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Passive Control for Bilinear Stochastic Systems with Bounded Control ⋆

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Abstract: This paper investigates the problem of passive control for a class of bilinear stochastic systems. It assumes that the multiple dimensional control is bounded. Based on storage function method, passivity condition for the bilinear stochastic systems is presented by introducing a bounded nonlinear feedback controller. Moreover, input and output strict stochastic passivity, the relationship between output strict stochastic passivity and stochastic L_2 gain from input to output, and stochastic KYP (Kalman-Yakubovich-Popov) property for bilinear stochastic systems have been considered. A numerical example is provided to show the effectiveness of the method.

Keywords: Bilinear stochastic systems, bounded control, passivity, finite L_2 gain, KYP property.

1. INTRODUCTION

A bilinear control system is described by the following dynamical equation $\dot{x} = Ax + \sum_{i=1}^{m} u_i N_i x + Bu$, where A, N_i, B are constant matrices of appropriate dimensions, and the control inputs $u = [u_1\ u_2\ \dots\ u_m]^T$ appear as coefficients of the state $x \in \mathbb{R}^n$. The bilinear control model, containing multiplicative controls, can approximate a wide class of engineering systems and natural phenomena, such as nuclear reactors, manufacturing processes, induction motor drives, heat exchanger with controlled flow, immune system, ecology processes, economic processes, and so on Mohler (1973); Pardalos and Yatsenko (2007). Since the concept of bilinear systems introduced in the early 1960s Mohler (1973), the study of such systems has been received considerable attention in the last half century Pardalos and Yatsenko (2007).

In fact, the bilinear control systems can be regarded as a class of special systems between nonlinear and linear systems. Many effective methods and algorithms for nonlinear control systems have been specialized for bilinear systems, and some approaches for linear control systems have also been extended to bilinear systems. Some earlier results on bilinear control systems can refer to Mohler (1973); Pardalos and Yatsenko (2007) and the references therein. Bounit (2003) has established some necessary and sufficient conditions for bilinear control systems via nonlinear feedback. The problem of global asymptotic stabilization for bilinear singular control systems has been investigated by Lu and Ho (2006) based on LaSalle invariant principle for nonlinear systems. For discrete-time uncertain bilinear fuzzy systems, Li et al. (2008) has proposed H_∞ controller by means of the parallel distributed compensation approach and linear matrix inequality (LMI) technique. Based on LMI optimization method, a static linear state-feedback control has been designed for bilinear control systems with parameters in polytopic region Amato et al. (2009).

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Apart from multiplicative controls in bilinear control systems, a large number of real-world systems described by bilinear models are frequently perturbed by multiplicative noises, for example, fluctuations in electrical circuits, diffusion processes in nuclear fission and heat transfer, immune system, etc. Mohler and Kolodziej (1980); Mao (1997); Sornette and Pisarenko (2008). The existence of multiplicative stochastic perturbations makes the analysis and design problems for bilinear stochastic systems more complex. The issues of realization and identification of bilinear stochastic systems have been considered in Desai (1986) and Favoreel et al. (1999), respectively. When control inputs are bounded, the bilinear stochastic systems can be transformed into linear ones with norm-bounded parametric uncertainties, and then by stochastic Lyapunov function method state observer and H_∞ filter have been constructed in terms of LMIs Ali et al. (2005); Halabi et al. (2009).

On the other hand, the passivity theory originating from circuit theory, has been widely applied in analysis and synthesis for dynamical systems, such as van der Schaft (2000); Byrnes et al. (1991); Lin and Shen (1999); Arcak and Sontag (2008); Zhao and Hill (2008). A system or circuit is passive means that it consumes energy, but does not produce energy. In other words, the motion of a passive system is accompanied by the loss of its storage energy. Thus, the passivity of a dynamical system implies its stability in Lyapunov sense. Most recently, passivity method has been used to control design for discrete-time singular bilinear systems with uncertainties Zhang and Zhang (2013). In Florchinger (1999), the passivity concept has been extended to nonlinear stochastic control systems. By nonlinear stochastic KYP Lemma, the relationship between a passive system and its zero-output dynamics for nonlinear stochastic systems has been presented in Lin and Lin (2009); Lin et al. (2012). The stochastic passivity approach has been employed to adaptive control Yaesh and Shaked (2009). Passivity analysis for stochastic neural networks and power systems has been studied recently in Chen et al. (2012); Zhu and Shen (2011); Odun-Ayo and Crow (2013). To the best of authors' knowledge, however, passive control for bilinear stochastic systems has not been investigated, which motivates the present paper.

In this paper, a passivity system approach is applied to feedback stabilization for a class of bilinear stochastic systems with multiple-dimensional bounded control. First, a sufficient condition, ensuring stochastic passivity of the closed-loop bilinear stochastic control systems under a nonlinear bounded feedback controller, has been provided using storage energy method. Compared with some existing results, the parameter of the controller needn't to be designed, and the controller relies only on the given systems parameters. Second, input strict stochastic passivity, output strict stochastic passivity, and stochastic finite L_2 gain from input to output of bilinear stochastic systems have also been considered. Third, a stochastic KYP condition for bilinear stochastic systems has been proposed by means of incorporating the bilinear term into state term. Finally, the validity of the proposed method is shown by an illustrative example.

Notations: The notations in this paper are standard. $|\cdot|$ refers to the Euclidean norm of a vector; $\text{tr}\{A\}$ is the trace of matrix A; $\mathbf{E}\{\cdot\}$ denotes the mathematical expectation; $\lambda_m(\cdot)$ and $\lambda_M(\cdot)$ designate the minimal and maximal eigenvalues of a symmetric matrix, respectively; $\langle *,*\rangle$ stands for usual inner product. $\mathbf{Pr}\{\cdot\}$ is the probability. $(\Omega,\mathscr{F},\mathscr{P})$ is a complete probability space, where Ω is the sample space, $\mathscr{F}$ is the σ-algebra of subsets of Ω, and $\mathscr{P}$ is the probability measure on $\mathscr{F}$. The symmetric term in a symmetric matrix is denoted as $*$.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following bilinear stochastic system

$$dx(t) =[Ax(t)+\sum_{i=1}^{m}u_i(t)N_ix(t)+Bu(t)]dt+Ex(t)dw(t) \quad (1a)$$

$$y(t) =Cx(t)+Du(t) \quad (1b)$$

where $x(t)\in\mathbb{R}^n$ is the state vector; $u(t)=[u_1(t)\ u_2(t)\ \ldots\ u_m(t)]^T$ is the control input satisfying $|\ u_i(t)\ |\leq\alpha$, where $\alpha>0$ is a known constant presenting the upper bound of control input; $y(t)\in\mathbb{R}^m$ is the output vector; $w(t)$ is a scalar Brownian motion (or Wiener process) defined on the complete probability space $(\Omega,\mathscr{F},\mathscr{P})$ satisfying $\mathbf{E}\{dw(t)\}=0$ and $\mathbf{E}\{dw^2(t)\}=dt$. $A\in\mathbb{R}^{n\times n},N_i\in\mathbb{R}^{n\times n},B\in\mathbb{R}^{n\times m},E\in\mathbb{R}^{n\times n},C\in\mathbb{R}^{m\times n},D\in\mathbb{R}^{m\times m}$ are known constant matrices.

We now proceed to present some preliminaries as follows.

Definition 1. Consider the following system

$$dx(t)=f(x)dt+g(x)dw(t). \quad (2)$$

System (2) is said to be stochastically stable or stable in probability if for any scalars $\rho\in(0,1)$ and $r>0$, there exists a scalar $\delta(\rho,r)>0$ such that

$$\mathbf{Pr}\{|\ x(t)\ |<r,\forall t\geq 0\}\geq 1-\rho$$

when $|\ x(0)\ |<\delta(\rho,r)$. Additionally, system (2) is said to be stochastically asymptotically stable, if it is stochastically stable, and further, for every $\rho\in(0,1)$, there exists a scalar $\delta(\rho)>0$ such that

$$\mathbf{Pr}\{\lim_{t\to\infty}|\ x(t)\ |=0,\forall t\geq 0\}\geq 1-\rho$$

when $|\ x(0)\ |<\delta(\rho)$.

Lemma 2. (Itô Lemma) Assume $x(t)$ is an Itô process and $V(x)$ is a positive definite and radially unbounded real-valued function, which is continuously twice differentiable in x and once differentiable in $t\geq 0$, then the stochastic differential of $V(x)$ along system (2) is

$$dV(x)=\mathscr{L}V(x)dt+\frac{\partial V}{\partial x}g(x)dw(t)$$

where the infinitesimal generator is written as

$$\mathscr{L}V(x)=\frac{\partial V}{\partial t}+\frac{\partial V}{\partial x}f(x)+\frac{1}{2}\text{tr}\{g^T(x)\frac{\partial^2 V}{\partial x^2}g(x)\}.$$

Based on stochastic stability theory Mao (1997), $\mathscr{L}V(x)\leq 0$ implies the stochastic stability of system (2). Furthermore, if $\mathscr{L}V(x)<0$ then system (2) is stochastically asymptotically stable.

Definition 3. a) System (2) is said to be stochastically passive, if there exists a continuous positive definite function $V(x)$ satisfying

$$\mathscr{L}V(x)\leq 2y^T(t)u(t),\forall t\geq 0$$

for any input signal $u(t)$. Moreover, system (2) is said to be strictly stochastically passive, if there exists a positive definite function $Q(u,x,t)$ such that

$$\mathscr{L}V(x)\leq 2y^T(t)u(t)-Q(u,x,t),\forall t\geq 0.$$

b) System (2) is said to be input strictly stochastically passive, if for any input signal $u(t)$, there exist a continuous positive definite function $V(x)$ and a scalar $\gamma>0$ satisfying

$$\mathscr{L}V(x)\leq 2y^T(t)u(t)-\gamma u^T(t)u(t),\forall t\geq 0$$

i.e., the positive definite function $Q(u,x,t)$ in a) is $\gamma u^T(t)u(t)$.
c) System (2) is said to be output strictly stochastically passive, if for any input signal $u(t)$, there exist a continuous positive definite function $V(x)$ and a scalar $\varepsilon>0$ satisfying

$$\mathscr{L}V(x)\leq 2y^T(t)u(t)-\varepsilon y^T(t)y(t),\forall t\geq 0$$

i.e., $Q(u,x,t)=\varepsilon y^T(t)y(t)$.

Remark 4. The function $V(x)$ satisfying Definition 3 is called storage function of system (2), and the inequality in a) is called the dissipation inequality. Recalling $\mathbf{E}\{dw(t)\}=0$, integrating both sides of the first inequality in a) and taking mathematical expectation, we have

$$\mathbf{E}\{V(x)\}\leq 2\mathbf{E}\{\int_0^\tau y^T(t)u(t)dt\}+V(0),\forall\tau\geq 0.$$

It indicates the phenomenon that the expectation of the storage energy $V(x)$ at any time $\tau>0$ is not larger than the sum of the expectation of total externally supplied energy $2\mathbf{E}\{\int_0^\tau y^T(t)u(t)dt\}$ and the energy stored in the system at initial time during the time interval $[0,\tau]$. Thus, similar to van der Schaft (2000), we can say that the expectation of internal energy of a system (2) is stochastically dissipative, or there is no internal creation of energy.

The following inequality is useful to derive the main results.

Lemma 5. For any vectors $x,\ y\in\mathbb{R}^n$ and a symmetric positive definite matrix $P\in\mathbb{R}^{n\times n}$, then

$$2x^Ty\leq x^TPx+y^TP^{-1}y.$$

The main objective of this paper is to construct a multiple dimensional bounded feedback control law $u(t)$ such that the resulting closed-loop system of (1) is stochastic passive, input/output strictly stochastically passive, and it has stochastic L_2 gain and stochastic KYP characteristics.

3. MAIN RESULTS

Firstly, system (1) can be rewritten as

$$dx(t) =[Ax(t)+(\bar{\mathbf{N}}_x+B)u(t)]dt+Ex(t)dw(t) \quad (3a)$$

$$y(t) =Cx(t)+Du(t) \quad (3b)$$

where $\bar{\mathbf{N}}_x=[N_1x(t)\ N_2x(t)\ \ldots\ N_mx(t)]$.

In some recent papers, for instance Li et al. (2008), a bounded scalar state-feedback control law is designed as

$$u(t) = \alpha \frac{Kx(t)}{\sqrt{1+x^T(t)K^TKx(t)}} \tag{4}$$

where the gain matrix $K \in \mathbb{R}^{1\times n}$ is to be determined. However, it should be noted that the feedback controller in (4) is one-dimensional.

Inspired by the one-dimensional nonlinear controller in Bounit (2003), in this paper we construct the following multiple dimensional nonlinear feedback control law

$$\begin{aligned} u(t) =& [u_1(t)\ u_2(t)\ \ldots\ u_m(t)]^T \\ u_i(t) =& -\alpha \frac{\langle N_i x(t)+B, x(t)\rangle}{1+\langle N_i x(t)+B, x(t)\rangle^2},\ i=1,2,\ldots,m. \end{aligned} \tag{5}$$

It is obvious that $u(t)$ in (5) is bounded with its components $| u_i(t) | \le \alpha$. In contrast with Li et al. (2008), the feedback controller (5) does not depend on any design parameters, and $u(t)$ can be computed by the given parameters α, N_i, B and the current state value $x(t)$.

By adopting the nonlinear feedback controller (5), the main results of this paper are presented in what follows.

3.1 Stochastic Passivity

For bilinear stochastic system (1), we have the following conclusion.

Theorem 6. The closed-loop system of (1) under nonlinear feedback controller (5) is strictly stochastically passive, if there exists a matrix $P > 0$ with appropriate dimensions such that

$$\Gamma = \begin{bmatrix} \Gamma_1 & PB-C^T \\ * & -D-D^T \end{bmatrix} < 0 \tag{6}$$

where $\Gamma_1 = PA + A^TP + E^TPE + m\alpha^2P + \sum_{i=1}^m N_i^TPN_i$.

Proof. Select a storage function as

$$V(x) = x^TPx \tag{7}$$

with $P > 0$ to be determined.

Applying Lemma 2, the stochastic differential of (7) along (1a) shows as

$$dV(x) = \mathscr{L}V(x)dt + 2x^TPEx(t)dw(t) \tag{8}$$

where the weak infinitesimal generator is

$$\begin{aligned} \mathscr{L}V(x) =& x^T(t)[PA + A^TP + E^TPE]x(t) \\ &+ 2x^T(t)P(\bar{\mathbf{N}}_x + B)u(t). \end{aligned} \tag{9}$$

According to Lemma 5 and $|u_i(t)| \le \alpha$, we get

$$\begin{aligned} &2x^T(t)P\bar{\mathbf{N}}_x u(t) \\ =& 2x^T(t)u_1(t)PN_1x(t) + 2x^T(t)u_2(t)PN_2x(t) \\ &+ \cdots + 2x^T(t)u_m(t)PN_mx(t) \\ \le& x^T(t)(\alpha^2P + N_1^TPP^{-1}PN_1 + \alpha^2P + N_2^TPP^{-1}PN_2 \\ &+ \cdots + \alpha^2P + N_m^TPP^{-1}PN_m)x(t). \\ =& x^T(t)(m\alpha^2P + \sum_{i=1}^m N_i^TPN_i)x(t). \end{aligned} \tag{10}$$

Noting (10) and (1b), we have

$$\begin{aligned} &\mathscr{L}V(x) - 2y^T(t)u(t) \\ \le& x^T(t)[PA + A^TP + E^TPE + (m\alpha^2P + \sum_{i=1}^m N_i^TPN_i)]x(t) \\ &+ 2x^T(t)PBu(t) - 2x^T(t)C^Tu(t) - 2u^T(t)D^Tu(t) \\ =& \xi^T(t)\Gamma\xi(t) \end{aligned} \tag{11}$$

where $\xi(t) = [x^T(t)\ u^T(t)]$, and Γ is given in (6).

When $\Gamma < 0$, denote $\lambda_M(\Gamma) = -\mu < 0$. Then, it follows from (11) that there surely exists a positive scalar $0 < \mu_0 < \mu$ such that $\mathscr{L}V(x) \le 2y^T(t)u(t) - \mu x^T(t)x(t) < 2y^T(t)u(t) - \mu_0 x^T(t)x(t)$. As a consequence of (6), system (1) is strictly stochastically passive from Definition 3. The proof is thus completed. ■

Remark 7. If $D = 0$, i.e., the output is $y(t) = Cx(t)$, then (6) is not strictly feasible. It should be noted from (11) that, in such case, if we can find a positive definite matrix $P > 0$ satisfying

$$PA + A^TP + E^TPE + m\alpha^2P + \sum_{i=1}^m N_i^TPN_i < 0$$

with equality constraint $PB = C^T$, then system (1) is strictly stochastically passive. In this paper, we assume that $D + D^T > 0$ are always satisfied if not special specified.

When $B = 0$, the stochastic system (1) is still bilinear, and Theorem 6 reduces to the following one.

Corollary 8. System (1) with $B = 0$ is strictly stochastically passive, if there exists a matrix $P > 0$ with appropriate dimensions such that

$$\begin{bmatrix} \Gamma_1 & -C^T \\ * & -D-D^T \end{bmatrix} < 0 \tag{12}$$

where Γ_1 is given in Theorem 6.

When $N_i = 0$, the stochastic system (1) becomes linear, and then we obtain the following result readily.

Corollary 9. System (1) with $N_i = 0$ is strictly stochastically passive, if there exists a matrix $P > 0$ with appropriate dimensions such that

$$\begin{bmatrix} PA + A^TP + E^TPE & PB-C^T \\ * & -D-D^T \end{bmatrix} < 0. \tag{13}$$

3.2 Input and Output Strict Stochastic Passivity

This subsection deals with input strict stochastic passivity and output strict stochastic passivity of bilinear system (1).

Select the storage function (7), then from (11) there is

$$\begin{aligned} &\mathscr{L}V(x) - 2y^T(t)u(t) + \gamma u^T(t)u(t) \\ \le& x^T(t)[PA + A^TP + E^TPE + (m\alpha^2P + \sum_{i=1}^m N_i^TPN_i)]x(t) \\ &+ 2x^T(t)PBu(t) - 2x^T(t)C^Tu(t) \\ &- 2u^T(t)D^Tu(t) + \gamma u^T(t)u(t) \\ =& \xi^T(t)\Theta\xi(t) \end{aligned} \tag{14}$$

where $\Theta = \Gamma + \mathrm{diag}\{0, -\gamma I\}$.

It is clear from Definition 3 that $\Theta < 0$ ensures the input strict stochastic passivity of system (1).

Therefore, the input strict stochastic passivity of system (1) can be presented as follows.

Theorem 10. The closed-loop system of (1) under nonlinear feedback controller (5) is input strictly stochastically passive, if there exist a scalar $\gamma > 0$ and a matrix $P > 0$ with appropriate dimensions such that

$$\begin{bmatrix} \Gamma_1 & PB - C^T \\ * & \gamma I - D - D^T \end{bmatrix} < 0. \tag{15}$$

where $\Gamma_1 = PA + A^T P + E^T PE + m\alpha^2 P + \sum_{i=1}^m N_i^T PN_i$.

Similarly, we can derive the output strict stochastic passivity of system (1).

Theorem 11. The closed-loop system of (1) under nonlinear feedback controller (5) is output strictly stochastically passive, if there exist a scalar $\varepsilon > 0$ and a matrix $P > 0$ with appropriate dimensions such that

$$\begin{bmatrix} \Lambda_1 & PB - C^T + \varepsilon C^T D \\ * & -D - D^T + \varepsilon D^T D \end{bmatrix} < 0. \tag{16}$$

where $\Lambda_1 = PA + A^T P + E^T PE + m\alpha^2 P + \sum_{i=1}^m N_i^T PN_i + \varepsilon C^T C$.

3.3 Stochastic Stability and Stochastic Finite L_2 Gain

This subsection will show that the stochastic passivity of the bilinear stochastic system (1) implies stochastic stability of (1a) with $u(t) = 0$. Moreover, we will establish the relationship between the output strict stochastic passivity and stochastic finite L_2 gain from input to output for system (1).

Consider the following stochastic system

$$\begin{aligned} dx(t) =& [f(x) + k(x)u(t)]dt + g(x)dw(t) \\ y(t) =& c(x) + h(x)u(t) \end{aligned} \tag{17}$$

where $f(x), k(x), g(x), c(x), h(x)$ are functions with compatible dimensions, then the stochastic finite L_2 gain from input $u(t)$ to output $y(t)$ of system (17) is defined in what follows.

Definition 12. System (17) is said to have stochastic finite L_2 gain, if there exists scalars $\kappa_1 > 0, \kappa_2 > 0$ satisfying

$$\mathbf{E}\{|y_\tau|_2^2\} \le \kappa_1 \mathbf{E}\{|u_\tau|_2^2\} + \kappa_2 |x(0)|_2^2, \forall \tau > 0$$

where $|z_\tau|_2^2 = \int_0^\tau z^T(t)z(t)dt$ for any vector $z(t)$.

Remark 13. The stochastic finite L_2 gain of system (17) means that finite input $u(t)$ in the sense of mean square will lead to finite output $y(t)$ in the sense of mean square over the time interval $[0, \tau]$. Stochastic finite L_2 gain expresses the input-output feature of a stochastic system, and it reduces to BIBO stability in square for a system in deterministic setting. Thus, we can also say that system (17) is stochastically input-output stable in mean-square if the inequality in Definition 12 holds.

Theorem 14. The bilinear stochastic system (1a) with $u(t) = 0$ is stochastically stable, if system (1) is stochastically passive; furthermore, system (1a) with $u(t) = 0$ is stochastically asymptotically stable, if system (1) is strictly stochastically passive.

Proof. Noting Definition 3, when system (1) is stochastically passive then $\mathscr{L}V(x) \le 2y^T(t)u(t)$.

In addition, it can be seen that $u(t) = 0$ ensuring $\mathscr{L}V(x) \le 0$, which guarantees the stochastic stability of system (1a) from Mao (1997).

Moreover, if system (1) is strictly stochastically passive, the following can be obtained by setting $u(t) = 0$

$$\mathscr{L}V(x) \le -Q(u, x, t) < 0, \forall t \ge 0.$$

Hence, system (1a) is stochastically asymptotically stable. ■

Theorem 15. If there exist a scalar $\varepsilon > 0$ and a matrix $P > 0$ with appropriate dimensions satisfying (16), then the bilinear stochastic system (1) under the nonlinear feedback control (5) has stochastic finite L_2 gain from input to output in the sense of Definition 12, where $\kappa_1 = \frac{16}{3\varepsilon^2}$ and $\kappa_2 = \frac{4}{3\varepsilon}\lambda_M(P)$.

Proof. In view of Theorem 11, system (1) is output strictly stochastically passive when (16) holds. So, (16) will guarantee $\mathscr{L}V(x) \le 2y^T(t)u(t) - \varepsilon y^T(t)y(t), \forall t \ge 0$.

Subsequently, we can deduce that for a scalar $\varepsilon > 0$

$$\begin{aligned} \mathscr{L}V(x) \le& 2y^T(t)u(t) - \varepsilon y^T(t)y(t) \\ =& -[\frac{1}{2}\sqrt{\varepsilon}y(t) - \frac{2}{\sqrt{\varepsilon}}u(t)]^T[\frac{1}{2}\sqrt{\varepsilon}y(t) - \frac{2}{\sqrt{\varepsilon}}u(t)] \\ &+ \frac{4}{\varepsilon}u^T(t)u(t) - \frac{3\varepsilon}{4}y^T(t)y(t) \\ \le& \frac{4}{\varepsilon}u^T(t)u(t) - \frac{3\varepsilon}{4}y^T(t)y(t) \end{aligned} \tag{18}$$

or

$$y^T(t)y(t) \le \frac{16}{3\varepsilon^2}u^T(t)u(t) - \frac{4}{3\varepsilon}\mathscr{L}V(x). \tag{19}$$

Integrating both sides of (19) over $[0, \tau]$ leads to

$$\int_0^\tau y^T(t)y(t)dt \le \frac{16}{3\varepsilon^2}\int_0^\tau u^T(t)u(t)dt - \frac{4}{3\varepsilon}\int_0^\tau \mathscr{L}V(x)dt. \tag{20}$$

Noticing $\mathbf{E}\{dw(t)\} = 0$ and $\mathbf{E}\{V(x(\tau))\} \ge 0, \forall \tau > 0$, we obtain

$$\begin{aligned} \mathbf{E}\{\int_0^\tau \mathscr{L}V(x)dt\} =& \mathbf{E}\{\int_0^\tau dV(x)\} \\ =& \mathbf{E}\{V(x(\tau))\} - V(0) \\ \ge& -V(0). \end{aligned}$$

Taking mathematical expectation to both sides of (20), there is

$$\begin{aligned} \mathbf{E}\{|y_\tau|_2^2\} \le& \frac{16}{3\varepsilon^2}\mathbf{E}\{|u_\tau|_2^2\} + \frac{4}{3\varepsilon}V(0) \\ \le& \frac{16}{3\varepsilon^2}\mathbf{E}\{|u_\tau|_2^2\} + \frac{4}{3\varepsilon}\lambda_M(P)|x(0)|_2^2. \end{aligned} \tag{21}$$

Obviously, system (1) has stochastic finite L_2 gain from input $u(t)$ to output $y(t)$ in the sense of Definition 12. ■

3.4 Stochastic KYP Property

In this subsection, stochastic KYP property of bilinear system (1) is considered. The definition of stochastic KYP property of the stochastic system (17) is addressed as follows.

Definition 16. System (17) is said to have stochastic KYP property, if there exist positive definite function $V(x)$ and matrix functions $l(x)$, $\phi(x)$ such that

$$\begin{cases} \frac{\partial V}{\partial x}f + \frac{1}{2}\mathrm{tr}\{g^T \frac{\partial^2 V}{\partial x^2} g\} \le -l^T l \\ \frac{\partial V}{\partial x}k - 2c^T = -2l^T \phi \\ h + h^T = \phi^T \phi. \end{cases} \tag{22}$$

Theorem 17. The closed-loop system of (1) under nonlinear feedback controller (5) is stochastically passive and has stochastic KYP property, if exist matrices $P > 0$ and L, W with appropriate dimensions satisfying

$$\begin{cases} PA+A^TP+E^TPE+m\alpha^2P+\sum_{i=1}^{m}N_i^TPN_i \leq -L^TL \\ PB-C^T = -L^TW \\ D+D^T = W^TW. \end{cases} \tag{23}$$

Moreover, system (1) is strictly stochastically passive with stochastic KYP property, if the first inequality of (23) holds without equality.

Proof. Choose the storage function $V(x) = x^TPx$, then

$$\begin{aligned} \mathscr{L}V(x) =& x^T(t)[PA+A^TP+E^TPE]x(t) \\ &+2x^T(t)P\bar{\mathbf{N}}_xu(t)+2x^T(t)PBu(t) \\ \leq& x^T(t)[PA+A^TP+E^TPE \\ &+m\alpha^2P+\sum_{i=1}^{m}N_i^TPN_i]x(t)+2x^T(t)PBu(t). \end{aligned} \tag{24}$$

In light of (23), there amounts to

$$\begin{aligned} \mathscr{L}V(x) \leq& -x^T(t)L^TLx(t)+2x^T(t)(C^T-L^TW)u(t) \\ =& 2x^T(t)C^Tu(t)+u^T(t)W^TWu(t) \\ &-[Lx(t)+Wu(t)]^T[Lx(t)+Wu(t)] \\ \leq& 2x^T(t)C^Tu(t)+u^T(t)(D+D^T)u(t) \\ =& 2y^T(t)u(t). \end{aligned} \tag{25}$$

It is clear to see that (23) means that system (1) is stochastically passive from Definition 3.

Furthermore, if the first inequality of (23) holds without equality, i.e., $PA+A^TP+E^TPE+m\alpha^2P+\sum_{i=1}^{m}N_i^TPN_i < -L^TL$, then $\mathscr{L}V(x) < 2y^T(t)u(t)$. We can always find a positive definite function $Q(u,x,t)$ such that

$$\mathscr{L}V(x) \leq 2y^T(t)u(t) - Q(u,x,t), \forall t \geq 0$$

which means that system (1) is strictly stochastically passive.

At the same time, system (1) can be written as model (17) with

$$\begin{aligned} f(x) =& [A+\sum_{i=1}^{m}u_i(t)N_i]x(t), \\ k(x) =& B,\ g(x) = Ex(t), \\ c(x) =& Cx(t),\ h(x) = Dx(t). \end{aligned} \tag{26}$$

Considering (10) and (23), it can be seen that

$$\begin{aligned} \frac{\partial V}{\partial x}f+\frac{1}{2}\mathrm{tr}\{g^T\frac{\partial^2 V}{\partial x^2}g\} \leq& x^T(t)[PA+A^TP+E^TPE \\ &+m\alpha^2P+\sum_{i=1}^{m}N_i^TPN_i]x(t) \\ \leq& -x^T(t)L^TLx(t). \end{aligned}$$

Selecting matrix functions $l(x)$, $\phi(x)$ as

$$l(x) = Lx(t),\ \phi(x) = Wu(t). \tag{27}$$

then the fist inequality in Definition 16 can be established, and the other two equalities in Definition 16 are also satisfied. So, system (1) has stochastic KYP property from Definition 16. This completes the proof. ■

Remark 18. It deserves to notice that, the function $f(x)$ in (26) includes the bilinear term $\sum_{i=1}^{m}u_i(t)N_ix(t)$ of system (1). Based on this disposal, the first inequality in Definition 16 can be established by (10) and (27). Otherwise, if the the bilinear term $\sum_{i=1}^{m}u_i(t)N_ix(t)$ is incorporated by $k(x)$ as $k(x) = \bar{\mathbf{N}}_x + B$, then the second equality in Definition 16 is expressed as

$$P[N_1x(t)\ N_2x(t)\ \ldots\ N_mx(t)]+PB-C^T = -L^TW.$$

Consequently, one can not find constant matrices $P > 0$ and L, W satisfying the above equation, since it relies on the stochastic variable $x(t)$.

4. A NUMERICAL EXAMPLE

In this section, a numerical example is given to demonstrate the effectiveness of the approach.

Example 19. Consider the bilinear stochastic system (1) with

$$\begin{aligned} A =& \begin{bmatrix} -1.5 & 0 \\ 0 & -2 \end{bmatrix}, B = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \\ E =& \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, C = [\,0.5\ \ 1\,], D = 0.2 \\ N_1 =& \begin{bmatrix} 0.1 & 0.2 \\ 0.1 & 0.3 \end{bmatrix}, N_2 = \begin{bmatrix} -0.2 & 0 \\ 0.1 & -0.2 \end{bmatrix}. \end{aligned} \tag{28}$$

In this example, we assume that the control input satisfies $|u_i| < \alpha = 0.8$, $i = 1,2$.

Solving (6) with (28), we can obtain

$$P = \begin{bmatrix} 4.3633 & 4.1711 \\ 4.1711 & 5.0422 \end{bmatrix}.$$

Therefore, system (28) under bounded nonlinear feedback controller (5) with $\alpha = 0.8$ is strictly stochastically passive according to Theorem 6. Moreover, the corresponding closed-loop system of (28) is stochastically asymptotically stable from Theorem 14.

Applying Theorem 10 to system (28), we have

$$P = \begin{bmatrix} 3.5310 & 3.9239 \\ 3.9239 & 4.9909 \end{bmatrix}, \gamma = 0.1751.$$

This means that system (28) under controller (5) with $\alpha = 0.8$ is input strictly stochastically passive.

By using Theorem 11 to this example, we can find that system (28) under controller (5) with $\alpha = 0.8$ is output strictly stochastically passive. From Theorem 15, the closed-loop system of (28) under controller (5) with $\alpha = 0.8$ has stochastic finite L_2 gain from input $x(t)$ to output $y(t)$ in the sense of Definition 12.

For this example, we can compute $W = \sqrt{2D} = \sqrt{0.4}$. Substitute W into (23), $L^T = -(PB-C^T)/\sqrt{0.4}$ can be obtained. Then by using Schur completes, the first inequality of (23) can be rewritten as $\begin{bmatrix} \Gamma_1 & -(PB-C^T)/\sqrt{0.4} \\ * & -1 \end{bmatrix} \leq 0$, where Γ_1 is given in Theorem 10. By solving this LMI, we can see that system (28) strictly stochastically passive with stochastic KYP property.

5. CONCLUSIONS

This paper has considered stochastic passive control for bilinear systems with multiplicative noises and bounded inputs. Based on storage energy method and introducing a bounded nonlinear feedback control, which is dependent only on the given system parameters, a stochastic passivity condition for the resulting closed-loop system has been presented. In the meantime, some other properties for bilinear stochastic systems have also been investigated. The usefulness of the proposed method has been verified by an illustrative example.

REFERENCES

Ali, H.S., Rafaralahy, H., Zasadzinski, M., Halabi, S., and Darouach, M. (2005). Observer design for a class of stochastic bilinear systems with multiplicative noise. In *Proc. 2005 American Control Conference*, Portland, OR, USA, Jun., 2005, pp. 3641-3642.

Amato, F., Cosentino, C., Fiorillo, A.S., and Merola, A. (2009). Stabilization of bilinear systems via linear state-feedback control. *IEEE Trans. on Circuits and Systems-II: Express Briefs*, 56(1), 76-80.

Arcak, M. and Sontag, E.D. (2008). A passivity-based stability criterion for a class of interconnected systems and applications to biochemical reaction networks. *Mathematical Biosciences and Engineering*, 5(1), 1-19.

Bounit, H. (2003). Comments on the feedback stabilization for bilinear control systems. *Applied Mathematics Letters*, 16, 847-851.

Byrnes, C.I., Isidori, A., and Willems, J.C. (1991). Passivity, feedback equivalence, and the global stabilization of minimum phase nonlinear systems. *IEEE Trans. on Automatic Control*, 36(11), 1228-1240.

Chen, Y., Wang, H., Xue, A. and Lu, R. (2010). Passivity analysis of stochastic time-delay neural networks. *Nonlinear Dynamics*, 61, 71-82.

Desai, U.B. (1986). Realization of bilinear stochastic systems. *IEEE Trans. on Automatic Control*, 31(1), 189-192.

Favoreel, W., De Moor, B., and Van Overschee, P. (1999). Subspace identification of bilinear systems subject to white inputs. *IEEE Trans. on Automatic Control*, 44(6), 1157-1165.

Florchinger, P. (1999). A passive system approach to feedback stabilization of nonlinear control stochastic systems. *SIAM Journal on Control and Optimization*, 37(6), 1848-1864.

Halabi, S., Ali, H.S., Rafaralahy, H., and Zasadzinski, M. (2009). H_∞ functional filtering for stochastic bilinear systems with multiplicative noises. *Automatica*, 45, 1038-1045.

Li, T.-H.S., Tsai, S.-H., Lee, J.-Z., Hsiao, M.-Y., and Chao, C.-H. (2008). Robust H_∞ fuzzy control for a class of uncertain discrete fuzzy bilinear systems. *IEEE Trans. on Systems, Man, and Cybernetics-Part B: Cybernetics*, 38(2), 510-527.

Lin, W. and Shen, T. (1999). Robust passivity and feedback design for minimum-phase nonlinear systems with structural uncertainty, *Automatica*, 35, 35-47.

Lin, Z. and Lin, Y. (2009). Passivity and feedback design of nonlinear stochastic systems. In *Proc. Joint 48th Conf. Decision and Control and 28th Chinese Control Conference*, Shanghai, China, Dec. 2009, pp. 1575-1580.

Lin, Z., Liu, J., Lin, Y., Zhang W. (2012). Nonlinear stochastic passivity, feedback equivalence and global stabilization. *International Journal of Robust and Nonlinear Control*, 22, 999-1018.

Lu, G. and Ho, D.W.C. (2006). Continuous stabilization controllers for singular bilinear systems: the state feedback case. *Automatica*, 42, 309-314.

Mao, X. (1997). *Stochastic Differential Equations ans Their Applications*. Horwood Publishing, Chichester, UK.

Mohler, R. (1973). *Bilinear Control Processes*. Academic Press, New York, USA.

Mohler, R.R. and Kolodziej, W.J. (1980). An overview of stochastic bilinear control processes. *IEEE Trans. on Systems, Man, and Cybernetics*, 10(12), 913-918.

Odun-Ayo, T. and Crow, M.L. (2013). An analysis of power system transient stability using stochastic energy functions. *International Transactions on Electrical Energy Systems*, 23(2), 151-165.

Pardalos, P.M. and Yatsenko, V. (2007). *Optimization and Control of Bilinear Systems: Theory, Algorithms, and Applications.* Springer, New York, USA.

Sornette, D. and Pisarenko, V.F. (2008). Properties of a simple bilinear stochastic model: estimation and predictability. *Physica D*, 237, 429-445.

van der Schaft, A. J. (2000). *L_2-Gain and Passivity Techniques in Nonlinear Control*. Springer, New York, USA.

Yaesh, I. and Shaked, U. (2009). Stochastic passivity and its application in adaptive control. *IEEE Trans. on Automatic Control*, 54(1), 136-142.

Zhang, X.-H. and Zhang, Q.-L. (2013). Passive control for uncertain discrete singular bilinear systems. In *Proc. 25th Chinese Control and Decision Conference*, Guiyang, China, May 2013, pp. 3906-3909.

Zhao, J. and Hill, D.J. (2008). Passivity and stability of switched systems: a multiple storage function method. *Systems and Control Letters*, 57, 158-164.

Zhu, S. and Shen, Y. (2011). Passivity analysis of stochastic delayed neural networks with Markovian switching. *Neurocomputing*, 74(10), 1754-1761.