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A Comparative Evaluation of Human Motion Planning Policies

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Abstract: A recently proposed approach to human intention estimation in goal-oriented motion is further improved in this paper. The main features of the approach are the fact that the unicycle model has been rewritten with the natural coordinate as the independent variable, avoiding the explicit dependence from the walking velocity and lowering the number of input variables to one, the proposal of a novel cost function, weighting not only the energy needed to perform the path but also the current distance of the human with respect to the target, and the adoption of the Frechét metric to assess the similarity of experimental and estimated paths. The proposed cost function is compared with other two cost functions, adopted in previous works, with reference to experimental data. The performance improvements with respect to the previous approach are apparent, either from a qualitative point of view and from a statistical analysis of the error distances.

Keywords: Optimal control, mobile robots, human-centered design, vision, robot kinematics.

1. INTRODUCTION

In recent years, the problem of human intention estimation has received a great attention in the robotics research. The impact of this topic on safety issues is apparent, particularly in the field of industrial robotics, where the goal of a real and safe cooperation among robots and humans is still far from being a common practice (Bascetta et al. [2011]). In order to achieve this goal the control system should be able to prevent dangerous situations, by estimating the human intention in time to perform a safe robot reaction. Estimation of human intention is also very important in the field of service robotics, for example to proactively interact with humans (Kanda et al. [2009]), and in medical applications, particularly in rehabilitation and assistive robotics (Kuan et al. [2010]).

While the estimation of the human intention "tout court", of course, is quite a hard problem, some results have been achieved in predicting the human behavior with reference to a goal-oriented motion, based on a model of the human motion planning (Mombaur et al. [2010]). The classical way to approach the problem, first proposed by Flash and Hogan (Flash and Hogan [1985]) is to assume that humans plan their motions solving an optimal control problem, whose dynamic model and cost function have to be selected in such a way that planned paths are human-like, i.e. resemble the paths walked by a human. More properly, what is to be solved is an inverse optimal control problem (Jameson and Kreindler [1973], Casti [1980]), where the dynamic model can be reasonably assumed as known (unicycle model) while the cost function must be selected so as to best fit the motion data. This approach has been adopted in Arechavaleta et al. [2008], Mombaur et al. [2008] and, in particular, in Puydupin-Jamin et al. [2012], where optimal control problem has been solved by minimizing residual functions based on the necessary conditions for optimality, through an efficient least-squares minimization.

Very recently, significant improvements have been obtained by Papadopoulos et al. [2013] adopting a similar approach, while introducing the following modifications:

- the unicycle model has been rewritten with the natural coordinate as the independent variable, avoiding the explicit dependence from the walking velocity and lowering the number of input variables to one;
- a novel cost function has been considered, weighting not only the energy needed to perform the path but also the current distance of the human with respect to the target;
- the Frechét metric (Alt and Godau [1995]), used in computational geometry as a "similarity" metric between two curves, has been adopted to assess the similarity of experimental and estimated paths.

After showing that the unicycle model, either in time or space domain, is actually well suited to describe the human walking dynamics with reference to goal-oriented motion, further improvements are presented in this paper, essentially based on the adoption of a novel cost function. The new cost function reduces the number of parameters, by weighting the Euclidean distance from the current to the target position instead of separately weight the Cartesian

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planar coordinates, and by normalizing the length and angular distances with respect to their boundary values. The proposed cost function is compared with other two cost functions, adopted in previous works, with reference to experimental data. The performance improvements with respect to the previous approach are apparent, either from a qualitative point of view and from a statistical analysis of the error distances, showing a reduction of the Frechét distance between the optimal path and the experimental one of 20% in the case of the worse path and 80% for the best path.

2. EXPERIMENTAL SETUP

In this section we describe the experimental setup used to collect a dataset of human walking paths. About one thousand paths were recorded using a 6 cameras motion capture system (SMART system by BTS S.p.A.). Each subject was equipped with 3 light reflective markers, two located on the hips – anterior superior iliac spine (asis) –, and one located on the sacrum (see Fig. 1).

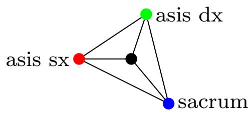

Fig. 1. Marker positions and barycentre.

The experimental protocol was inspired to the one adopted in Arechavaleta et al. [2008]. More specifically, we restrict the study to the "natural" forward locomotion, excluding goals located behind the starting position and goals requiring side-walk steps.
Goals are defined both in position and orientation, and in order to cover at best the accessibility region, the space for the experiments, a 4m×6m rectangle corresponding to the calibrated volume, was sampled with 144 points defined by 12 positions on a 2D grid and 12 orientations each. The final orientation varied from 0 to 2π with steps of $\pi/6$ at each final position (see Fig. 2). The starting position was always the same.

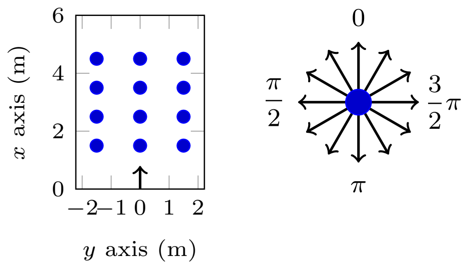

Fig. 2. Final porch positions (left) and orientations (right).

Locomotor trajectories of seven normal healthy people (both males and females), who volunteered for participation in the experiments, were recorded. Their ages, heights, and weights ranged from 24 to 50 years, from 1.60 to 1.85 m, and from 50 to 90 kg, respectively.
Each subject performed all the 144 trajectories. Subjects walked from the same initial configuration to a randomly selected final configuration (see Fig. 2). The target consisted of a porch that could be rotated around a fixed position in order to show the desired final orientation (see Fig. 3).
The subjects were instructed to freely cross over this porch without any spatial constraint relative to the path they might take. Further, they were allowed to choose their natural walking speed to perform the task.

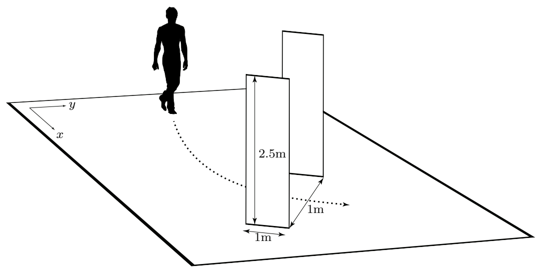

Fig. 3. An example of experiment.

A pre-processing phase on the paths collected by the optoelectronic system was required in order to remove the outliers, fill in the missing data and smooth the curves. The path of each marker was then interpolated with a smoothing spline and, considering the triangle that the three markers form (see again Fig. 1), the path of a unique "virtual" marker representing the human walking path was computed, as the barycentre of the triangle.

3. PROBLEM DEFINITION

As far as human trajectory planning is concerned, the complex activities performed during walking by muscles and brain in commanding and coordinating many elementary motor acts can be neglected, and the problem may be considered from a high-level kinematic model perspective. Further, the pioneering studies of Flash and Hogan [1985] introduced the idea that humans plan arm motion solving an optimal control problem. A similar approach was then adopted, many years later, in Arechavaleta et al. [2008], Mombaur et al. [2008], Puydupin-Jamin et al. [2012] to explain how humans plan walking trajectories.

In the framework just introduced, the problem of planning human walking paths can be thus formulated as an optimal control problem, whose dynamic model and cost function have to be selected in such a way that the planned paths are human-like, i.e. resemble the paths walked by a human. These two fundamental aspects, i.e. the selection of the walking model and of the cost function, are discussed in detail in Sections 4 and 5.

4. WALKING HUMAN MODEL

A walking human can be represented by a rectangular box (Fig. 4), that can translate and rotate around an axis parallel to the vertical dimension of the box, and crossing the base in its centre.
The pose of the human is thus completely described by the coordinate of the rectangular box base centre P, with respect to a reference frame fixed on the ground plane, and by the angle θ formed by the tangent to the walking path with the x-axis. Then, a human walking trajectory is defined as the path followed by the point P through the ground plane.

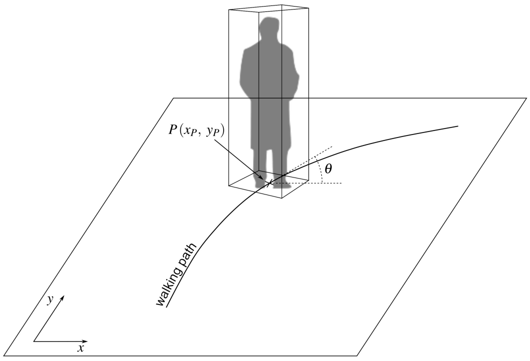

Fig. 4. Formalization of a human walking path.

Similarly to previous works (see e.g. Puydupin-Jamin et al. [2012]) we model the walking human with the following unicycle kinematic model

$$\begin{cases} \dot{x}_P = v\cos(\theta) \\ \dot{y}_P = v\sin(\theta) \\ \dot{\theta} = \omega \end{cases} \tag{1}$$

where x_P, y_P are the Cartesian coordinates of the point P, v is the linear (nonholonomic) velocity along the direction of motion, θ is the orientation, and ω is the angular velocity.

Consider now the forward velocity v. In principle it varies with time along the path and depends on a large number of factors (see Öberg et al. [1994] and Knoblauch et al. [1995]). In many cases, however, the variations of v are limited, in particular in the absence of obstacles and environmental stimuli that can trigger unpredictable human reactions. It can thus be considered constant (see e.g. Arechavaleta et al. [2008]). On the other hand, if one is interested in studying the geometry of the walking path only, instead of the complete trajectory as a function of time, it is possible to rewrite model (1) with the natural coordinate s as the independent variable, avoiding the explicit dependence of the model from the velocity v, and lowering the number of input variables to one. Thus, if $v > 0$ along the path, i.e. if the assumption of "natural walking" introduced in Arechavaleta et al. [2008] holds, the relation between the natural coordinate s and the time t is given by

$$s(t) = \int_0^t v(\tau)\,\mathrm{d}\tau$$

and can be inverted, defining $t = t(s)$. As a consequence, model (1) can be rewritten as

$$\begin{cases} x'_P = \cos(\theta) \\ y'_P = \sin(\theta) \\ \theta' = \sigma \end{cases} \tag{2}$$

where $\sigma = \omega/v$ is a new input variable, and the notation $'$ represents the derivative with respect to the natural coordinate s: $x' = \mathrm{d}x/\mathrm{d}s$.

Considering now how complex are the activities performed during walking, but how simple are models (1) and (2) herein introduced, a question naturally arises: is the unicycle model well-suited to describe the dynamics of a human that is walking in a free space?
In order to reply to this question, either the unicycle model in time domain (1) and in space domain (2) were simulated, fed by the velocities computed by the experimental data, comparing each simulated path with the corresponding experimental one. This comparison was based on the Fréchet metric [Alt and Godau, 1995], that the authors consider the best way to measure the geometrical difference between two curves, and on the Hausdorff metric, that, though being less reliable than the previous one, is definitely less computational expensive.
Fig. 5 shows the results of this validation, obtained using the dataset presented in Section 2. From the box plots it is apparent that independently of the time or space domain or of the chosen metric, the unicycle model is very suited to describe the dynamics of a walking human.

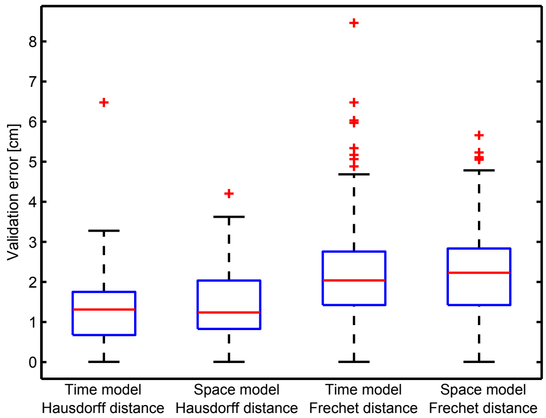

Fig. 5. Validation of the unicycle model.

5. CHOOSING THE COST FUNCTION

The approaches to human planning as an optimal control problem already mentioned in Section 3 reveal that the choice of the cost function is the most critical issue. In fact, apart from obvious criteria such as minimisation of the energy consumption or minimisation of the distance and the derivative of the curvature, the way humans plan walking paths depends in general from the situation, from environmental constraints, and so forth.
In this paper we will therefore focus on the definition of a cost function that, apart from obviously being experimentally validated, it should be physically grounded and as simple as possible. To this extent, three different cost functions, two of which have been already introduced in previous works, are presented in the following, comparing the paths they generate with the experimental dataset mentioned in Section 2.

Energy-based cost function In Puydupin-Jamin et al. [2012] an energy related cost function was proposed.
Considering the unicycle model in time domain (1), this cost function can be rewritten in continuous time as follows

$$J = \frac{1}{2}\int_0^T \left(\alpha v^2 + \omega^2\right)\,dt \tag{3}$$

where T is the duration of the trajectory, and α is an unknown parameter that has to be estimated through the solution of an inverse optimal control problem (for further details see Puydupin-Jamin et al. [2012]). This parameter governs how much we penalize control effort v relative to control effort ω.

As previously mentioned, the cost function introduced in Puydupin-Jamin et al. [2012] is related to the energy needed to perform the path, and the underlying rationale is that humans wants to minimize it.

Hybrid energy/goal-based cost function Following the same approach already introduced in Puydupin-Jamin et al. [2012], in Papadopoulos et al. [2013] the authors proposed a new cost function, that is based on the space domain unicycle model (2), and accounts either for the energy related to the control effort σ, and for the distance between the current state and the final state.
This cost function can be formulated in continuous time as follows

$$J = \frac{1}{2}\int_0^S \sigma^2 \left(1 + \beta^T \Delta^2\right)\, ds \tag{4}$$

where S is the length of the path,

$$\beta^T = [\beta_1 \ \beta_2 \ \beta_3]$$

is a set of unknown parameters that need to be estimated through the solution of an inverse optimal control problem, and

$$\left(\Delta^2\right)^T = \left[\left(x_P - x_{P_g}\right)^2 \ \left(y_P - y_{P_g}\right)^2 \ \left(\theta - \theta_g\right)^2\right]$$

$\left(x_{P_g}, y_{P_g}, \theta_g\right)$ being the final pose of the human.
The rationale behind this cost function is that the distance of the current state from the goal can be interpreted as a space-varying weight on the control effort σ.

Modified hybrid energy/goal-based cost function A new cost function, based on the one already introduced in Papadopoulos et al. [2013], is here considered, with the aim of simplifying the identification of the β parameters, and of improving the quality of the planned walking paths.
To this extent, two changes are introduced:

(1) a reduction of the number of parameters, weighting the Euclidean distance from the actual to the final human position instead of separately weight the x- and y-distances;
(2) a normalisation of the Euclidean and angular distances with respect to their boundary values.

The modified cost function can be thus formulated as follows

$$J = \frac{1}{2}\int_0^S \sigma^2 \left(1 + \gamma^T \tilde{\Delta}^2\right)\, ds \tag{5}$$

where

$$\gamma^T = [\gamma_1 \ \gamma_2]$$

is a set of unknown parameters that need to be estimated through the solution of an inverse optimal control problem, and

$$\left(\tilde{\Delta}^2\right)^T = \left[\frac{\left(x_P - x_{P_g}\right)^2 + \left(y_P - y_{P_g}\right)^2}{\left(x_{P_s} - x_{P_g}\right)^2 + \left(y_{P_s} - y_{P_g}\right)^2} \ \frac{\left(\theta - \theta_g\right)^2}{\left(\theta_s - \theta_g\right)^2}\right]$$

$(x_{P_s}, y_{P_s}, \theta_s)$ being the initial pose of the human.

The results achievable with the three cost functions here introduced, in reproducing a human walking path, are compared in Section 7.

6. SOLVING THE INVERSE OPTIMAL CONTROL PROBLEM

The basic assumption is that the problem formulation is assumed to be only "approximately optimal", while observations are assumed to be perfect (Puydupin-Jamin et al. [2012]). Accordingly, the discrete version of the cost function, say (5), can be rewritten as

$$J(\chi, \gamma) = \frac{1}{2}\sum_{k=0}^{N-1} \delta(k)\sigma^2(k)\left[1 + \gamma^T \tilde{\Delta}^2(k)\right]$$

where $\chi^T = [\, x_P \ y_P \ \theta \ \sigma \,]$ and $\delta(k) = s(k) - s(k-1)$, while model (2) gives rise to the following system of constraints

$$g(\chi) = \begin{bmatrix} \vdots \\ x_P(k+1) - [x_P(k) + \delta(k)\cos(\theta(k))] \\ y_P(k+1) - [y_P(k) + \delta(k)\sin(\theta(k))] \\ \theta(k+1) - [\theta(k) + \delta(k)\sigma(k)] \\ \vdots \end{bmatrix} = 0$$

for $k = 0, \ldots, N-1$.

Defining the Lagrangian of the problem as

$$\mathcal{L}(\chi, \gamma, \lambda) = J(\chi, \gamma) + \lambda^T g(\chi)$$

where $\lambda^T = \left[\, \lambda_x^0 \ \lambda_y^0 \ \lambda_\theta^0 \ \ldots \ \lambda_x^{N-1} \ \lambda_y^{N-1} \ \lambda_\theta^{N-1} \,\right]$, recalling the necessary conditions for equality constraint optimization

$$\nabla_{(\chi,\lambda)}\mathcal{L}(\chi, \gamma, \lambda) = 0$$

and the fact that the Lagrangian is linear with respect to γ and λ, the inverse optimal control problem can be solved by minimizing the residual function

$$\min_{\gamma,\lambda} \ \frac{1}{2}\|\nabla_{(\chi,\lambda)}\mathcal{L}(\chi, \gamma, \lambda)\|^2 = \min_{\gamma,\lambda} \ \frac{1}{2}\|Az - b\|^2 \tag{6}$$

where $z^T = \left[\, \gamma^T \ \lambda^T \,\right]$, while A and b depend only on the collected data.

Finally, in order to ensure that the value of γ resulting from (6) is actually positive, the solution of (6) is here taken as the initial guess for the solution of a new optimization problem, i.e., a constrained version of (6), which reads as

$$\begin{aligned} &\min_{\gamma,\lambda} \quad \frac{1}{2}\|Az - b\|^2 \\ &\text{s.t.} \quad \gamma \geq 0 \end{aligned}$$

7. EXPERIMENTAL RESULTS

This Section presents a comparison, based on the experimental set of paths introduced in Section 2, among the three cost functions described in Section 5.

First, the paths generated with the methods proposed in Puydupin-Jamin et al. [2012], herein called "time" method for brevity, and in Papadopoulos et al. [2013], herein called "space" method, are compared.
From Fig. 6 it is apparent that the "time" solution, whatever distance measure is selected (i.e., the Hausdorff distance, "HD", or the Frechét distance, "FD"), is not able to reliably reproduce the collected data. In fact, though Fig. 6 shows only the paths which are characterised by the minimum and the maximum error with respect to the experimental ones, this kind of behaviour is also present in many other optimised trajectories, omitted here for space

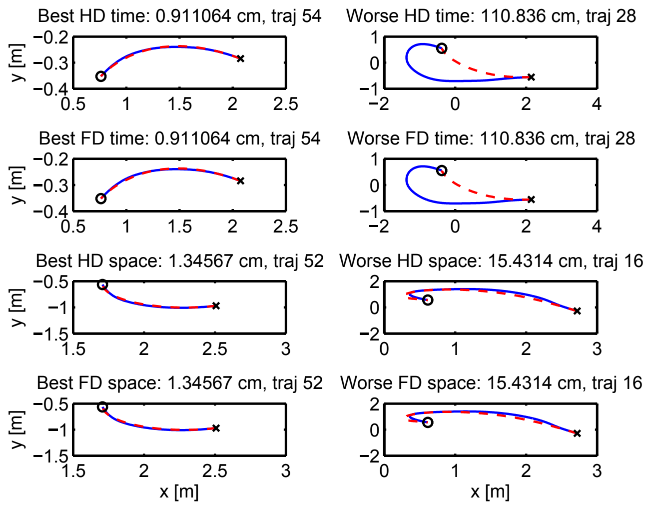

Fig. 6. The best and the worse path generated with the Puydupin-Jamin, here called "time", and with the Papadopoulos, here called "space", approach. The blue lines represent the experimental paths.

limitations. A concise representation of the performance of the method presented in Puydupin-Jamin et al. [2012], in reproducing the dataset herein considered, is given by the statistical analysis of the distance between each generated path and the corresponding experimental one (see Fig. 7) and by the comparison of the paths that give rise to the median distance (see Fig. 8). It is opinion of the authors

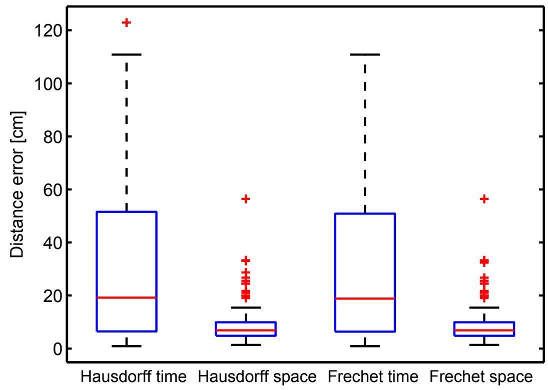

Fig. 7. Statistical analysis of the distance between each generated path and the corresponding experimental one.

that this kind of error in reproducing data is not only due to the fact that the chosen value of α may not be the optimal one, but also to the selected cost function (3) which is inherently not able to replicate the human way of planning paths.
In some cases both the time and the space methods manage to reproduce the human path - but also in those cases, the space method seems to be closer - but there are several other cases in which the time method fails. The performance improvement achieved by the cost function (4) is apparent, either from a qualitative comparison among the paths generated by the two approaches and the corresponding experimental ones (see Fig. 6), and from the statistical analysis of the error distances (see Fig. 7). Further, the Frechét distance between the generated path and the experimental one (see Fig. 6) shows that the space

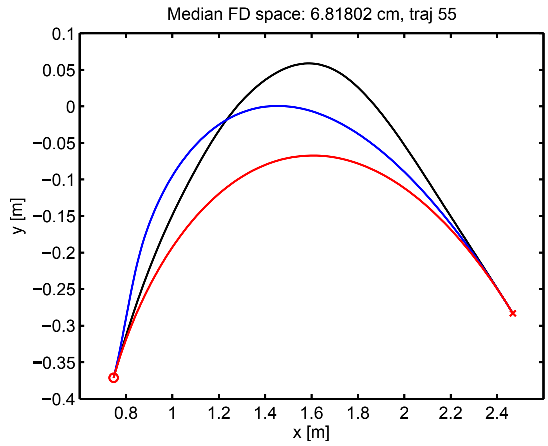

Fig. 8. A comparison among the paths, corresponding to the median distance error, generated with the time (red line) and the space (black line) approach and the corresponding experimental path (blue line).

method outperforms the time approach in the worse case and in the best case as well.

The results achieved with the approach presented in Papadopoulos et al. [2013] have been further improved by the cost function (5) herein proposed. The reduction of the distance error is apparent from the qualitative analysis of the best and the worse path (Fig. 9), and of the path corresponding to the median distance error (Fig. 11).

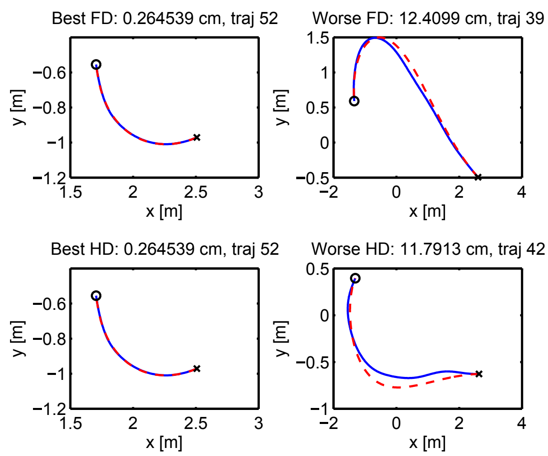

Fig. 9. The best and the worse path generated with the proposed approach. The blue lines represent the experimental paths.

Further, the quantitative analysis shows that the Frechét distance between the generated path and the experimental one has been reduced, with respect to the space method presented in Papadopoulos et al. [2013], of 20% in the case of the worse path and 80% for the best path (see Figs. 6 and 9).
Finally, the statistical analysis (Fig. 12) confirms that the previous conclusions hold for the whole dataset. As it is clearly shown by the comparison between the box plots obtained with the space approach and with the one herein proposed, whatever distance measure is considered, the last one yields a definite improvement in the reproduction of the human walking paths.

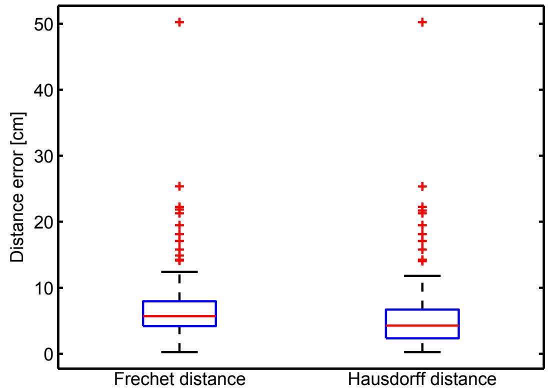

Fig. 10. Statistical analysis of the distance between each generated path and the corresponding experimental one.

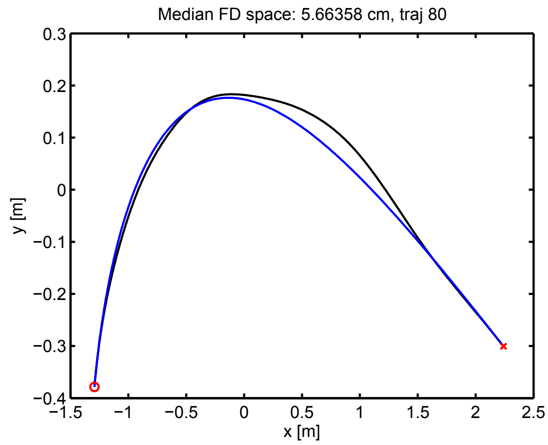

Fig. 11. A comparison between the path, corresponding to the median distance error, generated with the proposed approach (black line) and the corresponding experimental path (blue line).

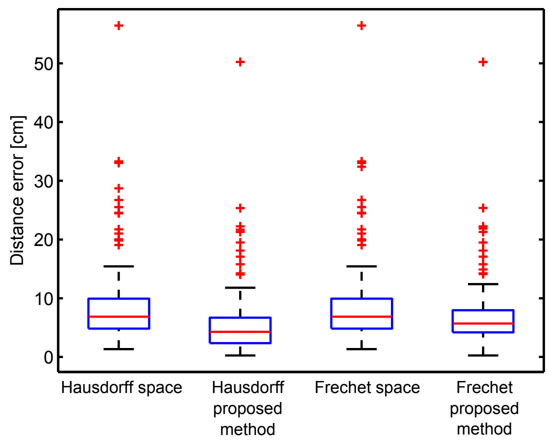

Fig. 12. Statistical analysis of the distance between each path, generated with the space method and the one here proposed, and the corresponding experimental one.

8. CONCLUSION

A comparative evaluation of human motion planning policies has been described in this paper. The basic assumption, widely adopted in the literature, is that humans plan their motions solving an optimal control problem, with the unicycle as the dynamic model and with an unknown cost function, to be selected in such a way that planned paths are human-like. In particular, a modification of a recently proposed approach has been compared with two approaches adopted in previous works, with reference to experimental data. The main features of the approach are the fact that the unicycle model has been rewritten with the natural coordinate as the independent variable, the proposal of a novel cost function, weighting not only the energy needed to perform the path but also the current distance of the human with respect to the target and the adoption of the Frechét metric to assess the similarity of experimental and estimated paths. The performance improvements with respect to the previous approaches are apparent, either from a qualitative point of view and from a statistical analysis of the error distances.

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