Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Balancing cyclic pursuit using proximity sensors with limited range

Pietro DeLellis [*,**] **Franco Garofalo** [*,**]
Francesco Lo Iudice [*,**] **Giovanni Mancini** [*,***]

[*] *Department of Electrical Engineering and Information Technology, University of Naples Federico II, Naples, 80125, Italy.*
[**] *email:*
{pietro.delellis}{franco.garofalo}{francesco.loiudice}@unina.it
[***] *email: g.mancini0@gmail.com*

Abstract: We propose a decentralized and non cooperative algorithm for estimation and control in a multi-agent system of oscillators to achieve a balanced circular formation. Each agent gathers an uncertain measurement of its phase distance from other agents only when they are in its proximity. Based on this uncertain and intermittent data and on the a priori knowledge of the nominal (e.g. uncontrolled) agent's velocities, we employ an estimation method to reconstruct the relative angular positions. Then, we develop a bang-bang controller to achieve a balanced platoon formation. The novelty of the approach is that the balanced formation is achieved by using proximity sensors rather than distance transducers. Moreover, the bang-bang control strategy is designed so that the control goal is achieved even when the range of the sensors is lower than the desired spacing distance. The effectiveness of the approach is illustrated through extensive numerical simulations.

Keywords: Bang-bang control, estimation algorithm, oscillators, discontinuous control, proximity measurements, multi-agent systems.

1. INTRODUCTION

Nowadays, proximity sensors are common devices in aeronautical, automotive, and manufacturing industry but also in civil applications as in garage and elevator doors, gates, vending machines, parking lots, ATMs. The widespread use of this kind of devices is motivated by their compactness, high reliability, and their suitability for harsh environments. Based on different technologies (e.g. eddy currents, Hall effect), they share the same functioning mechanism: when the distance of the target from the sensor's head is lower than the so-called detecting distance, a trigger signal is produced and is then passed through the output conditioning circuitry to give a high or low output, depending on the sensor application.

Coordination of multi-agent systems is a relevant issue in a plethora of applications (Ren et al. (2005)). Examples may be found in very diverse fields of science and engineering spanning from biology to robotics (Balch and Arkin (1997); Warburton and Lazarus (1991); Martinez et al. (2007)). Both in natural and artificial settings, proximity plays a prominent role. In biological networks, collective phenomena such as synchronization and consensus (Ren et al. (2005); DeLellis et al. (2010); Pikovsky et al. (2001); Mirollo and Strogatz (1990); DeLellis et al. (2013a)) are observed, and formations are achieved on the basis of proximity rules; this is the case, for instance, of migration phenomena, where the system moves towards a given target or when toroidal behaviors around a common center are observed (Couzin et al. (2002); Vicsek (2008)). These biological phenomena constitute a source of inspiration for designing the interaction rules for coordinating artificial multi-agent systems (Vicsek et al. (1995); DeLellis et al. (2013c)).

Controlling a system with intermittent data flow has motivated several researchers to investigate the paradigmatic problem of state estimation based on fleeting data, see (Le Bars et al. (2012)) and references therein. In the recent literature on the problem of coordinating a multi-agent system of oscillators to achieve a balanced circular formation (see Chen and Zhang (2011); Marshall et al. (2004) and references therein), a discontinuous control law is proposed to solve the control problem when the relative angular position between the agents is perfectly known. To analytically guarantee the achievement of a balanced circular formation, it is typically assumed that the graph describing the information exchange among the agents is connected or jointly connected. In this paper, we rely neither on the measurement of the relative angular position, nor on the agent's connectivity, as we model the case in which proximity sensors are used, whose range is lower than the desired distance.

In control technology, proximity sensors are mainly used as proximity switches, in combination with position control loops which verify if the desired positions is reached. In fact, their use in a feedback loop is limited as the data are gathered by the sensors only when their distance from the target is less than the detecting distance. For instance, in a pursuit problem (Marshall et al. (2004); Smith et al. (2005); Kim and Sugie (2007); Chen and Zhang (2013)), the relative agents' position needs to be measured, thus preventing the use of proximity sensors. In this paper, we propose a decentralized estimation and control strategies capable of coping with proximity measurements and of achieving a cyclic pursuit of agents on a circle. Specifically, to overcome this limitation and keep the loop closed even when the measurements are intermittent, we propose a model based estimator of the distance between the pursuer and his target, inspired by the estimation algorithm proposed in DeLellis et al. (2013b). When no data from proximity sensors are available, the estimation is made only on the basis of the model, while the model predicted estimation is corrected when a measurement is available. Then, a control action is exerted based on the estimated distance among the agents. Specifically, we propose a bang-bang controller: based on the estimation algorithm, each agent tries to identify its closest follower; then, the bang-bang controller is activated and the distance from the follower adjusted to obtain the desired spacing. The estimation and control strategy is based on the fact that each agent knows, with a certain degree of uncertainty, the nominal (i.e. the uncontrolled) angular velocity of the target. We emphasize that the controller is designed to be effective also when the range of the sensors is lower than the desired spacing. The validity of the approach is illustrated through extensive numerical simulations.

2. PROBLEM STATEMENT

Let us consider a multi-agent system of N oscillators with natural angular velocity ω. The set of the agents of the systems is denoted by $\mathcal{V} = \{1, \ldots, N\}$.The dynamics of the angular position $\theta_i(k)$ of agent i are described by

$$\theta_i(k+1) = \theta_i(k) + \omega + u_i(k), \tag{1}$$

for $i = 1, \ldots, N$, where $u_i(k)$ is the control input at time k.

Introducing the relative angular position $\theta_{ij}(k) := \theta_i(k) - \theta_j(k)$, we can write

$$\theta_{ij}(k+1) = \theta_{ij}(k) + u_{ij}(k), \tag{2}$$

where $u_{ij}(k) := u_i(k) - u_j(k)$.

Definition 1. We say that a control strategy u_i, $i = 1, .., N$, asymptotically leads the multi-agent system (1) towards a balanced circular formation if, for all $\theta_{ij}(0)$, $i, j = 1, \ldots, N, i \neq j$,

$$\lim_{t\to\infty} \theta_{ij}(k) = \frac{2\pi}{N} := \psi, \tag{3}$$

for all $(i, j) \in \{(1, 2), \ldots, (N-1, N), (N, 1)\}$, and ψ is the desired spacing distance.

Without loss of generality, in what follows we assume that $\theta_{ij}(0) \in [-\pi, \pi[$ for all $i, j = 1, ..., N$.

From $\theta_{ij}(k)$, it is possible to define the *phase distance* $\alpha_{ij}(k)$ between two agents i and j as

$$\alpha_{ij}(k) = d(\theta_{ij}(k)) := \min\{\mathrm{mod}(\theta_{ij}(k)), \mathrm{mod}(-\theta_{ij}(k))\}, \tag{4}$$

with $\mathrm{mod}(\cdot)$ being the modulo function [1]. From α_{ij}, it is possible to give the following definition

Definition 2. Let

$$\mathcal{E}(\theta_{\max}, k) := \{(i, j) : i, j \in \mathcal{V}, \alpha_{ij}(k) \leq \theta_{\max}\}.$$

Then, the pair $\mathcal{G}(k) = \{\mathcal{V}, \mathcal{E}(\theta_{\max}, k)\}$ is the proximity graph associated to multi-agent system (1) at time k.

Notice that $\theta_{\max} > 0$ in Definition 2 can be viewed as the detecting distance of a proximity sensor. Notice that, as the multi-agent systems evolves, the proximity graph changes over time. Le us denote $\mathcal{G}(k, k+r)$, $r \in \mathbb{N}$, the union of all proximity graph across a nonempty finite time interval $\{k, k+1, \ldots, k+r\}$, whose edges are the union of the edges of the proximity graphs at every discrete-time instants over this time interval, that is,

$$\mathcal{G}(k, k+\delta k) := \left\{\mathcal{V}, \cup_{\tau\in\{k,k+1,\ldots,k+\delta k\}} \mathcal{E}(\theta_{\max}, \tau)\right\}.$$

Now, we can give the following definition.

Definition 3. The multi-agent system (1) is jointly connected over $\{k, k+1, \ldots, k+r\}$ if $\mathcal{G}(k, k+r)$ is connected.

In a continuous time setting, and assuming that the multi-agent system was jointly connected for any k and δk, Chen and Zhang (2011) proved analytically that a balanced circular formation is achieved through the following class of controllers:

$$u_i(k) = \omega_0 + \sum_{j\in\mathcal{N}_i,\ j\neq i} \beta(\alpha_{ij}(k))\mathrm{sgn}^+(\sin(\theta_{ij}(k))), \tag{5}$$

where $\beta(\cdot)$ belongs to the so called class $\mathcal{S}$ functions, see Chen and Zhang (2011) for details, the function $\mathrm{sgn}^+(x)$ is defined as

$$\mathrm{sgn}^+(\sin(\mathrm{x})) = \begin{cases} 1 & \text{if } x \geq 0, \\ 0 & \text{if } x < 0, \end{cases} \tag{6}$$

and $\mathcal{N}_i$ is the set of neighbors of agent i.

The control law in (5) stabilizes the system towards the balanced formation (3) moving at the reference speed $\omega_r = \omega + \omega_0$.

Differently from Chen and Zhang (2011), here we consider a discrete time setting. Furthermore, we assume that only cheap proximity sensors with limited range are available. Mathematically, this implies that:

(a) We measure the phase distance $\alpha_{ij}(k)$ instead of $\theta_{ij}(k)$.
(b) For each pair of agents, the measurement $y_{ij}(k)$ of $\alpha_{ij}(k)$ is only available if $\alpha_{ij}(k)$ is lower than the detecting distance $\theta_{\max} > 0$ [2].
(c) The measurement y_{ij}, when available, is affected by a bounded uncertainty $\nu_{ij}(k)$.
(d) the detecting distance $\theta_{\max}$ is lower than the desired spacing distance ψ.

Assumptions (a)-(c) can be summarized into the following output equation for system (2):

$$y_{ij}(k) = \begin{cases} \alpha_{ij}(k) + \nu_{ij}(k) & \text{if } \alpha_{ij}(k) \in [0, \theta_{\max}] := I, \\ \text{no measure} & \text{otherwise}, \end{cases} \tag{7}$$

where $\nu_{ij}(k)$ is the measurement noise, with $|\nu_{ij}(k)| \leq \varphi$, for all $i, j = 1, \ldots, N$. We emphasize that, even in absence of noise, θ_{ij} could not be directly computed from y_{ij}, as

[1] $\mathrm{mod}(a) := c$ is the remainder of a modulo 2π, where c is the unique solution of the two equations $c = a - qb$ and $0 \leq c < |b|$, with q being a rational number.

[2] Note that the phase distance is biunivocally related to the magnitude of the linear distance, and to the absolute value of the relative orientation. Hence, different sensors can be employed in different application areas.

the inverse of the function d defined in (4) is a multivalued function such that:

(1) if y is a scalar, $d^{-1}(y)$ is the infinite set $\{(-y+z2\pi)\cup(y+z2\pi), z\in\mathbb{Z}\}$,
(2) if y is an interval, $d^{-1}(Y)$ is the infinite multi-interval set $\{[\underline{Y},\bar{Y}]\cup[-\bar{Y},-\underline{Y}]+z2\pi, z\in\mathbb{Z}\}$.

Assumption (d) implies that, when the desired spacing ψ is achieved, the proximity graph is not connected. Therefore, in our estimation and control design we are not going to rely on connectivity.

3. STRATEGY FOR ESTIMATION AND CONTROL

For the multi-agent system discussed in the previous section, we propose a decentralized estimation and control strategy capable of achieving a balanced circular formation. As distance measurements do not provide any information on the signs of the relative angular positions ($d^{-1}(\cdot)$ is multi-valued and non-smooth), and the measurement noise is bounded, we employ an estimation algorithm inspired to that proposed in DeLellis et al. (2013b), which is based on a technique known as Interval State estimation (Raissi et al. (2012)). Such strategy is then complemented with a bang-bang control law capable of equispacing the agents on the circle.

3.1 Estimation strategy

The estimation strategy combines in a non conventional way the information on the dynamics of the system, defined in equation (2), with that coming from the proximity measurements, defined in equation (7). Specifically, the idea is to predict the interval to which the true value of $\theta_{ij}(k)$ belongs by means of the state equation, and then reduce its width by intersecting it with the intervals obtained from the inversion of the output equation. Notice that local linearization-based approaches are inapplicable since the output function of the multi-agent system, that relates the states to the measurements, is non-smooth and non-injective. In our estimation strategy, we first exploit the information that each measure $y_{ij}(k)$ brings on the phase distance $\alpha_{ij}(k)$. Namely, at each time instant k, we know that

$$\alpha_{ij}(k)\in\begin{cases}\Upsilon_{ij}(k):=[\max\{y_{ij}(k)-\varphi,0\},\min\{y_{ij}(k)+\varphi,\\ \qquad\qquad\theta_{\max}\}]\subseteq I \quad \text{if a measure is available,}\\ I^c:=]\theta_{\max},\pi] \qquad\qquad \text{otherwise.}\end{cases} \tag{8}$$

Now, as $\theta_{ij}(k)$ is related to $\alpha_{ij}(k)$ through function $d(\cdot)$, at each time instant k, $d^{-1}(\Upsilon_{ij}(k))$ defines an infinite multi-interval set in which $\theta_{ij}(k)$ falls. Our strategy limits such infinite number of intervals by intersecting $d^{-1}(\Upsilon_{ij}(k))$ with the finite multi-interval computed from the a priori knowledge on the dynamics of agents i and j. To illustrate this point, consider the time instant $k=0$ in which we first perform the estimation. In this case, the a priori knowledge is represented by the assumption that the initial conditions $\theta_{ij}(0)$ belong to the interval $[-\pi,\pi[$ for all $i,j=1,...,N$. Hence, at time $k=0$ we know that

$$\theta_{ij}(0)\in J^{ij}(0)=\begin{cases}-\Upsilon(0)\cup\Upsilon_{ij}(0) & \text{if a measure is available}\\ -I^c\cup I^c & \text{otherwise.}\end{cases} \tag{9}$$

Then, for all $k\geq 0$, we project all the intervals of the set $J^{ij}(k)$ to time $k+1$ by means of equation (2), thus building what we call a priori uncertainty set $J^{ij}(k+1|k)$ on the relative angular position of agents i and j. To recursively reduce such uncertainty, we then compute the a posteriori uncertainty set $J^{ij}(k+1)$ by intersecting $J^{ij}(k+1|k)$ with $d^{-1}(\Upsilon_{ij}(k))$ if a measure is available, or with $d^{-1}(I^c)$, otherwise. As a result of the estimation strategy, at each time instant, we know that $\theta_{ij}(k)$ belongs to the multi-interval $J^{ij}(k)$. In DeLellis et al. (2013b), such estimation strategy is shown to be convergent, i.e. the multi-interval $J^{ij}(k)$ reduces to a singleton, under quite mild assumptions on the angular velocity of the oscillators. We remark that, in the application considered in this paper, the estimation strategy would require the knowledge of $u_{ij}(k)$ to compute the a priori uncertainty set $J^{ij}(k+1|k)$ by means of equation (2). In what follows, we show that our selection of the control law facilitate the estimation algorithm. In fact, in the assumption that all the agents share the same type of control law, it is possible to define an interval in which $u_{ij}(k)$ falls, allowing to perform an interval prediction of $\theta_{ij}(k)$.

Furthermore, as our control strategy only requires information on phase differences $\vartheta_{ij}(k):=\text{rem}(\theta_{ij}(k))$ and not relative angular positions, we define the interval

$$H^{ij}(k):\begin{cases}\inf\limits_{x}\{x\in\text{rem}(J^{ij}(k))\},\\ \sup\limits_{x}\{x\in\text{rem}(J^{ij}(k))\},\end{cases} \tag{10}$$

which represents an overestimate of the uncertainty on $\vartheta^{ij}(k)$. Finally, as a scalar estimate is required by our control law, we define it as

$$\hat{\vartheta}_{ij}(k)=\frac{\bar{H}^{ij}(k)-\underline{H}^{ij}(k)}{2}. \tag{11}$$

3.2 Bang-bang controller

As in Chen and Zhang (2011), we design our control strategy so that each agent is *pushed* by its followers. Specifically, in our case, each agent i is only influenced by its nearest follower $i-1$, defined as

$$i-1:\begin{cases}\underline{H}^{i,i-1}>0\\ \bar{H}^{i,i-1}<\underline{H}^{ij}_l\ \forall j,l|\underline{H}^{ij}_l>0, j\neq i-1\end{cases} \tag{12}$$

Furthermore, each agents is labeled, as is the case when, for instance, proximity measurements are made by means of RFID technology (Want (2006); Sanpechuda and Kovavisaruch (2008)). The agent L, which is randomly picked and denoted as *the leader*, which adopts the following control law:

$$u_L(k)=\omega_0, \tag{13}$$

while the control law of the remaining agents is described by

$$u_i(k)=\Big(\omega_0+K\text{sgn}^+(\psi-\text{mod}(\hat{\vartheta}_{i,i-1}(k)))\Big)\mathcal{I}(i), \tag{14}$$

where $\mathcal{I}(i)$ is the following indicator function:

$$\mathcal{I}(i)=\begin{cases}1 & \text{if } i-1 \text{ is univocally determined,}\\ 0 & \text{otherwise,}\end{cases} \tag{15}$$

for all $i=1,\ldots,N$, $i\neq L$. Notice that the selected control strategy is totally decentralized and non-cooperative as the action exerted by each agent depends only on the estimated distance from its closest follower. The key point of the proposed control law is that the bang-bang controller (14) is triggered only once the estimator determines the identity of the follower. ambiguity on the identity of the follower has been solved by the estimator. However, the selected control law facilitates the estimator, as it implies that

$$u_{ij}(k) \in \{-\omega_0 - K, -\omega_0, -K, 0, K, \omega_0, \omega_0 + K\}, \quad (16)$$

allowing to compute $J^{ij}(k+1|k)$ from $J^{ij}(k)$ as

$$\bigcup_{\lambda}[\underline{J}^{ij}_{\lambda}(k) - \omega_0 - K, \bar{J}^{ij}_{\lambda}(k) + \omega_0 + K]. \quad (17)$$

Nevertheless, to improve the performance of the estimator, it is possible to further restrict the set of allowed values for $u_{ij}(k)$ for selected cases of interest. To clarify this point, let us consider a generic agent $i \neq L$ and its follower $i-1$, as the control law u_i only requires information on the relative angular position of this pair of agents. In this case, we know that, before that agent i identifies its follower,

$$u_{i,i-1} \in \{-\omega_0 - K, -\omega_0, -K, 0\} \quad (18)$$

Furthermore, we observe that agent i may identify its follower only at a time instant $\bar{k}_i$ in which a measurement is available, and therefore $u_{i-1}(\bar{k}_i)$ cannot be zero. Given the previous considerations, as at time $\bar{k}_i$ the two agents perceive each other, and as $\theta_{\max} < \psi$, we know that $u_i(\bar{k}_i) = \omega_0 + K$. Furthermore, as we pointed out that $u_{i-1}(\bar{k}_i) \neq 0$, we know that the control law of agent $i-1$ has already been triggered. Hence, we have

$$u_{i,i-1} \in \{0, K\}, \ \forall k \geq \bar{k}_i. \quad (19)$$

Finally, if at time $\tilde{k}_i > \bar{k}_i$ agent i is able to push agent $i-1$ outside of its visual cone, then $u_{i,i-1}(\tilde{k}_i) = K$. This would imply that, at time $\tilde{k}_i$, agent $i-1$ has already reached the desired spacing with agent $i-2$, and its estimate $\hat{\vartheta}_{i-1,i-2}(k)$ of the angle $\vartheta_{i-1,i-2}(k)$ is greater than or equal to ψ. Therefore, $u_{i-1}(k) = \omega_0$ for all $k \geq \tilde{k}_i$. Then, for all $k \geq \tilde{k}_i$, agent i can estimate $\vartheta_{i,i-1}(k+1)$ on the basis of a scalar and unambiguous $u_{i,i-1}(k)$.

Summing up, when the detecting distance is lower than the desired spacing distance, and therefore, after the transient, the controller push the pairs of consecutive agents outside their mutual detecting distance, and the estimate must rely only on the predictive component of our estimator. For that reason, we choose an extremely simple control law, as the bang-bang action described in equation (14), so that the estimation of $\vartheta_{ij}(k)$ is complemented with a simple estimator of $u_{ij}(k)$, see the estimation and control scheme depicted in Figure 1.

4. NUMERICAL RESULTS

To validate our estimation and control strategy, we performed extensive numerical simulations. In all experimental conditions, we set

(a) the number of agents $N = 6$. Hence, the target spacing between consecutive agents is $\psi = 2\pi/N$;
(b) $\omega_0 = 0.01$;
(c) the simulation time $T = 3000$.

We test the effectiveness of our approach for different values of the detecting distance $\theta_{\max}$, the bound of the modulus of the measurement noise φ, and the control gain K. Specifically,

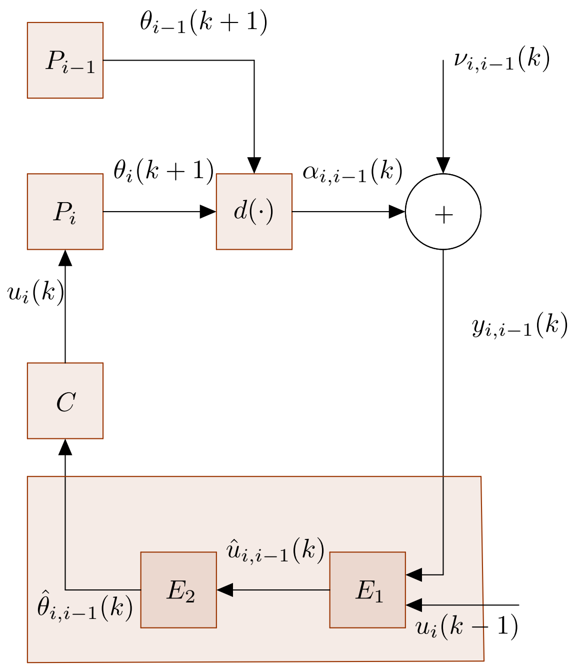

Fig. 1. Schematic of the estimation and control strategy: P_i is the i-th agent and P_{i-1} its follower; E_1 is the estimator of the relative control input $u_{i,i-1}$; E_2 is the estimator of the relative angular position $\theta_{i,i-1}$; C is the bang-bang controller.

(a) $\theta_{\max}$ is varied between 0.18ψ and 0.9ψ with step 0.18ψ;
(b) for each value of $\theta_{\max}$, φ is varied between $0.04\theta_{\max}$ and $0.20\theta_{\max}$ with step $0.04\theta_{\max}$;
(c) for each combination of $\theta_{\max}$ the control gain K is varied between 0.002 and 0.010 with step 0.002.

The result of this scheme is a total of 125 parameter combinations. For each parameter combination, we consider the same set of $R = 100$ randomly selected initial conditions for the angular positions. We remark that, in the parameter selection, we take $\psi < \theta_{\max}$ to remove the assumption of jointed connectivity. Moreover, we select φ as a function of $\theta_{\max}$ as it is typically related to the sensor's range. For the sake of clarity, we restrict the analysis to the case where the agents cannot overtake each other. Accordingly, the initial conditions for θ_{ij}, $i, j = 1, \ldots, N$, are selected in the interval $[2\varphi, \pi]$. As we took the same set of R initial conditions for each of the 125 simulated scenarios, the previous condition must be fulfilled considering the maximum value of φ, that is, 0.036ψ.

To test the performance of the algorithm, we introduce the definition of practical convergence. Namely, we say that the algorithm practically converges if there exists a time instant $k_c \leq T$ such that $(1/N)\sum_i |\vartheta_{i,i-1}(k) - \psi| \leq \delta$ for all $k \geq k_c$ [3], and we say that k_c is the convergence time. For our simulations, we set $\delta = 0.05\psi = 0.0524$rad, and we say that $k_c(K_i, \varphi_j, \theta_{\max,m}, s)$ is the convergence time of the

[3] We remind the reader that the follower of agent i is labeled as $i-1$. The follower of agent 1 is obviously agent 6.

s-th repetition of parameter combination corresponding to the i-th values of K, the j-th value of φ, and the m-th value of $\theta_{\max}$, for $i, j, m = 1, \ldots, 5$, and $s = 1, \ldots, R$.

Let us now describe the numerical results. Firstly, we underline that only in two simulations the specified tolerance was not fulfilled, and practical convergence is achieved in the 99.9984% of the runs. As for the convergence time k_c, its average

$$\langle k_c \rangle = \frac{1}{125R} \sum_{i,j,m=1}^{5} \sum_{s=1}^{R} k_c(K_i, \varphi_j, \theta_{\max,m}, s),$$

computed on the basis of all 12500 simulations, is 735 time instants. To have a fist insight on the effect of K, φ, and $\theta_{\max}$, we evaluated

$$\langle k_c(K) \rangle = \frac{1}{25R} \sum_{j,m=1}^{5} \sum_{s=1}^{R} k_c(K, \varphi_j, \theta_{\max,m}, s),$$

$$\langle k_c(\varphi) \rangle = \frac{1}{25R} \sum_{i,m=1}^{5} \sum_{s=1}^{R} k_c(K_i, \varphi, \theta_{\max,m}, s),$$

and

$$\langle k_c(\theta_{\max}) \rangle = \frac{1}{25R} \sum_{i,j=1}^{5} \sum_{s=1}^{R} k_c(K_i, \varphi_j, \theta_{\max}, s).$$

As expected, increasing the control gain K the convergence time decreases, see Fig. **??**, while an increase of the measurement noise φ produces a slight increase in the convergence time, see Fig. **??**. Finally, increasing $\theta_{\max}$, we experienced a steep increase in $\langle k_c(\theta_{\max}) \rangle$, as depicted in Fig. **??**.

To delve into the statistical significance of the observed variations, we performed a three-way ANOVA (analysis of variance), whose results are reported in Table 1. Such analysis tests the null hypothesis that each factor has no influence on the convergence time k_c. Hence, a low p-value implies that the null hypothesis must be rejected. The lowest p-value of 0 is obtained for the detecting distance $\theta_{\max}$ and the control gain K, while the null hypothesis may not be rejected for the effect of φ on k_c, as the p-value is 0.62. To further discuss the lack of significance of the variation of the convergence time as a function of φ, in Fig. 2 we display a box plot for a representative simulation scenario: as a result, the variability induced by φ, which is represented by the difference between the medians of the distributions (red horizontal lines) is negligible if compared to the natural variability of k_c (the width of the blue boxes). Therefore, we can conclude that the inter-class sampled variance is much smaller than the intra-class sampled variance. This means that the effect of measurement noise φ on k_c is too small to be statistically significant, if compared to the effect of the other parameters, and to the natural variability of k_c.

Finally, to further test the effectiveness of our control strategy, we consider the case in which the inertia is not negligible and the approximation of instantaneously switching the angular velocities is not acceptable. To model this scenario, we modify equation (1) and obtain the following expression for the dynamics of each agent:

$$\theta_i(k+1) = \theta_i(k) + \omega_i(k) \tag{20}$$

where $\omega_i(k)$ is

Factor	degrees of freedom	p-value
K	4	0
φ	4	0.62
$\theta_{\max}$	4	0

Table 1. Three-way ANOVA to test the influence on k_c of the factors K, φ, and $\theta_{\max}$.

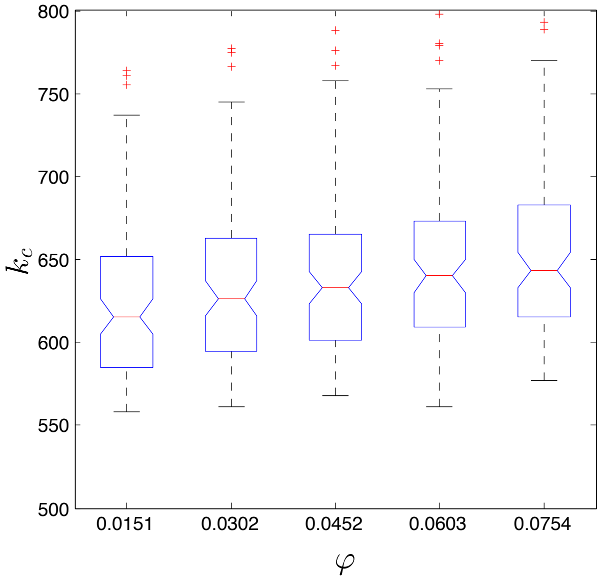

Fig. 2. Box plot of k_c as a function of φ for $\theta_{\max} = 0.3770$, $K = 0.008$. The central mark are the medians, the edges of the boxes are the 25-th and 75-th percentiles, the whiskers extend to the most extreme data points not considered outliers, and outliers are plotted individually.

$$\begin{cases} \omega_0 + \min\{\omega_i(k-1) + \alpha_1, u_i(k)\} & \text{if } u_i(k) \geq \omega_i(k-1) \\ \omega_0 + \max\{\omega_i(k-1) - \alpha_2, u_i(k)\} & \text{if } u_i(k) < \omega_i(k-1), \end{cases} \tag{21}$$

and $\omega_i(0) = \omega$. The parameters α_1 and α_2, possibly different, account for the inertias of the multi-agent system. In our preliminary analysis, based on a set of 100 simulations, with the same set of initial conditions considered above, and where we set $\theta_{\max} = 0.5864$ and $K = 0.01$, our approach successfully achieved practical convergence in all the repetitions.

5. CONCLUSIONS

In this paper, we tackled the problem of coordinating a multi-agent system of oscillators to achieve a balanced circular formation. Differently from the existing literature, we did not rely on the exact knowledge of the relative angular positions between the agents. This is motivated by the fact that, in many fields of application, the accuracy of the measurements is limited due to physical or economical constraints. Hence, we considered intermittent, uncertain and ambiguous measurements as those performed by cheap proximity sensors. Furthermore, we assumed that the sensor have a limited detecting distance, lower than the desired spacing among the agents. To cope with this reduced level of information, we developed a decentralized strategy for estimation and control. Inspired by the algorithm presented in DeLellis et al. (2013b), we proposed an estimator capable of reconstructing the relative angular position from the intermittent distance measurements. Implementing an appropriately designed bang-bang control law, each agent is capable of univocally identifying its closest follower and achieves an appropriate spacing from it. Extensive numerical simulations illustrated the effectiveness of the approach: the desired equispaced configuration is achieved and the convergence speed can be regulated with the control gain and is not significantly affected by the measurement noise. Moreover, preliminary results show how the approach can be successfully applied when the inertia is not negligible and the switches prescribed by the bang-bang control law cannot be instantaneous. A formal proof of convergence of the estimation and control strategy is subject of ongoing research.

ACKNOWLEDGEMENTS

This work was suported by the Reti di eccellenza Mastri, POR Campania FSE 2007/2013.

REFERENCES

Balch, T. and Arkin, R. (1997). Behavior-based formation control for multirobot teams. *IEEE Transactions on Robotics and Automation*, 4(6), 1–23.

Chen, Z. and Zhang, H.T. (2011). No-beacon collective circular motion of jointly connected multi-agents. *Automatica*, 47(9), 1929–1937.

Chen, Z. and Zhang, H.T. (2013). A remark on collective circular motion of heterogeneous multi-agents. *Automatica*, 49(5), 1236–1241.

Couzin, I.D., Krause, J., James, R., Ruxton, G.D., and Franks, N.R. (2002). Collective memory and spatial sorting in animal groups. *Journal of Theoretical Biology*, 218(1), 1–11.

DeLellis, P., di Bernardo, M., and Garofalo, F. (2013a). Adaptive pinning control of networks of circuits and systems in Lur'e form. *IEEE Transactions on Circuits and System I*, 60(11), 3033–3042.

DeLellis, P., di Bernardo, M., Garofalo, F., and Liuzza, D. (2010). Analysis and stability of consensus in networked control systems. *Applied Mathematics and Computation*, 217(3), 988–1000.

DeLellis, P., Garofalo, F., Lo Iudice, F., and Mancini, G. (2013b). State estimation of heterogeneus oscillators by mean of proximity measurements. Submitted to Automatica.

DeLellis, P., Porfiri, M., and Bollt, E.M. (2013c). Topological analysis of group fragmentation in multi-agent systems. *Physical Review E*, 87(2), 022818.

Kim, T.H. and Sugie, T. (2007). Cooperative control for target-capturing task based on a cyclic pursuit strategy. *Automatica*, 43(8), 1426–1431.

Le Bars, F., Sliwka, J., Jaulin, L., and Reynet, O. (2012). Set-membership state estimation with fleeting data. *Automatica*, 48(2), 381–387.

Marshall, J.A., Broucke, M.E., and Francis, B.A. (2004). Formations of vehicles in cyclic pursuit. *IEEE Transactions on Automatic Control*, 49(11), 1963–1974.

Martinez, S., Cortes, J., and Bullo, F. (2007). Motion coordination with distributed information. *Control Systems, IEEE*, 27(4), 75–88.

Mirollo, R.E. and Strogatz, S.H. (1990). Synchronization of pulse-coupled biological oscillators. *SIAM Journal on Applied Mathematics*, 50(6), 1645–1662.

Pikovsky, A., Rosenblum, M., and Kurths, J. (2001). *Synchronization: A Unified Approach to Nonlinear Science*. Cambridge University Press, Cambridge.

Raissi, T., Efimov, D., and Zolghadri, A. (2012). Interval state estimation for a class of nonlinear systems. *IEE Transactions on Automatic Control*, 57(1), 260–265.

Ren, W., Beards, R.W., and Atkins, E.M. (2005). A survey of consensus problems in multi-agent coordination. In *Proceedings of the 2005 American Control Conference*, volume 3, 1859–1864.

Sanpechuda, T. and Kovavisaruch, L. (2008). A review of RFID localization: Applications and techniques. In *5th International Conference on Electrical Engineering/Electronics, Computer, Telecommunications and Infomation Technology*, 769–772.

Smith, S.L., Broucke, M.E., and Francis, B.A. (2005). A hierarchical cyclic pursuit scheme for vehicle networks. *Automatica*, 41(6), 1045–1053.

Vicsek, T. (2008). Universal patterns of collective motion from minimal models of flocking. In *Second IEEE International Conference on Self-Adaptive and Self-Organizing Systems, SASO '08.*, 3–11.

Vicsek, T., Czirók, A., Ben-Jacob, E., Cohen, I., and Shochet, O. (1995). Novel type of phase transition in a system of self-driven particles. *Physical Review Letters*, 75(6), 1226–1229.

Want, R. (2006). An introduction to rfid technology. *IEEE Pervasive Computing*, 5(1), 25–33.

Warburton, K. and Lazarus, J. (1991). Tendency-distance models of social cohesion in animal groups. *Journal of Theoretical Biology*, 150(4), 473–488.