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Optimal feedforward compensators for integrating plants ⋆

Carlos Rodríguez * **José Luis Guzmán** * **Manuel Berenguel** *
Julio E. Normey-Rico **

* *Informatics Department, University of Almería, Almería, Spain (e-mail: {karlos.rc, joseluis.guzman, beren}@ual.es)*
** *Automation and Systems Department, Federal University of Santa Catarina, Florianópolis, SC 88040-970 Brazil (e-mail: julio.normey@ufsc.br)*

Abstract: This paper addresses the design of feedforward compensators for integrating processes. Initially, the disturbance rejection problem for a classic two degrees-of-freedom control scheme with feedfoward is analyzed to highlight the problem caused by integrating dynamics. Afterwards, two simple tuning rules are derived to obtain undershoot-free responses based only a desired settling time or by satisfying a tradeoff between maximum peak and settling time specifications. Finally, some simulations are shown to prove the advantages of the proposed controller. © *Copyright IFAC 2014*

Keywords: Process control; Feedforward control; PID control; Disturbance rejection.

1. INTRODUCTION

Feedforwarding measurable disturbance signals to compensate their effects before they affect the system is a classic strategy in process control [Hägglund, 2013]. Even though feedforward control is an old topic [Seborg et al., 2004], most existing tuning rules only consider the ideal cases or are only applied to very specific problems [Nisenfeld and Miyasak, 1973, Seborg et al., 2004].

The ideal feedforward compensator within a classic feedforward scheme is formed as the quotient of the reversed sign dynamics between the measurable disturbance and the process output divided by the dynamics between the control signal and the process output. However, in many cases this controller becomes non-realizable due to several causes: non-realizable delay inversion, non-minimum phase zeros, unstable poles, integrating dynamics or improper transfer function [Seborg et al., 2004, Guzmán et al., 2012].

In those cases where the perfect feedforward controller is not realizable, the effect of the measurable disturbance can not be totally rejected from feedback error using a classic feedforward scheme. In [Brosilow and Joseph, 2012], a non-interacting feedforward structure was introduced to cope with this problem by introducing a new block. This scheme greatly simplifies feedforward compensator design, as an independent nominal analysis can be done for both reference tracking and disturbance rejection even if the ideal compensator is not realizable. However, the main limitation of this scheme is that it cannot deal with unstable or integrating plants.

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Recently, feedforward controller tuning rules have appeared in the literature within classic and non-interacting feedforward schemes. [Guzmán and Hägglund, 2011] proposed a design based on the minimization of integral absolute error and the reduction of undershoot for the case when ideal feedforward is not realizable due to delay inversion problems. Similar results within a non-interacting feedforward scheme were also pointed in [Hast and Hägglund, 2012] and [Rodríguez et al., 2013], where the objective was the minimization of the integral squared error. All of these rules are based on simple first-order plus time delay systems, and their extension to higher-order dynamics is seldom achievable.

A different approach for stable systems is proposed in [Vilanova, 2007], where the authors establish a general design framework, in which a robust tuning procedure within an internal model control structure is used. This strategy was later extended to unstable processes in [Vilanova et al., 2009]. However, this control structure as well as those with feedforward made from the reference require a different design and are not treated in this work.

Within a classic feedforward scheme, a methodology to design feedforward compensators by shaping the disturbance rejection response for the case when ideal feedforward is not realizable due to plants with integrating dynamics is required. To suggest simple tuning rules for this case is the main contribution of this paper.

The paper is organized as follows. A brief overview of the classic feedforward scheme including closed-loop relationships is presented in section 2. Section 3 introduces the proposed design methodology for shaping the disturbance rejection response. Two simple rules to define the shape response according to settling time or as a tradeoff between maximum peak and settling time are obtained. In section

4, the proposed design is tested with some simulations. Finally, section 5 conducts the conclusions of the work.

2. CONTROL SCHEME

In this section, the classic feedforward control together with a two degrees-of-freedom (2DOF) structure is described. It is a well-known structure which allows to compensate measurable disturbance effect as soon as possible with an independent design for reference tracking and disturbance rejection. The main advantage with respect to classic feedback is that a control action is supplied even if there is no feedback error.

Fig. 1 presents the classic feedforward block diagram. There are two processes P_u and P_d relating the process output y with the control signal u and the measurable disturbance d, respectively. A primary controller C_{fb} and a reference filter F_r are used within a 2DOF closed-loop system for reference tracking purposes. Moreover, the feedforward compensator C_{ff} is connected in open-loop to counteract measurable disturbance effects.

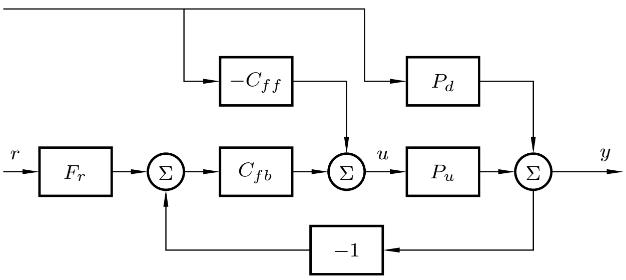

Fig. 1. Block diagram illustrating a 2DOF + feedforward control scheme

The relationships for reference tracking and disturbance rejection within this scheme are

$$\frac{y(s)}{r(s)} = \frac{F_r(s)L(s)}{1+L(s)} \tag{1}$$

$$\frac{y(s)}{d(s)} = \frac{P_{ff}(s)}{1+L(s)} \tag{2}$$

where $L(s) = C_{fb}(s)P_u(s)$ is the open-loop direct chain, and $P_{ff}(s) = P_d(s) - C_{ff}(s)P_u(s)$ is the open-loop disturbance rejection chain.

Note that within this scheme, perfect disturbance rejection is achieved for $C_{ff}(s) = P_d(s)/P_u(s)$. However, when the ideal compensator is not realizable, it can be observed that an interaction between $C_{fb}(s)$ and $C_{ff}(s)$ arises [Guzmán and Hägglund, 2011, Guzmán et al., 2012].

In what follows, the special case of integrating plants is presented and a procedure for shaping the disturbance rejection response based on a desired settling time is derived. Furthermore, an optimal controller which finds a satisfying tradeoff between maximum peak and settling time is proposed.

3. FEEDFORWARD DESIGN

In this section, the problem of integrating processes is presented and the controller design approach is addressed. Let us consider the following process descriptions

$$P_u(s) = \frac{\kappa_u}{D_u(s)s^{t_u}} \tag{3}$$

$$P_d(s) = \frac{\kappa_d}{D_d^-(s)} \tag{4}$$

such that t_u is the type of process $P_u(s)$, $D_u(s) = 1 + \sum_{i=1}^{n_u} a_u[i]s^i$ is a polynomial of degree n_u and $D_d^-(s) = 1 + \sum_{i=1}^{n_d} a_d[i]s^i$ is a polynomial of degree n_d with all its roots in the left half plane (LHP). Note that it is supposed without any loss of generality that $D_u(0) = D_d^-(0) = 1$ to ensure that κ_u and κ_d are process integrator and static gains, respectively.

As well-known, within a 2DOF control scheme, it is possible to shape the reference tracking response by correct tuning reference filter and feedback controller.

Let us consider

$$F_r(s) = \frac{1}{D_{fr}(s)} \tag{5}$$

$$C_{fb}(s) = \kappa_{fb}\frac{N_{fb}(s)}{D_{fb}(s)s^{t_{fb}}} \tag{6}$$

such that t_{fb} is the type of $C_{fb}(s)$ and $D_{fr}(0) = N_{fb}(0) = D_{fb}(0) = 1$ to ensure that 1 and κ_{fb} are $F_r(s)$ and $C_{fb}(s)$ static and integrator gains, respectively.

The reference tracking response can now be expressed as

$$\begin{aligned}\frac{y(s)}{r(s)} &= \frac{1}{D_{fr}(s)}\frac{N_{fb}(s)}{N_{fb}(s) + \frac{D_{fb}(s)D_u(s)s^{t_{fb}+t_u}}{\kappa_{fb}\kappa_u}} \\ &= \frac{1}{D_{fr}(s)}\frac{N_{fb}(s)}{D_{cl}(s)}\end{aligned} \tag{7}$$

where $D_{cl}(s)$ is a polynomial of degree n_{cl} that represents the closed-loop system dynamics. Note that since $D_{cl}(0) = 1$, if $t_{fb} + t_u \geq 1$, the reference tracking response has unitary static gain. In fact, to achieve zero steady-state error against reference signals with t_r poles in $s = 0$ ($r(s) = s^{-t_r}$), it is necessary to set $t_{fb} \geq t_r - t_u$.

Furthermore, if it is set $D_{fr}(s) = N_{fb}(s)$, the following final expression is obtained

$$\frac{y(s)}{r(s)} = \frac{1}{D_{cl}(s)} \tag{8}$$

Remember that since $D_{cl}(0) = 1$, expression (8) has unitary static gain.

3.1 Disturbance rejection

Within a classic feedforward scheme (see Fig. 1), it is possible to improve the disturbance rejection behaviour even if unstable dynamics exist in process $P_u(s)$. In fact, equation (2) can be expressed as

$$\begin{aligned}\frac{y(s)}{d(s)} &= \left(\frac{\kappa_d}{D_d^-(s)} - C_{ff}(s)\frac{\kappa_u}{D_u(s)}s^{-t_u}\right)\frac{D_u(s)s^{t_u}D_{fb}(s)s^{t_{fb}}}{D_{cl}(s)} \\ &= \left(\frac{\kappa_d D_u(s)s^{t_u}}{D_d^-(s)} - C_{ff}(s)\kappa_u\right)\frac{D_{fb}(s)s^{t_{fb}}}{D_{cl}(s)}\end{aligned} \tag{9}$$

where it can be observed that even unstable dynamics of $P_u(s)$ caused by its poles located in the right half plane (RHP) or at the origin have disappeared from the denominator.

However, when the process $P_u(s)$ has integrating behavior, pure derivative terms should be included in $C_{ff}(s)$ in order to achieve a perfect cancellation. Thus, non-realizable ideal controllers are possible in this case and tuning rules for the feedforward compensator are required. In this work, a proposal for the feedforward controller transfer function to obtain an undershoot-free response is presented. To this end, the feedforward compensator is defined as

$$C_{ff}(s) = \frac{\kappa_d}{\kappa_u} \frac{1}{D_{fb}(s)D_d^-(s)} \frac{1+\sum_{i=1}^{m_{ff}} \beta_{ff}[i]s^i}{(\tau_{ff}s+1)^{n_{ff}}} \quad (10)$$

such that $C_{ff}(0) = \kappa_d/\kappa_u$, and τ_{ff} will be the only tuning parameter.

Using (10) in (9), after some operations, equation (9) can be expressed as

$$\frac{y(s)}{d(s)} = G_{y/d}(s) = \frac{-\kappa_d s^{t_{fb}}}{(\tau_{ff}s+1)^{n_{ff}}} \frac{P(s)}{D_{cl}(s)D_d^-(s)} \quad (11)$$

with

$$P(s) = 1 + \sum_{i=1}^{m_{ff}} \beta_{ff}[i]s^i - (\tau_{ff}s+1)^{n_{ff}} D_{fb}(s)D_u(s)s^{t_u} \quad (12)$$

such that $P(0) = 1$.

Note that to achieve zero steady-state error against disturbance signals with t_d poles in $s = 0$ ($d(s) = s^{-t_d}$), it is necessary to set $t_{fb} \geq t_d$.

The idea is to cancel all stable roots of $D_{cl}(s)$ and $D_d^-(s)$ with $\beta_{ff}[i]$ coefficients, and therefore it is necessary to have the same number of $\beta_{ff}[i]$ coefficients that the sum of the degrees of both polynomials,

$$m_{ff} = n_{cl} + n_d \quad (13)$$

in order to solve a system of m_{ff} equations. Furthermore, it is considered that

$$\begin{aligned} n_{ff} &= \max(m_{ff} - (n_{fb} + n_d), 1) \\ &= \max(n_{cl} - n_{fb}, 1) \end{aligned} \quad (14)$$

to guarantee a realizable compensator, with n_{fb} the degree of $D_{fb}(s)$.

Moreover, the degree of $P(s)$ in (12) is desired to be equal to m_{ff} to avoid undesired zeros. Therefore, the constraint $n_{ff} + n_{fb} + n_u + t_u \leq m_{ff}$ must be satisfied. Note this is not a severe restriction since m_{ff} may be increased to cancel non-dominant poles — located far from the origin — in $D_d^-(s)$ or $D_{cl}(s)$.

As observed from (11), the resulting response will not present any undesired dynamics or undershoot (notice the transfer function with multiple real poles and a pure derivative term). This fact can be clearly observed by its consequent time response against unitary step [1] in d, which is given by

$$y(t) = \frac{-\kappa_d t^{n_{ff}-1}}{\tau_{ff}^{n_{ff}}(n_{ff}-1)!} e^{-\frac{t}{\tau_{ff}}} \quad (15)$$

[1] Expression (15) is also obtained for higher values of t_d if it is considered $t_{fb} = t_d$

Then, what remains is to determine the τ_{ff} value in order to obtain the desired response. In the following, two different solutions for τ_{ff} are given, resulting in simple tuning rules. First, τ_{ff} is used to obtain a desired settling time. Afterwards, an optimal solution for τ_{ff} is derived based on the computation of a satisfying tradeoff between maximum peak and settling time.

3.2 Settling time tuning rule

In (11), since $G_{y/d}(\infty)$ is the same as $G_{y/d}(0)$ — there is a pure derivative zero—, the settling time $t_{5\%}$ is defined as the time when the system reaches around 5% [2] of its maximum peak value. Therefore, it can be computed as

$$y(t_{5\%}) = 0.05M_{peak} \quad (16)$$

Thus, to calculate M_{peak}, first it is necessary to compute its peak time t_{peak} making the derivative of (15) with respect to t

$$\frac{dy(t)}{dt} = -\frac{\kappa_d e^{-\frac{t}{\tau_{ff}}}}{(n_{ff}-1)!\tau_{ff}^{n_{ff}}} \left((n_{ff}-1)t^{n_{ff}-2} - \frac{t^{n_{ff}-1}}{\tau_{ff}} \right) \quad (17)$$

equal to zero

$$e^{-\frac{t_{peak}}{\tau_{ff}}} t_{peak}^{n_{ff}-2} (\tau_{ff}(n_{ff}-1) - t_{peak}) = 0 \quad (18)$$

A solution for (18) is

$$t_{peak} = \tau_{ff}(n_{ff}-1). \quad (19)$$

Note that this solution includes $t = 0$ for the biproper case where $n_{ff} = 1$. Thus, the maximum peak M_{peak} is given by

$$M_{peak} = y(t_{peak}) = -\frac{\kappa_d}{\tau_{ff}} \frac{e^{1-n_{ff}}(n_{ff}-1)^{n_{ff}-1}}{(n_{ff}-1)!} \quad (20)$$

If we substitute (20) in (16), an analytical solution can be only found for $n_{ff} = 1$

$$t_{5\%} \approx 3\tau_{ff}. \quad (21)$$

For higher values of n_{ff}, explicit solutions can be found by solving numerically the equation

$$0.05 - \frac{x^{n_{ff}-1}}{(n_{ff}-1)^{n_{ff}-1}} e^{-x+n_{ff}-1} = 0 \quad (22)$$

with

$$x = t_{5\%}/\tau_{ff}. \quad (23)$$

Therefore, it is possible to shape the disturbance rejection response to meet a desired settling time, $t_{5\%}$, by using the following tuning rule for τ_{ff}

$$\tau_{ff} = \frac{t_{5\%}}{x} \quad (24)$$

3.3 Optimal tuning rule

A tradeoff arises from the fact that by making τ_{ff} small, the settling time is reduced (as observed in (23)) but the maximum peak (20) is increased.

[2] Other % values can be chosen without any change in the whole development.

One interesting characteristic of equation (15) is that its area does not depend on τ_{ff} and therefore it is not possible to find optimal shapes which minimize the integral error. So, a cost function to find a tradeoff between settling time and maximum peak can be proposed as follows

$$J = \alpha t_{5\%} + (1-\alpha)\left|M_{peak}\right| \quad \alpha \in (0,1) \tag{25}$$

where α is a weighting parameter.

Then, substituting (20) and (24) in (25), when (25) is derivative with respect to τ_{ff}

$$\frac{dJ}{d\tau_{ff}} = \alpha x - (1-\alpha)\frac{|\kappa_d|}{\tau_{ff}^2}\frac{e^{1-n_{ff}}\left(n_{ff}-1\right)^{n_{ff}-1}}{\left(n_{ff}-1\right)!} \tag{26}$$

and is taken equal to zero

$$\alpha x \tau_{ff}^2 - (1-\alpha)\left|\kappa_d\right|\frac{e^{1-n_{ff}}\left(n_{ff}-1\right)^{n_{ff}-1}}{\left(n_{ff}-1\right)!} = 0 \tag{27}$$

the following solution is obtained

$$\tau_{ff} = \sqrt{\left|\kappa_d\right|\frac{(1-\alpha)}{\alpha}\frac{e^{1-n_{ff}}\left(n_{ff}-1\right)^{n_{ff}-1}}{x\left(n_{ff}-1\right)!}} \tag{28}$$

Note this is the only feasible solution since τ_{ff} must be greater than zero to achieve a stable compensator, and α can be easily used as a tuning parameter to find a desired tradeoff between settling time and maximum peak values.

3.4 Guideline summary

The steps to tune the feedforward compensator according to the proposed methodology are:

(1) Set $n_{ff} = \max(n_{cl} - n_{fb}, 1)$.
(2) Set $m_{ff} = n_{cl} + n_d$.
(3) If the constraint $n_{ff} + n_{fb} + n_u + t_u \leq m_{ff}$ is not satisfied, increase the degree of $D_d^-(s)$ with a non-dominant pole and go to step 2. Otherwise, go to step 4.
(4) Set τ_{ff} according to the desired specification:

$$\begin{aligned} \text{Settling time}: &\quad \tau_{ff} = t_{5\%}/x \\ \text{Optimal}: &\quad \text{tuning rule (28)} \end{aligned}$$

(5) Obtain the coefficients $\beta_{ff}[i]$ such that

$$P(s) = D_d^-(s)D_{cl}(s) \tag{29}$$

(6) End of design.

4. RESULTS

In this section, low- and high-order process examples are presented to evaluate the proposed tuning rules. The results are compared to classic tuning rules presented in the literature, namely, static feedforward controllers and lead-lag compensators.

4.1 Settling time specifications

Consider the following process transfer functions

$$P_u(s) = \frac{1}{s\,(0.25s+1)} \tag{30}$$

$$P_d(s) = \frac{0.5}{0.9s+1} \tag{31}$$

To obtain a reference tracking response with the closed-loop dynamics given by $D_{cl}(s) = \left(0.25s^2 + 0.75s + 1\right)^2$, the feedback controller is selected as a PID controller with a filter in the derivative term such that

$$C_{fb}(s) = 2\frac{0.56s^2 + 1.5s + 1}{s\,(0.5s+1)} \tag{32}$$

and the reference filter $F_r(s)$ is set as

$$F_r(s) = \frac{1}{0.56s^2 + 1.5s + 1} \tag{33}$$

Then, the feedforward compensator is defined using the guideline presented in Subsection 3.4 resulting in

$$C_{ff}(s) = \frac{0.5}{(0.025s+1)\,(0.9s+1)\,(0.5s+1)}\frac{1+\sum_{i=1}^{6}\beta_{ff}[i]s^i}{\left(\tau_{ff}s+1\right)^3} \tag{34}$$

where a pole located at $s = -40$ was introduced in $D_d^-(s)$ to ensure that constraint $n_{ff} + n_{fb} + n_u + t_u \leq m_{ff}$ is satisfied.

Afterwards, τ_{ff} is set according to the following settling time specification

$$\tau_{ff} = 0.13t_{5\%} \tag{35}$$

Once the τ_{ff} is set, what remains is to calculate the β_{ff} coefficients. Several tunings are evaluated for different settling time values, such as presented in Table 1.

Fig. 2 shows the process output and control effort for classic (static gain compensator as $C_{ff}(s) = 0.5$ and lead-lag compensator as $C_{ff}(s) = 0.5(0.25s+1)/(0.9s+1)$) and the proposed feedforward compensators. In the simulation, a step disturbance of $d = 0.6$ is applied at time instant $t = 1$. Note that the feedforward compensators designed using the presented methodology satisfy the design specifications — undershoot-free response with fixed settling time —, while the controllers tuned using classic methods obtain oscillatory solutions. Numerical results are shown in Table 2 including norm-1 (integrated absolute error - IAE), norm-2 and the initial control peak ($\|y(t)\|_1$, $\|y(t)\|_2$, u_{init}) as performance measurements. A relation between low settling time and large control effort values can be observed. Note also how similar the areas given by $\|y(t)\|_1$ are with the proposed controller. This fact concludes that by using the proposed tuning rule, the desired settling time can be reached by keeping the same IAE value.

4.2 Optimal tuning rule

Consider the following process descriptions

$$P_u(s) = \frac{1}{s\,(s+1)} \tag{36}$$

$$P_d(s) = \frac{0.75}{(0.35s+1)^3} \tag{37}$$

Again, to obtain a closed-loop response given by $D_{cl}(s) = \left(0.25s^2 + 0.75s + 1\right)^2$, the feedback controller is selected as a PID controller with a filter in the derivative term such that

$$C_{fb}(s) = 3.2\frac{0.75s^2 + 1.5s + 1}{s\,(0.2s+1)} \tag{38}$$

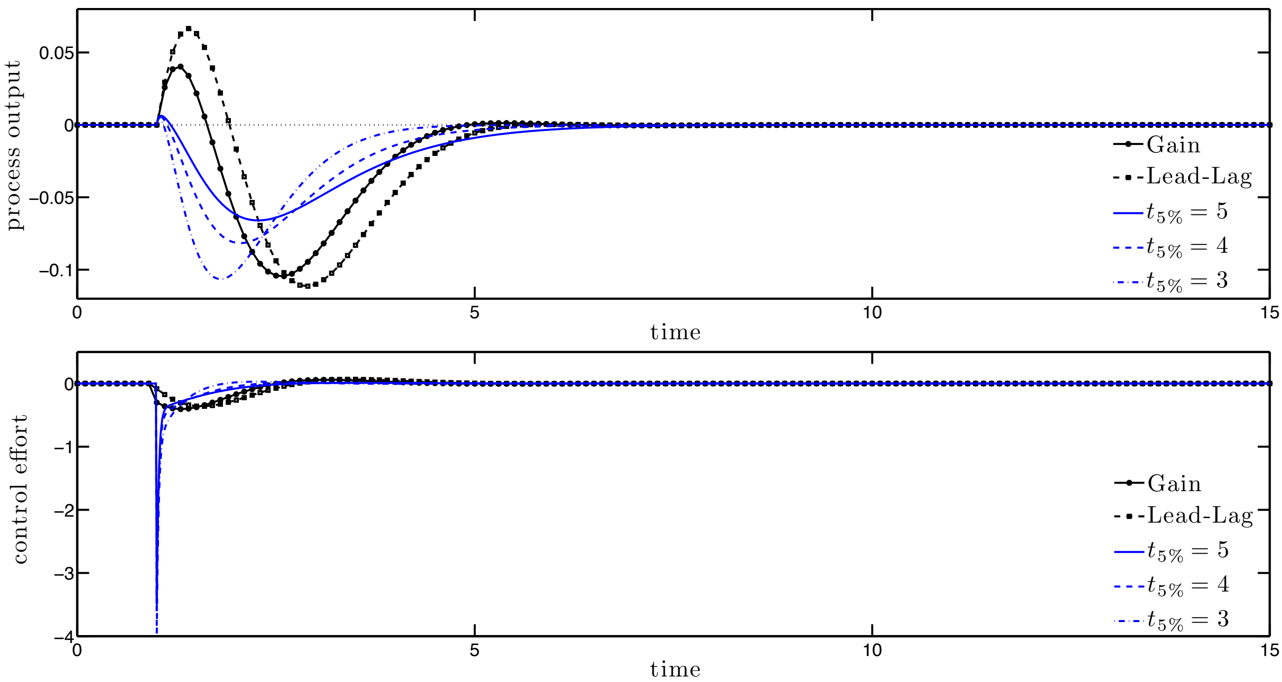

Fig. 2. Simulation for low-order process example and settling-time tuning rule

Table 1. Feedforward compensators parameters for low-order process example and settling-time tuning rule

Feedforward controller	$\beta_{ff}[1]$	$\beta_{ff}[2]$	$\beta_{ff}[3]$	$\beta_{ff}[4]$	$\beta_{ff}[5]$	$\beta_{ff}[6]$	τ_{ff}
$t_{5\%} = 5$	3.42	5.17	4.25	1.90	0.43	0.04	0.65
$t_{5\%} = 4$	3.42	4.78	3.50	1.38	0.27	0.02	0.52
$t_{5\%} = 3$	3.42	4.39	2.85	0.98	0.17	0.01	0.39

and the reference filter $F_r(s)$ is set as

$$F_r(s) = \frac{1}{0.75s^2 + 1.5s + 1} \tag{39}$$

The feedforward compensator is defined using the guideline presented in Subsection 3.4 resulting in

$$C_{ff}(s) = \frac{0.75}{(0.35s+1)^3(0.2s+1)} \frac{1 + \sum_{i=1}^{7} \beta_{ff}[i]s^i}{(\tau_{ff}s+1)^3} \tag{40}$$

In this case, the optimal feedforward tuning rule (28) is used for the different α values obtaining the compensator parameters presented in Table 3.

Table 2. Numerical results for the settling-time tuning rule example

Feedforward controller	$\|y(t)\|_1$	$\|y(t)\|_2$	u_{init}
Gain	18.57	1.16	−0.30
Lead-Lag	22.91	1.32	**−0.08**
$t_{5\%} = 5$	15.14	**0.83**	−3.47
$t_{5\%} = 4$	15.10	0.92	−3.60
$t_{5\%} = 3$	**15.05**	1.06	−3.96

Fig. 3 shows the process output and control effort for classic (static gain compensator as $C_{ff}(s) = 1$ and lead-lag compensator as $C_{ff}(s) = (s+1)/(0.35s+1)^3$) and the proposed optimal feedforward compensators. In the simulation, a step disturbance of $d = 0.6$ is applied at time instant $t = 1$. Note that a compromise between settling time and maximum peak is found since the area obtained with the proposed controllers is exactly the same as was already observed in section 3.1. Numerical results for the simulation are shown in Table 4. Moreover, classic tuning rules achieve a similar performance — both responses are almost overlapped — due to the aggressive tuning of the feedback controller. It can be observed that by increasing τ_{ff}, the initial control peak is reduced as well as the norm-2, while the norm-1 remains the same. Therefore, the parameter τ_{ff} could be considered as bigger as possible — α has to be small — to reduce both $\|y(t)\|_2$ and u_{init}, allowing higher settling times.

5. CONCLUSIONS

In this paper, the problem of disturbance compensation for the case when ideal feedforward compensator is not realizable due to the existence of integrating dynamics in the main process is addressed. Initially, a straightforward methodology to shape the desired response based on a desired settling time is derived. Later, a simple optimal tuning rule is proposed to establish a satisfying tradeoff between maximum peak and settling time.

Simulation examples were presented to illustrate the tuning procedure and to show that the proposed controller obtains better performance than classic tuning rules presented in the literature. A limitation of the proposed

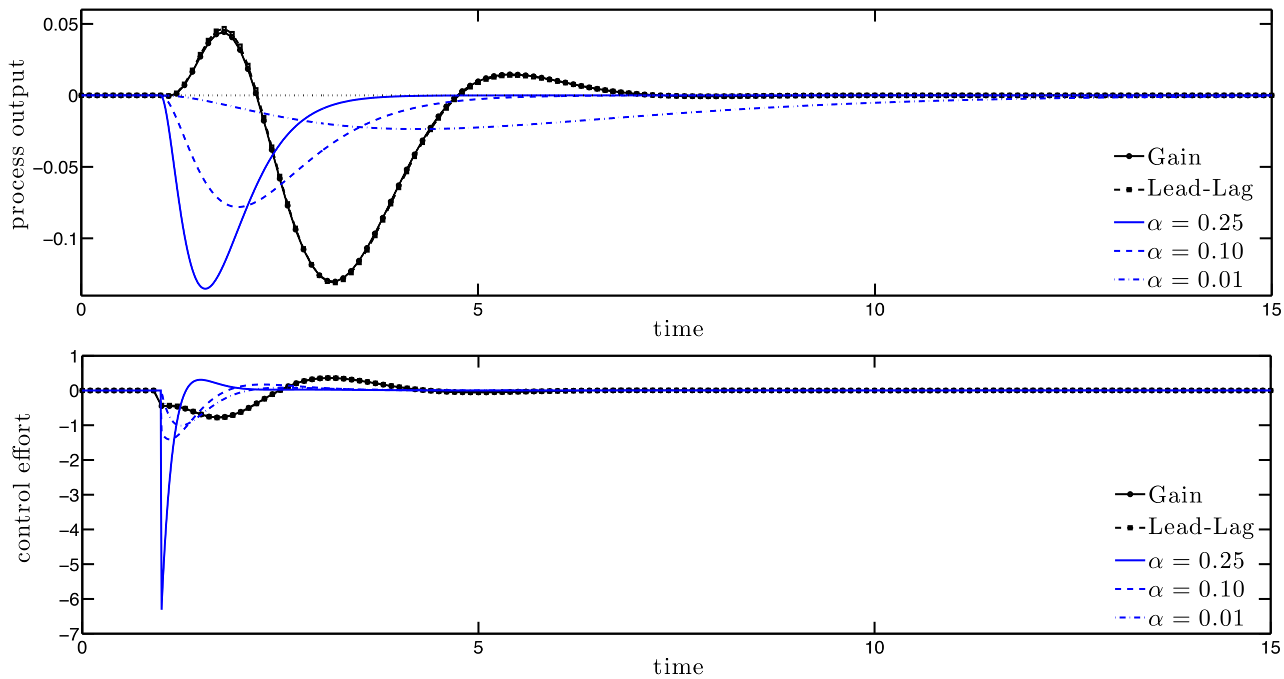

Fig. 3. Simulation for high-order process example and optimal tuning rule

Table 3. Feedforward compensators parameters for high-order process example and optimal tuning rule

Feedforward controller	$\beta_{ff}[1]$	$\beta_{ff}[2]$	$\beta_{ff}[3]$	$\beta_{ff}[4]$	$\beta_{ff}[5]$	$\beta_{ff}[6]$	$\beta_{ff}[7]$	τ_{ff}
$\alpha = 0.25$	3.55	5.05	3.54	1.39	0.32	0.04	0.01	0.28
$\alpha = 0.10$	3.55	5.67	4.75	2.17	0.53	0.06	0.01	0.49
$\alpha = 0.01$	3.55	9.06	15.95	15.52	6.89	6.88	0.01	1.62

Table 4. Numerical results for the optimal tuning rule example

Feedforward controller	$\|y(t)\|_1$	$\|y(t)\|_2$	u_{init}
Gain	23.35	1.40	-0.45
Lead-Lag	23.60	1.41	-0.43
$\alpha = 0.25$	**14.06**	1.15	-6.31
$\alpha = 0.10$	**14.06**	0.87	-1.21
$\alpha = 0.01$	**14.06**	**0.48**	$\mathbf{-0.03}$

methodology is that it is based on nominal processes without uncertainty and further robustness analysis for disturbance rejection are required. This point will be analyzed in future works.

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