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Collision Avoidance for Vessels using a Low-Cost Radar Sensor

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Abstract: Collision avoidance for vessels highly depends on a robust obstacle detection. This is commonly achieved by use of high precision radar sensing. For smaller vessels however, the use of low-cost sensors is typical. The idea of this work is to improve the robustness of collision avoidance by integrating a sensor model together with the collision avoidance algorithm in order to consider the accuracy of the measurements. Furthermore, a target tracking algorithm based on an interacting Multi-Model Filter (IMM) is used for robust obstacle detection.

Keywords: Collision avoidance, Ship navigation, Path planning, COLREGs, Raster grid, Interacting Multiple Model Filter,

1. INTRODUCTION

Collision avoidance for vessels is an ongoing topic in marine navigation research. In recent years, a lot of systems like Automatic Identification System (AIS) and Automatic Radar Plotting Aid (ARPA) have been developed to support the ship navigators. Further, a multitude of collision avoidance algorithms for scenarios on passages with high ship traffic like the Baltic Sea or the English Channel have been carried out. Most of these algorithms have been developed for land based applications, specifically used in Vessel Traffic Service (VTS) centers. These algorithms use the AIS information of all vessels in the region of interest to calculate optimal collision free routes for each vessel. However, in a lot of areas e. g. inland lakes or coastal regions, plenty of vessels like recreational crafts operate without an AIS system. For such scenarios, on-board systems like ARPA are necessary to detect and track other vessels. An ARPA system estimates the risk of collision and proposes avoidance maneuvers to support the navigator. Nevertheless, these systems are quit expensive, large, heavy, and have high power consumption. Thus, they are unqualified for smaller recreational crafts or small Unmanned Service Vehicles (USV). The aim of this work is to present an on-board collision avoidance approach, which detects and tracks other vessels based on the measurements of a low-cost radar sensor and to use these tracks to calculate a collision free path for close range encounter situations.

The presented approach has been developed and tested on the recreational craft "Korona", which is equipped with a Navico BR24 FMCW radar system, as shown in Fig. 1. The use of such a small, low-cost and low power consuming radar qualifies the approach for USVs and recreational crafts, particularly in regions where AIS carriage is not required. The test area is the Lake Constance and the upper Rhine river, where there is no obligation for AIS carriage. Because of the unavailability of AIS information, the position, course, speed, and dimensions of other vessels have to be estimated by radar measurements and additional algorithms to track the other vessels' course. For this purpose, an image preprocessing is used to detect objects in the radar data. To estimate tracks associated with these objects, a multiple model object tracking approach is applied. Based on these tracks, a collision avoidance algorithm calculates a collision free path in real time. The algorithm respects the turning circle of the own vessel and provides a drivable path according to the "Rules of the Road" (COLREGs, see (Benjamin et al., 2006)). This path could be used to support the vessel navigator or to guide the vessel automatically.

Fig. 1. The vessel Korona from the HTWG Konstanz equipped with the a Navico BR24 Radar Sensor.

In the last decades a multitude of algorithms concerning collision avoidance for vessels have been carried out. An overview of the approach until 2009 is given in (Statheros et al., 2008; Tam et al., 2009). Referring to Tam et al. (2009), the most promising approach is an evolutionary algorithm presented by Smierzchalski (1999) but the most practical and efficient approach is the grid-based method from Szlapczynski (2006). Nowadays, the algorithm for collision avoidance can by divided in two groups. The first

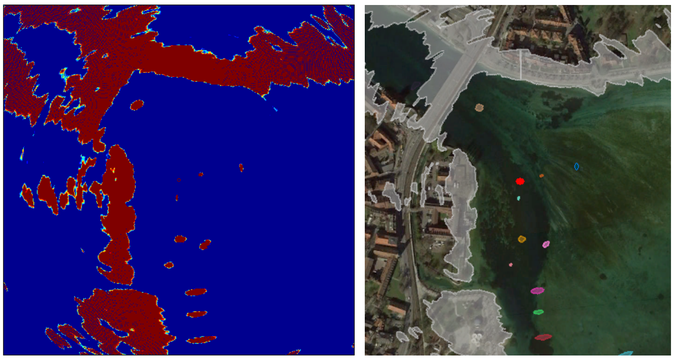


Fig. 2. Raw radar image (left) and extracted radar targets (right). The colored targets are dolphins or ships. The gray areas are considered to be part of the shore line and will not be used for target tracking.

one consists of approaches which are developed to optimize the vessel's routes by land-based stations like Vessel Traffic Services (VTS). Most of these algorithms are based on the approach of Smierzchalski (1999) and use heuristic optimization methods to find optimal collision free routes. The most promising approach, therefore, is the hybrid algorithm presented in (Szlapczynski and Szlapczynska, 2012). The second group comprises of methods that are used on-board the vessels. These methods use discrete grids to calculate a collision free path as presented in (Szlapczynski, 2006). Kuwata et al. (2013) presented an approach based on a velocity space grid. They also showed some results of actual on-water tests for multi vessel scenarios. In this work, the collision avoidance algorithm presented in (Blaich et al., 2012a,b) is used to perform real on-water test.

2. RADAR DATA PREPROCESSING

The Navico BR24 FMCW radar system is an imaging sensor which provides a 360° echo image of the environment every $2.5s$. The image consists of 2048 range scans (spokes), one spoke approx. every 0.2° but with an opening beam width of $5.2°(3dB)$. Each spoke itself is split in 1024 resolution cells, whose occupancy is indicated by a 4-bit value. The maximum range is configurable from 60m up to 32nmi. For the collision avoidance on inland waters we focused our work on a maximum range of about 800m. An example is shown in Fig. 2. Each time, an image has been received, the target extraction is performed in three steps: first an ego motion compensation, second a cell based occupancy likelihood determination and third a connected component labeling.

2.1 Ego Motion Compensation

In a first step, the effects of the own vessel's yaw rate is compensated. This is done by calculating the azimuth of each scan in respect to the corresponding own vessel heading. The own vessel moves with low speed, only. Changes in the absolute position of the sensor are small during one scan and will be corrected in the tracking system.

2.2 Occupancy Likelihood Determination

Second, the occupancy likelihood of each resolution cell is determined. Because of the rather large beam width, in one spoke, several cells are affected for each range step. The returned energy to the radar is the sum of the energy reflected from all those cells. However, the reported amplitude is zero or maximum most of the time, and therefore, of rather little use. Thus, for any value greater than zero, a cell is assumed to be reported as occupied. Based on the antenna beam width, we assume each cell becomes illuminated say n times during a 360°-scan. Furthermore, we assume that for each spoke the probability p of a cell being reported as occupied is independent and constant. Thus, the occupancy likelihood is binomially distributed and can be calculated by

$$\Pr(a_{i,j} = \text{occ}) = \sum_{k=0}^{m} \binom{n}{k} p^k (1-p)^{n-k}, \tag{1}$$

where

$$m = \sum_{l=\lfloor -n/2 \rfloor}^{\lfloor n/2 \rfloor} a_{i,j+l}. \tag{2}$$

Here, $a_{i,j} \in [0,1]$ denotes the reported occupancy value of each cell, i is the cell index in range and j in azimuth, respectively.

2.3 Connected Component Labeling

Finally, a cell is assumed to contain a target, if the occupancy probability is greater than 0.5. Since a target usually extends of several hundred cells, adjacent, occupied cells are grouped using connected component labeling (Gonzalez and Woods, 2006). Each group is considered to be a target of elliptical shape, for which range and azimuth of the center, its total size, cross range (distance between minimum and maximum bearing) and down range extends (distance between minimum range and maximum range) are calculated. The origin of each target is also verified using a map, which was obtained previously, by extracting the shore line from each radar scan (Greuter et al., 2012). Targets that are not located on the water are suppressed, and the remaining targets form the input of the Multi Object Tracking (Fig. 2).

3. TRACK FILTERING USING IMM

The extracted target positions are still very noisy, thus, strong low pass filtering is required to obtain an object's true position, heading, and velocity. Using just one dynamic model for straight line motion can lead to large estimation errors or even track loss during a turning maneuver. Using only one model for maneuvering targets can lead to poor state estimates when the object keeps its heading. Therefore, an interacting multiple model (IMM) filter is chosen here. The basic idea of this filter is to run several models in parallel. Based on the estimate of each model and the current measurement, a likelihood for each model to reflect the true motion state is determined. The output of the filter is a weighted sum of all model estimates. Since the IMM has been used in target tracking for many years, only the used models and parameters are explained here. For implementation details the reader is referred to Challa et al. (2011).

For each object on the water, three models of the form

$$\mathbf{x}_k^{(i)} = \mathbf{f}_k^{(i)}\left(\mathbf{x}_{k-1}^{(i)}\right) + \mathbf{G}_k^{(i)}\mathbf{w}_k^{(i)} \quad (3)$$

are assumed. Here $\mathbf{x}_k^{(i)}$ is the state vector of the ith model and $\mathbf{f}_k^{(i)}$ describes the corresponding system dynamics. $\mathbf{w}_k^{(i)}$ is a white noise sequence with zero mean and covariances $cov[\mathbf{w}_k^{(i)}] = \boldsymbol{Q}^{(i)}$, $\mathbf{G}_k^{(i)}$ serves as an input matrix and describes the interaction of the noise components with the states. For prediction and measurement update of each model, the Unscented Kalmanfilter is used (Julier and Uhlmann, 2004). The models become time dependent due to non constant sampling rate.

3.1 Dynamic Models

In the following, the dynamic models considered in this study are explained briefly. Details concerning these models can be found in Li and Jilkov (2003).

As straight line motion model the Constant Velocity(CV) model is applied. The state vector $\mathbf{x}^{(1)} = (x, \dot{x}, y, \dot{y})'$, where x denotes the position along the own vessel longitudinal axis and y the lateral axis. The process noise $\boldsymbol{Q}^{(1)} = \text{diag}[\sigma_x^2, \sigma_y^2]$ reflects very small acceleration in both direction.

The second model, used for maneuvering, is the Constant Turn Rate and Velocity (CTRV) model. Here, the state $\mathbf{x}^{(2)} = (x, y, \psi, \omega, V)'$ contains besides the position, the yaw angle ψ, turn rate ω. and the overall velocity V. It was already shown by Gertz (1989) that estimating the heading angle directly leads to better results during a maneuver than estimating the corresponding velocity directions. This model assumes that yaw rate and velocity are nearly constant, with $\boldsymbol{Q}^{(2)} = \text{diag}(\sigma_V^2, \sigma_\omega^2)$.

The last model is a Constant Acceleration (CA) model with state $\mathbf{x}^{(3)} = (x, \dot{x}, \ddot{x}, y, \dot{y}, \ddot{y})'$. This model serves as a transition model, when the motion changes from straight line to constant turn or fast velocity changes occur. It uses the same process noise expressions like the CV, but with higher values for the variances.

3.2 Measurement Model

Since the utilized radar does not provide any Doppler information, only the polar position measurements can be used for updating the dynamic models. The measurement equation is given by

$$\mathbf{z}_k = \begin{bmatrix} r \\ \theta \end{bmatrix} = h(\mathbf{x}_k) + \mathbf{v}_k = \begin{cases} \sqrt{x^2 + y^2} \\ \arctan(y/x) \end{cases} + \mathbf{v}_k \quad (4)$$

Here, $\mathbf{v}_k$ is also normally distributed with zero mean and covariance $\boldsymbol{R} = \text{diag}[\sigma_R^2, \sigma_\theta^2]$.

For a collision avoidance system the size of an object is of critical importance. In Salmond and Parr (2003) a fifth state is added to the CV model defining the length of an object. It is assumed that all objects are of elliptical shape with a fixed aspect ratio γ of major and minor axis. Thus, the relationship between the measured down range L_{DR} and the object length l is defined as

$$L_{DR}(\phi) = l\sqrt{\cos^2(\phi) + \gamma^2 \sin^2(\phi)}, \quad (5)$$

where the aspect angle $\phi = \psi - \theta$ is the difference of object heading and measured azimuth. However, for our system, the measured down range is overestimated most of the time and shows only little dependency on the aspect angle. Thus, we do not incorporate the object length in our dynamic models, but update the length estimate with a constant gain on every scan independently. The same is done for the extracted size of the target, which is used for data association only. Due to the high antenna beam width, cross range values are in general too large to be of any use. When a new measurement is available, each model is updated according to the rules of IMM. Based on the individual filter residuals, a new model likelihood is determined. The final state and uncertainty estimate is then given by the weighted sum of all models. Finally, the components of the object state necessary for the most likely model are provided for the path planning module, together with the length and the maneuvering likelihood.

4. TRACK HANDLING

One of the major challenge in a multi object tracking application still is the data association problem. In general, it is unknown which measurement was originated from which object, even the cardinality of objects is not known a priori. Especially when working with a radar on water, additional measurements occur which do not belong to any real object (clutter), or an existing object is not detected at all. To solve these problems, a large variety of algorithms have been proposed. These can be roughly divided into two groups. The first group is comprised of single instance filters. The states of all objects are collected in one set. This set is regarded as a new state space and is updated using one Bayes filter. The second group uses at least one filter instance for each object. For these filters, an explicit measurement to object association has to be performed. Here also a large number of algorithms have been designed for different applications and conditions.

For the presented application the following assumptions are valid:

- One object creates just a single measurement.
- Object trajectories can be in close proximity.

Because of these assumption, the Joint Probabilistic Data Association (JPDA) is selected, Bar-Shalom et al. (2005). The JPDA associates all measurements within a gate to a track subject to all other tracks in this gate. To use the JPDA, first the assignment matrix $\mathbf{A} \in \mathbb{R}^{M_m \times M_t}$ has to be calculated which contains for each measurement/track pair the conditional measurement data likelihood. M_m and M_t are the number of measurements that fall in the gate, number of tracks in the gate, respectively. For measurement i and track j the association likelihood is assumed to be Gaussian distributed:

$$a_{ij} = \mathcal{N}(\mathbf{z}_i, h(\mathbf{x}_j), \mathbf{S}_j) \quad i \in \{1, \dots, M_m\},\ j \in \{1, \dots, M_t\} \quad (6)$$

The innovation covariance $\mathbf{S}_j$ is the sum of the current track position covariance and the constant measurement covariance. If the value is below some gating threshold, the likelihood is zero. Then, $\mathbf{A}$ has to be expanded for each track by the likelihood of a missed detection, and for each measurement the likelihood of false alarm or new object likelihood has to be taken into account. Then, the joint association likelihood is given by

$$p(a_{ij}) = a_{ij} \frac{\text{per}(\mathbf{A}_{\bar{i}\bar{j}})}{\text{per}(\mathbf{A})} \tag{7}$$

where the permanent is the sum of products over all permutations:

$$\text{per}(\mathbf{A}) = \sum_{i=1}^{n} a_{ij} \text{per}(\mathbf{A}_{\bar{i}\bar{j}}) \tag{8}$$

and $\mathbf{A}_{\bar{i}\bar{j}}$ is the submatrix obtained by removing row i and column j. The computation of all permanents can still be very time consuming. For this reason, they are here replaced by an approximation. In Uhlmann (2008) various approximation schemes are compared with respect to their assignment correctness. Based on these results, here the approximation

$$\text{per}(A) \leq \prod_{i=1}^{n} \sum_{j=1}^{n} \frac{a_{ij} c_i}{c_j} \tag{9}$$

is adopted, where c_j is the sum of column j. Based on this aproximation, an optimal assignment approach is used to find the measurement to track update pairing. Not assigned measurements are used as candidates for new tracks in the next update step.

5. COLLISION AVOIDANCE

The collisions avoidance algorithm uses the track information and the most probable model provided by the IMM filter to predict the movement of the other vessels over a future time period. If the CTRV model is the most probable, the motion of the other vessel is predicted as an arc, otherwise it is predicted as a straight line. This information is used by the collision avoidance algorithm to find a safe path in close range encounter situations. The algorithm is divided in two steps. In the first step, a set of waypoints for a collision free path is estimated in the local area. The size of this local region depends on the sensor's range. This algorithm is presented in detail in (Blaich et al., 2012a,b), thus, only a short overview of the algorithm is given in this section. In a second step the waypoints are connected by Bezier splines to get a path that is navigable by the vessel.

The algorithm for the waypoint search uses the own vessel speed and course information to predict the own movement while traveling the local area. To predict the movement of the other vessels, the information provided by the tracking algorithm is used. Both informations are stored in a grid to calculate the cost of a path. If map data is available this information is also stored in the same grid. To find a collision free path in this grid an A^* search is used. To consider the own ship kinematic constraints, a special T-shape neighborhood is implemented. This neighborhood only considers those cells for the next search step that are actually reachable by the vessel. For the other ships, a ship domain as presented by Goodwin (1975) is used to prefer COLREGs compliant avoidance maneuvers. The usage of the grid and the A^* search enables the real-time capability of the waypoint search, which is necessary for on-board systems.

For connecting the waypoints, Bezier splines are used in such a way that the path segments are interconnected with continuous curvature. Thus, a smooth transition between the segments is assured. Moreover, since the path planning algorithm already takes into account the vessel kinematics, the Bezier splines provide a continuous path with admissible curvature.

6. EXPERIMENTAL RESULTS

In order to evaluate the performance of the tracking algorithm and to test the usability for collision avoidance, several live, on-water tests have been performed. The recreational craft Solgenia, as shown in Fig. 3, was used as target vessel. The target vessel performs different maneuvers in different environments for parameter estimation of the tracking system, Fig. 4. Therefore, both ships are equipped with low-cost GPS-receivers. The extracted target trace from the tracking system is compared with the GPS trace using the simple cost function

Fig. 3. The target vessel Solgenia used for on-water tracking tests.

$$\mathcal{J} = \frac{1}{N} \sum_{k=1}^{N} (\hat{\boldsymbol{x}}_k - \boldsymbol{x}_k)^T \boldsymbol{R}_{GPS}^{-1} (\hat{\boldsymbol{x}}_k - \boldsymbol{x}_k), \tag{10}$$

where N is the number of radar scans, $\hat{\boldsymbol{x}}_k$ the estimated and $\boldsymbol{x}_k$ the measured target state at time k in CV-representation, and $\boldsymbol{R}_{GPS}$ the GPS measurement noise. In the first step, the process noise parameters have been optimized using the target trajectory and simulated measurement data. For each parameter set, 50 Monte Carlo runs with fixed measurement noise have been performed, and the mean of the cost function has been evaluated.

In the second step, the measurement noise terms of the tracking system have been adjusted using the radar data from one data set. The obtained parameters have been validated against a second data set. The adjustment was made under the constraint that no track loss occurs.

CV	CTRV	CA	Sensor
σ_x=0.003m/s^2	σ_V=0.07m/s	σ_x=0.06m/s^2	σ_R=1.2m
σ_y=0.003m/s^2	σ_ω=0.5°$/s$	σ_y=0.06m/s^2	σ_θ=1.8°

The total position error is shown in Fig. 5. It has to be noted that for evaluation of the global coordinates of

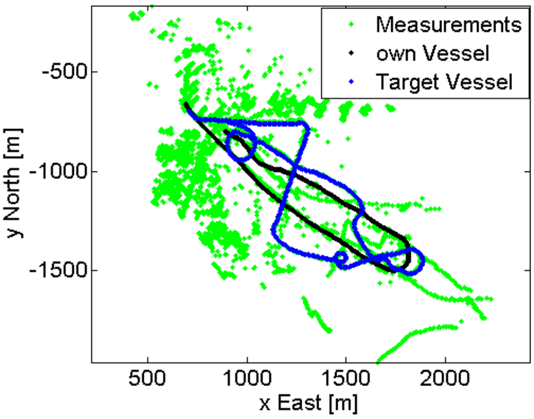


Fig. 4. Trajectory of target vessel and own vessel with all received radar detections

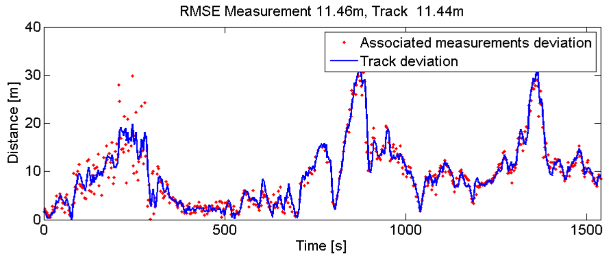


Fig. 5. Horizontal position deviation track trace to GPS trace

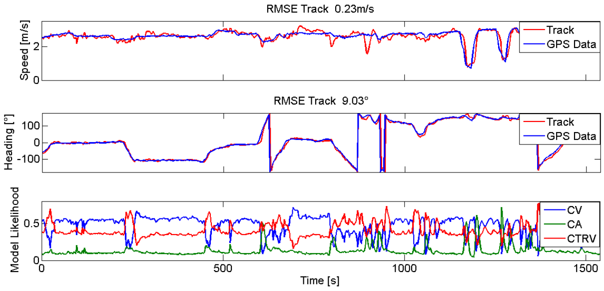


Fig. 6. Velocity and Heading of track and GPS with corresponding model likelihoods.

the track and measurement also very accurate information of the own ship heading is required. Since the utilized compass has a standard deviation of greater than two degree, the absolute position error inclines if the distance between the target vessel and the own vessel is large. This effect dominates the RMSE calculation, thus the deviation between measurements and filtered solution is small.

For a collision avoidance system, the accuracy of heading and velocity of an obstacle is as important as its position. These accuracies and the models' likelihoods are presented in Fig. 6. These results show that the CV-Model and the CTRV-Model have similar likelihoods in straight line motion. This may be due to the fact that in this case small yaw rates still exits (Fig. 7). The accuracy during a typical 90° turning maneuver is shown in Fig. 8.

When fast maneuvers occur, e.g. during a small circle Fig. 9, none of the models can estimate velocity and heading accurately. This is mainly due to the low sensor update rate and accuracy in comparison to the possible target

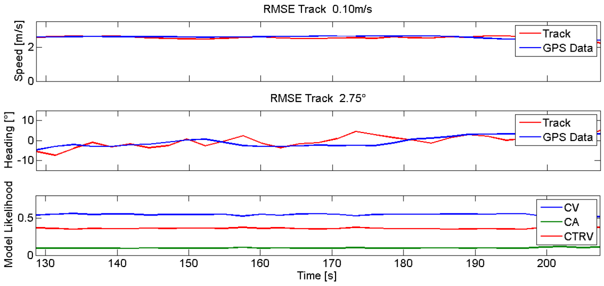


Fig. 7. States during straight line motion

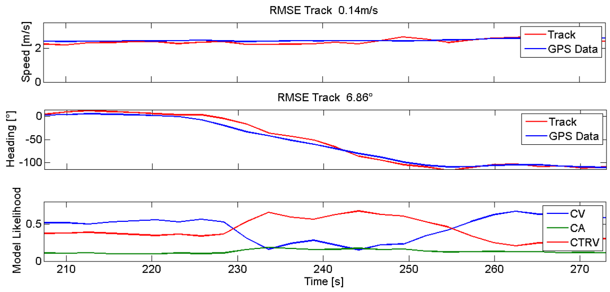


Fig. 8. States during typical turning maneuver

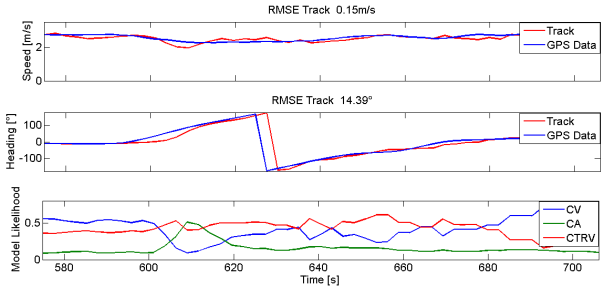
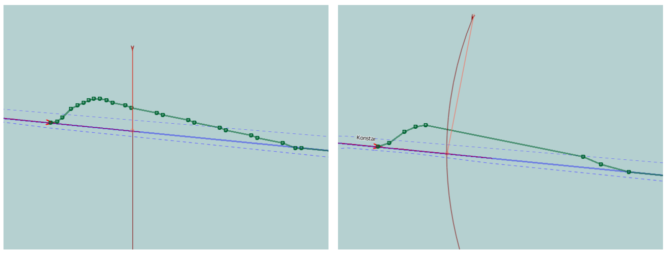


Fig. 9. States during fast circle

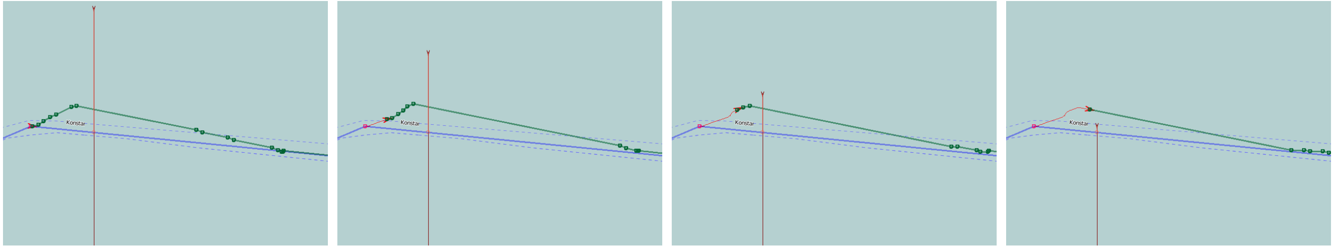
dynamics. The estimated length of the target vessel has a mean of $12.4m$, where the true length is $8.2m$. This accuracy is still sufficient for CA and a general distinction of target vessels into different groups like leisure boats or ferries.

The results of some close range encounter situations which will cause a collision if no avoidance maneuver is performed are shown in Fig. 10. The own vessel is depicted as red arrow, and the own course as blue line. For the other vessels and their predicted paths, a brown arrow and a brown line is used. The path provided by the algorithm to avoid a collision is plotted in green. An example for



(a) Straight target motion (b) Target in turning manoeuver

Fig. 10. Resulting path provided by the collision avoidance algorithm.



(a) (b) (c) (d)

Fig. 11. Sequence of calculated paths and executed trajectory for a complete evasive maneuver.

a scenario with a straight motion of the other vessel is shown in Fig. 10a, and an example with a turning motion is depicted in Fig. 10b. For the straight line motion, a sequence of calculated trajectories for the full evasive maneuver is shown in Fig. 11. The path taken by the own vessel is indicated in red. After the first encounter Fig. 11a, for each new measurement, a new trajectory is calculated, e.g. Fig. 11b,11c. Only small corrections from the initial path have to be made. The calculation of the collision point is stable, Fig. 11d.

7. CONCLUSION

In this paper, a case study of a collision avoidance system with low-cost sensors has been shown. The implemented tracking system is fast, and the track has proven to be stable over a long time. The achieved accuracy in position, heading and velocity is sufficient for the used collision avoidance system, however, it could be significantly improved if better heading estimates would be available. Although the absolute position accuracy is low, it should be noted that for the collision avoidance just the relative position to the own vessel is of importance, thus inaccurate compass and GPS resolution have minor impact on the performance of the algorithm. The suggested CA approach is quite insensitive to the remaining noise of the track inputs. Yet, further research is ongoing to incorporate also the covariances of the tracks in the target motion prediction.

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