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Consensus of Multi-Agent Systems: A Relative-Input-Output Approach

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Abstract: This paper studies the multi-agent consensus problem of complex networks consisting of general homogeneous continuous linear time-invariant systems with controls under a time-invariant directional communication topology. A new approach based on relative-input-output is proposed to solve the problem. In contrast to a popular method where each agent is equipped with an observer, and hence involving heavy observer dynamics, the proposed approach is static and utilizes relative-input and relative-output eliminating the observers aforementioned, therefore is practically much simpler and easier to implement significantly reducing computational and communicational burdens and costs. Extension of the framework is then made to tackle a dual problem: consensus of distributed filtering without controls. Two numerical examples are provided to validate the proposed approach.

1. INTRODUCTION

In the past several decades, an emerging trend that more and more control systems and their practical applications being distributed across networks located at different sites physically has been witnessed and networked control systems (NCS) have received a lot of research effort and attention. Among them, consensus and synchronization of networked agents are important subjects. Typical applications include coordinated air/ground vehicles, satellite formation flying, and air/ground traffic scheduling problems, networked robotic systems, to name a few. Earlier, important network phenomena such as data losses, data corruption, packet-reordering, and transmission delays had also been extensively studied in respect to their impacts on the NCS such as robustness, stability, and performance (Schenato 2008; 2009). We refer the reader to three review papers (Antonelli, 2013a; Cao *et al*., 2013; and Chen *et al*., 2013) for more recent accounts.

In the first part of this paper we study the consensus problem for a networked system consisting of homogeneous linear time-invariant multi-agents with controls. The topology of the underlying communication network is assumed to be directional, time-invariant, and having a spanning tree. The main objective of decentralized control in current context is that using distributed control strategies (a "local" concept) one expects to achieve the goal of consensus/synchronization (a "global" concept). As is well known, observer-based approaches are quite popular (Antonelli, *et al*., 2013b; Zhao *et al*., 2013; Zhang and Lewis, 2011), among which the relative-output plays an important role (Li *et al*., 2010). On top of that, we will introduce a new conceptual tool: relative-input in this paper, which is simple yet effective.

Along this line, a version based on reduced-order observers was presented (Li *et al*., 2011), which reduced the computational and communicational burdens and costs to a certain extent. A robust version allowing additive perturbations as uncertainties to the transfer matrices was also proposed (Trentelman *et al*., 2013).

In the second part of the paper we extend the results to address a dual problem: the consensus of distributed filtering. We limit our focus on linear stochastic systems without control input. Relative-output has been used (Olfati-Saber *et al*., 2012). We employ the new tool of relative-input to solve this problem. There are other works that have been developed to deal with more complex cases such as time-varying communication topology and dynamic controllers (Scardovi *et al*., 2009 and Wieland *et al*., 2011). Dynamic controllers are of course computationally more costly and more involved from implementation standpoint.

The rest of the paper is organized as follows. The system model is presented in Section 2, where a popular approach as exemplified by a recent work is also reviewed. In Section 3, a new approach based on relative-input-output is proposed. New elements such as agent-dependent coupling strength and relative-input are introduced. One of the main results, *Theorem 1*, is also presented. Section 4 presents a feasible method to determine the optimal gain. The results are then extended in Section 5 to the consensus of distributed filtering problem. Another main result of the paper, *Theorem 2* is introduced also. Two numerical examples and simulation results are given in Section 6 to validate the proposed approach. We conclude the paper in Section 7.

1.1 Notations

We adopt the following notations in this paper: x stands for the state of a dynamical system, $\hat{x}$ represents the observer's state, u is the control input, y is the output, L stands for observer gain, z refers to sensor measurement, J represents LQ cost functional, and K is the stabilizing feedback gain. Process noise and measurement noise are denoted by w and v for a stochastic system with covariance Q and R respectively. Eigenvalues are denoted as λ.

2. SYSTEM MODEL AND A POPULAR METHOD

2.1 System Model

Consider the following homogeneous multi-agent system consisting of n agents in which superscript " i " is used as the agent index

$$\dot{x}^{(i)} = Ax^{(i)} + Bu^{(i)} \tag{1}$$

$$y^{(i)} = Cx^{(i)} \tag{2}$$

$$\begin{aligned} &i \in \{1, 2, \cdots, n\}, \\ &(A, B, C) \text{ controllable and observable} \end{aligned} \tag{3}$$

where $x^{(i)}$ is the state, $u^{(i)}$ the control, and $y^{(i)}$ the output of agent i. The main objective is to reach consensus among all the networked agents. It is assumed that communication is occurring between neighboring agents only, which is represented by the so-called adjacency matrix. In addition, the directional graph corresponding to the communication topology does not have any disconnected components. Namely, we assume that the network topology has a directional spanning tree ensuring that the problem is well-posed, which is pivotal.

2.2 A Popular Method

Here we discuss a popular method: the observer-based approach, which could be exemplified by a recent work. Recall the definition of relative-output (Li *et al*., 2010)

$$\tilde{y}^{(i)} = \tau \sum_{j=1}^{n} a^{(ij)} (y^{(i)} - y^{(j)}), \;\; a^{(ij)} \in \{0, 1\} \tag{4}$$

where τ is the so-called coupling strength; $a^{(ii)} = 0$, $a^{(ij)} = 1$ if ith agent can get information from jth agent and $a^{(ij)} = 0$ otherwise. The matrix with entries of $a^{(ij)}$ is called the adjacency matrix.

The following protocol was proposed in a recent work (Li *et al*., 2010) to solve the consensus problem

$$u^{(i)} = K\hat{x}^{(i)} \tag{5}$$

$$\begin{aligned} \dot{\hat{x}}^{(i)} &= (A + BK)\hat{x}^{(i)} \\ &\quad - L\left[\tilde{y}^{(i)} - \tau \sum_{j=1}^{n} a^{(ij)} C(\hat{x}^{(i)} - \hat{x}^{(j)}) \right] \end{aligned} \tag{6}$$

where $\hat{x}$ represents the observer's state (also called protocol state (Li. *et al*., 2010)). The two matrices K and L in the above equations denote a stabilizing state feedback gain and the observer gain respectively. The above protocol possesses several advantages (Li. *et al*., 2010). First, unlike many existing methods which mostly handled integrator-type systems, the protocol (5)-(6) extends that to general systems. Second, the framework unifies consensus and synchronization treating them with the same approach. Third, it solves the consensus problem using relative-output rather than absolute-output or absolute-state. The latter is important in terms of its practicality (Li. *et al*., 2010; Antonelli *et al*., 2013b).

The above approach has some disadvantages, however. First, it utilizes as many full-order observers as the number of agents. Yet each observer has its own dynamics as depicted in (6). This results in increase of overall communicational and computational burdens. Reduced-order observer-based method has been reported (Li *et al*., 2011) to fix this problem but only to some extent. One major difference between our new method and the existing ones is that all the observers will be eliminated.

Second, if one follows their *Algorithm 1* closely, specifically the 3rd step (Li *et al*., 2010), one should find that in order to find a proper coupling strength such that the consensus problem is solvable by the protocol (5)-(6), one must compute the eigenvalues of the Laplacian matrix so as to find a thresh-hold value, which is computationally expensive. The latter will be a major problem if the dimension of that matrix becomes very large. Worse than that, one has to perform the expensive computation again if the underlying communication topology changes. Besides, the coupling strength in their approach is constant instead of being agent-dependent. As a measure of protocol robustness against parametric uncertainties (Li *et al*., 2010), one has to carefully tailor this coupling strength so as to enlarge the consensus region. The simple design for the coupling strength proposed in this paper makes also a major difference, especially when its adaptability is a big concern.

Third, the information of relative-output is used only for state estimation but not for control purpose, which means the agent receives useful information from neighbours without maximizing its use. Fourth, useful information such as relative-input (will be defined shortly) is not utilized. The above disadvantages motivate us to propose a new method.

Note that one starting assumption, which is critical for the agents to reach consensus at a nontrivial value, is that the eigenvalues of the given matrix A are all located along the imaginary axis (Li *et al*., 2010). The procedure of moving the eigenvalues of A to the imaginary axis will be covered in the present paper through the root agent.

3. A RELATIVE-INPUT-OUTPUT APPROACH

Before the main results are presented, some technical terms are introduced. First, let us define a normalization function for column-sum of the adjacency matrix

$$N^{(i)}\left(\sum_{m=1}^{n} a^{(mi)} \right) = \begin{cases} 0, & \text{if } \sum_{m=1}^{n} a^{(mi)} = 0 \\ 1, & \text{otherwise} \end{cases} \tag{7}$$

where $i = 1, 2, \cdots\cdots, n$. The reason why we call this a normalization function is because the value of column-sum is normalized to unity when it is nonzero, meaning that it is only counted once if the ith agent sends information to neighboring agents regardless of how many. For compact notations, we will omit the argument (i.e. the column-sum) of the normalization function hereafter. Next, we define the row-sum of the adjacency matrix as follows, which depicts how many neighboring agents send information to agent i

$$\phi^{(i)} = \sum_{j=1}^{n} a^{(ij)} \tag{8}$$

We may define the coupling strength as

$$\tau^{(i)} = \frac{1}{N^{(i)} + \phi^{(i)}} \tag{9}$$

Also introduced is the notion of local "centroid" variables

$$x^{(i*)} = \tau^{(i)} \left(N^{(i)} x^{(i)} + \sum_{j=1}^{n} a^{(ij)} x^{(j)} \right) \tag{10}$$

$$y^{(i*)} = \tau^{(i)} \left(N^{(i)} y^{(i)} + \sum_{j=1}^{n} a^{(ij)} y^{(j)} \right) \tag{11}$$

$$u^{(i*)} = \tau^{(i)} \left(N^{(i)} u^{(i)} + \sum_{j=1}^{n} a^{(ij)} u^{(j)} \right) \tag{12}$$

where i = 1, 2,⋯⋯, n. Note that we allow the coupling strength τ to be agent-dependent whose usefulness will become clear shortly. Corresponding to the local centroids we define the relative variables such as relative-state, relative-output, and relative-input respectively as

$$\tilde{x}^{(i)} = x^{(i)} - x^{(i*)} \tag{13}$$

$$\tilde{y}^{(i)} = y^{(i)} - y^{(i*)} \tag{14}$$

$$\tilde{u}^{(i)} = u^{(i)} - u^{(i*)} \tag{15}$$

One may easily verify that the relative-state, relative-input, and relative-output satisfy the following identities

$$\tilde{x}^{(i)} = \tau^{(i)} \sum_{j=1}^{n} a^{(ij)} (x^{(i)} - x^{(j)}) \tag{16}$$

$$\tilde{u}^{(i)} = \tau^{(i)} \sum_{j=1}^{n} a^{(ij)} (u^{(i)} - u^{(j)}) \tag{17}$$

$$\tilde{y}^{(i)} = \tau^{(i)} \sum_{j=1}^{n} a^{(ij)} (y^{(i)} - y^{(j)}) \tag{18}$$

$$\tilde{y}^{(i)} = C \tilde{x}^{(i)} \tag{19}$$

where i = 1, 2,⋯⋯, n. The relative-input, appearing in (15) and (17) as a conceptual tool, is new.

Now we are ready to present one of the main results.

Theorem 1: For a network consisting of n homogeneous agents (1)-(3) under a time-invariant communication topology that has a directional spanning tree without any disconnected components, static protocol (20) solves the consensus problem where K is such that A+BKC is stable.

$$u^{(i)} = \frac{1}{\tau^{(i)} \phi^{(i)}} \left(K \tilde{y}^{(i)} + \tau^{(i)} \sum_{j=1}^{n} a^{(ij)} u^{(j)} \right) \tag{20}$$

Proof: Let us define a positive definite Lyapunov function V for the system as follows where P is a symmetric and positive definite matrix.

$$V = \sum_{i=1}^{n} \tilde{x}^{(i)T} P \, \tilde{x}^{(i)} \tag{21}$$

Differentiating V with respect to time we get

$$\dot{V} = \sum_{i=1}^{n} \left[\dot{\tilde{x}}^{(i)T} P \, \tilde{x}^{(i)} + \tilde{x}^{(i)T} P \, \dot{\tilde{x}}^{(i)} \right] \tag{22}$$

Using the add-and-subtract technique we may rewrite (20) as

$$\begin{aligned} u^{(i)} &= \frac{1}{\tau^{(i)} \phi^{(i)}} \begin{bmatrix} K \tilde{y}^{(i)} - \\ \tau^{(i)} \sum_{j=1}^{n} a^{(ij)} \left(u^{(i)} - u^{(j)} - u^{(i)} \right) \end{bmatrix} \\ &= \frac{1}{\tau^{(i)} \phi^{(i)}} \left[K \tilde{y}^{(i)} - \tilde{u}^{(i)} + \tau^{(i)} \phi^{(i)} u^{(i)} \right] \end{aligned} \tag{23}$$

This expression further leads to the following relative-input

$$\tilde{u}^{(i)} = K \tilde{y}^{(i)} \tag{24}$$

Consider the time-derivative of relative-state (16). Using the relative-input (17) we obtain

$$\dot{\tilde{x}}^{(i)} = (A + BKC) \tilde{x}^{(i)} \tag{25}$$

where i = 1, 2,⋯⋯, n. The time-derivative of V now becomes

$$\dot{V} = \sum_{i=1}^{n} \tilde{x}^{(i)T} [(A + BKC)^T P + P(A + BKC)] \tilde{x}^{(i)} \tag{26}$$

Since A+BKC is stable, there exists a symmetric positive matrix P that satisfies the following two inequalities, in which the first implies the second,

$$(A + BKC)^T P + P(A + BKC) < 0 \tag{27}$$

$$\dot{V} < 0 \tag{28}$$

As a matter of fact, we took exactly this matrix P as the weighting matrix for the relative-state in the Lyapunov function (21). According to Lyapunov stability theory we know that V will converge to zero, that is, all relative-states will converge to zero asymptotically. Next, without loss of generality, consider the ith relative-state. According to (13) the following identity holds in the asymptotic sense.

$$x^{(i)} = x^{(i*)} = \tau^{(i)} \left(N^{(i)} x^{(i)} + \sum_{j=1}^{n} a^{(ij)} x^{(j)} \right) \tag{29}$$

Collecting $x^{(i)}$ in (29) together with some straightforward algebraic manipulations we get

$$\sum_{j=1}^{n} a^{(ij)}x^{(i)} = \sum_{j=1}^{n} a^{(ij)}x^{(j)} \tag{30}$$

This equality holds asymptotically regardless of what the values of individual state are. The following expression then must hold in the asymptotic sense for the ith agent and its associated neighbors

$$x^{(i)} = x^{(j)} \tag{31}$$

The above statement is true for any agent i. Recall the network has a directional spanning tree, which is a given condition for the problem to be well-posed. We conclude that asymptotic consensus will be reached. This completes the proof.

Note that even though locally each agent will apply control law (20), the global picture lies in (24)-(25) instead.

Remarks: Strictly speaking, protocol (20) does not apply to the root agent, for the latter has an important task to perform: generating a nontrivial trajectory for the rest agents to follow. We also note that some technical implementation issues are not covered here due to space limit such as how the input and output values of the neighbouring agents are transmitted (together or separately) through the communication channels.

Unlike that in (6) where all agents share the same coupling strength τ, rendering the analysis of consensus region difficult and requiring a multistep design procedure to obtain τ, involving computation of all the eigenvalues of Laplacian matrix (Li *et al*., 2010), our new approach (9) does not do so and the design is fairly straightforward and much simpler. The usefulness of using an agent-dependent coupling strength now becomes evident, as from the relative variable perspective it renders the resulting multi-agent system homogeneous regardless of how complex the underlying network's communication topology connecting all agents might be. It is worth pointing out that the coupling strength does not appear in the relative closed loop system (25).

Note that this new method dissociates the effect from the agent's and protocol's dynamics on the consensus stability from the network's communication topology (Li *et al*., 2010). One important consequence of this property is that one only needs to adjust the coupling strength according to (9), which could be done simply by hand, to solve the multi-agent consensus problem if the underlying topology of the network changes whereas the individual agent dynamics does not. This is also an advantageous feature of agent-dependent coupling strength proposed here. The latter potentially may also be extended to handle the time-varying case, which is not treated here as it is beyond the scope of this paper.

4. THE OUTPUT FEEDBACK GAIN

In previous section we require the output feedback gain K be stabilizing. In this section we present a feasible method to find that gain (Lewis, 1992). Given below are the governing equations for obtaining the gain

$$(A+BKC)^T P + P(A+BKC) + C^T K^T UKC + X = 0 \tag{32}$$

$$(A+BKC)W + W(A+BKC)^T + I = 0 \tag{33}$$

$$K = -U^{-1}B^T PWC^T (CWC^T)^{-1} \tag{34}$$

The matrices X and U in the above expressions represent weightings (which, for simplicity, could be chosen to be the identity matrices) in a linear-quadratic cost functional defined for the relative-state and relative-input respectively in

$$\tilde{J}^{(i)} = \int_0^\infty \left(\tilde{x}^{(i)T} X \tilde{x}^{(i)} + \tilde{u}^{(i)T} U \tilde{u}^{(i)}\right) dt \tag{35}$$

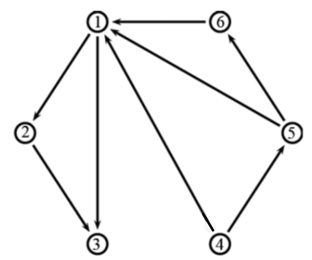

Fig. 1.An example of network topology (Li *et al*., 2010).

5. DISTRIBUTED FILTERING

Next, we extend the results to sensor networks, specifically to the consensus issue of distributed Kalman filtering. Again, the major difference between the proposed method and the existing ones (e.g. Olfati-Saber *et al*., 2012) is our utilization of relative-input.

Consider the following linear stochastic system of which some networked sensors are making measurement. Suppose the process and measurement noises are Gaussian and zero-mean with covariance Q and R respectively.

$$\dot{x} = Ax + w \tag{36}$$

$$z^{(i)} = Cx + v^{(i)}, \;\; i = 1, 2, \cdots\cdots, n \tag{37}$$

$$w : (0, Q), \quad v^{(i)} : (0, R) \tag{38}$$

The distributed filtering may be expressed as

$$\dot{\hat{x}}^{(i)} = A\hat{x}^{(i)} + L(z^{(i)} - C\hat{x}^{(i)}) + f^{(i)}, \; i = 1, 2, \cdots, n \tag{39}$$

where $f^{(i)}$ represents input from the network to be designed and L is the Kalman gain satisfying the following identities in which P is symmetric and positive definite (Lewis, 1992).

$$L = PC^T R^{-1} \tag{40}$$

$$\dot{P} = AP + PA^T + Q - PC^T R^{-1} CP \tag{41}$$

The objective of distributed filtering in current context is to achieve estimation consensus asymptotically among all the Kalman filters that are networked. Following the above framework we define, in the same spirit, the relative-state, relative-input, and relative-measurement before we present *Theorem 2*, another main result of this paper. Again, the relative-input plays an important role in tackling the problem.

$$\tilde{x}^{(i)} = \tau^{(i)} \sum_{j=1}^{n} a^{(ij)} (\hat{x}^{(i)} - \hat{x}^{(j)}) \tag{42}$$

$$\tilde{f}^{(i)} = \tau^{(i)} \sum_{j=1}^{n} a^{(ij)} (f^{(i)} - f^{(j)}) \tag{43}$$

$$\tilde{z}^{(i)} = \tau^{(i)} \sum_{j=1}^{n} a^{(ij)} (z^{(i)} - z^{(j)}) \tag{44}$$

Theorem 2: Given a stochastic system (36)-(38), and a communication network consisting of n sensors under a time-invariant communication topology that has a directional spanning tree, static protocol (45) solves the estimation consensus problem of the distributed Kalman filters.

$$f^{(i)} = \frac{1}{\phi^{(i)}} \left[\sum_{j=1}^{n} a^{(ij)} f^{(j)} - L\tilde{z}^{(i)} \right] \tag{45}$$

The following identity will be used in proving the theorem.

$$\frac{d}{dt} P^{-1} = -P^{-1} \dot{P} P^{-1} \tag{46}$$

Proof: Consider a Lyapunov function and its time-derivative

$$V = \sum_{i=1}^{n} \tilde{x}^{(i)T} P^{-1} \, \tilde{x}^{(i)} \tag{47}$$

$$\dot{V} = \sum_{i=1}^{n} \left[\begin{array}{l} \dot{\tilde{x}}^{(i)T} P^{-1} \, \tilde{x}^{(i)} + \tilde{x}^{(i)T} \dfrac{dP^{-1}}{dt} \tilde{x}^{(i)} \\ + \tilde{x}^{(i)T} P^{-1} \, \dot{\tilde{x}}^{(i)} \end{array} \right] \tag{48}$$

Using (42)-(46) and the same add-and-subtract technique that was applied in proving *Theorem 1* we get

$$\dot{V} = -\sum_{i=1}^{n} \left[\tilde{x}^{(i)T} (P^{-1} Q P^{-1} + C^T R^{-1} C) \, \tilde{x}^{(i)} \right] < 0 \tag{49}$$

$$\lim_{t \to \infty} V = 0 \tag{50}$$

The rest follows the same argument from *Theorem 1*.

We note that the proof for convergence of the error covariance is omitted. To deal with that, an augmented state containing both the estimation error and consensus error may be considered to start with. It is beyond the scope of this paper due to space limit, as our main focus is put on the filtering consensus itself. Intuitively, one may expect that the error covariance will converge asymptotically along with the convergence of the consensus error using the vanishing perturbation argument.

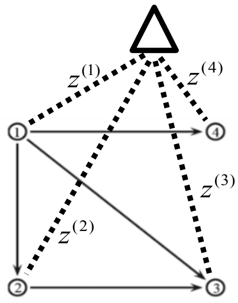

Fig. 2. An example of sensor and filter networks.

6. NUMERICAL EXAMPLES AND SIMULATIONS

The network topology of the first example is shown in Fig. 1. Agent 4 is the root agent ($K^{(4)}$ is designed separately), whose closed loop eigenvalues are placed at $+j$ and $-j$.

The second example is a stochastic plant of 4th order with topology shown in Fig. 2 (the solid lines) where the triangle on the top represents the plant and the dotted lines denote the measurement. The objective is to achieve estimation consensus.

Example 1:

$$A = \begin{bmatrix} -2 & 2 \\ -1 & 1 \end{bmatrix}, \; B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \; Q = I_2, \; R = 1,$$

$$K = 0.454, \; K^{(4)} = -1, \; \lambda(A + BK^{(4)}C) = \pm j$$

Example 2:

$$A = \begin{bmatrix} 0 & -2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 3 & 0 \end{bmatrix}, \; C^T = \begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 1 & -1 \\ 2 & 1 \end{bmatrix}, \; Q = 5 \cdot I_4, \; R = 3 \cdot I_2$$

It can be seen from Fig. 3 that the states from the six agents reach consensus asymptotically in Example 1. Controls are shown in Fig. 4.

Consensus of all the estimated states from the four Kalman filters can be observed in Example 2 as shown in Fig. 5 and Fig. 6. Fig. 7 shows how accurate the state estimation is. We chose Agent 2 as a reference agent in this example.

7. CONCLUSIONS

We addressed multi-agent consensus problem and proposed a relative-input-output approach to tackle the problem. New elements such as relative-input and agent-dependent coupling strength were utilized. The new approach improved the popular observer-based method. The results were then extended to solve the estimation consensus problem in distributed Kalman filtering. Two numerical examples were provided that validated the new method.

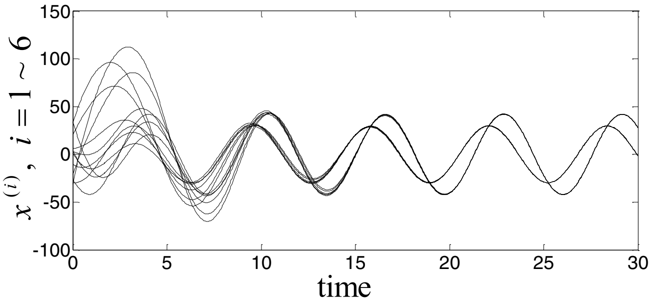

Fig. 3. Trajectories of states – Example 1.

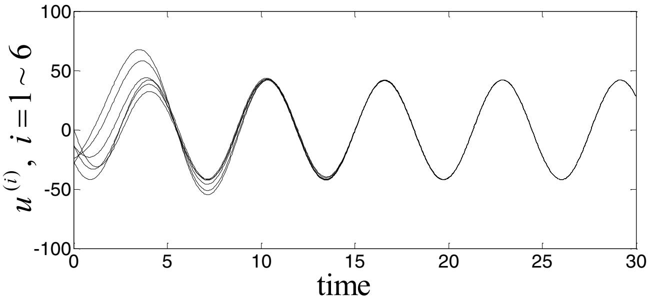

Fig. 4. Trajectories of control – Example 1.

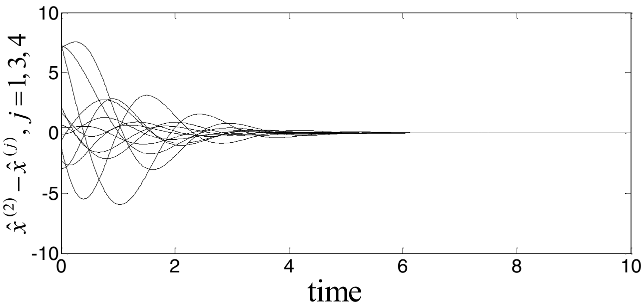

Fig. 5. Estimation difference – Example 2.

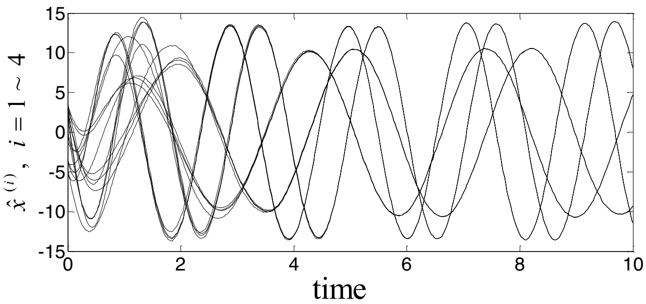

Fig. 6. State trajectories of all filters – Example 2.

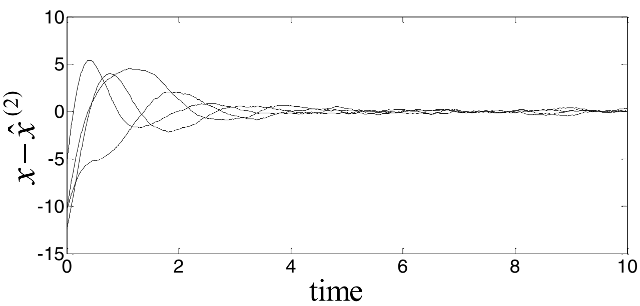

Fig. 7. Trajectories of estimation error – 2nd filter, Example 2.

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