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Finite-time stabilization for Markov jump systems governed by deterministic switches

Chongzhong Zhao*, Xiaoli Luan* and Fei Liu*

* Key Laboratory of Advanced Process Control for Light Industry, the Ministry of Education, and the Institute of Automation, Jiangnan University, Wuxi, 214122, P.R. China(e-mail: xiaoli_luan@126.com).

Abstract: The problem of controller synthesis in a fixed time interval for discrete-time switching Markov jump systems is dealt with in this paper. Compared with the existing results, the new proposed stabilization conditions are obtained by permitting the stochastic Lyapunov energy function to rise at each switching instant, but the switching signal is constraint by an average dwell time. Finally, the validity of the obtained results is demonstrated with an example.

1. INTRODUCTION

As an important institution of hybrid systems, Markov jump linear systems (MJLSs) have been widely investigated in the past few decades because they are very appropriate to model the physical systems with variable structures, which are caused by the stochastic abrupt changes such as component failure and parameters shifting, etc. Due to the existence of randomness, undesirable performance of MJLS can't be avoid. In order to make the dynamic of the system more controllable and optimize the system performance, a deterministic switching signal can be imposed on MJLSs. That is to say, the system has a hierarchical configuration, the MJLSs are to be controlled within it, the switching happens when the supervisor choose an appropriate controller among a finite number of alternatives. This hierarchical system is dubbed as switching MJLSs, and was firstly presented in (Bolzern, Colaneri & Nicolao, 2010). Some basic issues about MJLSs or switched systems have achieved lots of results (Feng, Loparo, Ji & Chizeck, 1992; Zhai, Hu, Yasuda & Michel,2001; Costa, Fragoso & Marques, 2005; Geromel &Colaneri, 2006; Shi, Xia, Liu & Ree, 2006; Zhang, Boukas & Lam, 2008; Luan, Liu & Shi, 2010; Luan, Zhao & Liu, 2013)respectively, but systems with both deterministic switching and stochastic jumps haven't gained much attention. The mean square stability and almost sure stability of continuous-time switching MJLSs have been analized in (Bolzern, Colaneri & Nicolao, 2010; Bolzern, Colaneri & Nicolao, 2013). Reference (Hou, Zong & Zheng, 2013) dealt with the exponential l_2-l_∞ stochastic stability problem and an l_2-l_∞ controller was designed for discrete-time switching MJLSs.

Up to now, most of the works about switching MJLSs relate to stability are defined over an infinite time interval, where the states of the systems converge to zero when time towards infinity. However, in many practical applications, such as aerospace control system, robot control system (see Weiss & Infante, 1967 and references there in), more attention should be paid on their behaviors over a fixed finite-time interval. Therefore, the definition of finite-time stability (FTS) was proposed by (Dorato, 1961) and the concept of FTS has been further extended to finite-time boundedness (FTB) (Amato, Ariola & Dorato, 2001; Amato & Ariola, 2005) when system possesses bounded exogneous disturbance. Then, a lot of results have been obtained for finite-time control problems particularly in recent years owning to the convenience of solving the linear matrix inequalities. The finite-time stability and control problems for switched systems and MJLS can be seen in (Moulay, Dambrine, Yeganefar & Perruquetti, 2008; Amato, Ambrosino & Ariola, 2009; Luan, Shi & Liu, 2010; Luan, Shi & Liu, 2013; Lin, Du & Li, 2011; Liu & Shen, 2011; Zhang, Feng & Sun, 2012). As far as we know, there has no result available yet on analysis control problems for switching MJLSs in the view of finite time, which motives our research.

The objective of this paper is to discuss the finite-time boundedness and stabilization problems for a class of hybrid systems subject to both stochastic jumps and deterministic switches. Some sufficient conditions in the form of liner matrix inequality for FTB and finite-time stabilization are established under an average dwell-time constraint on the switching signal.

2. PRELIMINARIES AND PROBLEM FORMULATION

Consider the system structure of discrete-time switching MJLS is as follows:

$$\begin{aligned} x(k+1) &= A(r_k,\sigma_k)x(k) + B(r_k,\sigma_k)u(k) \\ &\quad + B_w(r_k,\sigma_k)w(k) \end{aligned} \tag{1}$$

where $x(k) \in R^n$ is the state variable, $u(k) \in R^p$ is the control input, $w(k) \in l_2^q\left[0 \quad +\infty\right)$ is the exogenous disturbances satisfying

$$\|w\|_2^2 = \left[\sum_{k=0}^{N} w_k^{\mathrm{T}} w_k\right] < h^2 \tag{2}$$

r_k is the deterministic switching signal belonging to a fixed set $\mathbb{M}=\{1,2,\cdots,M\}$, σ_k is stochastic jumps belonging to $\mathbb{S}=\{1,2,\cdots,S\}$, and the transition probabilities (TP) satisfying

$$\Pr\{\sigma_{k+1}=j \,|\, \sigma_k=i, r_k=\alpha\}=\pi_{ij}^{\alpha} \tag{3}$$

where π_{ij}^{α} is the transition probabilities from mode i to mode j under switching signal $r_k=\alpha$ that satisfies

$$\pi_{ij}^{\alpha}\ge 0,\ \sum_{j=1}^{S}\pi_{ij}^{\alpha}=1,\ \forall i,j\in\mathbb{S}.$$

To simplify the denotation, when at times of k, $r_k=\alpha$, $\alpha\in\mathbb{M}$, $\sigma_k=i$, $i\in\mathbb{S}$, the system matrices can be denoted by

$$A(r_k,\sigma_k)=A_{\alpha,i},\, B(\mathrm{r}_k,\sigma_k)=B_{\alpha,i},\, B_w(r_k,\sigma_k)=B_{w\alpha,i}$$

For system (1), the state-feedback controller can be constructed as:

$$u(k)=K(r_k,\sigma_k)x(k) \tag{4}$$

When $r_k=\alpha$ and $\sigma_k=i$, we have $u(k)=K_{\alpha,i}x(k)$. Then the resulting closed-loop switching MJLS becomes

$$x(k+1)=\overline{A}_{\alpha,i}x(k)+B_{w\alpha,i}w(k) \tag{5}$$

Where

$$\overline{A}_{\alpha,i}=A_{\alpha,i}+B_{\alpha,i}K_{\alpha,i}$$

Before deducing our main results, the following definitions and lemmas will be introduced, which are needed for the derivation of the theorems.

Definition 1 (Amato, Ariola & Dorato, 2001)**.** The switching MJLS (1) with $u(k)=0$ is said to be stochastic FTB with respect to $(c_1 \quad c_2 \quad N \quad R \quad h)$, where $0<c_1<c_2$, $N>0$, $R>0$ if

$$x^{\mathrm{T}}(0)Rx(0)\le c_1^2 \Rightarrow E\left\{x^{\mathrm{T}}(k)Rx(k)\right\}<c_2^2,\quad k\in\{1,2\cdots N\}. \tag{6}$$

Definition 2 (Amato, Ariola & Dorato, 2001)**.** The switching MJLS (1) is said to be stochastic finite-time stabilization with respect to $(c_1 \quad c_2 \quad N \quad R \quad h)$, where $0<c_1<c_2$, $N>0$, $R>0$ if system (5) is stochastic FTB with the controller like (4).

Definition 3. For a switching signal r_k and any $k>k_0$, let $N_a(k_0,k)$ be the switching numbers of r_k from k_0 to k. If for any given $N_0>0$ and $\tau_a>0$ satisfied $N_a(k_0,k)\le N_0+(k-k_0)/\tau_a$, then τ_a and N_0 are called average dwell time and chatter bound, respectively. As used usually in the references, here we choose $N_0=0$.

Lemma 1 (Zong, Hou & Wu, 2011)**.** For the positive define matrix $M>0$ and the matrix N with compatible dimensions, the following inequality is established:

$$-NM^{-1}N^{\mathrm{T}}\le M-N^{\mathrm{T}}-N \tag{7}$$

3. MAIN RESULTS

The stochastic FTB criterion for switching MJLS (1) is developed firstly, then we designed a state feedback controller such that system (1) is stochastic finite-time stabilization based on linear matrix inequalities.

Theorem 1. For given scalars $\delta\ge 1$ and $\mu>1$, the discrete-time switching MJLS (1) is stochastic FTB with respect to $(c_1 \quad c_2 \quad N \quad R \quad h)$, if there exist a set of positive-definite matrices $P_{\alpha,i}>0,\ G_{\alpha,i}>0,\ \alpha\in\mathbb{M}, i\in\mathbb{S}$ such that the following inequalities hold:

$$\begin{bmatrix} A_{\alpha,i}^{\mathrm{T}}\overline{P}_{\alpha,i}A_{\alpha,i}-\mu P_{\alpha,i} & A_{\alpha,i}^{\mathrm{T}}\overline{P}_{\alpha,i}B_{w\alpha,i} \\ * & B_{w\alpha,i}^{\mathrm{T}}\overline{P}_{\alpha,i}B_{w\alpha,i}-G_{\alpha,i}\end{bmatrix}<0 \tag{8}$$

$$\overline{P}_{\alpha,i}\le\delta\overline{P}_{\beta,i} \tag{9}$$

$$\Delta_2<\Delta_1 \tag{10}$$

with average dwell time satisfying

$$\tau_a>\frac{N\ln\delta}{\ln\Delta_1-\ln\Delta_2}=\tau_a^* \tag{11}$$

Where

$$\overline{P}_{\alpha,i}=\sum_{j=1}^{S}\pi_{ij}^{\alpha}P_{\alpha,j},\ \tilde{P}_{\alpha,i}=R^{-1/2}P_{\alpha,i}R^{-1/2}$$

$$\Delta_1=\mathrm{c}_2^2\min_{\alpha\in\mathbb{M},\mathrm{i}\in\mathbb{S}}\lambda_{\min}(\tilde{P}_{\alpha,i})$$

$$\Delta_2=\mu^N\left[\max_{\alpha\in\mathbb{M},\mathrm{i}\in\mathbb{S}}\lambda_{\max}(\tilde{P}_{\alpha,i})c_1^2+\max_{\alpha\in\mathbb{M},\mathrm{i}\in\mathbb{S}}\lambda_{\max}(G_{\alpha,i})h^2\right]$$

Proof. Choose the stochastic Lyapunov function as follows:

$$V_{\alpha,i}(k)=x^{\mathrm{T}}(k)P_{\alpha,i}x(k) \tag{12}$$

From conditions (8), one has

$$\begin{aligned} E\left\{V_{\alpha,j}(k+1)|V_{\alpha,i}(k)\right\} &< \mu V_{\alpha,i}(k)+w^{\mathrm{T}}(k)G_{\alpha,i}w(k) \\ &\le \mu V_{\alpha,i}(k)+\max_{\alpha\in\mathbb{M},\mathrm{i}\in\mathbb{S}}\lambda_{\max}(G_{\alpha,i})w^{\mathrm{T}}(k)w(k) \end{aligned} \tag{13}$$

Assuming that $k_l,k_{l-1},k_{l-2},\cdots$ are the switching instants, and k_l is the latest switching instant, then for the same switching mode, formula (13) gives

$$
\begin{aligned}
E\left\{V(r_{k_l},\sigma_k,k)\right\} &< \mu V(r_{k_l},\sigma_{k-1},k-1) \\
&\quad + \max_{\alpha\in\mathbb{M},i\in\mathbb{S}} \lambda_{\max}(G_{\alpha,i}) w^{\mathrm{T}}(k-1)w(k-1) \\
&< \mu^{k-k_l} V(r_{k_l},\sigma_{k_l},k_l) \\
&\quad + \max_{\alpha\in\mathbb{M},i\in\mathbb{S}} \lambda_{\max}(G_{\alpha,i}) \sum_{\theta=k_l}^{k-1} \mu^{k-\theta-1} w^{\mathrm{T}}(\theta)w(\theta)
\end{aligned} \tag{14}
$$

Similar as (14), one has

$$
\begin{aligned}
& E\left\{V(r_{k_{l-1}},\sigma_{k_l},k_l)\right\} \\
& < \mu V(r_{k_{l-1}},\sigma_{k_l-1},k_l-1) \\
& \quad + \max_{\alpha\in\mathbb{M},i\in\mathbb{S}} \lambda_{\max}(G_{\alpha,i}) w^{\mathrm{T}}(k_l-1)w(k_l-1) \\
& < \mu^{k_l-k_{l-1}} V(r_{k_{l-1}},\sigma_{k_{l-1}},k_{l-1}) \\
& \quad + \max_{\alpha\in\mathbb{M},i\in\mathbb{S}} \lambda_{\max}(G_{\alpha,i}) \sum_{\theta=k_{l-1}}^{k_l-1} \mu^{k_l-\theta-1} w^{\mathrm{T}}(\theta)w(\theta)
\end{aligned}
$$

From the above two formulas and condition (9), it yields

$$
\begin{aligned}
& E\left\{V(r_{k_l},\sigma_k,k)\right\} < \mu^{k-k_l} V(r_{k_l},\sigma_{k_l},k_l) \\
& \quad + \max_{\alpha\in\mathbb{M},i\in\mathbb{S}} \lambda_{\max}(G_{\alpha,i}) \sum_{\theta=k_l}^{k-1} \mu^{k-\theta-1} w^{\mathrm{T}}(\theta)w(\theta) \\
& < \delta\mu^{k-k_{l-1}} V(r_{k_{l-1}},\sigma_{k_{l-1}},k_{l-1}) \\
& \quad +\delta \max_{\alpha\in\mathbb{M},i\in\mathbb{S}} \lambda_{\max}(G_{\alpha,i}) \sum_{\theta=k_{l-1}}^{k_l-1} \mu^{k-\theta-1} w^{\mathrm{T}}(\theta)w(\theta) \\
& \quad + \max_{\alpha\in\mathbb{M},i\in\mathbb{S}} \lambda_{\max}(G_{\alpha,i}) \sum_{\theta=k_l}^{k-1} \mu^{k-\theta-1} w^{\mathrm{T}}(\theta)w(\theta) \\
& < \delta^{N_a} \mu^{k-k_0} V(r_{k_0},\sigma_{k_0},k_0) + \max_{\alpha\in\mathbb{M},i\in\mathbb{S}} \lambda_{\max}(G_{\alpha,i}) \\
& \quad \left[\delta^{N_a} \sum_{\theta=k_0}^{k_1-1} \mu^{k-\theta-1} w^{\mathrm{T}}(\theta)w(\theta) \right. \\
& \quad + \delta^{N_a-1} \sum_{\theta=k_1}^{k_2-1} \mu^{k-\theta-1} w^{\mathrm{T}}(\theta)w(\theta) + \cdots \\
& \quad \left. + \delta^{0} \sum_{\theta=k_l}^{k-1} \mu^{k-\theta-1} w^{\mathrm{T}}(\theta)w(\theta) \right] \\
& < \delta^{k-k_0/\tau_a} \mu^{k-k_0} V(r_{k0},\sigma_{k0},k_0) \\
& \quad + \max_{\alpha\in\mathbb{M},i\in\mathbb{S}} \lambda_{\max}(G_{\alpha,i}) \delta^{k-k_0/\tau_a} \mu^{k-\theta-1} \sum_{\theta=k_0}^{k} w^{\mathrm{T}}(\theta)w(\theta) \\
& < \delta^{N/\tau_a} \mu^{N} \left[V(r_{k0},\sigma_{k0},k_0) + \max_{\alpha\in\mathbb{M},i\in\mathbb{S}} \lambda_{\max}(G_{\alpha,i}) h^2 \right]
\end{aligned} \tag{15}
$$

Note that

$$
\begin{aligned}
V(r_{k_0},\sigma_{k_0},k_0) &= x^{\mathrm{T}}(k_0)\bar{P}(r_{k_0},\sigma_{k_0},k_0)x(k_0) \\
&< \max_{\alpha\in\mathbb{M},i\in\mathbb{S}} \lambda_{\max}(\tilde{P}_{\alpha,i}) x^{\mathrm{T}}(k_0)Rx(k_0) \\
&< \max_{\alpha\in\mathbb{M},i\in\mathbb{S}} \lambda_{\max}(\tilde{P}_{\alpha,i}) c_1^2
\end{aligned} \tag{16}
$$

On the other hand

$$
E\left\{V_{\alpha,i}(k)\right\} > \min_{\alpha\in\mathbb{M},i\in\mathbb{S}} \lambda_{\min}(\tilde{P}_{\alpha,i}) x^{\mathrm{T}}(k)Rx(k) \tag{17}
$$

Therefore

$$
\begin{aligned}
& E\left\{x^{\mathrm{T}}(k)Rx(k)\right\} \\
& < \frac{\delta^{N/\tau_a} \mu^N \left(\max_{\alpha\in\mathbb{M},i\in\mathbb{S}} \lambda_{\max}(\tilde{P}_{\alpha,i}) c_1^2 + \max_{\alpha\in\mathbb{M},i\in\mathbb{S}} \lambda_{\max}(G_{\alpha,i}) h^2 \right)}{\min_{\alpha\in\mathbb{M},i\in\mathbb{S}} \lambda_{\min}(\tilde{P}_{\alpha,i})}
\end{aligned} \tag{18}
$$

Define

$$
\Psi = \frac{\mathrm{c}_2^2 \min_{\alpha\in\mathbb{M},i\in\mathbb{S}} \lambda_{\min}(\tilde{P}_{\alpha,i})}{\mu^N \left[\max_{\alpha\in\mathbb{M},i\in\mathbb{S}} \lambda_{\max}(\tilde{P}_{\alpha,i}) c_1^2 + \max_{\alpha\in\mathbb{M},i\in\mathbb{S}} \lambda_{\max}(G_{\alpha,i}) h^2 \right]}
$$

From the conditions (10) and (11), we can get

$$
\Psi > 1, \delta^{N/\tau_a} < \Psi
$$

which means

$$
E\left\{x^{\mathrm{T}}(k)Rx(k)\right\} < c_2^2
$$

Thus, the proof is completed.

Remark 1. Concerning the existence of both stochastic jumps and deterministic switching, we first gained the impact of jumping and switching signals on Lyapunov function recurrence from instant k_l to k_{l-1}, then the fundamental Lyapunov function relation between time k to 0 is derived, which forms the basis for guaranteeing FTB condition.

Remark 2. With the proposed average dwell time approach, the Lyapunov function energy can be allowed to increase at each switching instant, which leads to less conservativeness compared with other methods. Nevertheless, the switching frequency should be moderate, which is constrained with condition (11), or the state trajectory of the system will exceed the given bound c_2.

Theorem 2. The system (1) is stochastic finite-time stabilizable through controller (4) with respect to given $(c_1 \quad c_2 \quad N \quad R \quad h)$ and $\delta \ge 1, \mu > 1$ if there exist a set of matrices $X_{\alpha,i} > 0$, $G_{\alpha,i} > 0$, $Y_{\alpha,i}$ ($\alpha \in \mathbb{M}, i \in \mathbb{S}$) such that

$$\begin{bmatrix} -\mu X_{\alpha,i} & 0 & \tilde{M}_{\alpha,i}^{\mathrm{T}} \\ * & -G_{\alpha,i} & L_{\alpha,i}^{\mathrm{T}} \\ * & * & -X_{\alpha} \end{bmatrix} < 0 \tag{19}$$

$$\begin{bmatrix} \delta\sum_{j=1}^{s}\pi_{ij}^{\beta}X_{\beta,j}-2\delta X_{\alpha,i} & \sqrt{\pi_{i1}^{\alpha}}X_{\alpha,i} & \cdots & \sqrt{\pi_{is}^{\alpha}}X_{\alpha,i} \\ * & -X_{\alpha,1} & \cdots & 0 \\ * & * & \ddots & \vdots \\ * & * & * & -X_{\alpha,s} \end{bmatrix}$$

$$\le 0, \alpha, \beta \in \mathbb{M} \text{ and } \alpha \ne \beta \tag{20}$$

$$\begin{bmatrix} -c_2^2\mu^{-N}+\lambda_2 h^2 & c_1 \\ * & -\lambda_1 \end{bmatrix} < 0 \tag{21}$$

$$\lambda_1 I < R^{1/2}X_{\alpha,i}R^{1/2} < I \tag{22}$$

$$0 < G_{\alpha,i} < \lambda_2 I \tag{23}$$

with average dwell time satisfying

$$\tau_a > \frac{N\ln\delta}{\ln c_2^2\mu^{-N}-\ln\left(c_1^2/\lambda_1+\lambda_2 h^2\right)} = \tau_a^* \tag{24}$$

Where

$$\tilde{M}_{\alpha,i}^{\mathrm{T}} = \left[\sqrt{\pi_{i1}^{\alpha}}\tilde{A}_{\alpha,i}^{\mathrm{T}} \ \sqrt{\pi_{i2}^{\alpha}}\tilde{A}_{\alpha,i}^{\mathrm{T}} \ \cdots \ \sqrt{\pi_{iS}^{\alpha}}\tilde{A}_{\alpha,i}^{\mathrm{T}}\right]$$

$$\tilde{A}_{\alpha,i}^{\mathrm{T}} = (A_{\alpha,i}X_{\alpha,i}+B_{\alpha,i}Y_{\alpha,i})^{\mathrm{T}}$$

$$L_{\alpha,i}^{\mathrm{T}} = \left[\sqrt{\pi_{i1}^{\alpha}}B_{w\alpha,i}^{\mathrm{T}} \ \sqrt{\pi_{i2}^{\alpha}}B_{w\alpha,i}^{\mathrm{T}} \ \cdots \ \sqrt{\pi_{iS}^{\alpha}}B_{w\alpha,i}^{\mathrm{T}}\right]$$

$$X_{\alpha} = diag\{X_{\alpha,1} \ X_{\alpha,2}\cdots X_{\alpha,S}\}$$

Solving the above inequalities, then the controller can be constructed as $K_{\alpha,i} = Y_{\alpha,i}X_{\alpha,i}^{-1}$.

Proof. Using Schur complement, inequality (8) is equivalent to

$$\begin{bmatrix} -\mu P_{\alpha,i} & 0 & M_{\alpha,i}^{\mathrm{T}} \\ * & -G_{\alpha,i} & L_{\alpha,i}^{\mathrm{T}} \\ * & * & -P_{\alpha}^{-1} \end{bmatrix} < 0 \tag{25}$$

where

$$M_{1,i}^{\mathrm{T}} = \left[\sqrt{\pi_{i1}^{\alpha}}A_{\alpha,i}^{\mathrm{T}} \ \sqrt{\pi_{i2}^{\alpha}}A_{\alpha,i}^{\mathrm{T}} \ \cdots \ \sqrt{\pi_{iS}^{\alpha}}A_{\alpha,i}^{\mathrm{T}}\right]$$

$$P_{\alpha} = diag\{P_{\alpha,1} \ P_{\alpha,2}\cdots P_{\alpha,S}\}$$

Consider the state-feedback controller (4), replacing $A_{\alpha,i}$ by $A_{\alpha,i}+B_{\alpha,i}K_{\alpha,i}$ and performing a congruence to (25) by diag $\{P_{\alpha,i}^{-1}, I, I\}$ and denoting $X_{\alpha,i} = P_{\alpha,i}^{-1}$, $Y_{\alpha,i} = K_{\alpha,i}X_{\alpha,i}$, we can get LMI (19).

Inequality (24) can be written as

$$\begin{bmatrix} -\delta\sum_{j=1}^{S}\pi_{ij}^{\beta}P_{\beta,j} & \sqrt{\pi_{i1}^{\alpha}} & \cdots & \sqrt{\pi_{iS}^{\alpha}} \\ * & -P_{\alpha,1}^{-1} & \cdots & 0 \\ * & * & \ddots & \vdots \\ * & * & * & -P_{\alpha,S}^{-1} \end{bmatrix} \le 0 \tag{26}$$

Post and pre multiplying to (26) by diag $\{P_{\alpha,i}^{-1}, I, \cdots, I\}$, which leads to

$$\begin{bmatrix} -\delta\sum_{j=1}^{S}\pi_{ij}^{\beta}X_{\alpha,i}P_{\beta,j}X_{\alpha,i} & \sqrt{\pi_{i1}^{\alpha}}X_{\alpha,i} & \cdots & \sqrt{\pi_{iS}^{\alpha}}X_{\alpha,i} \\ * & -X_{\alpha,1} & \cdots & 0 \\ * & * & \ddots & \vdots \\ * & * & * & -X_{\alpha,S} \end{bmatrix}$$

$$\le 0 \tag{27}$$

Based on Lemma 1, we have

$$-X_{\alpha,i}P_{\beta,j}X_{\alpha,i} \le X_{\beta,j}-2X_{\alpha,i}$$

Then

$$-\delta\sum_{j=1}^{S}\pi_{ij}^{\beta}X_{\alpha,i}P_{\beta,j}X_{\alpha,i} \le \delta\sum_{j=1}^{S}\pi_{ij}^{\beta}X_{\beta,j}-2\delta X_{\alpha,i} \tag{28}$$

Formulas (27) and (28) lead to LMI (20) in Theorem 1.

On the other hand, we consider

$$\min_{\alpha\in\mathbb{M},i\in\mathbb{S}}\lambda_{\min}(\tilde{P}_{\alpha,i}) = \frac{1}{\max_{\alpha\in\mathbb{M},i\in\mathbb{S}}\lambda_{\max}(\tilde{X}_{\alpha,i})}$$

$$\max_{\alpha\in\mathbb{M},i\in\mathbb{S}}\lambda_{\max}(\tilde{P}_{\alpha,i}) = \frac{1}{\min_{\alpha\in\mathbb{M},i\in\mathbb{S}}\lambda_{\min}(\tilde{X}_{\alpha,i})}$$

where $\tilde{X}_{\alpha,i} = \tilde{P}_{\alpha,i}^{-1} = R^{1/2}X_{\alpha,i}R^{1/2}$.

From (22) and (23), one has

$\min_{\alpha\in\mathbb{M},i\in\mathbb{S}}\lambda_{\min}(\tilde{X}_{\alpha,i}) > \lambda_1$, $\max_{\alpha\in\mathbb{M},i\in\mathbb{S}}\lambda_{\max}(\tilde{X}_{\alpha,i}) < 1$,

$\max_{\alpha\in\mathbb{M},i\in\mathbb{S}}\lambda_{\max}(G_{\alpha,i}) < \lambda_2$.

Then (11) can be guaranteed by

$$c_2^2\mu^{-N} > c_1^2/\lambda_1+\lambda_2 h^2$$

Which, by Schur complementary, can be further turned to (21). Using matrix eigenvalue constraint, the dwell time constrain in theorem 1can be guaranteed by (24).

4. NUMERICAL EXAMPLE

In this subsection, one simulation is used to show the obtained controller can make the finite-time unstably free system finite-time stabilizable, which means the validity of our approach. Consider switching MJLS with $M=2,\ S=3$, and the following system parameters:

MJLS 1:

$$A_{1,1}=\begin{bmatrix}1 & -0.4\\ 2 & 0.81\end{bmatrix},\ A_{1,2}=\begin{bmatrix}0 & -0.26\\ 0.9 & 1.13\end{bmatrix}$$

$$A_{1,3}=\begin{bmatrix}0.2 & -1.1\\ 0.2 & 0.4\end{bmatrix},B_{1,1}=\begin{bmatrix}1 & 1\end{bmatrix}^{\mathrm{T}},B_{1,2}=\begin{bmatrix}1 & 1\end{bmatrix}^{\mathrm{T}}$$

$$B_{1,3}=\begin{bmatrix}2 & -1\end{bmatrix}^{\mathrm{T}},B_{w1,1}=\begin{bmatrix}-0.4 & 0.3\end{bmatrix}^{\mathrm{T}}$$

$$B_{w1,2}=\begin{bmatrix}0.2 & 0.26\end{bmatrix}^{\mathrm{T}},\ B_{w1,3}=\begin{bmatrix}0.5 & -0.3\end{bmatrix}^{\mathrm{T}}$$

The TP matrix is given as follows:

$$\Pi^1=\begin{bmatrix}0.3 & 0.6 & 0.1\\ 0.2 & 0.5 & 0.3\\ 0.2 & 0.2 & 0.6\end{bmatrix}$$

MJLS 2:

$$A_{2,1}=\begin{bmatrix}1 & -0.05\\ 0.4 & -0.72\end{bmatrix},A_{2,2}=\begin{bmatrix}0.8 & 0.8\\ 0.6 & 1\end{bmatrix}$$

$$A_{2,3}=\begin{bmatrix}-0.3 & 0.6\\ 0.4 & 0.34\end{bmatrix}$$

$$B_{2,1}=B_{1,1},\ B_{2,2}=B_{1,2},\ B_{2,3}=B_{1,3}$$

$$B_{w2,1}=B_{w1,1},\ B_{w2,2}=B_{w1,2},\ B_{w2,3}=B_{w1,3}$$

The TP matrix is given as follows:

$$\Pi^2=\begin{bmatrix}0.5 & 0.2 & 0.3\\ 0.7 & 0.1 & 0.2\\ 0.2 & 0.6 & 0.2\end{bmatrix}$$

Letting $c_2^2=5$, $R=I_2$, $N=10$, $h^2=0.5$, $\mu=1.01$, $\delta=1.2$ and by solving the matrix inequalities in Theorem 2, the corresponding controller can be derived:

$$K_{1,1}=\begin{bmatrix}-1.5019 & -0.2073\end{bmatrix}$$
$$K_{1,2}=\begin{bmatrix}-0.4674 & -0.4619\end{bmatrix}$$
$$K_{1,3}=\begin{bmatrix}-0.0421 & 0.5210\end{bmatrix}$$
$$K_{2,1}=\begin{bmatrix}-0.7078 & 0.3763\end{bmatrix}$$
$$K_{2,2}=\begin{bmatrix}-0.6965 & -0.9035\end{bmatrix}$$
$$K_{2,3}=\begin{bmatrix}0.1996 & -0.1730\end{bmatrix}$$

with $\lambda_1=0.6197$, $\lambda_2=1.7096$. Then according to (24), the minimum average dwell time is calculated as $\tau_a^*=3.0052$, so we choose the average dwell time $\tau_a=3.1$ satisfying $\tau_a>\tau_a^*$, then we can orchestrate 3 times switching in 10 steps.

For the purpose of facilitate comparison, we will analyze the state trajectory of the free system (1) with $u(k)=0$ and the controlled system with controller (4). The initial state, initial mode and disturbance signal are taken as $x_0=\begin{bmatrix}0.5 & 0.7\end{bmatrix}^{\mathrm{T}}$, $r_0=1$, $\sigma_0=1$ and $w(k)=0.6e^{-k}$ respectively. Figure 1 shows the jump modes and switching signal. The state trajectories of the free and controlled MJLS (1) are drawn in Figure 2 and Figure 3. It is obvious that the state trajectory of free system exceeds the given bound c_2, hence the original free system is not stochastic FTB. However, with the designed controller (4), the state trajectory of stabilized system is limited in the region.

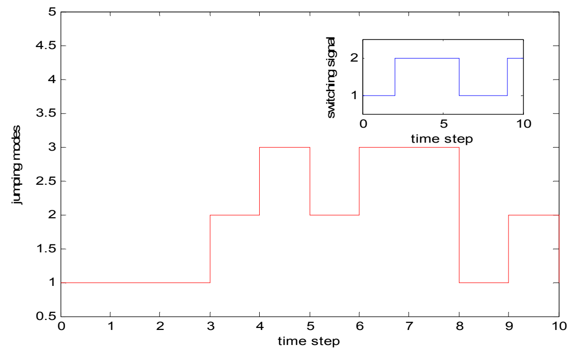

Fig.1. Jump modes and switching signal

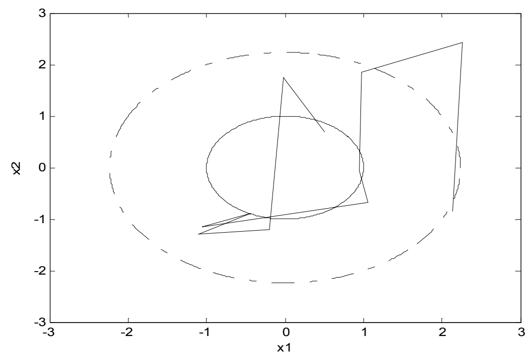

Fig.2. Trajectory of free system

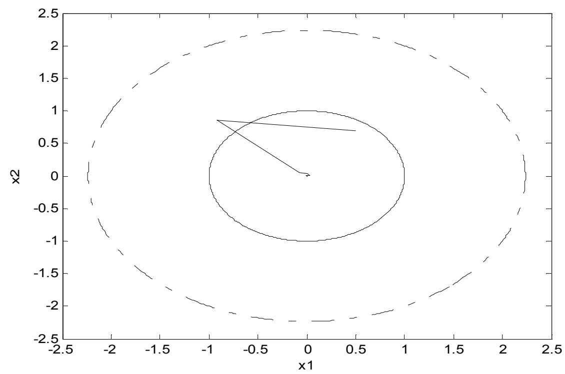

Fig.3. Trajectory of stabilized system

5. CONCLUSIONS

The control synthesis problems for discrete-time switching MJLSs in a finite time interval are studied here. The results guaranteeing finite-time boundedness have been provided firstly, then the state feedback controllers are designed such that the corresponding closed-loop system is finite-time bounded, finally one example is illustrated to show the validity of the obtained controllers in the end.

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REFERENCES

Amato, F., Ambrosino, R., Ariola, M. and Cosentino, C. (2009). Finite-time stability of linear time-varying systems with jumps, *Automatica*, 45(5), 1354-1358.

Amato, F., Ariola, M. and Dorato, P. (2001). Finite-time control of linear systems subject to parametric uncertainties and disturbances. *Automatica*, 37(9), 1459-1463.

Amato, F. and Ariola, M. (2005). Finite-time control of discrete-time linear systems. *IEEE Transactions on Automatic control*, 50(5), 724-729.

Bolzern, P., Colaneri, P. and Nicolao, G.D. (2013). Almost sure stability of Markov jump linear systems with deterministic switching. *IEEE Transactions on Automatic Control*, 58(1), 209-213.

Bolzern, P., Colaneri, P. and Nicolao, G.D. (2010). Markov jump linear systems with switching transition rates: Mean square stability with dwell-time. *Automatica*, 46, 1081-1088.

Costa, O.L.V., Fragoso, M.D. and Marques, R.P. (2005). Discrete time Markov jump linear systems. Springer, London.

Dorato, P. (1961). Short time stability in linear time-varying systems. *Proc of the IRE international Convention Record. Part 4*. New York, 83-87.

Feng, X.B., Loparo, K.A., Ji, Y.D. and Chizeck, H.J. (1992). Stochastic stability properties of jump linear systems. *IEEE Transactions on Automatic Control*, 37(1), 38-53.

Geromel, J.C. and Colaneri, P. (2006). Stability and stabilization of continuous-time switched linear systems. *SIAM Journal on Control and Optimization*, 45(5), 1915-1930.

Hou, L.L., Zong, G.D. and Zheng, W.X. (2013). Exponential l_2 -l_∞ control for discrete-time switching Markov jump linear systems. *Circuits Systems and Signal Processing*, DOI 10.1007/s00034-013-9588-1.

Lin, X.Z., Du, H.B. and Li, S.H. (2011). Finite-time boundedness and L2 gain analysis for switched delay systems with norm-bounded disturbance. *Applied Mathematics and Computation*, 217(12), 5982-5993.

Liu, H. and Shen, Y. (2011). H_∞ finite-time control for switched linear systems with time-varying delay. *Intelligent Control and Automation*, 2(2), 203-213.

Luan, X.L., Liu F. and Shi, P. (2010). Neural network based stochastic optimal control for nonlinear Markov jump systems. *International Journal of Innovative Computing Information and Control*, 6(8), 3715- 3723.

Luan, X.L., Liu, F. and Shi, P. (2010). Robust finite-time H_∞ control for nonlinear jump systems via neural networks. *Circuits systems and signal processing*, 29(3), 481-498.

Luan, X.L., Shi, P. and Liu, F. (2013). Finite-time stabilization for Markov jump systems with Gaussian transition probabilities. IET Control Theory & Applications, 7(2), 298-304.

Luan, X.L., Zhao, S.Y. and Liu, F. (2013). H_∞ control for discrete-time Markov jump systems with uncertain transition probabilities. *IEEE Transactions on Automatic Control*, 58(6), 1566-1572.

Moulay, E., Dambrine, M., Yeganefar, N. and Perruquetti, W. (2008). Finite-time stability and stabilization of time-delay systems. *Systems & Control Letters*, 57(7), 561-566.

Shi, P., Xia, Y., Liu, G. and Rees, D. (2006). On designing of sliding mode control for stochastic jump systems. *IEEE Transactions on Automatic Control,* 51(1), 97-103.

Weiss, L. and Infante, E. (1967). Finite time stability under perturbing forces and on product spaces. *IEEE Transactions on Automatic Control*, 12(1), 54-59.

Zhai, G.S., Hu, B., Yasuda, K. and Michel, A.N. (2001). Stability analysis of switched systems with stable and unstable subsystems: an average dwell time approach. *International Journal of Systems Science*, 32(8), 1055-1061.

Zhang, L.X., Boukas, E.K. and Lam, J. (2008). Analysis and synthesis of Markov jump linear systems with time-varying delays and partially known transition probabilities. *IEEE Transactions on Automatic Control*, 53(10), 2458-2464.

Zhang, X.F., Feng, G. and Sun, Y.H. (2012). Finite-time stabilization by state feedback control for a class of time-varying nonlinear systems. *Automatica*, 48(3), 499-504.

Zong, G.D., Hou L.L. and Wu Y.Q. (2011). Robust l_2-l_∞ guaranteed cost filtering for uncertain discrete-time switched system with mode-dependent time-varying delays. *Circuits, Systems, and Signal Processing*, 30(1), 17-33.