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# Iterative Learning Control with Time Domain Prediction using Laguerre Functions

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**Abstract:** This paper develops an iterative learning control algorithm starting from some recent results in the area of predictive repetitive control. The algorithm uses receding horizon control and Laguerre functions to parameterize the future control trajectory, where the Laguerre functions reduce the number of parameters requiring optimization on-line. Stability of the predictive iterative learning control system is analyzed and conditions on error convergence are established. Supporting experimental results from application to a robot arm are also given.

Keywords: iterative learning control, convergence, control law design.

## 1. INTRODUCTION

Many systems complete the same finite duration task over and over again. The sequence is that the task is completed, the system resets to the starting location, the next one is completed and so on. In this paper each execution is termed a trial and the duration the trial length. Such systems arise in many industrial applications, where a generic example is a gantry robot undertaking a pick and place task with the sequence of operations: i) collect the object from a fixed location, ii) transfer it over a finite duration, iii) place it at a static location or on a moving conveyor, iv) return to the starting location and v) repeat the previous four steps for as many times as required or until a halt is needed for maintenance or other reasons. Each execution is known as a trial and the execution time the trial length.

Once a trial is complete all information generated during its production is available for use in computing the control signal to be applied on the next trial. Iterative Learning Control (ILC), for which the first work is widely credited to [Arimoto et al., 1984], uses information generated on the previous trial, or a finite number thereof, in the computation of the input to be applied on the next trial and the survey papers [Bristow et al., 1984, Ahn et al., 2007] are one starting point for the literature. Repetitive control has been developed for cases where the process or plant output is required to track a given periodic signal with the novel feature is that information from previous periods or trials is used to modify the control signal [Hara et al., 1988]. The reference signal used in repetitive control is a periodic function in time.

One extensively studied class of ILC laws for linear dynamics is based on the minimization of a cost function constructed from the addition of two quadratic terms. The first of these is formed from the current trial error, i.e., the difference between the supplied reference signal and the current trial output and the second from the difference between the control signals used on successive trials. This class of algorithms is termed norm optimal and experimental verification of the performance of members of this class have also been reported, e.g., [Barton and Alleyne, 2011].

This paper develops a predictive ILC law that uses a similar cost function to the one in norm optimal ILC, but with the reference signal model embedded in the controller and use of the receding horizon control principle. The idea of embedding the reference signal information in the controller was successfully used in for, example, [Wang, 2009] and in other ILC related research [Moore and El-Sharif, 2009]. In this paper the wealth of information that the reference signal contains is embedded in the ILC design.

## 2. BACKGROUND

The design in this paper is based on a frequency domain decomposition of the supplied reference signal or vector in the single-input, single-output (SISO) and multiple-input, multiple-output (MIMO) cases respectively. Once these are selected they are embedded in the process state-space model in accordance with the internal model principle.

Consider first the SISO case and suppose that the frequency components of the reference signal to be included in the design have been selected, for the details see [Wang et al., 2012], and form the polynomial

$$\begin{aligned} D(z) &= (1 - z^{-1})\Pi_{i=1}^{l}(1 - 2cos(i\omega)z^{-1} + z^{-2}) \\ &= 1 + d_1 z^{-1} + d_2 z^{-2} + d_3 z^{-3} + \ldots + d_\gamma z^{-\gamma}, \end{aligned} \quad (1)$$

where 0 and $i\ \omega$, $i = 1, 2, \ldots, l$, for some chosen positive integer $l$ denotes the frequencies to be included. The control law is to be designed to track the reference signal and hence, by the internal model principle [Francis and Wonham, 1975], $D(z)$ must be embedded in the denominator of the controller transfer-function and one way of doing this is as follows.

In the MIMO case, suppose that the plant to be controlled has input, output and state vectors $u(k) \in \mathbb{R}^{m_u}$, $y(k) \in \mathbb{R}^{m_y}$ and $x_m(k) \in \mathbb{R}^{n_1}$, respectively, and state-space model matrices $\{A_m, B_m, C_m\}$. Then the frequency components of the reference vector can, as one approach, be embedded in the design by adding a vector $\mu(k) \in \mathbf{R}^{n_1}$ to the state dynamics to obtain

$$\begin{aligned} x_m(k+1) &= A_m x_m(k) + B_m u(k) + \mu(k), \\ y(k) &= C_m x_m(k), \end{aligned} \tag{2}$$

where each entry in $\mu(k)$ is the inverse $z$-transform of $\frac{1}{D(z)}$ defined by (1). (This approach extends naturally to examples where it is required to embed a different number of frequencies for some of the entries in the reference vector.)

Let $q^{-1}$ denote the backward shift operator and $D(q^{-1})$ the shift operator interpretation of $D(z)$. Applying $D(q^{-1})$ to $x_m(k)$ and $u(k)$ of (2) gives

$$x_s(k) = D(q^{-1})x_m(k), \;\; u_s(k) = D(q^{-1})u(k),$$

Also $D(q^{-1})\mu(k) = 0$ and from (2)

$$\begin{aligned} x_s(k+1) &= A_m x_s(k) + B_m u_s(k), \\ D(q^{-1})y(k+1) &= C_m A_m x_s(k) + C_m B_m u_s(k). \end{aligned} \tag{3}$$

Introducing the state vector

$$x(k) = \left[\, x_s^T(k) \;\; y^T(k) \;\ldots\; y^T(k-\gamma+1) \,\right]^T,$$

gives the following augmented state-space model for design

$$\begin{aligned} x(k+1) &= Ax(k) + Bu_s(k) \\ y(k) &= Cx(k) \end{aligned} \tag{4}$$

where

$$A = \begin{bmatrix} A_m & 0 \\ \hat{C} & A_d \end{bmatrix}, \;\; \hat{C} = \begin{bmatrix} C_m A_m \\ 0 \end{bmatrix},$$

$$A_d = \begin{bmatrix} -d_1 I & -d_2 I & \ldots & -d_{\gamma-1} I & -d_\gamma I \\ I & 0 & 0 & \ldots & 0 \\ 0 & I & 0 & \ldots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \ldots & I & 0 \end{bmatrix}$$

and 0 and $I$ denote the zero and identity matrices, respectively, of compatible dimensions. Also

$$\begin{aligned} B &= \left[\, B_m^T \;\; (C_m B_m)^T \;\; 0 \;\ldots\; 0 \;\; 0 \,\right]^T, \\ C &= [\, 0 \;\; I \;\; 0 \;\ldots\; 0 \;\; 0 \,]. \end{aligned}$$

The poles of (4) are the union of those for plant model and that representing the embedded reference frequencies.

## 3. ILC DESIGN

In the ILC setting the trial number is denoted by the superscript $j = 0, 1, 2, \ldots$. In this setting the process state-space model (4) is

$$\begin{aligned} x^j(k+1) &= Ax^j(k) + Bu_s^j(k), \\ y^j(k) &= Cx^j(k), \end{aligned} \tag{5}$$

where $x^j(k)$, $u_s^j(k)$ and $y^j(k)$ denote the state, filtered control and output vectors, respectively, at sampling instant $k$ on trial $j$. The state vector $x^j(k)$ is formed from two sub-vectors, the first of which is $x_s^j(k)$ and for stable systems is equal to the process state vector $x_m^j(k)$ filtered by the operator $D$. In the steady-state this vector is zero. If it is assumed that the process has reached the steady-state before a trial commences, the state initial vector on each trial can be assumed to be zero. The second sub-vector is formed from samples of the output vector, which is a measured output and hence its initial condition on trial $j$ is known.

To complete the ILC problem formulation, let $r(k)$ denote the reference vector to be tracked. Then the error on trial $j$ is

$$e^j(k) = y^j(k) - r(k)$$

and it is a straightforward step to write the dynamics in terms of the current trial error as a state-space model with the structure of (5) where $e^j(k)$ replaces $y^j(k)$ and the state vector $x^j(k)$ is replaced by

$$x^j(k) = \left[\, (x_s^j)^T(k) \;\; (e^j)^T(k) \;\ldots\; (e^j)^T(k-\gamma) \,\right]^T$$

On trial $j+1$ and sampling instant $k$, the future state vector along this trial, denoted $x^{j+1}(k+m \mid k)$ is, given $x^{j+1}(k)$ and the required filtered control inputs, obtained from the state equation in (5) as

$$x^{j+1}(k+m \mid k) = A^m x^{j+1}(k) + \sum_{i=0}^{m-1} A^{m-i-1} B u_s^{j+1}(i),$$

where $m$ is a future sampling instant.

In control problems that require the modeling of the future control trajectory, one approach is to embed an integrator in the design and the incremental control trajectory is then directly computed within an optimization window. For the ILC design considered in this paper, the signal to be optimized is the filtered control signal $u_s^j(k)$ on trial $j$ and the design could be undertaken by modeling this signal using pulse functions. The main drawback is the requirement to optimize a large number of parameters if fast sampling is required and/or the system has a relatively complex dynamic response.

Fast sampling is typically required for mechanical and electro-mechanical systems because the time constants arising in the various sub-components can vary in duration and a smaller sampling interval $\Delta t$ is required to capture the effects of the smaller of these. One approach to reduce the number of parameters requiring optimization on-line is to parameterize the future trajectory of the filtered control signal using a set of Laguerre functions, where a scaling factor is used to reflect the time scale of the predictive control system.

The use of Laguerre functions in model predictive control, including identification based models of the system dynamics, is detailed in, e.g., Wang [2004] and the following is a summary relevant to the new results in this paper, focusing on the SISO case with the natural MIMO extension noted For ease of notation, the ILC trial variable

$j$ is omitted in this presentation of required background material.

The basis of the design is the use of a set of discrete orthonormal functions to describe the filtered future control signal $u_s(m)$ within a moving horizon window, $0 \leq m \leq N_p$. Assume that $N$ is the number of terms in the expansion and let $l_i(m)$, $1 \leq i \leq N$, be a set of Laguerre functions, which are orthonormal. Then

$$u_s(m) \approx \sum_{h=1}^{N} c_h l_h(m), \tag{6}$$

where, in general, the coefficients $c_h$ are functions of $k$ but the notation here is used for simplicity.

In this application, the $z$ transfer-function of the $h$ th Laguerre function is given by

$$\Gamma_h(z) = \frac{\sqrt{1-a_h^2}}{1-a_h z^{-1}} \left( \frac{z^{-1}-a_h}{1-a_h z^{-1}} \right)^{h-1}, \tag{7}$$

where $0 \leq a_h < 1$ is the scaling factor. Also the network structure of the $z$ transfer-function representation (7) can be used to show that the set of discrete Laguerre functions satisfies the difference equation

$$L(m+1) = \Omega L(m), \tag{8}$$

where $L(m) = [\, l_1(m) \ l_2(m) \ l_N(m) \,]^T$

$$\Omega = \begin{bmatrix} a & 0 & \ldots \ldots & 0 \\ \beta & a & \ddots \ \vdots & 0 \\ -a\beta & \beta & \ddots \ 0 & 0 \\ \vdots & \vdots & \ddots \ \ddots & 0 \\ -a^{N-2}\beta & a^{N-3}\beta & \ldots \ \beta & a \end{bmatrix}, \tag{9}$$

$\beta = (1-a^2)$ and

$$L(0) = \sqrt{\beta} \left[\, 1 \ -a \ a^2 \ -a^3 \ \ldots \ (-1)^{N-1}a^{N-1} \,\right]^T.$$

Setting $a = 0$ and $\delta_i(m) = \delta(i)$, where $\delta(i)$ is the Dirac delta function recovers the standard formulation of model predictive control, i.e., the number of parameters to be optimized is not approximated. Hence the solution of the design problem given below is a computationally less demanding approximation that approaches the true problem under a well defined limiting operation.

Continuing with the MIMO case, given the state vector $x(p)$ the prediction of the future state at time $\tau$, written $x(p+\tau|k)$ can be written as

$$x(p+\tau \mid p) = A^\tau x(p) + \phi(\tau)\eta \tag{10}$$

where, if $B_i$ is the $i$th column of the state-space model input matrix

$$\eta = \left[\, \eta_1^T \ \eta_2^T \ \ldots \ \eta_{m_u}^T \,\right]^T$$

and

$$\phi(\tau) = \sum_{j=0}^{\tau-1} A^{\tau-j-1} \left[\, B_1 L_1^T(j) \ \ldots \ B_{m_u} L_{m_u}^T(j) \,\right]$$

Also the $i$th input is given by $L_i^T \eta_i$. where $L_i$ is generated by applying (8) for this input. Moreover, the number of terms and the scaling factor used in this last construction can be chosen independently for each input.

The basic idea in Laguerre function based design is to represent $u_s(i)$ by a set of Laguerre functions and associated coefficients. This is illustrated in the SISO case by

$$u_s(i) = L^T(i)\eta, \tag{11}$$

where the Laguerre function vector

$$L(i) = [\, l_1(i) \ l_2(i) \ \ldots \ l_N(i) \,]^T$$

and the Laguerre coefficient vector

$$\eta = [\, c_1 \ c_2 \ \ldots \ c_N \,]^T.$$

Moreover, $N$ is the dimension of the Laguerre function vector and is also the number of terms used in the approximation. The Laguerre functions are pre-determined in the design once the scaling factor $0 \leq a < 1$ and the number of terms $N$ are chosen.

In the SISO case, for simplicity, suppose that the control trajectory is approximated by a Laguerre polynomial Then the future state vector at time $m$ on trial $j+1$, i.e., $x^{j+1}(k+m|k)$, can be written as

$$x^{j+1}(k+m \mid k) = A^m x^{j+1}(k) + \phi^T(m), \eta^{j+1}, \tag{12}$$

where $\phi^T(m) = \sum_{i=0}^{m-1} A^{m-i-1} B L^T(m)$ and this term is independent of the trial number $j$.

The cost function for the MIMO ILC design is

$$\begin{aligned} J = & \sum_{m=1}^{N_p} x^{j+1}(k+m \mid k)^T Q x^{j+1}(k+m \mid k) \\ & + \sum_{m=0}^{N_p} (u_s^{j+1}(m) - u_s^j(m))^T R (u_s^{j+1}(m) - u_s^j(m)), \end{aligned} \tag{13}$$

where $Q$ and $R$ are symmetric positive definite matrices and also the difference between the control signals on the current trial previous trials is penalized. The motivation for this last choice is to achieve trial-to-trial error reduction without unduly large changes in the amplitudes of the control signals required.

The previous trial filtered input vector is also parameterized in the form detailed above with a long prediction horizon $N_p$ and hence

$$\sum_{m=0}^{N_p-1} u_s^j(m)^T R u_s^j(m) = (\eta^j)^T R_L \eta^j, \tag{14}$$

$$\sum_{m=0}^{N_p-1} u_s^{j+1}(m)^T R u_s^j(m) = (\eta^{j+1})^T R_L \eta^j, \tag{15}$$

$$\sum_{m=0}^{N_p-1} u_s^{j+1}(m)^T R u_s^{j+1}(m) = (\eta^{j+1})^T R_L \eta^{j+1}, \tag{16}$$

where the orthonormal property of the Laguerre functions has been used, i.e., $\sum_{m=0}^{N_p} L(m)^T L(m) = I$, and $R_L$ is an $N \times N$ diagonal matrix.

Substituting (12) and (14)–(16) into (13) gives

$$\begin{aligned} J = & (\eta^{j+1})^T \Omega \eta^{j+1} + 2(\eta^{j+1})^T \Psi x^{j+1}(k) \\ & - 2(\eta^{j+1})^T R_L \eta^j + (\eta^j)^T R_L \eta^j, \end{aligned} \tag{17}$$

where

$$\Omega = \sum_{m=1}^{N_p} \phi(m) Q \phi^T(m) + R_L, \;\; \Psi = \sum_{m=1}^{N_p} \phi(m) Q A^m.$$

The minimum value of this cost function occurs when

$$\eta^{j+1} = -\Omega^{-1}(\Psi x^{j+1}(k) - R_L \eta^j). \tag{18}$$

Under receding horizon control, only the first sample of the optimal control trajectory is implemented, which is constructed as the filtered control signal on trial $j+1$ at sample $k$, in the SISO case with an obvious extension to MIMO,

$$u_s^{j+1}(k) = L^T(0)\eta^{j+1}. \tag{19}$$

By combining (18) and (19), the predictive iterative learning control law is

$$u_s^{j+1}(k) = -L^T(0)\Omega^{-1}\Psi x^{j+1}(k) + L^T(0) R_L \eta^j. \tag{20}$$

This control law is the sum of two terms in the current state vector $x^{j+1}(k)$ and the previous trial term $\eta^j$, where the first term can be written as $-K_{mpc}x^{j+1}(k)$ with

$$K_{mpc} = L^T(0)\Omega^{-1}\Psi$$

and is a current trial state feedback control law. The second term is a feed-forward function, i.e., use of previous trial information in the construction of the next trial input.. Moreover, the state feedback provides stabilization of the along the trial dynamics.

### *3.1 Convergence and Performance Analysis*

The core problem in ILC design is to enure trial-to-trial error convergence but since the trial length is finite convergence can occur where the along the trial dynamics are unstable, i.e., the state matrix has eigenvalues on or outside the unit circle in the complex plane. In the lifted approach to ILC design for discrete-time linear systems, a stabilizing control loop is first designed and then ILC is applied to the resulting dynamics to enforce trial-to-trial error convergence. For a detailed treatment of the lifted approach to ILC design, one starting point are the relevant papers cited in [Bristow et al., 1984] and [Ahn et al., 2007].

Another alternative is to use a repetitive process setting. Repetitive processes are a class of 2D systems and can be used [Paszke et al., 2013] to design in one step an ILC law to stabilize and/or regulate the transient dynamics along the trials and enforce trial-to-trial error convergence. Design in this setting also allows for the inclusion of only selected frequencies from the reference signal or vector. Hence the new results in this paper can be considered as an alternative way to do such designs.

To investigate the properties of this new ILC design, the fact [Wang, 2009] that, for standard linear systems, the use of a suitably large prediction horizon and large $N$ in the Laguerre expansion the control law is identical to the solution of an Linear Quadratic Regulator (LQR) problem is used. In particular, the LQR problem as one way of designing a stabilizing state feedback control law is exploited, including the extension to a weighted design and hence increased relative stability and/or regulation of the transient dynamics along the trials.

Substituting for $\eta^{j+1}$ in the control law (18) gives

$$u_s^{j+1}(k) = -L(0)^T\Omega^{-1}\Psi x^{j+1}(k) + L(0)^T\Omega^{-1}R_L\eta^j. \tag{21}$$

Consider $j = 0$, with the assumption that $\eta$ is the zero vector. Then filtered control signal is

$$u_s^1(k) = -K_{mpc}x^1(k). \tag{22}$$

and on applying the control law

$$x^1(k+1) = (A - BK_{mpc})x^1(k) = A_{cl}x^1(k). \tag{23}$$

For $j = 1$, the filtered control signal is

$$u_s^2(k) = -K_{mpc}x^2(k) - K_1 x^1(k) \tag{24}$$

where $K_1 = L(0)^T\Omega^{-1}R_L\Omega^{-1}\Psi$, and

$$x^2(k+1) = A_{cl}x^2(k) - BK_1x^1(k) \tag{25}$$

and so on for $j = 2, 3, \ldots$.

If all eigenvalues of $(A - BK_{mpc})$ have modulus strictly less than unity then for an induced matrix norm $||\cdot||$ there exist constants $0 < M < \infty$ and $0 < \lambda < 1$ such that $||(A - BK_{mpc})^k|| \le M\lambda^k$ and hence

$$||x^1(k)|| \le M\lambda^k||x^1(0)||. \tag{26}$$

Returning to the case of $j = 1$ gives

$$\begin{aligned} x^2(k+1) &= (A - BK_{mpc})x^2(k) - BK_1x^1(k) \\ &= (A - BK_{mpc})x^2(k) - BK_1(A - BK_{mpc})^k x^1(0), \end{aligned} \tag{27}$$

where (23) has been used. Also for given $x^2(0)$, it follows from (27) that

$$\begin{aligned} x^2(k) &= (A - BK_{mpc})^k x^2(0) \\ &\quad - \sum_{i=0}^{k-1}(A - BK_{mpc})^{k-i-1}BK_1(A - BK_{mpc})^i x^1(0) \end{aligned}$$

and hence

$$\begin{aligned} ||x^2(k)|| &\le M\lambda^k||x^2(0)|| \\ &\quad + \sum_{i=0}^{k-1} M\lambda^{k-i-1}||BK_1||M\lambda^i||x^1(0)|| \\ &= M\lambda^k||x^2(0)|| + M^2k\lambda^{k-1}||BK_1||||x^1(0)||. \end{aligned}$$

By induction, for any value of $j$,

$$\begin{aligned} ||x^j(k)|| &\le M\lambda^k||x^j(0)|| \\ &\quad + M^2k\lambda^{k-1}||BK_1||||x^{j-1}(0)|| + \ldots \end{aligned} \tag{28}$$

This last expression shows that the initial conditions for each trial affect the error. Also the performance along the trial and the rate of trial-to-trial error convergence is determined by the value of $\lambda$. To improve performance, a smaller value of $\lambda$ can be used, i.e., require that the eigenvalues of $A - BK_{mpc}$ lie inside a circle of radius $\lambda < 1$ in the complex plane, where the choice of $\lambda$ is application dependent. Once $\lambda$ is selected, LQR theory, see, e.g., [Wang, 2009] and the relevant cited references, can be used in the form of the following procedure.

(1) For the selected $\lambda < 1$, solve the following steady-state Riccati equation for a given $Q > 0$ and $R > 0$

$$\begin{aligned} &\frac{A^T}{\lambda}[P_\infty - P_\infty\frac{B}{\lambda}(R + \frac{B^T}{\lambda}P_\infty\frac{B}{\lambda})^{-1}\frac{B^T}{\lambda}P_\infty]\frac{A}{\lambda} \\ &\quad + Q - P_\infty = 0 \end{aligned} \tag{29}$$

(2) Select $\alpha \geq 1$ such that all eigenvalues of the matrix $\alpha^{-1}A$ lie in the open unit circle in the complex plane and form

$$Q_\alpha = \gamma^2 Q + (1-\gamma^2)P_\infty,\ \gamma = \frac{\lambda}{\alpha}$$
$$R_\alpha = \gamma^2 R. \tag{30}$$

(3) Use the $Q_\alpha$ and $R_\alpha$ in the design of model predictive control with the chosen $\alpha$ and a sufficiently large $N_p$. Also replace $A$ and $B$ by $\alpha^{-1}A$ and $\alpha^{-1}B$ and compute ILC solution with cost function

$$J = \sum_{m=1}^{N_p} x^{j+1}(k+m \mid k)^T Q_\alpha x^{j+1}(k+m \mid k) + W$$
$$W = \sum_{m=0}^{N_p} (u_s^{j+1}(m) - u_s^j(m))^T R_\alpha (u_s^{j+1}(m) - u_s^j(m)). \tag{31}$$

## 4. EXPERIMENTAL CASE STUDY

Figure 1 shows an anthropomorphic robot arm undertaking a 'pick and place' task in a horizontal plane using two joints. The robots end-effector travels from the 'pick' to the 'place' location in a straight line using joint reference trajectories that minimize the end-effector acceleration. During the movement, the arm stops at two intermediate points, chosen such that there is a change in the direction of travel along the path after reaching each of them. Having reached the 'place' location, the robot repeats the movement in reverse, arriving back at the 'pick' location. Positional and velocity control loops have been implemented around each joint to provide baseline performance and the control scheme operates at 20Hz ($\Delta t = 0.05$ *sec*).

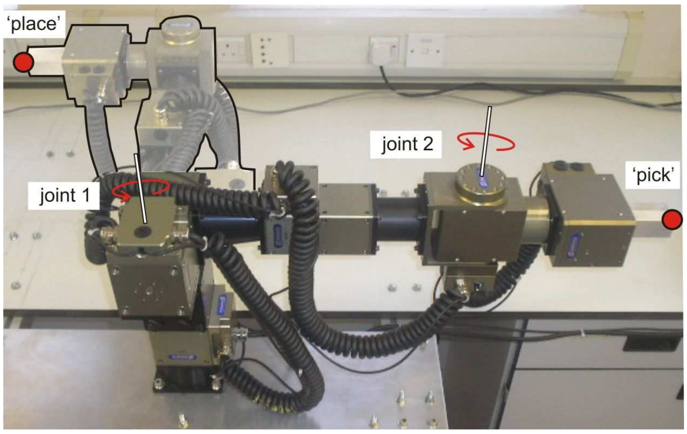


Fig. 1. Photograph of the robot arm showing pick and place locations.

A model for this robot has been estimated using frequency response tests and is given in Wang et al. [2012]. Two sets of operational tasks are considered in this paper, both of which are mapped to the coordinates of thee axes. These reference signals, termed the $r_1$ and $r_2$, respectively, are shown in Fig. 2, in the 1st and 3rd plots.

The experimental verification of this design requires many tests to evaluate the effects of choices, including relative aspects where appropriate, of the design parameters, e.g., the Laguerre functions, the prediction horizons and the weighting matrices in the design. Due to limited space, only a selection of the test results can be given for which the control design variables are given in Table 1. Figure 2 gives the experimental results obtained, where

| | |
|---|---|
| $a_1 = a_2 = a$ | 0.6 |
| $N_1 = N_2 = N$ | 8 |
| $N_p$ | 100 |
| $Q$ | $I$ |
| $R$ | $I$ |
| $\alpha$ | 1.1 |
| $\lambda$ | 0.7 |

Table 1. Design Parameters used to generate the experimental results given in Fig. 2.

the first plot gives $r_1$ reference signal and the resulting output for trials 1, 3 and 5 and the second plot gives the corresponding errors generated. The third and fourth plots in this figure gives the corresponding results for the $r_2$ reference signal. Another issue is the level of control signals used and the plots for the first and second cases, respectively, are shown in the fifth and six plots in Fig. 2. These are judged to be acceptable.

## 5. CONCLUSIONS

This paper has developed an iterative learning control law design starting from some recent results in the area of predictive repetitive control. The algorithm uses receding horizon control and Laguerre functions to parameterize the future control trajectory. Stability and error analysis has also been undertaken. Supporting experimental results from application to a robot confirms the basic potential of the design. Due to space limitations, only a subset of the experimental testing to enable a detailed evaluation of the performance of this new design have been reported. One critical aspect not treated at this stage is comparison with alternative designs and this should be addressed in any future research.

The results in this paper establish the basic feasibility of the design and the use of Laguerre functions to parameterize the future control trajectory reduces the computational burden. Online computational costs, in particular, is an ever increasing problem as applications of ILC expand beyond the domain of industrial robotics, including multiple-input multiple-output systems. Applying control along the trials should also have a role in extending the design to deal with model uncertainty and disturbances and this should also be investigated in future research. Further research should also be directed to using the reset time between trials to estimate and pre-compensate for the effects of uncertainty in the trial state initial vector.

In some cases, the required control effort required may be outside the capabilities or safe operating ranges of the actuators employed and hence there is a need to consider constrained design. Such designs for other ILC algorithms are beginning to emerge, e.g., Chu et al. [2010] and further research is required to extend the design in this paper to allow constraints to be placed on the magnitude of the control signals allowed and their rates of change. The model predictive control setting should extend naturally to this case with on-line computations.

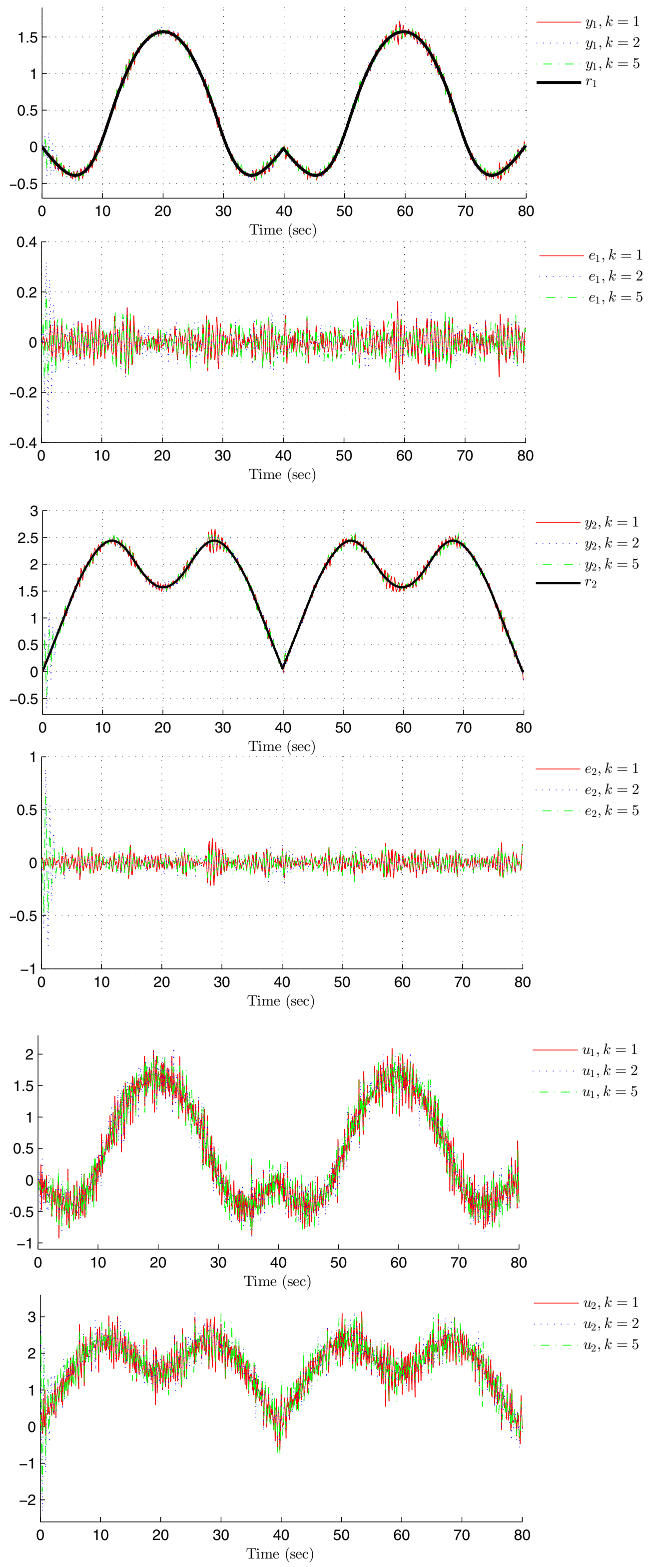


Fig. 2. Experimental results: in descending order from the top are output 1, error 1, output 2, error 2, input 1 and input 2.

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