Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Crawling Gait Planning for a Quadruped Robot with High Payload Walking on Irregular Terrain

Nan Hu* Shaoyuan Li* Dan Huang Feng Gao*****

**Department of Automation, Key Laboratory of System Control and Information Processing, Ministry of Education, Shanghai Jiao Tong University, Shanghai,*
200240, China (e-mail: syli@sjtu.edu.cn)
*** School of Aeronautics and Astronautics, Shanghai Jiao Tong University, Shanghai,*
200240, China
**** School of Mechanical Engineering, Shanghai Jiao Tong University, Shanghai*
200240, China

Abstract: Walking on irregular terrain is usually a common task for a quadruped robot. It is however difficult to control the robot in this situation as undesirable impulse force by collision between the foot of robot and obstacles makes the robot unstable. This paper presents a hybrid Force-Posture Feedback Compensation (FPFC) Controller for a quadruped robot with high payload walking on irregular terrain. In order to make the robot walk stably on irregular terrain, the proposed controller utilizes the feedback signals detected by force sensor and gyroscope to adjust every leg of the robot in real-time. The foot trajectory is scheduled based on the Bezier curve method in order to improve the stability of quadruped robot. Simulations of crawling gait on irregular terrain have been performed. The results have verified that the proposed methods have better stability and performance for walking on the irregular terrain.

Keywords: Quadruped, robot, gait planning, trajectory planning, kinematics

1. INTRODUCTION

We study the problem of gait planning for a quadruped robot walking on irregular terrain in this paper. The research on walking on uneven terrain is very crucial as this is the case for robots to walk in most circumstances, especially in disaster relief sites. It is however difficult to control the robot in this situation as undesirable impulse force by collision between the foot of robot and an obstacle makes the robot unstable. Conventional position control can not be used to solve this problem as the position control only utilize the foot trajectory designed in advance to control the robot's motion. When walking on even terrain, the robot is stable. But when walking on uneven terrain, the robot is unstable because the designed trajectory of robot's foot will be interrupted by obstacles or irregular terrain.

The work described here is in pursuit of cyclic locomotion of robots walking on irregular terrain stably. Many researches refer to gait planning of quadruped robots walking on even terrain (Hua et al, 2012; Bin et al, 2011; Zamani et al,2011; Chuangfeng, 2010). The methods in these papers are not applicable when the robots run into an obstacle or walking on uneven terrain. Quadruped walking on irregular terrain has been pursued recently by several labs using the LittleDog robotic platform (Shkolnik et al., 2011; Kolter et al., 2011). The main goal of these projects was always path planning and foot placement on large-scale obstacles. Many of the circumstances traversed by these robots were also modeled in advance, and full state of robots was given to the robot through offboard sensing. Therefore the autonomy of robot is weak. Impedance control is applied to the quadruped robot in (Jaehwan et al., 2012). Where an impedance controller is designed for trotting on irregular terrain. Though the speed of the robot was higher, the stability was deteriorated with the trotting gait. A force threshold-based position controller was designed in (Luther et al., 2012). The method only used the force feedback signal to adjust the length of the robot's legs. So the robot may not be stable in highly complex terrain.

In order to solve the problems above, we design a controller to counteract the disturbance produced by the collision between the foot of robot and the obstacle. The biggest challenge we face in this problem is that the obstacle's size is random and unknown. So the controller we design should be robust to adjust the compensation value in real-time in the light of the obstacle's size.

The robot we study should have the ability of high payload as it is always used to carry many goods compared with other robots. Therefore another challenge we face is the robot we design should exhibit higher stability than usual ones to ensure goods safety. That is to say we should make the roll angle, pitch angle of the robot as small as possible. For instance, a robot without high payload on it may be considered to be stable when the roll angle and pitch angle of it are all less than 10°. But in our case, we have to make those angles less than 5°.

In order to address the aforementioned problems, we present a Force-Posture Feedback Compensation (FPFC) Controller in this paper. The FPFC utilizes the feedback signals detected by force sensor and gyroscope at the same time to adjust every leg of the robot in real-time. With FPFC, whatever the size of the obstacle is, the robot can traverse the irregular terrain stably. We also present crawling gait generation method as this gait has higher stability than other gaits. The foot trajectory is scheduled based on the Bezier curve method in order to improve the stability of quadruped robot.

The paper is structured as follows. Kinematic model of the robot is presented in Section 2; Section 3 describes the trajectory planning method and the reasons for choosing the crawling gait; The designed FPFC Controller is given is in Section 4. The simulations are proposed in Section 5. Finally, The conclusion and future work are given in Section 6.

2. KINEMATIC MODEL OF THE ROBOT

The quadruped robot used in this paper consists of a body frame and four legs as show in Fig. 1. Each leg of the robot we designed has three degrees of freedom, one on the Coronal Plane (CP) and two along the Sagittal Plane (SP).

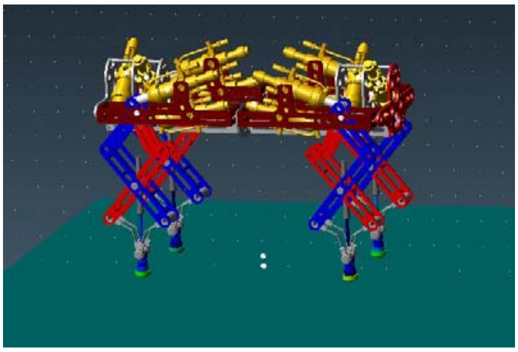

Fig. 1. The robot model built in RecurDyn.

Next, the forward kinematics (FK) and inverse kinematics (IK) equations of single leg will be built. The Coronal Plane and Sagittal Plane of the kinematic structure of the robot's right front leg are shown in Fig.2 and Fig.3 respectively.

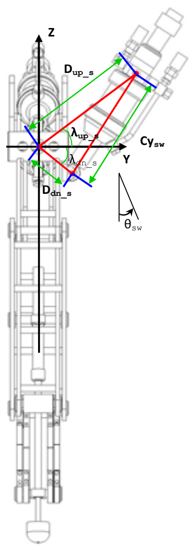

Fig. 2. Coronal plane of the right front leg.

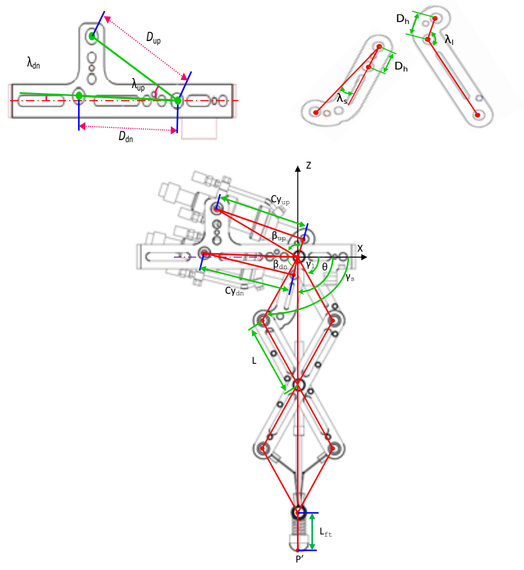

Fig. 3. Sagittal plane of the right front leg.

2.1 *The solution of inverse kinematics*

Given the foot tip position $P_b(x_p, y_p, z_p)$, the coordinate of the foot tip on the sagittal plane can be derived from the transform matrix below.

$$\theta_{sw} = -\arctan\left(\frac{y_p}{z_p}\right), \tag{1}$$

$$P(x, 0, z) = R_x(\theta_{sw}) \cdot P_b(x_p, y_p, z_p), \tag{2}$$

Where R_x is the transform matrix from the frame fixed with the body to the frame rotate with the sagittal plane.

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{sw} & \sin\theta_{sw} \\ 0 & -\sin\theta_{sw} & \cos\theta_{sw} \end{bmatrix}, \tag{3}$$

On the sagittal plane in Fig.3, we can get the equations below based on simple geometric relationships.

$$L_p = \sqrt{x^2 + z^2}, \tag{4}$$

$$L_{p'} = L_p - L_{ft}, \tag{5}$$

$$\theta = \frac{\pi}{2} - \arctan\frac{x}{z}, \tag{6}$$

$$\alpha = \arccos\left(\frac{L_{p'}}{4L}\right), \tag{7}$$

$$\begin{cases} \gamma_l = \theta - \alpha \\ \gamma_s = \theta + \alpha, \end{cases} \tag{8}$$

$$\beta_{up} = \pi - (\lambda_l - \gamma_l) - \lambda_{up}, \tag{9}$$

$$\beta_{dn} = \pi + \lambda_s - \gamma_s + \lambda_{dn}, \tag{10}$$

$$\beta_{sw} = \lambda_{up\_s} + \lambda_{dn\_s} - \theta_{sw}, \tag{11}$$

Then, we can get the length of the cylinder as follows:

$$Cy_{up}^{2} = D_{up}^{2} + D_{h}^{2} - 2D_{up}D_{h}\cos(\beta_{up}), \tag{12}$$

$$Cy_{dn}^{2} = D_{dn}^{2} + D_{h}^{2} - 2D_{dn}D_{h}\cos(\beta_{dn}), \tag{13}$$

$$Cy_{sw}^{2} = D_{dn\_s}^{2} + D_{up\_s}^{2} - 2D_{dn\_s}D_{up\_s}\cos(\beta_{sw}), \tag{14}$$

Therefore, by inputting the foot-tips value, the length of the cylinder will be solved by the aid of IK equations.

2.2 *The solution of forward kinematics*

Given the length of the three cylinders Cy_{up}, Cy_{dn}, Cy_{sw} we can obtain β_{up}, β_{dn}, β_{sw} of the Fig.3 by simple geometric relationships.

$$\beta_{up} = \arccos\left(\frac{D_{up}^{2} + D_{h}^{2} - Cy_{up}^{2}}{2D_{up}D_{h}}\right), \tag{15}$$

$$\beta_{dn} = \arccos\left(\frac{D_{dn}^{2} + D_{h}^{2} - Cy_{dn}^{2}}{2D_{dn}D_{h}}\right), \tag{16}$$

$$\beta_{sw} = \arccos\left(\frac{D_{dn\_s}^{2} + D_{up\_s}^{2} - Cy_{sw}^{2}}{2D_{dn\_s}D_{up\_s}}\right), \tag{17}$$

We can also obtain the equations by simple geometric relationships as follows:

$$\gamma_{l} = -\pi + \lambda_{up} + \lambda_{l} + \beta_{up}, \tag{18}$$

$$\gamma_{s} = \pi + \lambda_{dn} + \lambda_{s} - \beta_{dn}, \tag{19}$$

$$\theta_{sw} = \lambda_{up\_s} + \lambda_{dn\_s} - \beta_{sw}, \tag{20}$$

$$\theta = \frac{\gamma_{l} + \gamma_{s}}{2}, \tag{21}$$

$$\alpha = \theta - \gamma_{l}, \tag{22}$$

$$L_{p} = 4L\cos\alpha + L_{ft}, \tag{23}$$

$$\begin{cases} x = L_{p}\cos\theta \\ y = 0 \\ z = L_{p}\sin\theta, \end{cases} \tag{24}$$

Then we can get the foot tip position $P_b(x_p, y_p, z_p)$:

$$P_b(x_p, y_p, z_p) = R_x^{T}(\theta_{sw}) \cdot P(x, 0, z). \tag{25}$$

Therefore, by formulating the FK equations and knowing the foot tips position of the quadruped robot, it will be able to construct the pattern of gait planning for crawling.

3. GAIT AND FOOT TRAJECTORY PLANNING

3.1 *Crawling gait planning*

Gait is a pattern of discrete foot placements performed in a given sequence. The gaits of the quadruped robots are classified into static gaits and dynamics gaits. Static gaits which contain crawl and wave mean that the vertical projection of center of mass always remains inside the polygon formed by the supporting legs of quadruped robot. Dynamic gaits which include trot, pace and gallop occur when the vertical projection of center of mass is not necessary to remain inside the polygon formed by the supporting legs of quadruped robot with the dynamic balance to be maintained.

Stability is more important than velocity when the robot walks on irregular terrain. So in this paper, we choose crawling gait for the quadruped robot because it has superior stability, redundancy and high traction to other gaits. (Carlos et al., 2004). When crawling, the robot's legs follow this sequence: left front leg, left back leg, right front leg, right back leg, in a regular 1-3-2-4 beat. At the crawl, the robot will always have one foot swung and the other three feet on the ground, save for a brief moment when weight is being transferred from one foot to another.

When describing the crawling gait, the support phase of a leg is the period in which the foot is on the ground while the swing phase of a leg is the period in which the foot is not on the ground. The sequence diagram of crawl is shown in Fig.4. In order to increase the robot stability, the quadruped robot's duty factor is taken as 0.85, which means that it exists four foots on the ground at the same time in one period of walking.

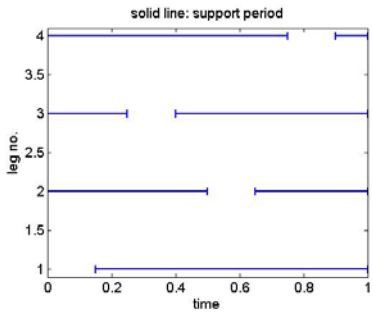

Fig. 4. Sequence diagram of crawl gait.

3.2 *Foot trajectory planning*

Two factors should be considered to design the foot trajectory. The first is that the foot trajectory can't make a great impact when the robot's feet land on the ground. The second is that the shape of foot trajectory should be similar to trajectory of an animal.

In the Fig.4, if we let the T be the time of one whole period it shows a leg will take only 0.15T to complete its own motion. In order to have a high stability, the foot trajectory of quadruped robot must meet the requirements as follows: the leg trajectory should be continuous; the velocity and acceleration of foot must be zero when leg starts to leave and land on the floor. Therefore, we use Bezier curve to design the trajectory of foot which can satisfy the requirements above perfectly. There are also some other methods to plan the trajectory of quadruped robot like Cubic Trajectory and Sinusoidal Trajectory and so on. However, none of these trajectories take the acceleration into account which may result in a fierce collision when the foot of robot lands on the ground. The equation of Bezier curve is defined as (Matthias et al., 1995).

$$B(t)=\sum_{i=0}^{n} C_{n}^{i} P_i (1-t)^{n-i} t^{i}, \tag{26}$$

Then we can get a 5 order Bezier curve to define the foot trajectory on X orientation from equation (1) as follows:

$$\begin{aligned} X(t) = {} & P_0(1-t)^5 + 5P_1(1-t)^4 t + 10P_2(1-t)^3 t^2 \\ & + 10P_3(1-t)^2 t^3 + 5P_4(1-t)t^4 + P_5 t^5, \end{aligned} \tag{27}$$

It must meet the following requirements:

Position: $$\begin{cases} X(0) = X_0 \\ X(0.15T) = X_0 + \Delta X, \end{cases} \tag{28}$$

Velocity: $$\begin{cases} \dot{X}(0) = 0 \\ \dot{X}(T) = 0, \end{cases} \tag{29}$$

Acceleration: $$\begin{cases} \ddot{X}(0) = 0 \\ \ddot{X}(T) = 0, \end{cases} \tag{30}$$

where X_0 is the start position of robot's foot on X orientation while ΔX is the step pitch on X orientation. So we can get the coefficient P_0 to P_5 from equations (27) to (30). Let X_0 =0.5m, ΔX =0.33m, T=0.65s , then we have $X(t)$ as follows:

$$\begin{aligned} X(t) = {} & 0.5 + 3560.41t^3 - 54775.55t^4 + 224720.2t^5 \\ & (0 \le t \le 0.0975), \end{aligned} \tag{31}$$

Similarly, we can get a 5 order Bezier curve to define the foot trajectory on Z orientation from equation (1) as follows:

$$\begin{aligned} Z(t) = {} & P_0'(1-t)^5 + 5P_1'(1-t)^4 t + 10P_2'(1-t)^3 t^2 \\ & + 10P_3'(1-t)^2 t^3 + 5P_4'(1-t)t^4 + P_5' t^5, \end{aligned} \tag{32}$$

It must meet the following requirements:

Position: $$\begin{cases} Z(0) = Z_0 \\ Z(0.075T) = Z_0 + \Delta Z, \end{cases} \tag{33}$$

Velocity: $$\begin{cases} \dot{Z}(0) = 0 \\ \dot{Z}(T) = 0, \end{cases} \tag{34}$$

Acceleration: $$\begin{cases} \ddot{Z}(0) = 0 \\ \ddot{Z}(T) = 0, \end{cases} \tag{35}$$

where Z_0 is the start position of robot's foot on Z orientation while ΔZ is the step height on Z orientation. So we can get the coefficient P_0' to P_5' from equations (32) to (35). Let Z_0 =0, ΔZ =0.2m, then we can get the foot trajectory $Z(t)$ as follows:

$$Z(t) = \begin{cases} 17262.6t^3 - 531156.84t^4 + 4358209.93t^5 \\ \qquad (0 \le t \le 0.04875) \\ \\ -17262.6(t-0.04875)^3 + 531156.84(t-0.04875)^4 \\ -4358209.93(t-0.04875)^5 + 0.2 \\ \qquad (0.04875 \le t \le 0.0975) \end{cases} \tag{36}$$

The X , Y and Z direction displacements curves and velocity curves of robot's four legs are in Fig. 5 and Fig.6 respectively.

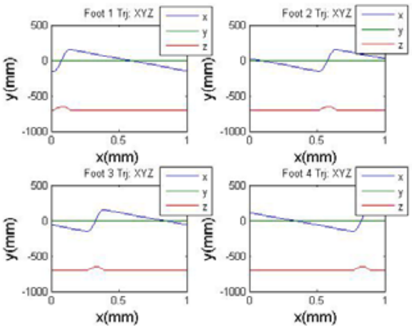

Fig. 5. Position curves of planning trajectory.

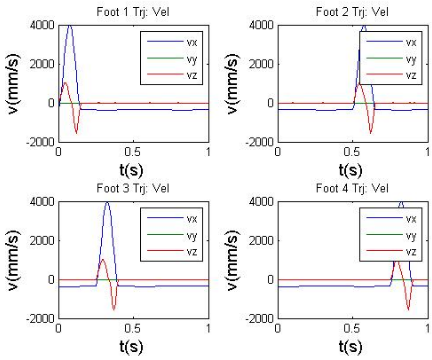

Fig. 6. Velocity curves of planning trajectory.

4. FORCE-POSTURE FEEDBACK COMPENSATION CONTROLLER

Fig.7 shows the model of quadruped robot walking on an obstacle. As shown in the figure, we suppose the robot's heading direction is to the right, and the right front leg runs into the obstacle firstly. In the Fig.7, $\theta_{roll}, \theta_{pitch}, \theta_{yaw}$ is the roll angle, pitch angle and yaw angle of the robot respectively, which represents the rotation angel about *X*-axis, *Y*-axis, *Z*-axis respectively, and the counterclockwise direction is positive. $L_{rf}, L_{rb}, L_{lf}, L_{lb}$ represents right front leg, right back leg, left front leg and left back leg respectively.

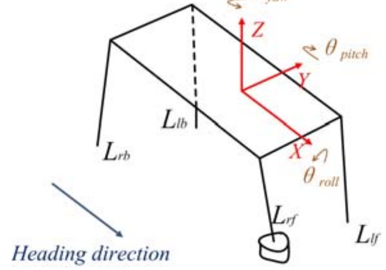

Fig. 7. Quadruped robot walking on an obstacle.

The FPFC controller detects $\theta_{roll}, \theta_{pitch}, \theta_{yaw}$ and the force of four foots $F_{rf}, F_{rb}, F_{lf}, F_{lb}$ in real-time. In this paper, we let the robot walk a straight line, so the θ_{yaw}=0. We set θ_1 and θ_2 is the threshold of θ_{pitch} and θ_{roll} respectively, $\theta_1 > 0$, $\theta_2 > 0$. When $|\theta_{pitch}| < \theta_1$ and $|\theta_{roll}| < \theta_2$ happens simultaneously, the robot walks very stably, so we regard the terrain is even. When $|\theta_{pitch}| \geq \theta_1$ or $|\theta_{roll}| \geq \theta_2$ happens, the robot walks unstably, so we regard the terrain is uneven. θ_1, θ_2 are always set with experience and practical requirement. Considering our robot with high payload, in order to make the goods on the robot never drop, we usually set θ_1=5°, θ_2=5°. The design of FPFC controller is divided into three cases as follows:

Case 1 When $|\theta_{pitch}| < \theta_1$, $|\theta_{roll}| < \theta_2$, the quadruped robot walking on even terrain or the relief amplitude is very low. In this case, we just utilize equations (31) and (32) as the foot trajectory curve of the robot. We don't use the feedback signal to adjust the length of the robot's foot here because in this case, the pitch angle and roll angle are all very small, the robot is very stable. If we use the feedback compensation, we can just improve the stability of robot by decreasing the pitch angle and roll angle a little, but it will cost more time to compute the trajectory curve.

Case 2 When $\theta_{pitch} < -\theta_1$, $\theta_{roll} < -\theta_2$, in this case, there are two subcases: the robot's right front foot may run into an obstacle or robot's left back foot may drop into a hole. We need to judge which subcase happens at first.

Subcase 1 As shown in Fig.3, when walking on even terrain, one foot takes 0.15T to swing. During this period of time, the foot doesn't contact the ground, then $F_{rf} = 0$. If we detect $F_{rf} \neq 0$ during this period, it shows that the right front foot runs into an obstacle. If we still just utilize the equations (31) and (32) as the foot trajectory curve of the robot, when the right front foot is running into the obstacle, the right leg of the robot will go on extending, then the left front foot of the robot will leave the ground, and the robot will become unstable or might even tumble. In order to maintain the stability of the robot, we should adjust every leg of the robot. Set the length of the L_{rf} , L_{rb} , L_{lf} , L_{lb} is l_{Lrf} , l_{Lrb} , l_{Llf} , l_{Llb} respectively, then we adjust every leg of the robot as follows:

$$l'_{Lrf} = l_{Lrf} - \delta_1\theta_{roll} - \delta_2\theta_{pitch} \quad (37)$$

$$l'_{Lrb} = l_{Lrb} - \delta_1\theta_{roll} + \delta_2\theta_{pitch} \quad (38)$$

$$l'_{Llf} = l_{Llf} + \delta_1\theta_{roll} - \delta_2\theta_{pitch} \quad (39)$$

$$l'_{Llb} = l_{Llb} + \delta_1\theta_{roll} + \delta_2\theta_{pitch} \quad (40)$$

Where, δ_1 , δ_2 are PD gains on joint controllers, We adjust them based on experience.

Subcase 2 If the left back leg have swung by 0.15T, but we still detect $F_{lf} = 0$, it shows that the left back foot runs into a hole. If we still just utilize the equations (31) and (32) as the foot trajectory curve of the robot, when the left back foot running into a hole, the back leg of the robot will stop extending, and the robot will become unstable or might even tumble. In order to maintain the stability of the robot, we adjust every leg of the robot as equations (37)-(40).

Case 3 When $\theta_{pitch} > \theta_1$, $\theta_{roll} > \theta_2$, there are also two subcases: the robot's left front foot may run into an obstacle or robot's right front foot may drop into a hole.

Subcase 1 Similar as case 2, If during swing period of left front foot, we detect $F_{lf} \neq 0$, it shows that the left front foot runs into an obstacle. In order to maintain the stability of the robot, we adjust every leg of the robot as follows:

$$l'_{Llb} = l_{Llb} - \delta_1\theta_{roll} - \delta_2\theta_{pitch} \quad (41)$$

$$l'_{Llf} = l_{Llf} - \delta_1\theta_{roll} + \delta_2\theta_{pitch} \quad (44)$$

$$l'_{Lrb} = l_{Lrb} + \delta_1\theta_{roll} - \delta_2\theta_{pitch} \quad (43)$$

$$l'_{Lrf} = l_{Lrf} + \delta_1\theta_{roll} + \delta_2\theta_{pitch} \quad (44)$$

Subcase 2 If the right front leg have swung by 0.15T, but we still detect $F_{lf} = 0$, it shows that the right front foot runs into a hole. In order to maintain the stability of the robot, we adjust every leg of the robot as equations (41)-(44).

We don't consider the case $|\theta_{pitch}| > \theta_1$, $|\theta_{roll}| < \theta_2$, and $|\theta_{pitch}| < \theta_1$, $|\theta_{roll}| > \theta_2$, because when walking on irregular terrain, $|\theta_{pitch}| > \theta_1$, $|\theta_{roll}| > \theta_2$ usually happen simultaneously.

5. SIMULATION RESULTS

To test the effectiveness and superiority of FPFC controller we designed, we make several simulations in RecurDyn. Combined simulation is divided into two steps. Firstly, building a quadruped robot model in RecurDyn. Secondly, the corresponding control strategy is built by simulation which can be used to drive the model in RecurDyn. Then the crawl gait of the quadruped robot walking on irregular terrain is realized.

We designed the terrain with random two humps. One is 4cm tall and the other is 6cm tall. Then we made a simulation that the quadruped robot traversed them. The simulation is shown in Fig.8 which is the snapshots of simulation in RecurDyn.

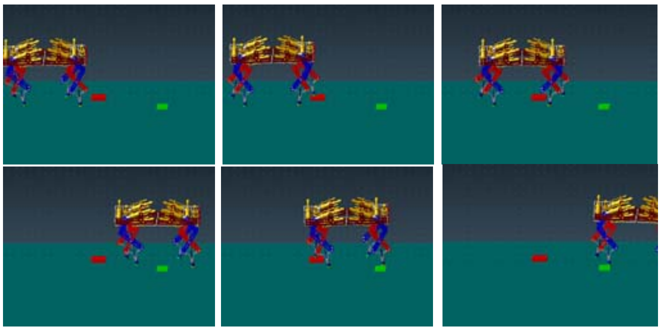

Fig. 8. Traversing a complex terrain.

From Fig. 8, we can see that the quadruped with FPFC controller has traversed a complex terrain very stably.

In order to verify the superiority of FPFC, we also made a simulation of robot without FPFC. The body pitch angle and body roll angle are shown in Fig. 9 and Fig. 10.

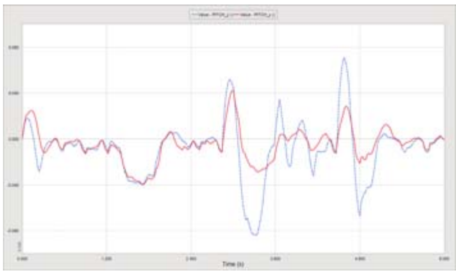

Fig. 9. The body pitch angle of robot with and without FPFC.

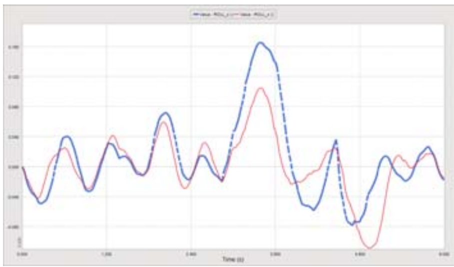

Fig. 10. The body roll angle of robot with and without FPFC.

In Fig.16 and Fig.17, the solid red line is the pitch angle and roll angle of robot with FPFC while the dotted blue line is the pitch angle and roll angle of robot without FPFC respectively. We can see from it that, when traversing a complex terrain, the peak value of pitch angle of robot with FPFC is about approximately 2.3°, while that of robot without FPFC is approximately 4.8°. And the peak value of pitch angle of robot with FPFC is approximately -5.7°, while that of robot without FPFC is approximately 9.5°. So FPFC controller can improve the stability of the robot substantially when traversing a complex terrain.

6. CONCLUSIONS AND FUTURE WORKS

In this paper, we design a FPFC controller to realize stable walking of quadruped robot on irregular terrain. We utilize the feedback signals detected by force sensors and gyroscope to adjust every leg of robot. Then we can improve the stability of robot by decreasing the pitch angle and roll angle of robot. We have made several simulations under different environment, which verify the effectiveness and superiority of FPFC controller.

Currently our FPFC controller is just verified in RecurDyn simulations. We will do our future text on real quadruped robot platform.

ACKONWLEDGEMENT

This work was supported by the National Basic Research Program of China (973 Program-2013CB035500), the National Nature Science Foundation of China (61233004, 61221003, 61074061, 61104091), partly sponsored by the International Cooperation Program of Shanghai Science and Technology Commission (12230709600), the Higher Education Research Fund for the Doctoral Program of China (20120073130006 & 20110073110018), and the Scientific Research Foundation for the Returned Overseas Chinese Scholars, State Education Ministry.

REFERENCES

Bin L, Yibin L., Xuewen R., and Jian M.(2011). Trotting Gait Planning and Implementation for a Little Quadruped Robot.In *the 2011 International Conference on Electric and Electronics*, 195-202.

Carlos S.M. and Sabri T.(2004). Design of a crawling gait for a modular robot. In *Proceedings of the 17th Florida Conference on Recent Advances in Robotics.* Orlando, Florida, USA.

Chuangfeng H. and Pingan L.(2010). Reliable Gait Planning for a Quadruped Walking Robot. In *2010 Chinese Control and Decision Conference*, 1783-1787.

Hua N., Ronglei S., Xiaofeng H., and Xi H.(2012). Static Gait Control for Quadruped Robot. In t*he 15th International Conference on Intelligent Robotics and Applications*, 376-384.

Jaehwan P. and Jonghyeon P.(2012). Impedance control of quadruped robot and its impedance characteristic modulation for trotting on irregular terrain. In *2012 IEEE/RSJ International Conference on Intelligent Robots and Systems*, 175-180.

Kolter J. Z. and Ng A. Y.(2011). The stanford littledog: A learning and rapid replanning approach to quadruped locomotion. *International Journal of Robotics Research*, 30(2):150-174.

Luther P. and Caitrin E.(2012). Toward innate leg stability on unmodeled and natural terrain: quadruped walking.

Matthias E.(1995). Least squares degree reduction of Bezier curves. *Computer-Aided Design*, 27(11): 845-851. *2012 IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2775-2780.

Shkolnik A. C., Levashov M., Manchester I. R., and R.Tedrake.(2011). Bounding on rough terrain with the littledog robot. *International Journal of Robotics Research*, 30(2):192– 215.

Zamani A., Khorram M., and Moosavian S. A. A.(2011). Dynamics and Stable Gait Planning of a Quadruped Robot.In *the 11th International Conference on Control, Automation and Systems*, 25-30.