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Robust Attitude Control with Improved Transient Performance ⋆

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Abstract: This paper aims to present a robust attitude control strategy with guaranteed transient performance. Firstly, a Lyapunov-based control law is designed to achieve high-performance attitude control in the absence of disturbance and parameter variation. The proposed control law uses small feedback gains to suppress the control torque at large attitude error, and increases those gains with the convergence of attitude error to accelerate the system response. The overshooting phenomenon is also avoided by imposing a restriction on the parameter selection. Then, the integral sliding mode control technique is employed to improve the robustness, where the Lyapunov-based control law is used as the equivalent control part. Theoretical analysis and simulation results verify the effectiveness of the proposed strategy.

1. INTRODUCTION

Controlling the rotational motion of rigid spacecraft is a challenging issue. The difficulty lies in the highly nonlinear and coupled governing equations, as well as the undesired torque caused by disturbance and parametric uncertainty (Schaub and Junkins [2009]). Therefore, for achieving desired control performance, nonlinear control techniques with strong robustness should be utilized.

Since the first systematic study in (Meyer [1971]), Lyapunov-based control technique has been extensively investigated in the attitude control literature (Wie et al. [1989],Wen and Kreutz-Delgado [1991],Suk et al. [2001],Schlanbusch et al. [2010]). By finding some energy-like Lyapunov functions, the associated attitude controllers are constructed by two parts, the attitude variable feedback terms and the nonlinearity compensation terms (Wie et al. [1989],Wen and Kreutz-Delgado [1991]). The closed-loop dynamics can be approximated using a simple damped harmonic oscillator model, which makes the controller very convenient to validate, tune and implement. Nonetheless, only a boundedness conclusion can be obtained in the presence of disturbance and parametric uncertainty (Schlanbusch et al. [2010]). As a result, the control accuracy is unacceptable for space missions such as rendezvous and docking, where a highly accurate pointing or slewing is required. Moreover, there is a tradeoff between accelerating system response and suppressing the peak control torque, which will degrade the control performance if unsuitable parameters are selected.

In order to address those shortcomings, various strategies have been adopted. On the one hand, the robustness issue has been considered in many research works. In (Lizarralde and Wen [1996]) and (Tsiotras [1998]), the inertia matrix is not required in the attitude controller design by exploiting the passivity properties of the attitude control system. Hence, the control precision will not be affected by the uncertain inertia matrix and the robustness is therefore enhanced. However, such a conclusion only holds for the case of attitude reorientation. With respect to attitude tracking, an exact knowledge of inertia matrix is still required. In (Akella [2001]), a simple adaptive law was designed to estimate the slow varying inertia matrix. However, the disturbance torque is not taken into account. Integrating with disturbance observer is another effective approach of improving the robustness of Lyapunov-based attitude controller (Yamashita et al. [2004],Sun and Li [2013],Sun and Li [2011]). Nonetheless, the control accuracy depends directly on the disturbance observer and a rigorous stability analysis under the composite controller is generally absent due to the challenging separation principle issue.

On the other hand, in order to ensure high performance, the backstepping method has been applied to attitude control (Krstić and Tsiotras [1999],Kim and Kim [2003],I.Ali et al. [2010]). Compared with conventional Lyapunov-based control, in backstepping, required specifications can be considered during the design procedure, instead of a careful parameter tuning after the controller design. In (Krstić and Tsiotras [1999]), an inverse optimal attitude control law, which is optimal with respect to a meaningful cost function, was proposed. By virtue of the backstepping design, the task of solving Hamilton-Jacobi equation has been avoided. Aiming to address the tradeoff problem between excessive control torque and the sluggish motion, a nonlinear virtual control law (also termed as tracking function) was employed in (Kim and Kim [2003]). Similar strategy was developed in (I.Ali et al. [2010]) to handle the input saturation problem. As is well known, in the backstepping based attitude controller design, desired system response is characterized by the virtual control and is realized by the tracking of virtual control output by the actual control input. Nonetheless, such a tracking can only be achieved asymptotically or in finite time. In

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other words, the expected performance cannot be globally realized throughout the control action.

In this paper, the attitude control problem of rigid spacecraft is firstly addressed in the absence of disturbance and parameter variation. The related object is improving the transient performance, e.g., accelerating the attitude tracking evolution while avoiding excessive control torque. Unlike the backstepping strategy using a nonlinear virtual control law in (Kim and Kim [2003]), a simple Lyapunov-based attitude control law with state-dependent feedback gains is presented. By restricting the damping ratio at the equilibrium point, the overshooting phenomenon can be avoided. Furthermore, with respect to the robustness issue, the integral sliding mode (ISM) control technique is utilized to redesign the Lyapunov-based control law. As a result, a robust attitude controller with improved transient performance is developed. The effectiveness of the proposed strategy will be verified by theoretical analysis and numerical simulation.

2. MATHEMATICAL MODEL AND PROBLEM STATEMENT

Consider a thruster-controlled rigid spacecraft, whose governing equations are described by

$$\hat{\mathcal{J}}\dot{\boldsymbol{\omega}}_b + \boldsymbol{\omega}_b^\times \hat{\mathcal{J}}\boldsymbol{\omega}_b = \boldsymbol{T}_c + \boldsymbol{T}_d + \boldsymbol{T}_p \tag{1}$$

$$\dot{\boldsymbol{\sigma}}_b = \boldsymbol{M}(\boldsymbol{\sigma}_b)\boldsymbol{\omega}_b \tag{2}$$

where $\hat{\mathcal{J}} = \mathrm{diag}(J_1, J_2, J_3)$ is the nominal part of the inertia matrix $\mathcal{J} \in \mathbb{R}^{3\times 3}$, and $\boldsymbol{\omega}_b \in \mathbb{R}^3$ denotes the inertial angular velocity. The superscript $(\cdot)^\times$ is the skew-symmetric matrix operator on any 3×1 vector $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \alpha_3]^{\mathrm{T}}$ such that

$$\boldsymbol{\alpha}^\times = \begin{bmatrix} 0 & -\alpha_3 & \alpha_2 \\ \alpha_3 & 0 & -\alpha_1 \\ -\alpha_2 & \alpha_1 & 0 \end{bmatrix}$$

$\boldsymbol{T}_c \in \mathbb{R}^3$ is the control torque provided by the reaction control thrusters. $\boldsymbol{T}_d \in \mathbb{R}^3$ stands for the disturbance torque, including the environmental and non-environmental torques. $\boldsymbol{T}_p \in \mathbb{R}^3$ is the torque induced by the parametric uncertainty. Let $\Delta\mathcal{J} = (\mathcal{J} - \hat{\mathcal{J}}) \in \mathbb{R}^{3\times 3}$ denote the inertia matrix uncertainty, then $\boldsymbol{T}_p = -\Delta\mathcal{J}\dot{\boldsymbol{\omega}}_b - \boldsymbol{\omega}_b^\times \Delta\mathcal{J}\boldsymbol{\omega}_b$. $\boldsymbol{\sigma}_b \in \mathbb{R}^3$ denotes the Modified Rodriguez Parameters (MRP) representation for the inertial attitude of the spacecraft. $\boldsymbol{M}(\cdot) : \mathbb{R}^3 \to \mathbb{R}^{3\times 3}$ is the Jacobian matrix operator such that

$$\boldsymbol{M}(\boldsymbol{\sigma}_b) = \frac{(1 - \|\boldsymbol{\sigma}_b\|^2)\mathbf{I}_3 + 2\boldsymbol{\sigma}_b^\times + 2\boldsymbol{\sigma}_b\boldsymbol{\sigma}_b^{\mathrm{T}}}{4} \tag{3}$$

where $\mathbf{I}_3$ is the 3×3 identity matrix and $\|\cdot\|$ is the vector 2-norm. Moreover, $\boldsymbol{M}^{-1}(\boldsymbol{\sigma}_b) = \boldsymbol{M}^{\mathrm{T}}(\boldsymbol{\sigma}_b)/m(\boldsymbol{\sigma}_b)$ with $m(\boldsymbol{\sigma}_b) = (1 + \|\boldsymbol{\sigma}_b\|^2)^2/16$.

Let $\boldsymbol{\sigma}_d, \boldsymbol{\omega}_d \in \mathbb{R}^3$ denote the desired attitude variables, which also satisfy the attitude kinematics in (2), i.e., $\dot{\boldsymbol{\sigma}}_d = \boldsymbol{M}(\boldsymbol{\sigma}_d)\boldsymbol{\omega}_d$. It is assumed that $\boldsymbol{\sigma}_d$ and $\boldsymbol{\omega}_d$ together with $\dot{\boldsymbol{\omega}}_d$ are all bounded. Subsequently, the attitude error variables are defined as

$$\boldsymbol{\sigma}_e = \boldsymbol{\sigma}_b \oplus \boldsymbol{\sigma}_d^* \tag{4}$$

$$\boldsymbol{\omega}_e = \boldsymbol{\omega}_b - \boldsymbol{R}(\boldsymbol{\sigma}_e)\boldsymbol{\omega}_d \tag{5}$$

where $\boldsymbol{\sigma}_e, \boldsymbol{\omega}_e \in \mathbb{R}^3$ represent the MRP error and the angular velocity error. $\oplus$ is the MRP addition operator, characterizing the successive rotations. For two MRPs, e.g., $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$, it is calculated as follows:

$$\boldsymbol{\sigma}_1 \oplus \boldsymbol{\sigma}_2 = \frac{(1 - \|\boldsymbol{\sigma}_2\|^2)\boldsymbol{\sigma}_1 + (1 - \|\boldsymbol{\sigma}_1\|^2)\boldsymbol{\sigma}_2 - 2\boldsymbol{\sigma}_1^\times\boldsymbol{\sigma}_2}{1 + \|\boldsymbol{\sigma}_1\|^2\|\boldsymbol{\sigma}_2\|^2 - 2\boldsymbol{\sigma}_1^{\mathrm{T}}\boldsymbol{\sigma}_2}$$

The superscript $(\cdot)^*$ denotes the complex conjugate of MRP and $\boldsymbol{\sigma}_d^* = -\boldsymbol{\sigma}_d$. $\boldsymbol{R}(\cdot) : \mathbb{R}^3 \to \mathbb{R}^{3\times 3}$ is the rotation matrix operator. For $\boldsymbol{\sigma}_e$, one has

$$\boldsymbol{R}(\boldsymbol{\sigma}_e) = \mathbf{I}_3 + \frac{8\boldsymbol{\sigma}_e^\times\boldsymbol{\sigma}_e^\times - 4(1 - \|\boldsymbol{\sigma}_e\|^2)\boldsymbol{\sigma}_e^\times}{(1 + \|\boldsymbol{\sigma}_e\|^2)^2}$$

By substituting (4) and (5) into (1) and (2), the governing equations in terms of $\boldsymbol{\omega}_e$ and $\boldsymbol{\sigma}_e$ can be described as

$$\begin{aligned}\hat{\mathcal{J}}\dot{\boldsymbol{\omega}}_e =& \hat{\mathcal{J}}\left(\boldsymbol{\omega}_e^\times \boldsymbol{R}\boldsymbol{\omega}_d - \boldsymbol{R}\dot{\boldsymbol{\omega}}_d\right) - \boldsymbol{\omega}_e^\times \hat{\mathcal{J}}\left(\boldsymbol{\omega}_e + \boldsymbol{R}\boldsymbol{\omega}_d\right) \\ &- (\boldsymbol{R}\boldsymbol{\omega}_d)^\times \hat{\mathcal{J}}\left(\boldsymbol{\omega}_e + \boldsymbol{R}\boldsymbol{\omega}_d\right) + \boldsymbol{T}_c + \boldsymbol{T}_d + \boldsymbol{T}_p\end{aligned} \tag{6}$$

$$\dot{\boldsymbol{\sigma}}_e = \boldsymbol{M}\boldsymbol{\omega}_e \tag{7}$$

where the related arguments in $\boldsymbol{M}(\boldsymbol{\sigma}_e)$ and $\boldsymbol{R}(\boldsymbol{\sigma}_e)$ are ignored for clarity.

From a practical point of view, the disturbance torque and the inertia matrix uncertainty are both bounded. Following the same line of (Huang et al. [2008]), it is reasonable to assume that $\|\boldsymbol{T}_d + \boldsymbol{T}_p\|_\infty \le c_0 + c_1\|\boldsymbol{\sigma}_e\|_\infty + c_2\|\boldsymbol{\omega}_e\|_\infty$, where c_i $(i = 1, 2, 3)$ are known positive constants and $\|\cdot\|_\infty$ is the vector infinity norm. Thus, the control object can be summarized as follows. Find an attitude controller such that 1) $\boldsymbol{\sigma}_e$ and $\boldsymbol{\omega}_e$ can be globally stabilized in the presence of bounded disturbance and inertia matrix uncertainty; 2) transient performance of the closed-loop system is guaranteed.

3. MAIN RESULTS

In this paper, the above-mentioned control object is realized by two steps. Firstly, high-performance attitude control in the absence of disturbance and inertia matrix uncertainty is guaranteed by an enhanced Lyapunov-based control law. Then, the proposed control law is redesigned by the ISM control technique to ensure the robustness. Before moving on, current Lyapunov-based control scheme is briefly reviewed.

3.1 Current Lyapunov-based control

The basic idea of Lyapunov-based control is to design a feedback control law that renders the derivative of a specified Lyapunov function negative definite or negative semi-definite. To this end, consider the following energy-like Lyapunov function

$$V = \frac{1}{2}\boldsymbol{\omega}_e^{\mathrm{T}}\boldsymbol{\omega}_e + 2k_p \ln(1 + \boldsymbol{\sigma}_e^{\mathrm{T}}\boldsymbol{\sigma}_e) \tag{8}$$

where $k_p > 0$ is a constant scalar.

With respect to the nominal attitude control system, i.e., assuming $\boldsymbol{T}_d = \boldsymbol{T}_p = \mathbf{0}$, taking the derivative of (8) gives

$$\begin{aligned}\dot{V} &= \boldsymbol{\omega}_e^{\mathrm{T}}\hat{\mathcal{J}}^{-1}\hat{\mathcal{J}}\dot{\boldsymbol{\omega}}_e + 4k_p\frac{\boldsymbol{\sigma}_e^{\mathrm{T}}\dot{\boldsymbol{\sigma}}_e}{1 + \boldsymbol{\sigma}_e^{\mathrm{T}}\boldsymbol{\sigma}_e} \\ &= \boldsymbol{\omega}_e^{\mathrm{T}}\hat{\mathcal{J}}^{-1}\left[\hat{\mathcal{J}}\left(\boldsymbol{\omega}_e^\times \boldsymbol{R}\boldsymbol{\omega}_d - \boldsymbol{R}\dot{\boldsymbol{\omega}}_d\right) - \boldsymbol{\omega}_e^\times \hat{\mathcal{J}}\left(\boldsymbol{\omega}_e + \boldsymbol{R}\boldsymbol{\omega}_d\right)\right] \\ &\quad + \boldsymbol{\omega}_e^{\mathrm{T}}\hat{\mathcal{J}}^{-1}\left[\boldsymbol{T}_c - (\boldsymbol{R}\boldsymbol{\omega}_d)^\times \hat{\mathcal{J}}\left(\boldsymbol{\omega}_e + \boldsymbol{R}\boldsymbol{\omega}_d\right)\right] + k_p\boldsymbol{\sigma}_e^{\mathrm{T}}\boldsymbol{\omega}_e\end{aligned}$$

As reported in (Wie et al. [1989]) and (Wen and Kreutz-Delgado [1991]), the corresponding feedback control law can be designed as

$$\begin{aligned} \boldsymbol{T}_c =& \boldsymbol{\omega}_e^{\times} \hat{\boldsymbol{\mathcal{J}}} \left(\boldsymbol{\omega}_e + \boldsymbol{R}\boldsymbol{\omega}_d\right) - \hat{\boldsymbol{\mathcal{J}}} \left(\boldsymbol{\omega}_e^{\times} \boldsymbol{R}\boldsymbol{\omega}_d - \boldsymbol{R}\dot{\boldsymbol{\omega}}_d\right) \\ &+ \left(\boldsymbol{R}\boldsymbol{\omega}_d\right)^{\times} \hat{\boldsymbol{\mathcal{J}}} \left(\boldsymbol{\omega}_e + \boldsymbol{R}\boldsymbol{\omega}_d\right) - k_p \hat{\boldsymbol{\mathcal{J}}} \boldsymbol{\sigma}_e - k_d \hat{\boldsymbol{\mathcal{J}}} \boldsymbol{\omega}_e \end{aligned} \tag{9}$$

where $k_d > 0$ and $k_p > 0$ are constant feedback gains.

By substituting the attitude control law (9) into the derivative of the Lyapunov function, one has

$$\dot{V} = -k_d \boldsymbol{\omega}_e^{\mathrm{T}} \boldsymbol{\omega}_e$$

which implies that the Lyapunov function (8) possesses a negative semi-definite time derivative under the attitude control law (9).

Then, based on Matrosov theorem, one can conclude that the closed-loop system is globally uniformly asymptotically stable. If the disturbance and parametric uncertainty are taken into account, a practical asymptotic stability, the global uniform ultimate boundedness, can be obtained as shown in (Schlanbusch et al. [2010]). The practical stability is suitable for the case where the requirement of high-precision attitude control is not strict. When involving orbit transfer or other pointing missions, such a bounded control precision is not acceptable. In (Schlanbusch et al. [2010]), it has been shown that improving the control precision of (9) needs to increase the feedback gains. However, excessive control torque will be induced for large attitude error.

On the other hand, the parameter turning problem of the attitude controller (9) has also been considered in (Wie et al. [1989]). Based on small angle approximation and the characteristics of eigenaxis rotation, a connection is established between the feedback gains and the damping ratio/natural frequency. Generally speaking, the damping ratio determines the system behavior, e.g., overdamped, underdamped, or critically damped. Once the damping ratio is selected, the natural frequency should be carefully tuned to find balance between decreasing settling time and suppressing peak control torque. Such a tradeoff problem can also be found in the backstepping design. As discussed in (Kim and Kim [2003]), if the feedback gains are poorly tuned, the sluggish motion or excessive control input would present.

3.2 Enhanced Lyapunov-based control

In this subsection, an enhanced Lyapunov-based control law is presented, which can address the tradeoff problem in the feedback gain selection and consequently improve the transient response. Similar to the previous study, the attitude controller is designed for the nominal attitude control system and the robustness issue will be handled in the next subsection.

From (9), one can see that the tradeoff problem is actually induced by the fixed-value feedback gains. For example, if larger k_p and k_d are selected, the required peak control torque becomes larger but the settling time becomes shorter, and vice versa. Thus, to address such a problem, it is a natural choice to utilize self-tuning feedback gains instead of the fixed ones. Consider the following attitude control law:

$$\begin{aligned} \boldsymbol{T}_c = \boldsymbol{\omega}_e^{\times} \hat{\boldsymbol{\mathcal{J}}} \left(\boldsymbol{\omega}_e + \boldsymbol{R}\boldsymbol{\omega}_d\right) - \hat{\boldsymbol{\mathcal{J}}} \left(\boldsymbol{\omega}_e^{\times} \boldsymbol{R}\boldsymbol{\omega}_d - \boldsymbol{R}\dot{\boldsymbol{\omega}}_d\right) \\ + \left(\boldsymbol{R}\boldsymbol{\omega}_d\right)^{\times} \hat{\boldsymbol{\mathcal{J}}} \left(\boldsymbol{\omega}_e + \boldsymbol{R}\boldsymbol{\omega}_d\right) - k_p^* \hat{\boldsymbol{\mathcal{J}}} \boldsymbol{\sigma}_e - k_d^* \hat{\boldsymbol{\mathcal{J}}} \boldsymbol{\omega}_e \end{aligned} \tag{10}$$

where

$$\begin{aligned} k_p^* =& k_\sigma \mathrm{e}^{-k_1 \|\boldsymbol{\sigma}_e\|^2} (1 + \|\boldsymbol{\sigma}_e\|^2) \\ k_d^* =& k_\omega \mathrm{e}^{k_2 \|\boldsymbol{\sigma}_e\|^2} \end{aligned} \tag{11}$$

with $k_1 \geq 1$, $k_2 < -0.5k_1$, $k_\sigma > 0$, and $k_\omega = \sqrt{k_\sigma} > 0$.

To begin with, the associated stability analysis is shown in the following theorem.

Theorem 1. For the nominal attitude control system described in (6) and (7) with $\boldsymbol{T}_d = \boldsymbol{T}_p = \boldsymbol{0}$, by applying the attitude control law (10), the closed-loop system is globally uniformly asymptotically stable.

Proof 1. By substituting the attitude control law (10) into the nominal attitude control system, the closed-loop system is characterized by

$$\begin{aligned} \hat{\boldsymbol{\mathcal{J}}} \dot{\boldsymbol{\omega}}_e &= -k_p^* \hat{\boldsymbol{\mathcal{J}}} \boldsymbol{\sigma}_e - k_d^* \hat{\boldsymbol{\mathcal{J}}} \boldsymbol{\omega}_e \\ \dot{\boldsymbol{\sigma}}_e &= \boldsymbol{M} \boldsymbol{\omega}_e \end{aligned}$$

Let $\dot{\boldsymbol{\sigma}}_e = \dot{\boldsymbol{\omega}}_e = \boldsymbol{0}$. Then, one can conclude that the unique equilibrium point is $(\boldsymbol{\sigma}_e, \boldsymbol{\omega}_e) = (\boldsymbol{0}, \boldsymbol{0})$, where the fact that the Jacobian matrix $\boldsymbol{M}$ is invertible is used.

Consider the following Lyapunov function

$$V = \frac{1}{2} \boldsymbol{\omega}_e^{\mathrm{T}} \boldsymbol{\omega}_e + \frac{2k_\sigma}{k_1} \left(1 - \mathrm{e}^{-k_1 \boldsymbol{\sigma}_e^{\mathrm{T}} \boldsymbol{\sigma}_e}\right) \tag{12}$$

Taking the time derivative of the above Lyapunov function along the trajectory of the closed-loop system yields

$$\begin{aligned} \dot{V} =& \boldsymbol{\omega}_e^{\mathrm{T}} \hat{\boldsymbol{\mathcal{J}}}^{-1} \hat{\boldsymbol{\mathcal{J}}} \dot{\boldsymbol{\omega}}_e - \frac{2k_\sigma}{k_1} \mathrm{e}^{-k_1 \boldsymbol{\sigma}_e^{\mathrm{T}} \boldsymbol{\sigma}_e} \left(-2k_1 \boldsymbol{\sigma}_e^{\mathrm{T}} \dot{\boldsymbol{\sigma}}_e\right) \\ =& -k_d^* \boldsymbol{\omega}_e^{\mathrm{T}} \boldsymbol{\omega}_e \leq 0 \end{aligned}$$

As the time derivative is negative semi-definite, one can conclude that $\boldsymbol{\sigma}_e$ and $\boldsymbol{\omega}_e$ are uniformly bounded. Then, $W = \boldsymbol{\sigma}_e^{\mathrm{T}} \boldsymbol{\omega}_e$ is also bounded, whose time derivative along the closed-loop trajectory is given by

$$\dot{W} = \boldsymbol{\omega}_e^{\mathrm{T}} \boldsymbol{M}^{\mathrm{T}} \boldsymbol{\omega}_e - k_p^* \boldsymbol{\sigma}_e^{\mathrm{T}} \boldsymbol{\sigma}_e - k_d^* \boldsymbol{\sigma}_e^{\mathrm{T}} \boldsymbol{\omega}_e$$

Let $\boldsymbol{\Omega} = \{(\boldsymbol{\sigma}_e, \boldsymbol{\omega}_e) : \dot{V} = 0\} = \{(\boldsymbol{\sigma}_e, \boldsymbol{\omega}_e) : \boldsymbol{\omega}_e = \boldsymbol{0}\}$. When $(\boldsymbol{\sigma}_e, \boldsymbol{\omega}_e) \in \boldsymbol{\Omega}$, the time derivative of W becomes

$$\dot{W} = -k_p^* \boldsymbol{\sigma}_e^{\mathrm{T}} \boldsymbol{\sigma}_e$$

which means that $\dot{W}$ is non-zero definite on $\boldsymbol{\Omega}$.

Then, based on Matrosov theorem, one can conclude that the closed-loop system is globally uniformly asymptotically stable.

In the following, the transient performance of the attitude control law (10) is evaluated. Inspired by the work in (Wie et al. [1989]), the feedback gains, k_p^* and k_d^*, will be firstly related to the response characteristics of damping ratio and natural frequency. To this end, consider the closed-loop attitude dynamics, which is described by

$$\hat{\boldsymbol{\mathcal{J}}} \dot{\boldsymbol{\omega}}_e + k_d^* \hat{\boldsymbol{\mathcal{J}}} \boldsymbol{\omega}_e + k_p^* \hat{\boldsymbol{\mathcal{J}}} \boldsymbol{\sigma}_e = \boldsymbol{0} \tag{13}$$

Before moving on, following lemma is introduced.

Lemma 1. If the initial attitude variables are collinear, i.e., $\boldsymbol{\sigma}_e(t_0) \times \boldsymbol{\omega}_e(t_0) = 0$, an eigenaxis rotation will be performed by the attitude control law (10), i.e., $\boldsymbol{\sigma}_e(t) \times \boldsymbol{\omega}_e(t) = 0$ for $t \in [t_0, +\infty]$, where $t_0 \geq 0$ is the initial time.

The proof is similar to that in (Wie et al. [1989]) and is omitted here.

When the attitude maneuver is performed as an eigenaxis rotation, following relationships can be obtained

$$\boldsymbol{\omega}_e(t) = \dot{\theta}(t)\boldsymbol{e}, \quad \dot{\boldsymbol{\omega}}_e(t) = \ddot{\theta}(t)\boldsymbol{e}, \quad \boldsymbol{\sigma}_e(t) = \tan(\theta(t)/4)\boldsymbol{e}$$

with $\theta(t)$ being the Euler principal angle and $\boldsymbol{e} \in \mathbb{R}^3$ denoting the unit eigenaxis vector (Schaub and Junkins [2009]).

Then, (13) can be rewritten as

$$\ddot{\theta}(t) + k_d^*\dot{\theta}(t) + k_p^* \tan\frac{\theta(t)}{4} = 0 \tag{14}$$

Using the small angle approximation, one has

$$\ddot{\theta}(t) + k_d^*\dot{\theta}(t) + \frac{k_p^*}{4}\theta(t) = 0 \tag{15}$$

which implies that the closed-loop dynamics can be approximated by a simple damped harmonic oscillator.

Let ξ and ω_n respectively denote the damping ratio and the natural frequency of the closed-loop system, i.e., let $k_p^* = 4\omega_n^2$ and $k_d^* = 2\xi\omega_n$. Then, one has $\xi = (k_d^*)^2/k_p^*$ and $\omega_n = \sqrt{k_p^*}/2$. For clarity, let $x = \|\boldsymbol{\sigma}_e\|^2$ in the following discussion. Taking the derivative of ξ with respect to x gives

$$\frac{\mathrm{d}\xi}{\mathrm{d}x} = \mathrm{e}^{(2k_2+k_1)x}\frac{(2k_2+k_1)(1+x)-1}{(1+x)^2} \tag{16}$$

Because $k_2 < -0.5k_1$ and $x \geq 0$, one has $\mathrm{d}\xi/\mathrm{d}x < 0$, which means ξ is monotonically decreasing on $x \in [0, +\infty)$. Further, let $\underline{\xi}$ and $\bar{\xi}$ stand for the damping ratio at the initial time and the equilibrium point. According to (11), one has

$$\begin{aligned} \underline{\xi}|_{t=t_0} =& \frac{(k_d^*)^2}{k_p^*} = \frac{k_\omega^2 \mathrm{e}^{2k_2\|\boldsymbol{\sigma}_e(t_0)\|^2}}{k_\sigma \mathrm{e}^{-k_1\|\boldsymbol{\sigma}_e(t_0)\|^2}(1+\|\boldsymbol{\sigma}_e(t_0)\|^2)} \\ =& \frac{\mathrm{e}^{(2k_2+k_1)\|\boldsymbol{\sigma}_e(t_0)\|^2}}{1+\|\boldsymbol{\sigma}_e(t_0)\|^2} \leq 1 \\ \bar{\xi}|_{t\to+\infty} =& \frac{(k_d^*)^2}{k_p^*} = \frac{k_\omega^2}{k_\sigma} = 1 \end{aligned} \tag{17}$$

Recalling the monotonicity of ξ, one can conclude that the behavior of closed-loop dynamics varies from the initial underdamped system (for $\boldsymbol{\sigma}_e(t_0) \neq \boldsymbol{0}$) to the final critically damped system. It is common knowledge that the underdamped system is characterized by fast system response with severe oscillations, and the critically damped system presents no overshooting but has a slower response. Hence, the immediate advantage of such a damping ratio varying scheme lies in that fast system response can be achieved without overshooting.

On the other hand, based on the monotonicity judgment of k_p^*, one can easily find that the natural frequency ω_n is also monotonically decreasing on $x \in [0, +\infty)$. Therefore, a small ω_n can be initially utilized to suppress the control torque amplitude at large attitude error. With the convergence of the MRP error, the natural frequency, as well as the damping ratio mentioned above, will get larger. As the settling time is inversely proportional to the product of damping ratio and natural frequency, it is obvious that the attitude control law (10) can speed up the system response without inducing excessive control torque.

3.3 ISM redesign

Based on the preceding analysis, one can conclude that the attitude control performance can be significantly improved by the proposed attitude control law. However, the attitude controller is designed for the nominal attitude control system, where the undesired torques caused by disturbance and parameter variation are not taken into account. In order to preserve the desired dynamic response and guarantee the robustness in the presence of disturbance and inertia matrix uncertainty, the attitude control law (10) is redesigned via ISM control technique.

To begin with, define the integral sliding function as

$$\boldsymbol{s}_\mathrm{I} = \boldsymbol{\omega}_e + \boldsymbol{z} \tag{18}$$

where $\dot{\boldsymbol{z}} = k_p^*\boldsymbol{\sigma}_e + k_d^*\boldsymbol{\omega}_e$ with $\boldsymbol{z}(t_0) = -\boldsymbol{\omega}_e(t_0)$

To make the related sliding surface (i.e., $\boldsymbol{s}_\mathrm{I} = \boldsymbol{0}$) attractive, the ISM attitude control law is designed as

$$\begin{aligned} \boldsymbol{T}_c =& \boldsymbol{T}_{eq} + \boldsymbol{T}_{sw} \\ =& \boldsymbol{\omega}_e^\times \hat{\boldsymbol{\mathcal{J}}}\left(\boldsymbol{\omega}_e + \boldsymbol{R}\boldsymbol{\omega}_d\right) - \hat{\boldsymbol{\mathcal{J}}}\left(\boldsymbol{\omega}_e^\times \boldsymbol{R}\boldsymbol{\omega}_d - \boldsymbol{R}\dot{\boldsymbol{\omega}}_d\right) \\ &+ (\boldsymbol{R}\boldsymbol{\omega}_d)^\times \hat{\boldsymbol{\mathcal{J}}}\left(\boldsymbol{\omega}_e + \boldsymbol{R}\boldsymbol{\omega}_d\right) - k_p^*\hat{\boldsymbol{\mathcal{J}}}\boldsymbol{\sigma}_e \\ &- k_d^*\hat{\boldsymbol{\mathcal{J}}}\boldsymbol{\omega}_e - \eta\frac{\boldsymbol{s}_\mathrm{I}}{\|\boldsymbol{s}_\mathrm{I}\|} \end{aligned} \tag{19}$$

where $\boldsymbol{T}_{eq} \in \mathbb{R}^3$ denotes the equivalent control derived from $\dot{\boldsymbol{s}}_\mathrm{I} = \boldsymbol{0}$, which is identical to the Lyapunov-based control law (10). $\boldsymbol{T}_{sw} = -\eta\frac{\boldsymbol{s}_\mathrm{I}}{\|\boldsymbol{s}_\mathrm{I}\|} \in \mathbb{R}^3$ is the switching control dealing with disturbance and inertia matrix uncertainty. $\eta = c_1 + c_2\|\boldsymbol{\sigma}_e\|_\infty + c_3\|\boldsymbol{\omega}_e\|_\infty + \delta$ is the switching gain with $\delta > 0$.

Subsequently, following conclusion can be drawn.

Lemma 2. For the attitude control system described in (6) and (7), the closed-loop system trajectory will move along the sliding surface throughout the entire control action. In other words, $\boldsymbol{s}_\mathrm{I} = \boldsymbol{0}$ for $t \in [t_0, +\infty)$.

Proof 2. Chose the Lyapunov function as

$$V = \frac{1}{2}\boldsymbol{s}_\mathrm{I}^\mathrm{T}\hat{\boldsymbol{\mathcal{J}}}\boldsymbol{s}_\mathrm{I} \tag{20}$$

Taking the time derivative along the closed-loop trajectory yields

$$\begin{aligned} \dot{V} =& \boldsymbol{s}_\mathrm{I}^\mathrm{T}\hat{\boldsymbol{\mathcal{J}}}\dot{\boldsymbol{s}}_\mathrm{I} \\ =& \boldsymbol{s}_\mathrm{I}^\mathrm{T}(\boldsymbol{T}_d + \boldsymbol{T}_p) - \eta\boldsymbol{s}_\mathrm{I}^\mathrm{T}\frac{\boldsymbol{s}_\mathrm{I}}{\|\boldsymbol{s}_\mathrm{I}\|} \\ \leq& \|\boldsymbol{s}_\mathrm{I}\|_2\left(c_1 + c_2\|\boldsymbol{\sigma}_e\|_\infty + c_3\|\boldsymbol{\omega}_e\|_\infty\right) - \eta\|\boldsymbol{s}_\mathrm{I}\| \\ \leq& -\delta\|\boldsymbol{s}_\mathrm{I}\| \end{aligned}$$

As $\boldsymbol{s}_\mathrm{I}(t_0) = \boldsymbol{0}$, it follows that $V(t_0) = 0$. Then, the non-positive time derivative of V implies $V(t) \leq 0$. Based on the fact that $V \geq 0$, one can conclude that $V(t) = 0$ for $t \in [t_0, +\infty)$, which means $\boldsymbol{s}_\mathrm{I} = \boldsymbol{0}$ for $t \in [t_0, +\infty)$.

Combining the above lemma with the sliding function definition in (18) yields

$$\boldsymbol{\omega}_e = -\int_{t_0}^{t}(k_p^*\boldsymbol{\sigma}_e + k_d^*\boldsymbol{\omega}_e)\mathrm{d}\tau \tag{21}$$

Noticing that such a relationship is established in the presence of disturbance and inertia matrix uncertainty, thus it can be employed for the stability analysis, which is expressed as follows.

Theorem 2. The ISM attitude control law (19) can globally uniformly asymptotically stabilize the attitude control system described in (6) and (7).

Proof 3. Recall the Lyapunov function in (12). As the closed-loop system trajectory satisfies the relationship in (21), the time derivative of V becomes

$$\begin{aligned}\dot{V} =& \boldsymbol{\omega}_e^{\mathrm{T}} \hat{\mathcal{J}}^{-1} \hat{\mathcal{J}} \dot{\boldsymbol{\omega}}_e - \frac{2k_\sigma}{k_1} \mathrm{e}^{-k_1 \boldsymbol{\sigma}_e^{\mathrm{T}} \boldsymbol{\sigma}_e} \left(-2k_1 \boldsymbol{\sigma}_e^{\mathrm{T}} \dot{\boldsymbol{\sigma}}_e\right) \\ =& - k_d^* \boldsymbol{\omega}_e^{\mathrm{T}} \boldsymbol{\omega}_e \leq 0\end{aligned}$$

The rest proof goes parallel with Theorem 1 and is thus omitted here.

From Theorem 2, it is obvious that the robustness against the disturbance and inertia matrix uncertainty is ensured. Here, the associated transient performance is examined, which is illustrated in the following corollary.

Corollary 1. For the attitude control system in (6) and (7), by applying the ISM attitude control law (19), the equivalent dynamics of closed-loop system is same as the nominal one controlled by the attitude control law (10).

4. NUMERICAL SIMULATION

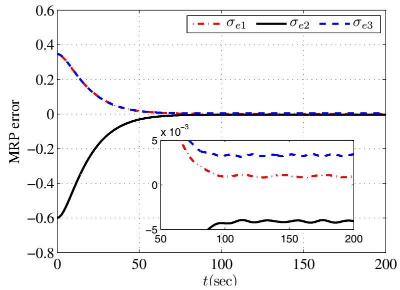
In this section, the numerical simulation of a large angle attitude maneuver is employed to verify the effectiveness of the proposed attitude controller (19) by comparing it with the existing Lyapunov-based attitude controller (9).

Supposing the nominal inertia matrix of the spacecraft is $\hat{\mathcal{J}} = \mathrm{diag}(3472, 2280, 2992)$ (kg.m) and the uncertainty is 10% of the nominal value. The disturbance torque is $\boldsymbol{T}_d = [1 + \sin(0.2t), -2 + 2\cos(0.3t), 1 + \sin(0.4t)]^T \times 10^{-1}$ (N.m). The initial attitude variables of the spacecraft are $\boldsymbol{\sigma}_b(t_0) = \mathbf{0}$ and $\boldsymbol{\omega}_b(t_0) = \mathbf{0}$ (rad/s), i.e., the spacecraft is inertially stabilized. The desired attitude variables are the MRP representation and the angular velocity of a local vertical local horizontal (LVLH) frame whose orbit parameters are

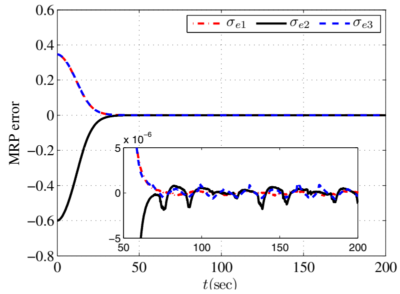
Parameter	Value
Semi-major axis	6899807 (m)
Eccentricity	0
Orbital inclination	30(deg)
Argument of perigee	60(deg)
Right ascension of ascending node	0(deg)
The initial true anomaly	90(deg)

For comparison, the control parameters are tuned such that those two controllers possess a similar peak control torque. Then, the settling time as well as the steady accuracy will be compared. To this end, the parameters of current Lyapunov-based attitude control law in (9) are selected as $k_p = 0.0289$ and $k_d = 0.17$. For the proposed attitude control law in (19), the related control parameters are $k_\sigma = 0.0292$, $k_\omega = 0.16$, $k_1 = 5$, $k_2 = -3$, and the switching gain coefficients are $c_1 = 0.5$, $c_2 = 5$, $c_3 = 30$ and $\delta = 0.00001$.

The simulation results are shown in Fig.1–Fig.3, where the superscripts $1, 2, 3$ denote the triaxial components of related vectors.

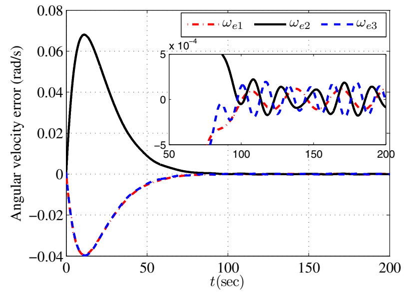


(a) attitude control law (9)

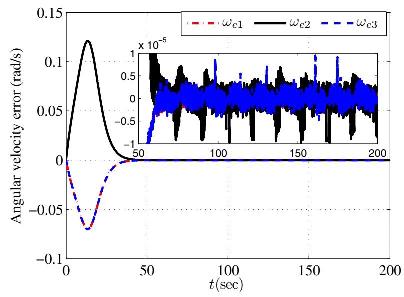


(b) attitude control law (19)

Fig. 1. MRP error response comparison



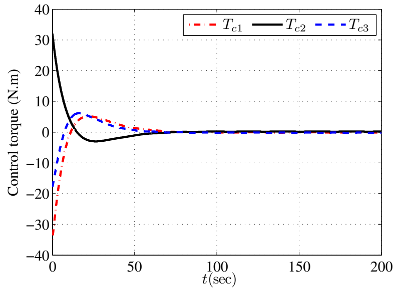
(a) attitude control law (9)



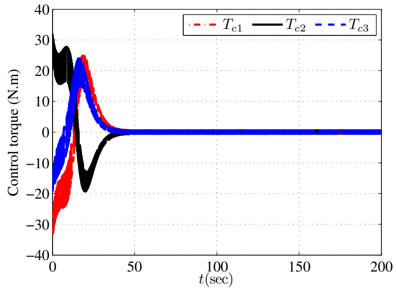
(b) attitude control law (19)

Fig. 2. Angular velocity error response comparison

Fig.1 shows the MRP errors controlled by the attitude controllers (9) and (19). It can be seen that the overshooting phenomenon has been successfully avoided by those two controllers. However, the transient performances as well as the control precisions are quite different between them. On the one hand, one can see that a faster convergence of the MRP error is achieved by the proposed attitude controller, where the settling time related to (19) is about 26 (sec) while the one associated with (9) is about 54 (sec). On the other hand, the control accuracy has been significantly improved by the proposed attitude controller. Specifically,



(a) attitude control law (9)



(b) attitude control law (19)

Fig. 3. Control torque evolution comparison

the magnitude of steady-state error controlled by (19) is nearly 1/1000 of that controlled by (9). Similar conclusion can be obtained for the angular velocity error comparison, which is shown in Fig.2.

Fig.3 illustrates the control torque comparison, where one can find that the peak values of those two controllers are almost the same. Therefore, combining with the preceding discussion about the transient performance, it can be concluded that the response acceleration by (19) will not result in excessive control torque. Nonetheless, it is worth mentioning that the undesired chattering occurs in the control torque generated by (19). As the chattering reduction is not the topic in this paper, it will not be discussed here and the interested reader should consult (Utkin and Lee [2006]).

5. CONCLUSION

In this paper, a high-performance control strategy is proposed for rigid spacecraft undergoing large angle attitude maneuvers. Besides the strong robustness against disturbance and inertia matrix uncertainty, desired transient response is also achieved by the proposed control strategy. A Lyapunov-based attitude control law with state-dependent feedback gains is firstly designed for the nominal attitude control system. On the basis of an in-depth analysis of the closed-loop dynamics, it has been shown that the acceleration of the system response can be realized without increasing the peak control torque. Then, the Lyapunov-based attitude control law is redesigned by the integral sliding mode control technique, where the desired transient performance is preserved and the robustness is improved. The advantages of the proposed control strategy is verified by numerical simulation.

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