov et al. (1970)) - see appendix A.

An application of this result to our wind estimation problem is presented in next section.

4. APPLICATION TO EFFECTIVE WIND SPEED ESTIMATION

Let us propose here some simple control strategy for an effective wind speed estimator according to proposition 3.1, in the case of our wind turbine model (3)-(4)-(5). It can indeed be checked here that the model admits a single zero which is stable, and even makes η very small.
As a result, any control scheme for an estimator of the form (7) ensuring a tracking of the measured output y with a convergence rate fast enough will provide some estimate for the unknown input v.
The chosen structure for the estimator is shown in Fig. 1: it has a control loop that feeds the turbine model with the necessary wind speed so that its output (the high-speed shaft rotational speed) matches the one of the real system.

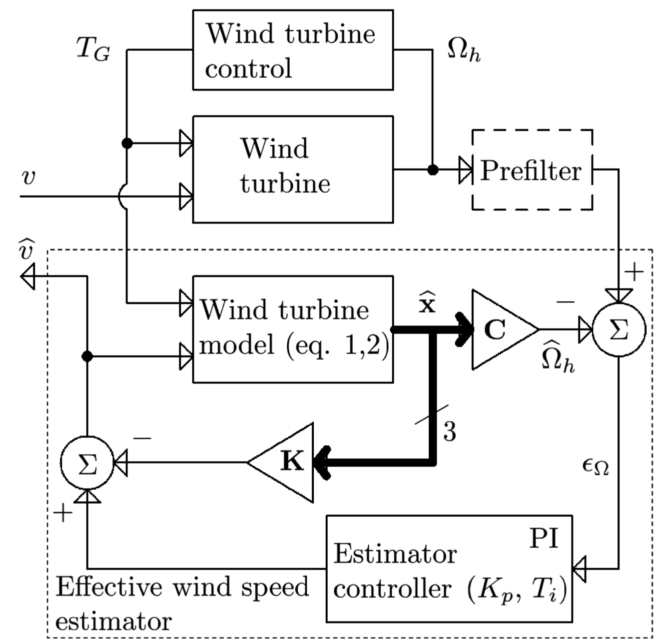

Fig. 1. Effective wind speed estimator structure

This special structure, highlighting a PI feedback, is inspired by the one presented in Xu et al. (2012). However, there are some major differences which are listed hereafter:

First, it does not use the anemometer information, as the estimator must provide an independent alternative wind speed measure.

Second, the system output is the high-speed shaft rotational speed and not the output power (notice that effective wind speed must be related to the output power). This choice is possible because it is considered that the wind turbine is controlled at variable speed, using a power loop. This basically means that the power reference is built by using an algebraic function of rotational speed cubed (Burton et al. (2001)), meaning that these variables are dynamically linked. So if the model is fed with the known control input (the generator torque), and has the same rotational speed as the real system (by means of the estimator control loop), the output power of these systems will be equal. To conclude, even if the model outputs the high-speed shaft rotational speed $\widehat{\Omega}_h$, the provided unknown input, $\widehat{v}$, still relates to the output power.

A third difference is a second (inner) loop consisting of state feedback that relocates the model poles to convenient positions: this allows for the 'large convergence rate' which is required by proposition 3.1. Outputs of these loops are summed to form the estimation of the unknown input of the system, which is the effective wind speed.

Typically, complex-conjugated poles of the model (*i.e.*, given by matrix (4)) are highly undamped with quite large natural frequency and the real one corresponds to a very slow dynamic (see Fig. 2). One cannot use directly a controller with a significant integral component over the model without rendering unstable the closed-loop system.

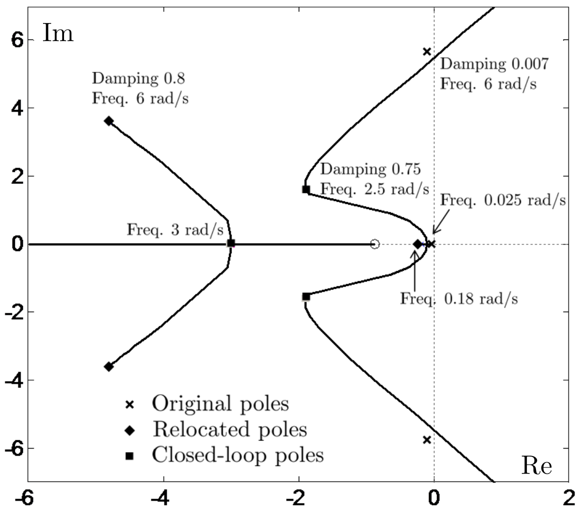

Fig. 2. Pole positions and root locus for the closed-loop estimator. Representations correspond to parameters in Appendix B

Hence the relocatation of those complex-conjugated poles becomes necessary, chosen here to have about the same frequency and a more regular damping. The real pole is moved at a frequency five-to-ten times larger with respect to the initial one (see Fig. 2). Note that these choices are not critical. These relocations allow to use effectively a proportional-integral controller with high gains and consequently quite high closed-loop bandwidths.

Root locus depicted in Fig. 2 corresponds to the closed-loop estimator behavior. The chosen controller gains – hence the pole positions on the root locus – correspond to smooth behavior in terms of damping. Because wind speed variations are in the same frequency domain as the estimator, a high damping reduces dynamical excitation of the estimator, hence reducing the estimation errors.

Note that the closed-loop structure has four poles; it is imposed by design that the poles situated in high frequency to be at the same location on the abscissa. It is obvious that to zero's position corresponds a certain value of the PI controller time constant, T_i, and to poles' positions on the root locus a certain value of controller gain, K_p (see also Appendix B).

5. SIMULATION RESULTS

Numerical simulations have been carried out using MATLAB/ Simulink® for both deterministic (step and sinusoidal variations) and band-limited stochastic wind speed variations, whose values determine turbine partial load operation. Both plant (wind turbine) and the model included into estimator have been represented by the nonlinear model given by equations (1) and (2). The plant is classically controlled at variable speed by a two-loop cascaded control structure, employing in the outer loop a power controller and in the inner loop a rotational speed controller. This structure outputs the generator torque which also feeds the model within the estimator. Estimator control structure of Fig. 1 is completed with a prefilter on the high-speed shaft rotational speed, in order to cancel the zero introduced by PI controller (also visible in Fig. 2).

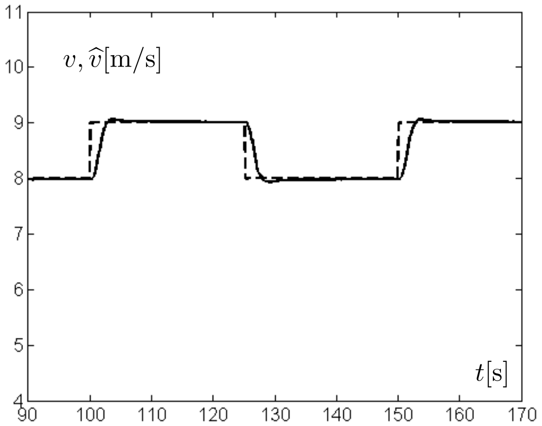

Fig. 3. Effective wind speed estimator response (solid trace) at wind speed step variations (dotted trace)

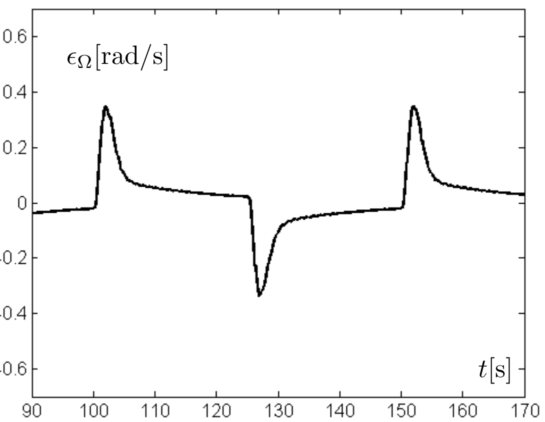

Fig. 4. Speed error at estimator controller: $\epsilon_\Omega = \Omega_h - \widehat{\Omega}_h$

Fig. 3 comparatively presents the estimator output when the wind speed at the plant evolves with step variations. Settling time is about 2.5 s and the overshoot is small. There is a slightly different behavior between rising and falling steps due to system nonlinearity. The corresponding rotational speed error, $\Omega_h - \widehat{\Omega}_h$ is plotted in Fig. 4. Note the quite reduced speed error with respect to the high-speed shaft rotational speed rated value – $\Omega_r \approx 150$ rad/s.

Figs. 5 and 6 show the estimator performance under sinusoidal wind speeds with different frequencies. Note in Fig. 5 that the lag between the wind speed and its estimate significantly increases with the frequency. As expected, the error peak value – visible in Fig. 6 – decreases as the frequency decreases by almost the same ratio. Based upon this harmonic analysis, the estimation bandwidth (at 3 dB) may be established at 1.5 rad/s.

Next results correspond to stochastic wind speed variations. The synthetic wind speed has been obtained by passing a random variable through a shaping filter (see Nichita et al. (2002)). This latter corresponds to von Karman spectrum and the IEC standard. The resulted wind speed sequence has 7 m/s as an average value and a turbulence intensity of 0.15.

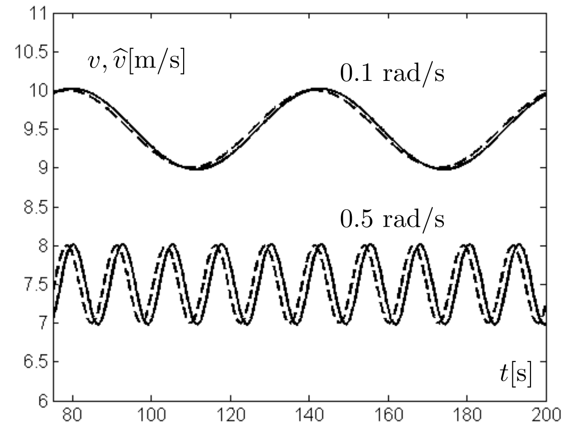

Fig. 5. Effective wind speed estimator response (solid trace) at sinusoidal wind speed variations (dotted trace) for two different frequencies

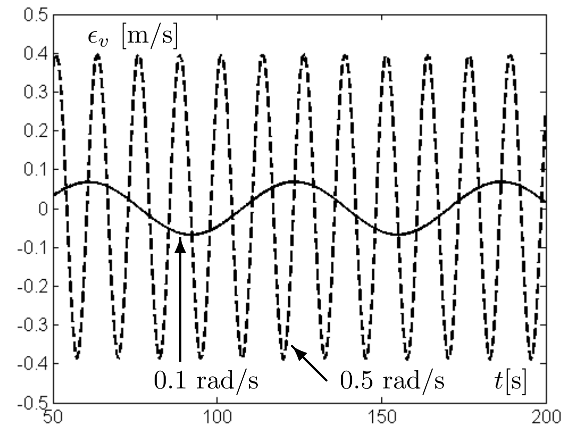

Fig. 6. Wind speed estimation error ($\epsilon_v = v - \widehat{v}$) for different wind speed frequencies: 0.1 rad/s (solid line) and 0.5 rad/s (dotted line)

Note in Fig. 7 that the estimated wind speed value fairly follows the wind speed, with a delay of about 2 s. Its error has a variance around 0.45. Fig. 8 shows the relative errors

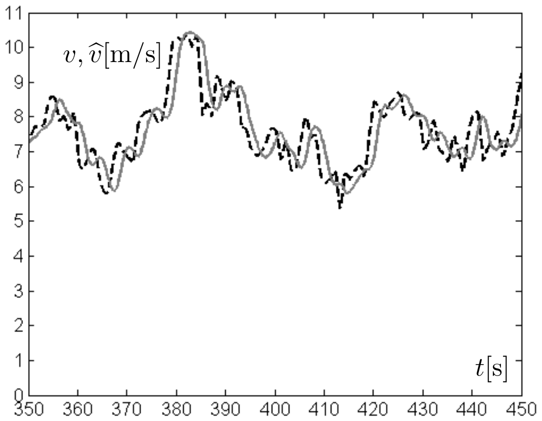

Fig. 7. Stochastic variations of wind speed: real (dotted trace) and estimated (solid)

between plant state variables and pseudo-system state variables at stochastic wind speed variations resuming those in Fig. 7, but for a larger time interval, that is 5 minutes. Their averages are zero and their peak values do not overpass 0.02.

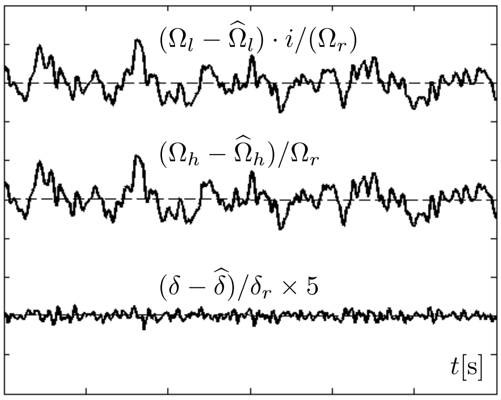

Fig. 8. Relative errors between plant state variables and pseudo-system state variables. Horizontal axis has 50 s/div and vertical axis has 0.02 /div

Next, Fig. 9 shows error between the wind speed and its estimation when the former has a unity step variation, at different average values, hence at different steady-state operation points of the pseudo-system. The estimator dynamic performance has been set by design (see Section 4) at an operating point corresponding to a wind mean speed $\bar{v} = 8$ m/s. Note that the estimation dynamical performances are slightly changing with the operation point, and become more undamped as the mean wind speed, $\bar{v}$, is smaller, while the canceling time does not differ significantly.

6. CONCLUSION

This paper has presented a control-based observer structure used for unknown input estimation purpose. Different from classical estimation structures, it contains a

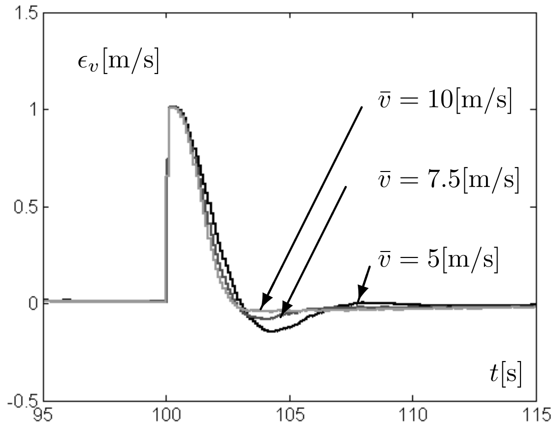

Fig. 9. Wind speed estimation error at different pseudo-system operation points, $\epsilon_v = v - \widehat{v}$

controller that cancels the difference between plant and observer outputs. This controller feeds the system model with the input estimation sought for.

The basic idea of the proposed structure is that, by means of a high-gain control, the estimation error can be rendered arbitrarily small – within an interval bounded by the gradient of the estimated variable – in an arbitrarily small time. One of the consequences is that the state estimation error can also be arbitrarily decreased.

This strategy has been applied for effective wind speed reconstruction in wind turbines operating in partial load and has resulted in a simple easy to tune control structure. It has been preliminarily validated though numerical simulation with promising results. Indeed, effective wind speed is reconstructed with a good dynamic – with respect to the one of the wind turbine – thus being able to replace anemometer information for diagnosis and output power assessment purposes.

The proposed observer structure presumably exhibits good robustness properties as ensured by the intrinsic robustness of PI controller. Thorough robustness analysis should make the object of future work. Another points to investigate further will consist in assessing the estimator operation in real-world wind turbines, to compare its performance against other types of effective wind speed estimation algorithms and to generalize it for the full-load operation also. Extension to some other plant types having unmeasurable inputs is also envisaged.

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Appendix A. PROOF OF PROPOSITION 3.1 (SKETCH)

From assumptions (a1) and (a2), system (3) admits some irreducible transfer function between v and y of the form:

$$G_{yv}(s) = \frac{b_m s^v + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \cdots a_1 s + a_0}$$

for some coefficients a_i's b_i's and some degree m of the numerator.

From it, one can easily derive a new representation for extended system (6) of the following canonical form:

$$\dot{\mathbf{z}} = \mathbf{A}_c\mathbf{z} + \mathbf{B}_c \sum_{i=0}^{m} b_i v^{(i)} + \bar{\mathbf{B}}u \qquad \text{(A.1)}$$

with

$$\mathbf{A}_c = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & & \vdots \\ & & & & 0 \\ 0 & \cdots & & 0 & 1 \\ 0 & -a_0 & \cdots & & -a_{n-1} \end{pmatrix}; \ \mathbf{B}_c = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix};$$

and $\bar{\mathbf{B}}$ a matrix of $\mathbb{R}^{(n+1)\times m}$.

Hence one can take F_c such that $A_c - B_c F_c$ is stable.

Then, consider $\bar{v}$ given by:

$$\begin{aligned} \bar{v} &= -\mathbf{F}_c \boldsymbol{\Lambda} \hat{z} \\ \dot{\hat{z}} &= \mathbf{A}_c \hat{z} + \mathbf{B}_c \hat{v} + \bar{\mathbf{B}}u - \mathbf{G}y \end{aligned} \qquad \text{(A.2)}$$

with $\mathbf{G} = (1\ 0\ \ldots\ 0)^T \in \mathbb{R}^{n+1}$ and $\boldsymbol{\Lambda}$ an $(n+1)\times(n+1)$ diagonal matrix with λ^{n+2-i} as entry (i,i) for $i = 1$ to $n+1$, and $\lambda > 1$.

Consider now $\mathbf{e}$ the vector of components $e_i := \frac{\hat{z}_i - z_i}{\lambda^{i-1}}$ for $i = 2$ to $n+1$, and $e_1 := \hat{z}_1$. Then it can be checked that:

$$\dot{\mathbf{e}} = \lambda(\mathbf{A}_c - \mathbf{B}_c\mathbf{F}_c)\mathbf{e} - \frac{1}{\lambda^n} B_c \sum_{i=0}^{m} b_i v^{(i)} - \mathbf{B}_c \sum_{i=2}^{n+1} f_i \frac{z_i}{\lambda^{i-2}} \qquad \text{(A.3)}$$

with f_i the ith compenent of F_c.

At this point, dividing equation (A.3) by λ and appending the resulting equation to (A.1), we clearly get a representation with slow and fast dynamics, satisfying the conditions of Tikhonov's theorem (see Khalil (2002)).

As a result, and since the $v^{(k)}$'s and z are bounded (assumption (a3)), $\mathbf{e}$ can be made arbitrarily small in arbitrarily short time by choosing λ large enough, and from the definition of $\mathbf{e}$, this is clearly also true for $\hat{z}_1 - z_1$.

Now consider $e_{n+2} := \dot{e}_{n+1}$ and the dynamics of λe_i for $i = 2$ to $n+2$: it can be checked that one recovers the same equation structure as (A.3), with a shift in indexes of e_i's and one degree of derivation more in v, z. Hence, with the same arguments as before, the same result as the one for $\hat{z}_1 - z_1$ is obtained for $\hat{z}_2 - z_2$.

By iterating the procedure, it follows that all errors $\hat{z}_i - z_i$'s can be made arbitrarily small in arbitrarily short time, up to $n+2$, where by definition the error is $-\sum_{i=2}^{n+1} a_{i-2}(\hat{z}_i - z_i) + \bar{v} - \sum_{i=0}^{m} b_i v^{(i)}$.

By defining $\hat{v} := \frac{\bar{v}}{b_0}$, it results that $\forall \varepsilon > 0$ one can get:

$$|\hat{v}(t) - v(t)| \leq \varepsilon + \sum_{i=1}^{m} \frac{b_i}{b_0} V$$

in arbitrarily short time, which gives (8).

Finally, it is clear that if the zeros of G_{yv} are stable, $\tilde{v}$ as:

$$\sum_{i=0}^{m} b_i \tilde{v}^{(i)} = b_0 \hat{v}$$

will approach $\hat{v}$ after some transient time given by those zeros, which ends the proof. ∇

Appendix B. WIND TURBINE PARAMETERS

Wind turbine parameters:
$R_b = 40$ m, $P_{rated} = 2$ MW, $v_{rated} = 11$ m/s, $\Omega_r = 150$ rad/s, $\delta_r = \pi/4$, $i = 100$, $J_l = 9e6$ kgm^2, $J_h = 990$ kgm^2, $k_A = 1.73e8$ kgm^2/s^2, $d_h = 1.22$ kgm^2/s, $d_l = 5.9e3$ kgm^2/s, $k_1 = -4.5e5$ Ns/rad, $k_2 = 2.36e5$ Ns/m

Estimator parameters:
$\mathbf{K} = [368.7\ \ -3.57\ \ 36.68]$, $K_p = 1.44$, $T_i = 1.2$ s