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Adaptive Controller for Linear Plant with Parametric Uncertainties, Input Delay And Unknown Disturbance

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Abstract: We present a new stabilization approach for unstable linear plants with long input delay, unknown parameters and disturbance. The predictor-based algorithm providing plant identification and stabilization is proposed. Also the extension is considered with estimation and cancellation of an unknown disturbance. A numerical examples are given to illustrate the efficiency of our adaptive controller.

Keywords: input delay, parametric uncertainties, disturbance rejection.

1. INTRODUCTION

Control of systems with delays is one of fundamental problems of modern control theory. For now we skip a comprehensive review of the results obtained recently in the field (see, e.g., Arstein [1982], Bobtsov and Pyrkin [2010, 2012], Krstic and Smyshlyaev [2008], Krstic [2009], Kwon and Pearson [1980], Manitus and Olbrot [1979], Niculescu and Annaswamy [2004], Parsheva and Tsykunov [2001], Pyrkin et al. [2010 a,b], Pyrkin [2010], Smith [1959]) within the bounds of a single conference paper. Lyapunov-Krasovskii functionals became more common for stability analysis and replaced the traditional Lyapunov functions approach in this sense (see, e.g., Pyrkin et al. [2011], Pyrkin and Bobtsov [2011]). Initially this technique was adapted mostly for state delays, while more complex problems related to time-delays in the control signal was not covered. The landmark work of Otto Smith (Smith [1959]) introduced a novel control design approach providing invariance of system stability and performance with respect to time-delays. However, applicability of this result was restricted by a class of asymptotically stable systems with known parameters. In subsequent years researchers across the world concentrated their efforts on extensions to more generic algorithms, e.g. for parametrically uncertain (see Parsheva and Tsykunov [2001]) and unstable plants (Krstic and Smyshlyaev [2008], Krstic [2009], Pyrkin et al. [2010 a,b]). Nevertheless, works (Parsheva and Tsykunov [2001]) considered only open-loop stable cases, while (Krstic and Smyshlyaev [2008], Krstic [2009]) didn't offer any solution for the actual problem of simultaneous external disturbance rejection. In (Pyrkin et al. [2010 a,b]) the complex task of stabilization and disturbance rejection was solved only for systems with known parameters.

This work describes the stabilization algorithm for a parametrically uncertain linear time-invariant plant under unknown external disturbance. This algorithm is based on synthesis of the predictor analogous to one described in (Krstic and Smyshlyaev [2008], Krstic [2009]). Proof of the exponential stability of the closed-loop system with such predictor based on the Lyapunov functions analysis allows us extend this result for the cases of unknown delays as well as for parametric, signal and structural disturbances in the system. In contrast to the well known Smith predictor the approach introduced in (Kwon and Pearson [1980]) and (Manitus and Olbrot [1979]) can be effectively implemented for open-loop unstable systems.

2. PROBLEM FORMULATION

Consider the LTI plant

$$\dot{x}(t) = Ax(t) + Bu(t-D) + Bf, \quad y(t) = Cx(t), \quad (1)$$

where $x \in R^n$ is a measurable vector of state variables, $u(t)$ is a scalar input, $y(t)$ is a scalar output, $D \geq 0$ is a known

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constant delay, A, B, C are matrices with corresponding dimensions involving unknown parameters, $f = const$ is an unknown disturbance.

We assume that $u(t-D) = 0$ for $t < D$.

The control goal is to design a state-feedback controller that provides asymptotic stability of the equilibrium $x = 0$ and cancellation the unknown disturbance f.

3. PRELIMINARY RESULTS

We start from a classical result dealing with stabilization of unstable systems with input delay (Krstic and Smyshlyaev [2008], Krstic [2009], Kwon and Pearson [1980], Manitus and Olbrot [1979]).

It is well known that for the system (1) with known parameters, under condition of full controllability, for $f = 0$ and $D = 0$ it is possible to design the control law as

$$u = Kx(t), \tag{2}$$

where the vector K such that state matrix of the closed-loop $A + BK$ is Hurwitz, i.e. all its eigenvalues have a negative real part.

For the case $D > 0$ the control law (2) may be rewritten as

$$u(t) = Kx(t + D), \tag{3}$$

where $x(t+D)$ is predicted value of the state $x(t)$ after D seconds. It is clear that the controller (3) is unrealisable, because the vector $x(t + D)$ is not available for direct measuring. However the vector $x(t+D)$ can be calculated basing on a priori plant information.

The fundamental solution of (1) for $f \equiv 0$ is as follows

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-s)}Bu(s-D)ds. \tag{4}$$

From (4) we get a value $x(t + D)$

$$\begin{aligned} x(t+D) &= e^{A(t+D)}x(0) + \int_D^{t+D} e^{A(t+D-s)}Bu(s-D)ds \\ &= e^{AD}x(t) + \int_{t-D}^{t} e^{A(t-s)}Bu(s)ds. \end{aligned} \tag{5}$$

It is known that $x(t + D)$ can be calculated by (5) if elements of matrices A and B are known and all values of the control signal u on the interval $[t - D;\ t]$ are known as well. From (5) it is easy to get the control law, providing stabilization of unstable systems with the input delay (see Krstic [2009])

$$u(t) = Ke^{AD}x(t) + K\int_{t-D}^{t} e^{A(t-s)}Bu(s)ds. \tag{6}$$

For the particular case $D = 0$ the control law (6) is the same as (2).

Restrictions of the strategy (6) are the matrix exponential e^{AD} and the infinite-dimensional integral. One more strong assumption is required because for the control law (6) calculation plant parameters are necessary.

In the next section the adaptive controller is presented for the problem when all parameters of the plant are unknown.

4. ADAPTIVE CONTROLLER FOR PLANT WITH PARAMETRIC UNCERTAINTIES

In this section we present the controller design technique and one numerical example.

4.1 Control design

We consider the control problem for the plant (1) with matrices A, B and C that can have unknown parameters. Assume that state vector $x(t)$ is available for measuring and defined in the canonical observable basis. Let the unknowns present only in matrix A

$$A = \begin{bmatrix} \theta_1 & 1 & 0 & \cdots & 0 \\ \theta_2 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \theta_n & 0 & 0 & \cdots & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}, C^{\mathrm{T}} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

To design the predictor-based controller (6) it is necessary to know all plant parameters. Rewrite (1) as the system of differential equations.

$$\dot{x}_i(t) = x_{i+1}(t) + \theta_i\, x_1(t), \quad i = 1, \ldots, n-1, \tag{7}$$
$$\dot{x}_n(t) = u(t - D) + \theta_n\, x_1(t). \tag{8}$$

Equations (7), (8) include unmeasurable variables, therefore, these equations could not be used directly to design adaptation law for unknown parameters. Consider n linear first-order filters for the each state variable and one filter for the delayed control

$$\dot{\xi}_i(t) = -\lambda\, \xi_i(t) + \lambda\, x_i(t), \quad i = 1, \ldots, n, \tag{9}$$
$$\dot{\xi}_u(t) = -\lambda\, \xi_u(t) + \lambda\, u(t - D), \tag{10}$$

where $\lambda > 0$ is a positive number.

After direct and inverse Laplace transformation in (7) and (8) with respect to (9) and (10) we obtain the following system of equations:

$$\dot{\xi}_i(t) = \xi_{i+1}(t) + \theta_i\, \xi_1(t) + \varepsilon_i(t), \tag{11}$$
$$\dot{\xi}_n(t) = \xi_u(t) + \theta_n\, \xi_1(t) + \varepsilon_n(t), \tag{12}$$

where exponentially decaying functions of time ε tend to zero faster with increasing the coefficient λ.

From (11) and (12) it is possible to design the adaptive update law for unknown parameters estimates (Ioannou [1996])

$$\dot{\hat{\theta}}_i = k_i\xi_1\left(\dot{\xi}_i - \xi_{i+1} - \hat{\theta}_i\xi_1\right), k_i > 0, \tag{13}$$
$$\dot{\hat{\theta}}_n = k_n\xi_1\left(\dot{\xi}_n - \xi_u - \hat{\theta}_n\ \xi_1\right), k_n > 0. \tag{14}$$

It is not difficult to show that the adaptation algorithm (13), (14) provides convergence for estimates $\hat{\theta}_i$ to true values θ_i if the initial state is nonzero. If $\xi_1 \equiv 0$ then the algorithm (13), (14) does not work since the system is frozen at the equilibrium state.

Consider estimate errors for the each parameter $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$. After differentiation

$$\tilde{\theta}_i = \theta_i - \hat{\theta}_i$$

and substitution (13), (14), and then (11), (12) we have model of errors

$$\dot{\tilde{\theta}}_i = -k_i\xi_1^2\tilde{\theta}_i + k_i\xi_1\varepsilon_i, k_i > 0. \qquad (15)$$

From (15) it is straightforward to show that all estimation errors $\tilde{\theta}_i$ go to zero that guarantees that all estimates $\hat{\theta}_i$ converge to plant parameters θ_i.

Basing on $\hat{\theta}_i$ we can collect the state matrix estimate as

$$\hat{A}(t) = \begin{bmatrix} \hat{\theta}_1(t) & 1 & 0 & \cdots & 0 \\ \hat{\theta}_2(t) & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \hat{\theta}_n(t) & 0 & 0 & \cdots & 0 \end{bmatrix}.$$

The control law is formed using (6) as

$$u(t) = \hat{K}(t)\left[e^{\hat{A}(t)D}x(t) + \int_{t-D}^{t} e^{\hat{A}(t)(t-s)}Bu(s)ds\right], \qquad (16)$$

where the vector-row $\hat{K}(t)$ is such that matrix $\hat{A}(t) + B\hat{K}(t)$ is Hurwitz for all time instants t.

Since all estimates $\hat{\theta}_i$ tend to true values then the matrix $\hat{A}$ converges to the original matrix A. Parameters $\hat{K}$ are also go to some constant corresponding to the matrix A. Thus, we obtain the control law (9), (10), (13), (14), (16) for LTI plants with unknown parameters and the constant input delay.

4.2 A numerical example I

Consider the unstable plant (1) with following parameters $n = 3$, $\theta_1 = 1$, $\theta_2 = 0$, $\theta_3 = -1$, $D = 1$, $x^{\mathrm{T}}(0) = [\,1\ 0\ 1\,]$.

Take four filters (9), (10) with coefficients $\lambda = 10$. Then we design three update laws (13), (14) with parameters $k_1 = 10$, $k_2 = 10$, $k_3 = 10$.

To design the control law as (16) it is necessary to choose the algorithm to calculate the vector $\hat{K} = [\,\hat{K}_1\ \hat{K}_2\ \hat{K}_3\,]$.

Consider characteristic polynomial $Q(p)$ of the matrix $\hat{A} + B\hat{K}$

$$\begin{aligned} Q(p) &= \det\left[pI - (\hat{A} + B\hat{K})\right]^{-1} \\ &= \det\left(\begin{bmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{bmatrix} - \begin{bmatrix} \hat{\theta}_1 & 1 & 0 \\ \hat{\theta}_2 & 0 & 1 \\ \hat{\theta}_3 + \hat{K}_1 & \hat{K}_2 & \hat{K}_3 \end{bmatrix}\right) \\ &= p^3 + (-\hat{\theta}_1 - \hat{K}_3)p^2 + (-\hat{\theta}_2 - \hat{K}_2 + \hat{\theta}_1\hat{K}_3)p \\ &\quad + (-\hat{\theta}_3 - \hat{K}_1 + \hat{\theta}_1\hat{K}_2 + \hat{\theta}_2\hat{K}_3). \end{aligned}$$

To provide desired quality of transients one can take the etalon polynomial like

$$Q_*(p) = p^3 + 3\omega\, p^2 + 3\omega^2\, p + \omega^3,$$

where $\omega > 0$ is a positive number which defines transient time and also a sensitivity with respect to external disturbances. This parameter is set by a designer of a control system.

Controller's gain $\hat{K} = [\,\hat{K}_1\ \hat{K}_2\ \hat{K}_3\,]$ we get from an equality between $Q(p)$ and $Q_*(p)$

$$\hat{K}_3 = -3\omega - \hat{\theta}_1, \qquad (17)$$

$$\hat{K}_2 = -3\omega^2 - \hat{\theta}_2 + \hat{\theta}_1\hat{K}_3, \qquad (18)$$

$$\hat{K}_1 = -\omega^3 - \hat{\theta}_3 + \hat{\theta}_1\hat{K}_2 + \hat{\theta}_2\hat{K}_3 \qquad (19)$$

It is well-known that the definite integral in the control law (16) can be found as an area of the region bounded by a graph of an integrand.

In fig. 1 results of a numerical simulation of the closed-loop with $\omega = 3$. As one can see the adaptive controller successfully solves the given problem.

5. ADAPTIVE CONTROLLER WITH DISTURBANCE CANCELLATION FOR UNKNOWN PLANT WITH INPUT DELAY

5.1 Main result

In this section we present the main result, i.e. the adaptive controller for the plant with the input delay, parametric uncertainties and an unmeasurable disturbance.

We consider the following mathematical model of the plant

$$\dot{x}(t) = Ax(t) + Bu(t - D) + Bf, y(t) = Cx(t), \qquad (20)$$

where $f = const$ is an unknown external disturbance.

To derive the main result for the plant (20) we use the algorithm proposed in previous section.

Firstly, we consider the auxiliary filters, then we have to design the update law for unknown parameters and disturbance, and eventually we can form the control law.

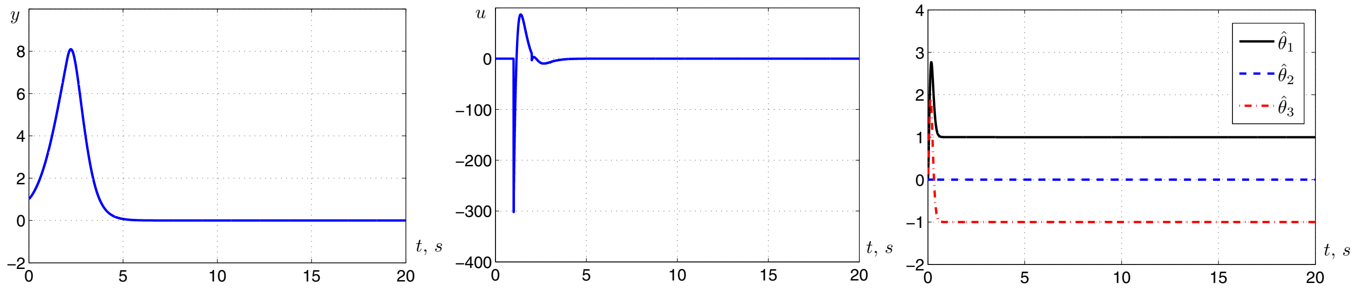
Let suppose that we already introduced filters (9), (10). Taking in account that the disturbance associated with variable $x_n(t)$, one can check that we can get all formulas (11), while the equation (12) should be modified as follows

$$\dot{\xi}_n(t) = \xi_u(t) + \theta_n\,\xi_1(t) + f + \varepsilon_n(t), \qquad (21)$$

where there is already used the fact that the reaction of the filter with a transfer function $\frac{\lambda}{s+\lambda}$ on a constant disturbance f is equal to the same value f, while transient process associated with the function $\varepsilon_n(t)$.

Hence, the algorithm to estimate parameters θ_1, ..., θ_{n-1} is given by (13). And the update laws for the last parameter θ_n and an unknown disturbance presented below.

After differentiation (21) and neglecting the exponential decayng term $\varepsilon_n(t)$ we have



(a) Output variable $y(t)$ (b) Control signal u (c) Parameters estimates $\hat{\theta}_1(t)$, $\hat{\theta}_2(t)$, $\hat{\theta}_3(t)$

Fig. 1. Transients for the closed-loop system

$$\ddot{\xi}_n(t) = \dot{\xi}_u(t) + \theta_n \, \dot{\xi}_1(t). \tag{22}$$

From (22) and (14) we get the identification algorithm for θ_n

$$\dot{\hat{\theta}}_n = k_n \dot{\xi}_1 \left(\ddot{\xi}_n - \dot{\xi}_u - \hat{\theta}_n \, \dot{\xi}_1 \right). \tag{23}$$

The algorithm (23) is not realizable in this way since the function $\ddot{\xi}_n$ is unmeasurable. Introduce an auxiliary variable

$$\chi = \hat{\theta}_n - k_n \dot{\xi}_1 \dot{\xi}_n. \tag{24}$$

Differentiation (24) yields

$$\begin{aligned} \dot{\chi} &= k_n \dot{\xi}_1 \left(\ddot{\xi}_n - \dot{\xi}_u - \hat{\theta}_n \, \dot{\xi}_1 \right) - k_n \ddot{\xi}_1 \dot{\xi}_n - k_n \dot{\xi}_1 \ddot{\xi}_n \\ &= -k_n \dot{\xi}_1 \dot{\xi}_u - k_n \hat{\theta}_n \, \dot{\xi}_1^2 - k_n \ddot{\xi}_1 \dot{\xi}_n. \end{aligned} \tag{25}$$

From (9) we obtain the expression to calculate $\ddot{\xi}_1$

$$\begin{aligned} \ddot{\xi}_1(t) &= -\lambda \, \dot{\xi}_1(t) + \lambda \, \dot{x}_1(t) \\ &= -\lambda \, \dot{\xi}_1(t) + \lambda \, x_2(t) + \theta_1 x_1. \end{aligned} \tag{26}$$

From (26) we get the update law for the variable χ

$$\begin{aligned} \dot{\chi} = &-k_n \dot{\xi}_1 \dot{\xi}_u - k_n \hat{\theta}_n \, \dot{\xi}_1^2 \\ &- k_n \dot{\xi}_n \left(-\lambda \, \dot{\xi}_1(t) + \lambda \, x_2(t) + \hat{\theta}_1 x_1 \right). \end{aligned} \tag{27}$$

The realizable algorithm for estimation the unknown parameter θ_n follows from (24)

$$\hat{\theta}_n = \chi + k_n \dot{\xi}_1 \dot{\xi}_n. \tag{28}$$

Basing on the update law for the parameterθ_n one can design the observer for the external disturbance in the following way

$$\hat{f}(t) = \dot{\xi}_n(t) - \xi_u(t) - \hat{\theta}_n \, \xi_1(t). \tag{29}$$

The control law that provides the disturbance rejection looks like

$$u(t) = u_0(t) - \hat{f}(t), \tag{30}$$

where u_0 is the first control loop that is designed to stabilize the plant. The function u_0 is calculated in the same form as (16) by the following formula

$$\begin{aligned} u_0(t) = &\hat{K}(t) e^{\hat{A}(t)D} x(t) \\ &+ \hat{K}(t) \int_{t-D}^{t} e^{\hat{A}(t)(t-s)} B u_0(s) ds. \end{aligned} \tag{31}$$

Since the all parameters are estimated with errors that tend to zero asymptotically we conclude that the disturbance observer provides precise estimate of the signal f

$$\lim_{t \to \infty} \left(f - \hat{f}(t) \right) = 0. \tag{32}$$

It is straightforward to show that the adaptive controller (30) allows to cancel the disturbance completely in the closed-loop system and choosing controller's gain $\hat{K}(t)$ provides stabilization the equilibrium $x = 0$ for the plant (1).

5.2 A numerical example II

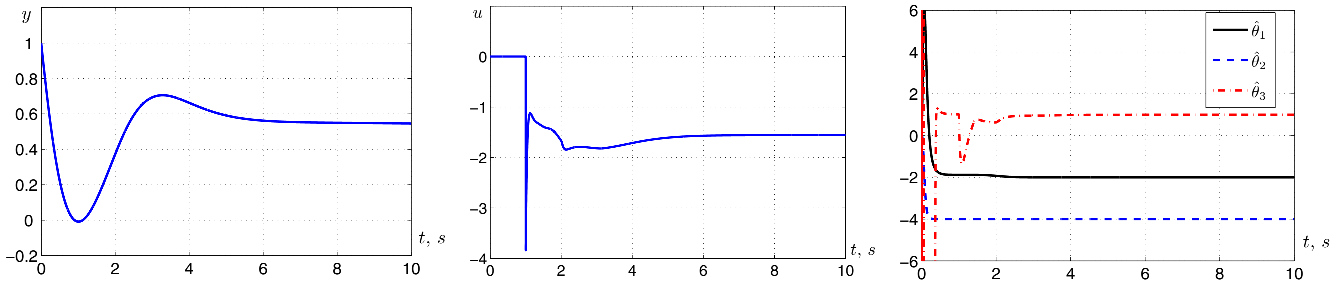
Consider the unstable plant (20) with the following parameters: $n = 3$, $\theta_1 = -2$, $\theta_2 = -4$, $\theta_3 = 1$, $D = 1$, $f = 1$, $x^{\mathrm{T}}(0) = [\,1\;\;0\;\;1\,]$.

Take four filters (9), (10) with coefficient $\lambda = 15$. Design two update laws of the view (13) with gains $k_1 = 50$, $k_2 = 50$. The parameter θ_3 is estimated by the algorithm (27), (28) with the gain $k_3 = 50$.

The disturbance observer is given by (29). Eventually, the control law is defined by (30), where gain $\hat{K}(t)$ is chosen from (17)-(19) with coefficient $\omega = 2$.

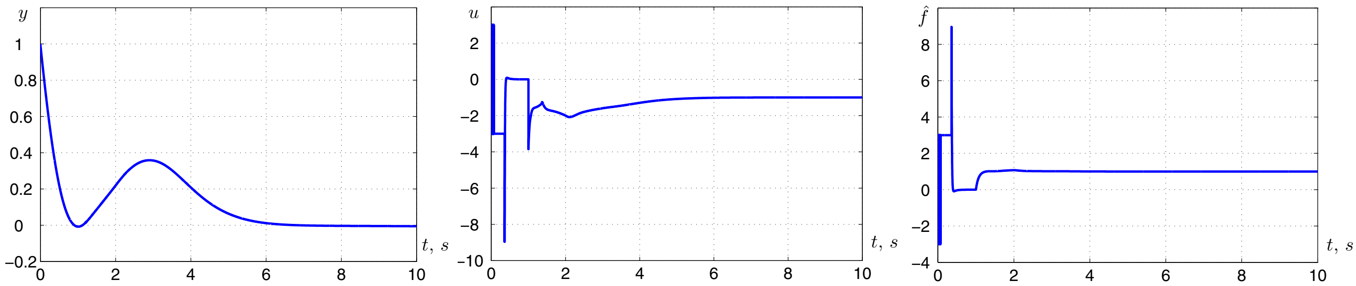
In fig. 2 the simulation results for the closed-loop without disturbance cancellation scheme are presented. In this example the stabilization loop works only. From fig. 2 one can see that unstable plant is stabilized but the output doesn't converge to zero due to the disturbance.

In fig. 3 the simulation results for the closed-loop system with disturbance compensation scheme are shown. In fig. 3 one can see that algorithm (30) provides stabilization of the plant and the disturbance is cancelled such hat the output variable asymptotically goes to zero.



(a) Output variable $y(t)$ (b) Control signal u (c) Parameters estimates $\hat{\theta}_1(t)$, $\hat{\theta}_2(t)$, $\hat{\theta}_3(t)$

Fig. 2. Transients for the closed-loop system: stabilization only



(a) Output variable $y(t)$ (b) Control signal u (c) Disturbance estimate $\hat{f}(t)$

Fig. 3. Transients for the closed-loop system with disturbance cancellation

6. CONCLUSION

In the paper the adaptive state-feedback controller based on Smith predictor (Smith [1959]) and results Krstic [2009] is proposed. The controller (30), (31) provides rejection of the constant disturbance f and stabilization the unstable plant (1) with unknown parameters in the state matrix. The possible extension of presented approach is the case of output-feedback controller for the same problem and also more complicated forms of the external disturbance.

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