Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Consensus-based Approach for the Economic Dispatch Problem

Giulio Binetti*[◊], David Naso*, Biagio Turchiano*, Ali Davoudi[◊], and Frank L. Lewis[◊]

**Department of Electrical and Information Engineering, Polytechnic of Bari, Bari, Italy (email: giulio.binetti@poliba.it; david.naso@poliba.it; biagio.turchiano@poliba.it)*

[◊]Electrical Engineering Department, University of Texas at Arlington, Arlington, USA (e-mail: davoudi@uta.edu; lewis@uta.edu)

Abstract: This paper presents a distributed consensus-based approach to solve the economic dispatch problem with power generator constraints and transmission losses. Buses and transmission lines in the power system are modeled as nodes and edges in a communication graph, respectively. Each node exchanges information with its neighbors and runs two consensus algorithms in parallel, without relying on a centralized decision maker. A consensus algorithm plus a correction term is run to reach consensus on a Lagrangian variable to satisfy the generation-demand equality constraint, while another consensus algorithm is used to estimate the power mismatch in the network. Thus, each generating unit computes its output power according to its cost function. Advantages and limitations of the proposed approach are discussed. Finally, the algorithm is validated by means of numerical simulations on several benchmarks.

Keywords: Consensus algorithm, Distributed control, Economic Dispatch, Power system.

1. INTRODUCTION

The transformation of the legacy electric grid into a smart grid demands novel control techniques to solve both classic and emerging problems in power systems. One important challenge for power systems is the Economic Dispatch (ED) problem (Han et al. 2011) that deals with the allocation of power among the generating units, while minimizing the cost, matching the load demand and satisfying the operational constraints (Wood and Wollenberg 1996). A well-known centralized numerical method to solve the ED is the lambda-iteration algorithm (Lin and Viviani 1984). When considering non-convex ED formulation with losses, valve-point loading effects and multiple-fuel options (Cai and Liu 2005; Min et al. 2008), optimization techniques such as genetic algorithms (Chiang 2005) and particle swarm optimization (Neyestani et al. 2009) leads to satisfactory solutions, as in many other engineering fields, e.g., the design of non-linear motor drives (Cupertino et al. 2003).

Distributed algorithms are becoming increasingly important in the domain of intelligent decision-making and control to overcome the main drawbacks of centralized supervision in large scale systems (see, e.g., (Giordano et al. 2005)). Distributed approaches appear very promising also in the context of future power system since they can be more robust and immune to topological variations and can better accommodate the “plug-and-play” feature of the generating units and loads. However, it is more difficult to include the ED operational constraints in such a distributed formulation.

Distributed consensus-based approaches have been proposed in recent literature to solve the ED, since it can be stated as the search of the value for a Lagrangian variable. Among the recent papers, Binetti et al. (2013a) propose a consensus approach for ED without losses and lower and upper power bounds, while Dominguez-Garcia and Hadjicostis (2011) propose a double-iteration algorithm considering also power bounds. The algorithm in (Zhang and Chow 2012) assumes a priori knowledge of the power mismatch to a leader unit, while a variant in (Zhang et al. 2011) uses an additional level of consensus to estimate the power mismatch in a distributed fashion. Mudumbai et al. (2012) present a distributed iterative procedure for load-frequency control and ED that requires adjusting some parameters with an initial centralized intervention. Finally, Lorenzen et al. (2013) present a distributed optimization algorithm based on the ideas of cutting-plane consensus algorithm and adjustable robust counterparts. The cited papers do not consider losses in their models and, to the best knowledge of the authors, no distributed approach has yet been proposed to solve the ED in the more realistic case with non-negligible losses.

This paper proposes an approach to solve the ED problem in a distributed fashion, considering losses as well as lower and upper bounds for the generated power. The power system is supported by a sparse communication network, resulting in a more flexible, scalable, and reliable system. The proposed approach assumes that each node runs two algorithms in parallel. The first one is a first-order consensus algorithm designed to find the value of a Lagrangian variable that specifies the solution. A correction term is added to such algorithm to act as a proportional controller which drives the power mismatch to zero and fulfills the generation-demand equality constraint. The second algorithm is based on a consensus idea used for task allocation in multi-robot system (Binetti et al. 2013b), which relies on the association of a timestamp to the node state. This algorithm lets each node estimate locally the power mismatch in the system by storing the most up-to-date information about the other nodes.

This work was supported in part by NSF grant ECCS-1137354, NSF grant ECCS-1128050, ONR grant N00014-13-1-0562, and ARO grant W911NF-11-D-0001.

The rest of the paper is organized as follows. In Section 2, the ED is formulated and centralized solutions are discussed. Section 3 presents preliminaries of graph theory and basic consensus algorithms. In Section 4 the distributed approach is described. Several case studies in Section IV verify the effectiveness of the proposed approach.

2. ECONOMIC DISPATCH PROBLEM

In this section, the general formulation of the ED problem with both losses and generator constraints is introduced, and some aspects of centralized solutions are discussed.

2.1) Notations

To distinguish the variables available locally from those available centrally in standard ED approaches, the following notation is adopted in the paper: x_i denotes scalar x for node i; $\mathbf{x}_i$ denotes element i of vector $\mathbf{x}$; X_{ij} denotes element (i,j) of matrix X; $x^{(i,j)}$ denotes scalar x whose meaning is related to the specific meaning of the pair (i,j); $x^{[i]}$ denotes scalar x carried by node i; $\mathbf{x}^{[i]}$ denotes vector x carried by node i; $\mathbf{x}_j^{[i]}$ denotes element j of vector x carried by node i; $x(k)$ means that x is function of variable k; $x[k]$ refers to variable x at discrete time k.

2.2) Economic Dispatch Formulation

The economic dispatch determines the output of a number of generating units matching the load demand at the lowest cost. Denote by $\mathcal{S}_G$, $\mathcal{S}_D$, and $\mathcal{S}_B$, the sets of generator buses, load buses, and all the buses in the power systems, respectively, with cardinality $n_G = |\mathcal{S}_G|$, $n_D = |\mathcal{S}_D|$, and $n_B = |\mathcal{S}_B|$. It results $\mathcal{S}_B = \mathcal{S}_G \cup \mathcal{S}_L$ and, without loss of generality, it is also assumed $\mathcal{S}_G \cap \mathcal{S}_D = \varnothing$. Each load bus i is characterized by a local load demand p_{Di}.

Solving the ED problem requires to minimize the power generation cost given by

$$f(\mathbf{p_G}) = \sum_{i \in \mathcal{S}_G} f_i(p_{Gi}) \tag{1}$$

where $\mathbf{p_G} = \left[p_{G1}, \dots, p_{Gn_G}\right]^T$ is the power vector with p_{Gi} the power generated at bus i, and $f_i(p_{Gi})$ is the cost function usually expressed (Wood and Wollenberg 1996) as

$$f_i(p_{Gi}) = \alpha_i + \beta_i p_{Gi} + \gamma_i p_{Gi}^2 \tag{2}$$

with α_i, β_i, and γ_i the cost coefficients for generator i.

The ED is subject to several operational constraints. The following generation-demand equality constraint

$$\Delta p = \sum_{i \in \mathcal{S}_D} p_{Di} + p_L - \sum_{i \in \mathcal{S}_G} p_{Gi} = 0 \tag{3}$$

states that the sum of demand and losses, p_L, has to be equal to the generated power. Thus, Δp denotes the total power mismatch in the system which ideally should be equal to zero. The losses can be expressed as

$$p_L = \sum_{(i,j) \in \mathcal{S}_B \times \mathcal{S}_B} r^{(i,j)} \left(\iota^{(i,j)}\right)^2 / 2 \tag{4}$$

where $r^{(i,j)}$ is the line resistance between buses i and j, and $\iota^{(i,j)}$ is the current flowing in the line between buses i and j (with $\iota^{(i,j)} = 0$ if there is no direct connection between the two buses). The power generator constraints

$$p_{Gi}^{\min} \le p_{Gi} \le p_{Gi}^{\max}, \forall i \tag{5}$$

state that the generated power p_{Gi} has to be chosen in the range between minimum, $p_{Gi}^{\min}$, and maximum, $p_{Gi}^{\max}$, power limits for each generator i.

It is possible to formulate the ED using the Lagrangian operator L (Bronson and Naadimuthu 1997) such that

$$L = f(\mathbf{p_G}) + \lambda \Delta p + \sum_{i=1}^{2n_G} \mu^{(i)} \psi^{(i)} \tag{6}$$

where λ and $\mu^{(i)}$'s are the Lagrange multipliers associated with equality constraint (3) and inequality constraints in (5), respectively (Bronson and Naadimuthu 1997). Using this notation, the minimization of the Lagrangian operator also encompasses the satisfaction of the constraints.

2.3) Centralized Solution of the ED

A key concept of the centralized solutions for ED is the incremental cost, that is, the derivative of the cost function. When power generator constraints and losses are neglected, the Lagrangian operator provides a set of equations

$$\frac{\partial L}{\partial p_{Gi}} = \frac{df_i(p_{Gi})}{dp_{Gi}} - \lambda = 0 \tag{7}$$

or, equivalently,

$$\lambda = \frac{df_i(p_{Gi})}{dp_{Gi}} \tag{8}$$

Thus, the necessary condition for the existence of a minimum-cost operating point is that all incremental costs must be equal to λ. In this case, the optimal incremental cost and the output powers are given (Binetti et al. 2013a) by

$$\lambda = \left(\sum_{i \in \mathcal{S}_D} p_{Di} + \sum_{i \in \mathcal{S}_G} \frac{\beta_i}{2\gamma_i}\right) \Bigg/ \left(\sum_{i \in \mathcal{S}_G} \frac{1}{2\gamma_i}\right) \tag{9}$$

$$p_{Gi} = (\lambda - \beta_i)/(2\gamma_i) \tag{10}$$

When the generator constraints are considered, the necessary conditions for the existence of a minimum-cost operating condition may be expanded (Wood and Wollenberg 1996) as

$$\begin{aligned} df_i(p_{Gi})/dp_{Gi} &= \lambda \quad \text{for } p_{Gi}^{\min} < p_{Gi} < p_{Gi}^{\max} \\ df_i(p_{Gi})/dp_{Gi} &\le \lambda \quad \text{for } p_{Gi} = p_{Gi}^{\max} \\ df_i(p_{Gi})/dp_{Gi} &\ge \lambda \quad \text{for } p_{Gi} = p_{Gi}^{\min} \end{aligned} \tag{11}$$

This implies that all the generators operating within their bounds have the same incremental cost, while the other generators operate on the limits $p_{Gi}^{\min}$ or $p_{Gi}^{\max}$.

When the losses are considered, the Lagrangian operator gives the set of equations

$$\frac{\partial L}{\partial p_{Gi}} = \frac{df_i}{dp_{Gi}} - \lambda \left(1 - \frac{\partial p_L}{\partial p_{Gi}}\right) \tag{12}$$

or, equivalently,

$$\lambda = \frac{1}{\left(1 - \partial p_L / \partial p_{Gi}\right)} \frac{df_i}{dp_{Gi}} \tag{13}$$

where $\partial p_L / \partial p_i$ is called the incremental loss for unit i, and $Pf_i = 1/\left(1 - \partial p_L / \partial p_i\right)$ is called the penalty factor for unit i. In this case, the ED problem becomes analytically intractable even with simplified cost functions (Mudumbai et al. 2012).

3. GRAPH THEORY AND CONSENSUS ALGORITHMS

In this section, some preliminaries of graph theory are recalled, and then the consensus algorithms used to design the proposed distributed solution are introduced.

3.1) Graph Theory

A graph is expressed as $\mathcal{G} = \left(\mathcal{V}, \mathcal{E}, A\right)$ with $\mathcal{V} = \{v_1, v_2, \ldots, v_n\}$ the set of nodes, $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ the set of edges, and $A = [A_{ij}] \in R^{n \times n}$ the adjacency matrix. An edge from j to i is denoted by (v_j, v_i), which means that node i receives information from node j. A_{ij} is the weight of edge (v_j, v_i), and $A_{ij} > 0$ if $(v_j, v_i) \in \mathcal{E}$, otherwise $A_{ij} = 0$. Node j is called neighbor of node i if $(v_j, v_i) \in \mathcal{E}$. The set of node i neighbors, denoted by $\mathcal{N}_i = \{ j \,|\, (v_j, v_i) \in \mathcal{E} \}$, has cardinality $\bar{d}_i$. The in-degree d_i of node i is defined as the i-th row sum of A, i.e., $d_i = \sum_{j \in \mathcal{N}_i} A_{ij}$. A path from i to j is a sequence $\left\{ (v_i, v_k), (v_k, v_l), \ldots, (v_m, v_l) \right\}$. The distance $l^{(i,j)}$ between nodes i and j is the number of edges in the shortest path connecting them. The graph diameter is the greatest minimum distance between any pair of nodes.

3.2) First-order Consensus Algorithm

Consider a group of nodes, distributed on a communication graph $\mathcal{G}$, with identical dynamics. Suppose that each node has scalar discrete-time dynamics given by

$$x^{[i]}[k+1] = x^{[i]}[k] + u^{[i]}[k] \tag{14}$$

where $x^{[i]}, u^{[i]} \in R$ are the state and control input for node i.

Assume the communication graph $\mathcal{G}$ is strongly connected and satisfies the property of bidirectional equal-neighbor weight topology (Olshevsky and Tsitsiklis 2009), i.e.,

$$(v_i, v_j) \in \mathcal{E} \Rightarrow (v_j, v_i) \in \mathcal{E}, \quad (v_i, v_i) \in \mathcal{E} \tag{15}$$

with the elements a_{ij} of the adjacency matrix defined as

$$A_{ij} = \begin{cases} 1/\bar{d}_i & \text{if } j \in \mathcal{N}_i \\ 0 & \text{if } j \notin \mathcal{N}_i \end{cases} \quad \forall i \tag{16}$$

where $\mathcal{N}_i$ is the set of neighbors and $\bar{d}_i$ is its cardinality. Note that each node has a loop such that $i \in \mathcal{N}_i$. This topology for the graph $\mathcal{G}$ results in an adjacency matrix that is non-negative (all its elements are non-negative), row-stochastic (all row-sums are 1), irreducible (since the associated graph is strongly connected), and primitive (it has only one eigenvalue with maximum modulus).

Considering the local control algorithm

$$u^{[i]}[k] = \sum_{j \in \mathcal{N}_i} A_{ij} \left(x^{[j]}[k] - x^{[i]}[k] \right) \tag{17}$$

with a_{ij} denoting the graph edge weights, the closed-loop system becomes

$$x^{[i]}[k+1] = \sum_{j \in \mathcal{N}_i} A_{ij} x^{[j]}[k] \tag{18}$$

since $\sum_{j \in \mathcal{N}_i} A_{ij} = 1$ for the equal-neighbor weight topology.

Considering the properties of the adjacency matrix A, note that it has right eigenvector $\mathbf{1}$ associated to the eigenvalue $\lambda_1 = 1$ since $A\mathbf{1} = \mathbf{1}$ and $(I - A)\mathbf{1} = 0$. Moreover, by Perron-Frobenious theorem, the dynamics (18) reach asymptotically consensus and the group decision value is

$$x_c = \sum_i \mathbf{w}_i x^{[i]}[0] \tag{19}$$

with $\mathbf{w}$ the normalized left eigenvector associated to $\lambda_1 = 1$ (Jadbabaie et al. 2003; Olfati-Saber and Murray 2004).

3.3) Consensus on the most up-to-date information

Consider a group of nodes, distributed on a communication graph $\mathcal{G}$, with identical dynamics. Suppose that each node has vector discrete-time dynamics represented by state vector $\mathbf{x}^{[i]} \in R^n$ and timestamp vector $\mathbf{s}^{[i]} \in R^n$. $\mathbf{x}_j^{[i]}$ and $\mathbf{s}_j^{[i]}$ are the j-th element of $\mathbf{x}^{[i]}$ and $\mathbf{s}^{[i]}$, respectively. The element $\mathbf{x}_i^{[i]}$ is the node i state evaluated by the same node with a local function, say $g_i(\cdot)$, while the elements $\mathbf{x}_j^{[i]}$ are the most up-to-date information that node i has about the state of node j. Each element $\mathbf{s}_j^{[i]}$ is the timestamp associated to the corresponding state information $\mathbf{x}_j^{[i]}$, that is, the instant time at which the state information was generated. Going into details, the node dynamics is given by

$$\begin{aligned} &\mathbf{s}_i^{[i]}[k+1] = k+1 && \mathbf{s}_j^{[i]}[k+1] = \max_{h \in \mathcal{N}_i \setminus \{i\}} \left(\mathbf{s}_j^{[h]}[k] \right), \; j \neq i \\ &\mathbf{x}_i^{[i]}[k+1] = g_i(\cdot) && \mathbf{x}_j^{[i]}[k+1] = \mathbf{x}_j^{\left[\tilde{j}^{(i)}\right]}[k], \; j \neq i \end{aligned} \tag{20}$$

with

$$\tilde{j}^{(i)}[k+1] = \underset{h \in \mathcal{N}_i \setminus \{i\}}{\operatorname{argmax}} \left(\mathbf{s}_j^{[h]}[k] \right) \tag{21}$$

Eq. (21) identifies node $\tilde{j}^{(i)}$ in the neighborhood of node i with the most up-to-date information about node j according to the timestamps. Node i stores the information about node j received by node $\tilde{j}^{(i)}$ which is the node with minimum distance from j. Thus, the dynamics (20)-(21) lets each node update its state and save the states of all other nodes with a certain delay depending on the graph structure. The following Lemmas derive directly from (20)-(21).

Lemma 1. *Let the graph $\mathcal{G}$ be strongly connected. Under algorithm (20)-(21) each node has*

$$\mathbf{x}_j^{[i]}[k] = \mathbf{x}_j^{[j]}\left[k - l^{(i,j)}\right] \tag{22}$$

where $l^{(i,j)}$ is the finite distance between nodes i and j.

Lemma 2. *Let the graph $\mathcal{G}$ be strongly connected. Assume that, under algorithm (20)-(21), after a certain time instant $\bar{k}$ a node i does not change its local value $\mathbf{x}_i^{[i]}$, that is, $g_i[k+1]=g_i[k]$ for $k \geq \bar{k}$. Then*

$$\mathbf{x}_i^{[j]}[k]=\mathbf{x}_i^{[i]}[\bar{k}] \quad \forall k \geq \bar{k}+\phi \tag{23}$$

with ϕ being the diameter of the communication graph $\mathcal{G}$.

Lemma 1 states that a node i knows the state of the node j with a delay equal to the finite distance $l^{(i,j)}$, since the graph is strongly connected. Lemma 2 states that if a node i does not change its local value $\mathbf{x}_i^{[i]}$, then all other nodes reach the consensus on such a fixed value $\mathbf{x}_i^{[i]}$ within a number of iterations equal to the communication graph diameter, ϕ.

4. DISTRIBUTED SOLUTION

The communication network supporting the power system has same topology of the power network. Based on this model, a distributed consensus-based solution is proposed to solve the ED with losses and power generator constraints.

4.1) Distributed Algorithm

The proposed method for the ED uses two consensus algorithms, running in parallel, to reach consensus on the Lagrangian variable λ and to estimate the total power mismatch Δp.

Each node runs the following consensus algorithms

$$\lambda^{[i]}[k+1]=\sum_{j\in\mathcal{N}_i} A_{ij}\lambda^{[j]}[k]+\kappa_P 2\gamma_i\delta^{[i]}[k] \quad i\in\mathcal{S}_G \tag{24}$$

$$\lambda^{[i]}[k+1]=\sum_{j\in\mathcal{N}_i} A_{ij}\lambda^{[j]}[k] \quad i\in\mathcal{S}_D \tag{25}$$

where A is the adjacency matrix in (16), γ_i is the generator cost coefficient in (2) for bus i, κ_P is a proportional control gain, and $\delta^{[i]}[k]$ is the estimation of total power mismatch of bus i. The algorithm (24) for the generator buses is a first-order consensus algorithm plus the correction term $\kappa_P 2\gamma_i\delta^{[i]}[k]$ that lets each generator bus i change the Lagrange multiplier according to the estimation $\delta^{[i]}[k]$ of the power mismatch. The algorithm (25) is a consensus algorithm for the load buses used only to propagate the information about the Lagrange multiplier in the network.

The rationale of algorithms (24)-(25) is that the global dynamics of the system without correction term can be described by a marginally stable system with one pole $z=1$ in the z-plane (Jadbabaie et al. 2003; Olfati-Saber and Murray 2004). Therefore, the correction term in (24) acts as a proportional controller in closed loop and can effectively drive the power mismatch to zero.

The local estimation $\delta^{[i]}[k]$ of the total power mismatch is performed as follows. Consider that each bus i can compute its local power mismatch $\Delta p_i[k]$ at each time step as

$$\Delta p_i[k]=p_{Di}[k]+p_{Li}[k]-p_{Gi}[k] \tag{26}$$

where the losses associated to each bus can be expressed by

$$p_{Li}[k]=\sum_{j\in\mathcal{N}_i} r^{(i,j)}\left(\iota^{(i,j)}[k]\right)^2/2 \tag{27}$$

and the generated power associated to each generator bus can be expressed by

$$p_{Gi}[k]=\begin{cases}\dfrac{\lambda^{[i]}[k]-\beta_i}{2\gamma_i} & p_{Gi}^{\min}<\dfrac{\lambda^{[i]}[k]-\beta_i}{2\gamma_i}<p_{Gi}^{\max}\\ p_{Gi}^{\min} & \dfrac{\lambda^{[i]}[k]-\beta_i}{2\gamma_i}<p_{Gi}^{\min}\\ p_{Gi}^{\max} & \dfrac{\lambda^{[i]}[k]-\beta_i}{2\gamma_i}>p_{Gi}^{\max}\end{cases}, \quad i\in\mathcal{S}_G \tag{28}$$

$$p_{Gi}[k]=0, \quad i\in\mathcal{S}_D \tag{29}$$

In Eq. (28) the generator buses fulfill also the constraints on minimum and maximum output powers, while in Eq. (29) the load buses set the output power to 0. In Eq. (27), each bus is required to know the resistances $r^{(i,j)}$ of the transmission lines with its neighbors, which is local information, and the current $\iota^{(i,j)}$ flowing in such lines, which can be locally obtained by the protective relay devices (Perez et al. 1994). Thus, the local power mismatch Δp_i in (26) is actually local information.

Assume now that each node carries two vectors to perform the estimation of the power mismatch: the vector of local power mismatches, $\mathbf{x}^{[i]}\in R^{n_B}$, and the timestamp vector, $\mathbf{s}^{[i]}\in R^{n_B}$. Each bus runs the following consensus algorithm on the most up-to-date information:

$$\begin{aligned}&\mathbf{s}_i^{[i]}[k+1]=k+1 && \mathbf{s}_j^{[i]}[k+1]=\max_{h\in\mathcal{N}_i\setminus\{i\}}\left(\mathbf{s}_j^{[h]}[k]\right),\ j\neq i\\ &\mathbf{x}_i^{[i]}[k+1]=\Delta p_i[k+1] && \mathbf{x}_j^{[i]}[k+1]=\mathbf{x}_j^{\left[\tilde{j}^{(i)}\right]}[k],\ j\neq i\end{aligned} \tag{30}$$

with

$$\tilde{j}^{(i)}[k+1]=\operatorname*{argmax}_{h\in\mathcal{N}_i}\left(\mathbf{s}_j^{[h]}[k]\right) \tag{31}$$

Then, the estimation of the total power mismatch at each bus i is defined as

$$\delta^{[i]}[k]=\sum_{j=1}^{n_B}\mathbf{x}_j^{[i]}[k] \tag{32}$$

Each generator bus can use the above estimation to apply the correction in (24), modify the local Lagrangian variable, and drive the power mismatch to zero.

4.2) Heuristics for Proportional Gain Design

Considering the consensus procedure in (24)-(25), note that the parameter κ_P has a similar role to learning rates in iterative algorithms used for adaptive or learning control schemes, which can trigger oscillations or even instability when set too high. Thus, the parameter κ_P should not be set too large to avoid excessively fast updates of the local λ_i's. Moreover, the larger is the network diameter, the slower is the information propagation for the consensus algorithm in (30)-(31). Thus, smaller networks can use larger κ_P without causing undesired oscillating behavior. This suggests to design κ_P as a function of the topological properties of the communication graph. A possible heuristic candidate is

$$\kappa_P = 1/n_B \tag{33}$$

where n_B is the number of buses in the power system. Each bus can locally compute n_B as the size of timestamp vector exploiting the property of the consensus on the most up-to-date information. This heuristic choice will be shown in the following sections to be an effective choice for both small and large scenarios.

5. NUMERICAL RESULTS

In this section, two case studies with 6 and 300 buses show the effectiveness of the proposed algorithm on a small and a large network.

5.1) Simulation Setting

The 6-bus system is described in the book of Wood and Wollenberg (1996), while the details of the standard IEEE 300-bus system can be found in the archive of Matpower (Zimmerman et al. 2011). The simulations have been run in Matlab and the current values for the losses computation have been estimated by running the power flow routine of Matpower (Zimmerman et al. 2011).

5.2) Case Study with 6 buses

In this case study, the initial load demand is $210\,\text{MW}$, with three loads of $70\,\text{MW}$ for each load bus. At the time step $k = 50$ and $k = 100$, all the loads are first increased by 20% and then reduced by 10%, respectively. This simulation considers an initial output power distribution with all the generator buses providing their minimum output power with the corresponding local incremental cost. Fig. 1 shows that the system responds automatically to all the changes in the load demands and converges to the new solutions. Fig. 1(d) shows also that the algorithm can properly handle the power generator constraints; in fact, the minimum output power $p_{G1}^{\min} = 50\,\text{MW}$ for generator 1 is active for both the first and third total load demand during the simulation. The Lagrange multiplier is updated according to the estimation of the power mismatch and the consensus is reached after a few iterations. The total power mismatch goes to zero and the generation-demand equality constraint is satisfied. Finally, note that the losses in the system are properly considered in Fig. 1(c), where the generated power is greater than the demanded power to take into account for the losses.

5.3) Case Study with 300 buses

In this case study, the IEEE 300-bus system is considered to show the effectiveness of the proposed approach for a large network. The simulation starts with the initial condition given by the Matpower case study (Zimmerman et al. 2011) and a total load demand of $23526\,\text{MW}$. Then, at the time steps $k = 400$ and $k = 800$, the total load demand is first increased by 20% and then reduced by 10%, respectively. As shown in Fig. 2, the proposed algorithm converges automatically to a new solution, fulfilling the generation-demand equality constraint including the transmission losses. Moreover, each iteration requires about 0.007 seconds per bus running on a Personal Computer with a Core2 Duo processor (3 GHz) and 4 GB RAM. Since the algorithm converges in a few hundred iterations, assuming a conservative time step of 0.02 seconds for a practical application, the algorithm converges in a few seconds. Thus, the algorithm is sufficiently fast and suitable for actual implementation considering that the deployment time for the ED solution is usually from 5 to 15 minutes.

CONCLUSIONS

The economic dispatch problem is addressed in a distributed fashion considering losses and power generator constraints. The communication network has same topology of the power system, resulting in a sparse graph. The proposed solution is based on consensus algorithms that let each generating bus to decide autonomously its output power in response to load demand changes. The effectiveness of the proposed algorithm to properly handle losses and power generator constraints has been verified in simulation on both small and large networks. Moreover, the proposed algorithm converges in times that are suitable for actual implementation. Some heuristic design guidelines for the main algorithm parameter, namely the proportional gain, have been discussed. However, the theoretical characterization of the proportional gain limits corresponding to good performances will be the main focus of future research.

REFERENCES

Binetti, G., Abouheaf, M.I., Lewis, F.L., Naso, D., Davoudi, A., and Turchiano, B. (2013). Distributed solution for the economic dispatch problem. *Proc. of 21st Mediterranean Conference on Control and Automation*, pp. 243-250.

Binetti, G., Naso, D., and Turchiano, B. (2013). Decentralized task allocation for surveillance systems with critical tasks. *Robotics and Autonomous Systems*, 61(12), pp. 1653-1664.

Bronson, R. and Naadimuthu, G. (1997). *Operations Research*. In Schaum's Outline, McGraw-Hill, New York.

Cai, Y. and Liu, D. (2005). An orthogonal array optimization for the economic dispatch with nonsmooth cost functions. *Proc. of 44th IEEE Conference on Decision and Control*, pp. 1264-1269.

Chiang, C.L. (2005). Improved genetic algorithm for power economic dispatch of units with valve-point effects and multiple fuels. *IEEE Transactions on Power Systems*, 20(4), pp. 1690-1699.

Cupertino, F., Giordano, V., Naso, D., Turchiano, B., Salvatore L. (2003). On-line genetic design of fuzzy controllers for DC drives with variable load. *Electronics Letters*, 39(5), pp. 479-480.

Dominguez-Garcia, A.D. and Hadjicostis, C.N. (2011). Distributed algorithms for control of demand response and distributed energy resources. *Proc. of 50th IEEE Conference on Decision and Control*, pp. 27-32.

Giordano, V., Zhang, J.B. Naso, D., Lewis F. (2008). Integrated supervisory and operational control of a warehouse with a matrix-based approach. *IEEE Transactions on Automation Science and Engineering*, 5(1), pp. 53-70.

Han, Y., Young, P., and Zimmerle, D. (2011). Optimal generation units dispatch for fuel consumption minimization. *Proc. of 50th IEEE Conference on Decision and Control*, pp. 7206-7211.

Jadbabaie, A., Lin, J., and Morse, A.S. (2003). Coordination of groups of mobile autonomous agents using nearest neighbor rules. *IEEE Transactions on Automatic Control*, 48(6), pp. 988-1001.

Lin C.E. and Viviani, G.L. (1984). Hierarchical economic dispatch for piecewise quadratic cost functions. *IEEE Transactions on Power Apparatus and Systems*, PAS-103(6), pp. 1170-1175.

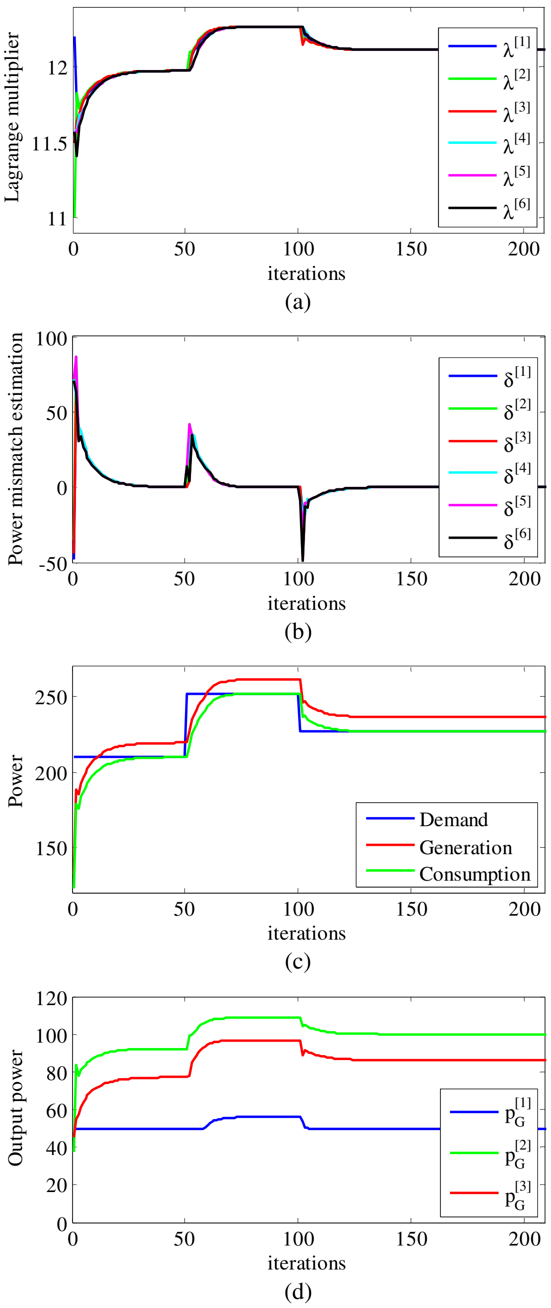

Fig. 1. Case study with 6 buses: (a) Lagrange multiplier, (b) power mismatch estimation, (c) demand, generation, and consumption, (d) output power.

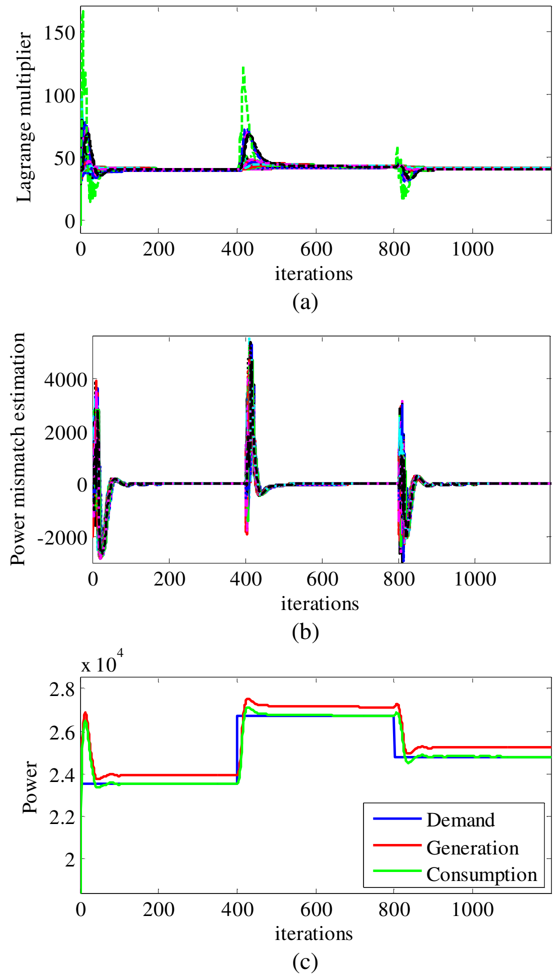

Fig. 2. Case study with 300 buses: (a) Lagrange multiplier, (b) power mismatch estimation, (c) demand, generation, and consumption.

Lorenzen, M., Burger, M., Notarstefano, G., and Allgower, F. (2013). A distributed solution to the adjustable robust economic dispatch problem. *Proc. of 4th IFAC Workshop on Distributed Estimation and Control in Networked Systems*, pp. 75-80.

Min, K.I., Moon, Y.H., and Lee, S.W. (2008). A global solution to economic dispatch with multiple fuel units using a function merger. *Proc. of 17th IFAC World Congress*, pp. 11064-11069.

Mudumbai, R., Dasgupta, S., and Cho, B.B. (2012). Distributed control for optimal economic dispatch of a network of heterogeneous power generators. *IEEE Transactions on Power Systems*, 27(4), pp. 1750-1760.

Neyestani, M., Farsangi, M.M., Nezamabadi-Pour, H., and Lee, K.Y. (2009). A modified particle swarm optimization for economic dispatch with nonsmooth cost functions. *Proc. of 6th IFAC Symposium on Power Plants and Power Systems Control*, pp. 267-272.

Olfati-Saber, R. and Murray R. (2004). Consensus problems in networks of agents with switching topology and time-delays. *IEEE Transactions on Automatic Control*, 49(9), pp. 1520-1533.

Olshevsky, A. and Tsitsiklis, J.N. (2009). Convergence speed in distributed consensus and averaging. *SIAM Journal on Control and Optimization*, 48(1), pp. 33-55.

Perez, L.G., Flechsig, A.J., and Venkatasubramanian, V. (1994). Modeling the protective system for power system dynamic analysis. *IEEE Transactions on Power Systems*, 9(4), pp. 1963-1973.

Wood, A.J. and Wollenberg, B.F. (1996). *Power Generation, Operation, and Control*. Wiley, New York.

Zhang, Z., Ying, X., and Chow, M.Y. (2011). Decentralizing the economic dispatch problem using a two-level incremental cost consensus algorithm in a smart grid environment. *Proc. of 2011 North American Power Symposium*, pp. 1-7.

Zhang, Z. and Chow, M.Y. (2012). Convergence analysis of the incremental cost consensus algorithm under different communication network topologies. *IEEE Transactions on Power Systems*, 27(4), pp. 1761-1768.

Zimmerman, R.D., Murillo-Sanchez, C.E., and Thomas, R.J. (2011). MATPOWER: Steady-state operations, planning and analysis tools for power systems research and education. *IEEE Transactions on Power Systems*, 26(1), pp. 12-19.