Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Distributed Consensus Charging for Current Unbalance Reduction ⋆

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Abstract: Electric Vehicle (EV) technology has developed rapidly in recent years, with the result that increasing levels of EV penetration are expected on electrical grids in the near future. The increasing electricity demand due to EVs is expected to provide many challenges for grid companies, and it is expected that it will be necessary to reinforce the current electrical grid infrastructure to cater for increasing loads at distribution level. However, by harnessing the power of Vehicle to Grid (V2G) technologies, groups of EVs could be harnessed to provide ancillary services to the grid. Current unbalance occurs at distribution level when currents are unbalanced between each of the phases. In this paper a distributed consensus algorithm is used to coordinate EV charging in order to minimise current unbalance. Simulation results demonstrate that the proposed algorithm is effective in rebalancing phase currents.

Keywords: Decentralized Control; Electric Vehicles; Smart Power applications.

1. INTRODUCTION

By definition, in a balanced three-phase power network, the magnitudes of the voltage and currents on individual phases are equal and each phase is separated by 120 degrees (Kim et al.,2005). However, the phases are typically not balanced like this in reality, e.g., when a residential distribution network connects mixed three-phase loads and uneven single phase loads. Uneven power consumption on each phase can lead to unbalanced power flow from the transformer's perspective and this can result in unbalanced current and voltage issues. For example, unbalanced loads can result in zero sequence currents in transformers, which causes overheating of devices and results in increased power losses (Gruzs,1990). In addition, unbalanced voltages can reduce the efficiency of 3-phase connected devices, such as motors, and are undesirable for utilities in terms of safety concerns and economic efficiency. Thus, it is desirable to develop methods which will automatically adjust voltages and currents along individual phases in order to minimise imbalances (Kütt et al.,2013).

Current and voltage unbalance are intrinsically linked. However, the issue of balancing currents is less complex than that of balancing voltages. This is because the current magnitudes and angles along phases can easily be measured at the transformer, and current demand can then potentially be altered to balance phases. The voltage on the other is related to the distribution of individual loads along the phases, and it can be less certain as to where the controllable loads, e.g. EVs, might be placed.

The benefit of current balancing is obvious, in that by reducing the neutral current flow, the total losses incurred during power transmission can be minimised. If voltage issues are not catered for too, some three-phase devices on the network (e.g. mostly induction motors) may not always work properly (Jouanne et al.,2001). Also, voltage unbalance can lead to current unbalance, which can result in losses for both customers and the grid company (Kütt et al.,2013). Typically power electronics devices, such as Static Synchronous Compensators (STATCOMs) and the Unified Power Flow Controller (UPFC), can be used for reactive power compensation to regulate voltages and power flow (Xu et al.,2010).

The motivation of this work is to use groups of EVs for the provision of a grid compensation ancillary service in order to alleviate current unbalance between phases. To do this, a distributed EV charging strategy is developed, based on techniques from the cooperative control literature, to balance currents along the individual phases. Many cooperative control algorithms have been used for power system applications (Xin et al.,2011) (Hatta et al.,2010). Here the distributed consensus method developed in (Knorn, 2011) is applied to mitigate current unbalance using EV charging. In the case of current balancing, it is more advantageous to use a distributed algorithm rather than a centralised solution because global knowledge of the system may not be available to a centralised controller, and a distributed approach allows flexibility in terms of what EVs are controlled at a given time, i.e., it allows

⋆ Funding by Science Foundation Ireland (grants 11/PI/1177 and 09/SRC/E1780) for this research is gratefully acknowledged.

a more "plug and play" type approach. To the authors' knowledge, this is the first time that a distributed control algorithm has been used to manage current unbalance issues via controlled EV charging.

The paper is organised as follows: in Section 2, the current balancing objective is formulated. Section 3 introduces the distributed control algorithm. In Section 4 a real distribution level power system simulation is developed which uses the proposed charging strategy. The results from the simulation are discussed in Section 5. In Section 6 conclusions and future work are discussed.

2. CURRENT UNBALANCE

2.1 Current Unbalance Factor

There have been many different methods proposed for quantifying the degree of voltage unbalance on a network, such as the *IEEE* and *NEMA* standards (Meyer et al.,2011), (Pillary et al.,2001). However, as of yet, there is no widespread agreement as to which of the proposed Standards for the Current Unbalance Factor (SCUF) should be used. To be consistent with the previous work in (Fernandez et al, 2013), in this paper it is assumed that the minimum acceptable SCUF on the grid is 10%. The SCUF is given as follows (Putrus et al.,2009):

$$\%\mathrm{SCUF} = (\frac{|i_\mathrm{n}|}{|i_\mathrm{p}|}) \times 100\%, \tag{1}$$

where i_n and i_p are the negative sequence and positive sequence component of the current i. If we denote the zero sequence current as i_0, and the current in the individual phases as i_a, i_b, and i_c then:

$$\begin{bmatrix} i_0 \\ i_\mathrm{p} \\ i_\mathrm{n} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} i_\mathrm{a} \\ i_\mathrm{b} \\ i_\mathrm{c} \end{bmatrix}, \tag{2}$$

where α is defined as follows:

$$\alpha = e^{j\frac{2}{3}\pi}. \tag{3}$$

To this end, we can easily calculate the current unbalance factor using the measured current on each phase.

2.2 Current Balance Equation

The current balance equation can be formulated in a general framework by considering the EV charging loads on each phase (Fernandez et al, 2013). The balance equations are given as follows:

$$\begin{aligned} i_\mathrm{n} &= i_\mathrm{nr} + i_\mathrm{ni}\mathrm{i}, \\ i_\mathrm{p} &= i_\mathrm{pr} + i_\mathrm{pi}\mathrm{i}, \end{aligned} \tag{4}$$

where

$$\begin{aligned} i_\mathrm{pr} &= \frac{1}{3}\left(i_\mathrm{d1} + i_\mathrm{d2} + i_\mathrm{d3}\right) \\ i_\mathrm{pi} &= -\frac{1}{3}\left(i_\mathrm{q1} + i_\mathrm{q2} + i_\mathrm{q3}\right) \\ i_\mathrm{nr} &= \frac{1}{3}i_\mathrm{d1} - \frac{1}{6}\left(i_\mathrm{d2} + i_\mathrm{d3}\right) + \frac{\sqrt{3}}{6}\left(i_\mathrm{q2} - i_\mathrm{q3}\right) \\ i_\mathrm{ni} &= \frac{1}{3}i_\mathrm{q1} - \frac{1}{6}\left(i_\mathrm{q2} + i_\mathrm{q3}\right) - \frac{\sqrt{3}}{6}\left(i_\mathrm{d2} - i_\mathrm{d3}\right) \end{aligned} \tag{5}$$

And also with:

$$\begin{bmatrix} i_\mathrm{d1} \\ i_\mathrm{d2} \\ i_\mathrm{d3} \end{bmatrix} = \begin{bmatrix} \dfrac{P_\mathrm{L1} + P_\mathrm{EV1}}{V_\mathrm{s1}} \\ \dfrac{P_\mathrm{L2} + P_\mathrm{EV2}}{V_\mathrm{s2}} \\ \dfrac{P_\mathrm{L3} + P_\mathrm{EV3}}{V_\mathrm{s3}} \end{bmatrix}, \begin{bmatrix} i_\mathrm{q1} \\ i_\mathrm{q2} \\ i_\mathrm{q3} \end{bmatrix} = \begin{bmatrix} \dfrac{Q_\mathrm{L1} + Q_\mathrm{EV1}}{V_\mathrm{s1}} \\ \dfrac{Q_\mathrm{L2} + Q_\mathrm{EV2}}{V_\mathrm{s2}} \\ \dfrac{Q_\mathrm{L3} + Q_\mathrm{EV3}}{V_\mathrm{s3}} \end{bmatrix} \tag{6}$$

In (6), the variables $P_{\mathrm{EV}j}$ and $Q_{\mathrm{EV}j}$ denote the total active and reactive powers consumed by the EV loads on the j^th phase. Similarly, the variables $P_{\mathrm{L}j}$ and $Q_{\mathrm{L}j}$ are used to represent the total active and reactive powers consumed by all household loads on the j^th phase. In addition, $V_{\mathrm{s}j}$ is used to represent the voltage at the transformer on the j^th phase. In (4), i_nr and i_ni denote the real and imaginary components of the negative sequence current. Similarly, i_pr and i_pi are used to represent the real and imaginary components of the positive sequence current. In (5), $i_{\mathrm{d}j}$ and $i_{\mathrm{q}j}$ represent the in-phase and quadrature component on the j^th phase of current i. It is worth noting that in a balanced system, zero sequence component i_0 should always equals to zero. Since the zero sequence component is not related to the SCUF defined in (1), this expression is not considered in the context of this paper.

According to the definition in (1), the value of i_n has a significant influence on the level of current unbalance on the grid. Referring to (5), if the following holds:

$$i_{d1} = i_{d2} = i_{d3} > 0 \tag{7}$$

$$i_{q1} = i_{q2} = i_{q3} = 0 \tag{8}$$

then both i_nr and i_ni will be minimised while i_p will still be a positive value. Therefore, $|i_\mathrm{n}| = \sqrt{i_\mathrm{nr}^2 + i_\mathrm{ni}^2}$ will also be minimised. Referring to (6), the active power flow on each phase can be made equivalent via manipulation of $P_{\mathrm{EV}j}$, for $j = 1, 2, 3$ (ignoring the small differences between phase voltages at the head of transformer V_s), so as to satisfy (7), and (8) is satisfied by letting:

$$Q_{\mathrm{L}j} = -Q_{\mathrm{EV}j}, \text{ for } j = 1, 2, 3. \tag{9}$$

It is possible to achieve this goal using a Vehicle to Grid (V2G) framework where each EV has the flexibility to adjust their charge rates as required (Shahnia et al.,2009). Based on this idea, a distributed control framework can be designed as in the following section.

3. DISTRIBUTED CONSENSUS ALGORITHM

3.1 Preliminaries

It is assumed that each EV charger has a maximum allowable apparent power S^i_max, a maximum active power draw, C^i_max, and a maximum reactive power draw, R^i_max, such that:

$$S^i_\mathrm{max} = \sqrt{C^{i2}_\mathrm{max} + R^{i2}_\mathrm{max}} \tag{10}$$

The load power consumption in the network is discretized, with a sample time Δ. The measured active and reactive power consumptions on the j^th phase of the transformer at sample step k are denoted by $T^j_\mathrm{a}(k)$ and $T^j_\mathrm{r}(k)$, respectively. The index set of all EVs connected at the j phase at sample step k is denoted by $\Theta_j(k)$. To protect the transmission line from overloading, the maximum power flow that can be tolerated at each phase is defined as $P_{\max}$. The active and reactive charge rates for the i^{th} EV at sample step k are denoted by $C_i(k)$ and $R_i(k)$, respectively. Here, $R_i(k) > 0$, such that the i^{th} EV can inject the same amount of reactive power as the reactive power consumed by the houses. The variables $G_{\mathrm{a}}^j(k)$ and $G_{\mathrm{r}}^j(k)$, which denote the total chargeable capacity of active and reactive powers on the j_{th} phase, are given as follows:

$$\begin{aligned} G_{\mathrm{a}}^j(k) &= \sum_{i\in\Theta_j(k)} \left(C_{\max}^i - C_i(k)\right), \text{ for } j = 1,2,3, \\ G_{\mathrm{r}}^j(k) &= \sum_{i\in\Theta_j(k)} \left(R_{\max}^i - R_i(k)\right), \text{ for } j = 1,2,3. \end{aligned} \tag{11}$$

3.2 Cooperative Control

Here the basic cooperative control algorithm given in (Knorn, 2011) is applied to the EV charging strategy outlined above. The following lemma relates to a consensus algorithm in the case of a common input (Knorn, 2011):

Lemma 1: *Let $P_k \in \Re^{n\times n}$ be a sequence of matrices from a finite set of primitive, row-stochastic matrices with strictly positive main diagonal entries, and $\vartheta(x_k, k)$ a sequence of real numbers.*

Then, if $x_k = (x_k^1, \ldots, x_k^n)^{\mathrm{T}}$ evolves for some $x_{k=0} = x_0 \in \Re^n$ according to:

$$x_{k+1} = P_k x_k + \vartheta(x_k, k)\mathbf{1} \tag{12}$$

then the elements of x_k will approach each other over time, which is:

$$\lim_{x\to\infty} x_k^p - x_k^q = 0 \tag{13}$$

for all $p, q \in \{1, 2, \ldots, n\}$

This leads to the following distributed consensus algorithm for the charging of EVs on the individual phases:

Algorithm 1 Distributed Consensus algorithm

while Battery not charged **do**
  for each $i \in \Theta_j(k)$ **do**
    $\delta_i(k) = \eta_{\mathrm{a}} \sum_{h\in N_k^i} (C_i(k) - C_h(k)) + \mu_a.E_{\mathrm{a}}^j(k)$
    $C_i(k+1) = \min\left(C_{\max}^i, C_i(k) + \delta_i(k)\right)$
  end for
end while

while Charger is active **do**
  for each $i \in \Theta_j(k)$ **do**
    $\lambda_i(k) = \eta_{\mathrm{r}} \sum_{h\in N_k^i} (R_i(k) - R_h(k)) + \mu_r.E_{\mathrm{r}}^j(k)$
    $R_i(k+1) = \min\left(R_{\max}^i, R_i(k) + \lambda_i(k)\right)$
  end for
end while

Here, N_k^i represents the set of neighbour agents communicating to the ith EV on the same phase. The variables η_{a} and λ_{r} are the parameters determining the convergence rate for active and reactive power and μ_{a} and μ_{r} affect the stability of the algorithm. With a proper choice of parameters, consensus will be achieved for the both active

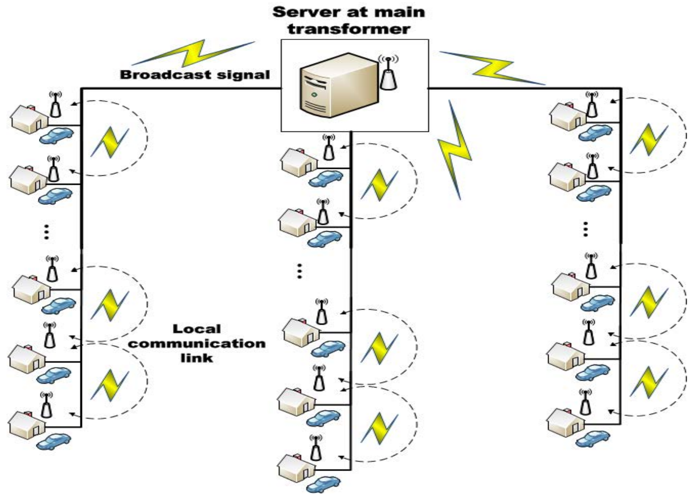

Fig. 1. Schematic diagram of the communication topology

and reactive charge rates for each EV connected on the same phase. The variable $E_{\mathrm{a}}^j(k)$ is the difference between the allowable power on phase j and the current power being drawn on phase j. Effectively this is the amount of extra power that can be absorbed by the available EVs along each phase j. A similar definition applies to $E_{\mathrm{a}}^j(k)$ as regards the reactive power, where it is only required to compensate for the reactive power consumed on each phase j by the household loads.

To apply this theory, it is also assumed that each EV has the ability to exchange information about its charge rate with its connected neighbours, such that the constructed communication graph covers all connected EVs at each sample step. This graph can be time-varying and adaptive such that it is always valid over time unless only one or zero EVs are connected in the network. Beyond that, it is also required that each EV has the ability to receive a broadcast signal from the transformer. This signal is used to coordinate the charge rates of EVs and achieve some common goal (e.g. current balancing).

It should be noted that, for the algorithm presented here, it is not required to know the number of connected EVs at each time slot. The values of $G_{\mathrm{a}}^j(k)$ and $G_{\mathrm{r}}^j(k)$ in (11) are communicated in a similar fashion to the charge rates, but the result is accumulated as it is transmitted along the communication nodes. In practice, this process is performed before the broadcast signals are updated each time step on the transformer side. It is also required that at least one of the EVs connected to the j^{th} phase is able to send the $G_{\mathrm{a}}^j(k)$ and $G_{\mathrm{r}}^j(k)$ signals back to the transformer.

It is then necessary for the transformer to recalculate both the reactive power required for compensation at each sample step, and also the active power charge rates required for the maximisation of EV charging whilst maintaining balanced phases. To this end, the signals related to the active, $E_{\mathrm{a}}^j(k)$, and reactive power, $E_{\mathrm{r}}^j(k)$, broadcast from the transformer to each EV connected to the j^{th} phase, are formulated as follows:

$$\begin{aligned} P_{\text{tot}}^j(k) &= \min\left(P_{\max}, G_{\mathrm{a}}^j(k) + T_{\mathrm{a}}^j(k)\right), \\ P_{\min}(k) &= \min\left(P_{\text{tot}}^1(k), P_{\text{tot}}^2(k), P_{\text{tot}}^3(k)\right), \\ E_{\mathrm{a}}^j(k) &= P_{\min}(k) - T_{\mathrm{a}}^j(k), \\ E_{\mathrm{r}}^j(k) &= \min\left(G_{\mathrm{r}}^j(k), T_{\mathrm{r}}^j(k)\right). \end{aligned} \tag{14}$$

Here $P_{\mathrm{tot}}^{j}(k)$ is the total potential power that could be drawn by phase j at sample step k. The variable $P_{\min}(k)$ is defined as the minimum of the $P_{\mathrm{tot}}^{j}(k)$ values that can be drawn amongst the three phases at sample step k. This is then the setpoint for the total power to be drawn in each of the individual phases. The allowable power to be drawn in phase j, $E_{\mathrm{a}}^{j}(k)$, is then given by the difference between the total power phase j can draw and the current power being drawn in phase j. The reactive power, $E_{\mathrm{r}}^{j}(k)$, is chosen on each phase to equal the reactive power being drawn by household loads at sample step k.

4. RESIDENTIAL DISTRIBUTION SYSTEM SIMULATION

4.1 Distribution Level Simulation

In this section a distribution level simulation is developed. Here, the authors adopted a similar topology to a distribution network modelled previously for a pure EV charging scenario (Liu et al.,2013b). For this paper, this model has been revised for the purposes of demonstrating the current balancing algorithm on a residential network.

As illustrated in Fig. 2, a 10kV/400V(400kVA) delta/wye (grounded) transformer was connected at the head of the feeder. The percentage resistance of each winding on the transformer was set to 0.5, and the percentage reactance of the transformer from primary to secondary side was set to 2. The voltage from the external grid was set to 1.05pu. The distribution network was modelled with the TN-C-S earthing system, where the three-phase transmission line was split clearly with a neutral line connected back to the neutral point. The variable Z denotes the total power consumption at the location to which it is connected, which includes both household loads and EV charging loads. Each load Z was modelled as a constant current load and each household load was associated with a power factor. For simplicity, we assumed this factor was equal to 0.95 lagging for each house during the day. The load profile for each house was randomly chosen from the dataset given in (CER, 2012). Then each load profile was resampled every minute such that the load consumption was constant within each one minute interval. In addition, the sampling time for the charging algorithm was set to one second. In other words, it was assumed that each EV was able to finish exchanging charge rate information with their neighbours in this one second interval.

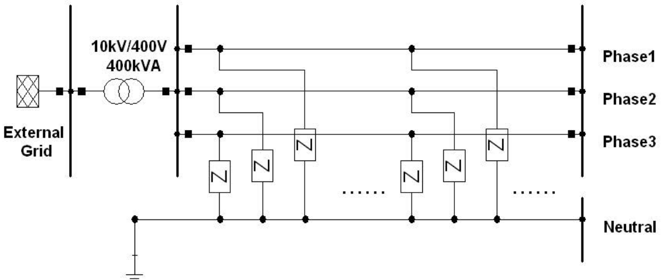

Fig. 2. Schematic diagram of the distribution network.

It is assumed that there were 20 houses located evenly on each phase. The distance between each house was 10 meters. The power limit $P_{\max}$ for each phase line was defined as 80kW. The maximum apparent power $S_{\max}^{i}$ for ith EV was set to 4kW, and the maximum active charge rate $C_{\max}^{i}$ was set to 3.7kW. Thus, the maximum reactive charge rate $R_{\max}^{i}$ was calculated as 1.52kVar, which corresponds to a maximum of 6.6A quadratic component of charging current under a nominal single phase voltage of 230V. The battery capacity for each EV was assumed to be 20kWh and the initial energy required by each EV was normally distributed with a mean of 10kWh and a standard deviation of 1.5kWh. EV arrival times were also based on a normal time distribution with mean 6pm and a standard deviation of 1 hour, and EVs left between 7am and 10am with the following time distribution: 7am-8am (21%), 8am-9am (47%) and 9am-10am (32%) based on (CSO, 2009). These consumption patterns are consistent with the authors' previous work in (Liu et al.,2013a,b). The power system simulation was conducted on a customized OpenDSS-Matlab simulation platform (EPRI, 2013).

5. RESULTS AND DISCUSSION

5.1 Algorithm Dynamics

To demonstrate the dynamics of the basic consensus algorithm, a simple Matlab based simulation was devised. In this simulation 10 EVs are connected to each phase and each was given a random initial charge rate. The maximum number of sample steps was set to 100. Here EVs only sought to balance the active power flow on each phase. At the initial time step, the total measured power flow on each phase was 50kW, 40kW and 60kW in phases 1, 2 and 3, respectively, and the maximum active charge rate for each EV was 4kW. The simulation was run and the results are given in Figs. 3-6.

The results show that the EVs on each phase coordinate their actions and achieve consensus on their charge rates within 50 time steps. By adjusting η, the convergence speed can be made faster, as illustrated in Fig. 4. On the other hand, it is also shown that improper selection of μ, can result in instability, as illustrated in Fig. 6. Therefore, a suitable choice of both parameters is essential to maintain stable and fast control of the system. A more detailed discussion on this is given in (Knorn, 2011).

5.2 Power System Simulation

The simulation results are shown in Figs.7-11. It can be seen in Figs.7 and Fig.8 that by using the distributed consensus technique both active power and reactive power consumptions were balanced on each phase. The active power flow is well balanced until around 10pm, when most of the EVs have finished charging. The reactive power remains balanced while most of the EVs remain plugged in. As a result, the neutral current and current unbalance factor were minimized during EV charging times (the SCUF is almost zero in this case).

While the EVs are connected both the active and reactive powers are well balanced as desired across the 3 phases. This is dependent on there being critical levels of EVs in place on the grid. Outside of the times when there are critical levels of EV penetration on the grid, it may not

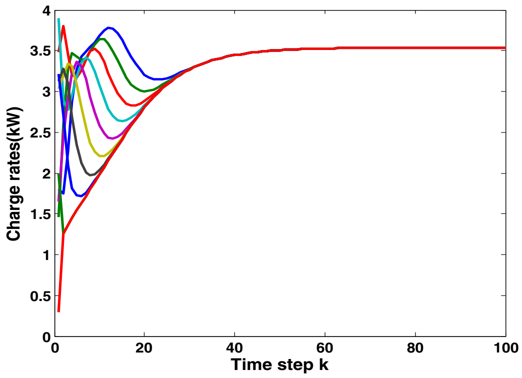

Fig. 3. Active charge rates for EVs connected to phase one with $\eta = 0.5$ and $\mu = 0.01$.

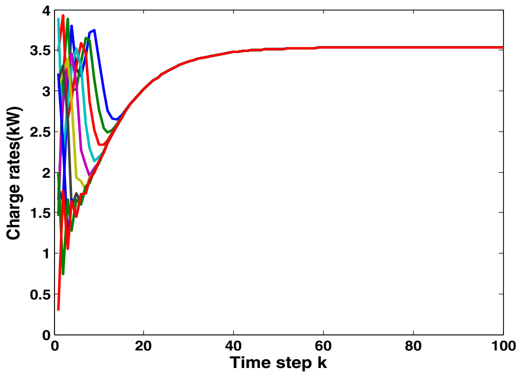

Fig. 4. Active charge rates for EVs connected to phase one with $\eta = 0.8$ and $\mu = 0.01$.

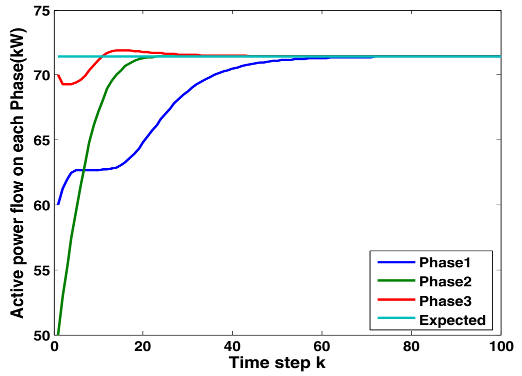

Fig. 5. Total power consumptions on each phase compared with desired result with $\eta = 0.5$ and $\mu = 0.01$.

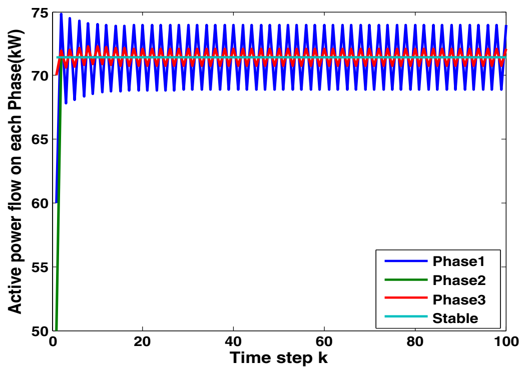

Fig. 6. Active charge rates for EVs connected to phase one with $\eta = 0.5$ and $\mu = 0.2$.

be possible to provide active power phase balancing using the available EVs alone. Thus, it may also be necessary to have other sources which could provide this ancillary service.

6. CONCLUSION

Increasing penetration of EVs poses major challenges to the electricity grid, but also offers opportunities for enhanced flexibility and operating efficiency if charging is appropriately coordinated. This paper has proposed a distributed consensus control framework and algorithms that can deliver the required coordination in an efficient manner. The proposed solution seeks to regulate EV charging in order to prevent overloading of the grid, while at the same time coordinating charging activity across phases so as to reduce power losses. Simulation results using realistic power system simulations confirm the efficacy of the consensus control algorithms and demonstrate, in particular, that provided sufficient numbers of EVs are connected to the grid a substantial positive impact can be achieved.

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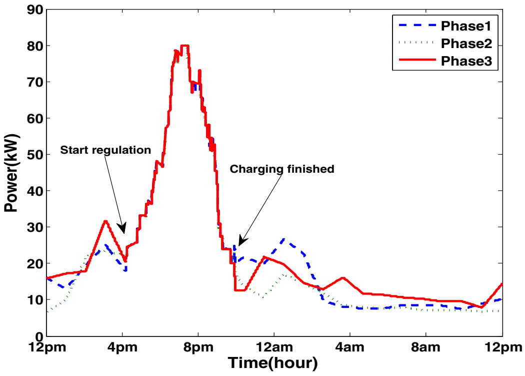

Fig. 7. Active power consumption on each phase with distributed consensus algorithm.

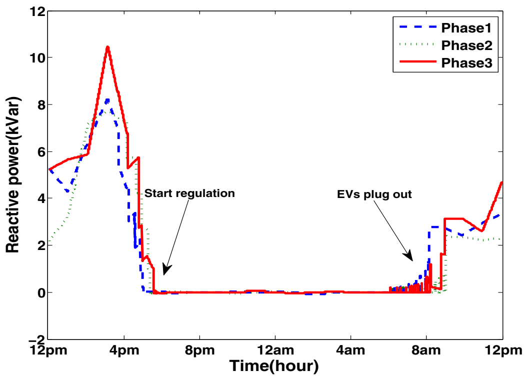

Fig. 8. Reactive power consumption on each phase with distributed consensus algorithm.

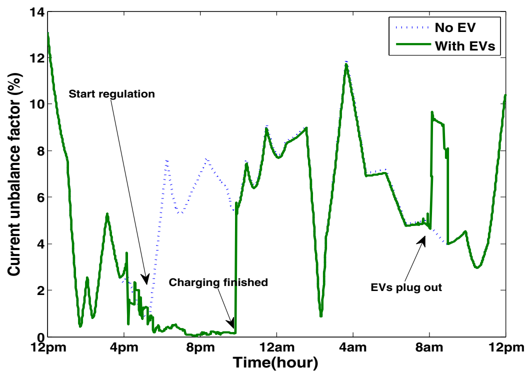

Fig. 9. Comparison of current balance factor

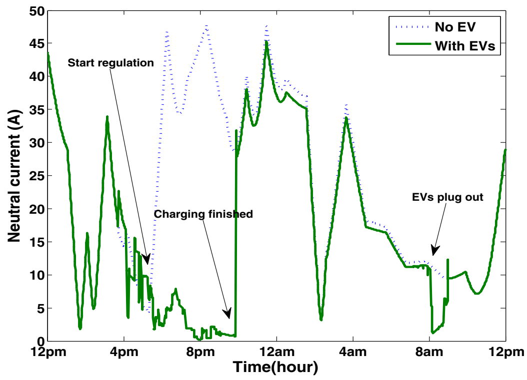

Fig. 10. Comparison of neutral current magnitude

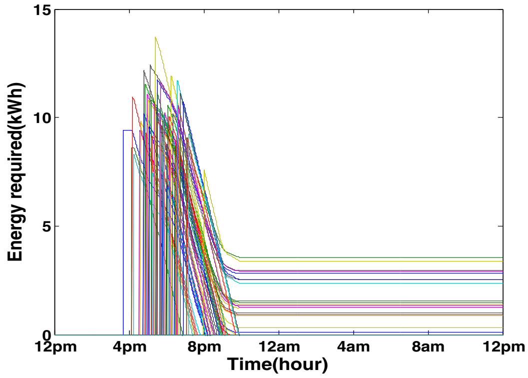

Fig. 11. Energy requirements for all EVs

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