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Cooperative Adaptive Fuzzy Control of High-Order Nonlinear Multi-Agent Systems with Unknown Dynamics ⋆

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Abstract: This paper focuses on the cooperative adaptive fuzzy control of high-order nonlinear multi-agent systems. The communication network is a undirected graph with a fixed topology. Each agent is modeled by a high-order integrator incorporating with unknown nonlinear dynamics and an unknown disturbance. Under the backstepping framework, a robust adaptive fuzzy controller is designed for each agent such that all agents ultimately achieve consensus. Moreover, these controllers are distributed in the sense that the controller design for each agent only requires relative state information between itself and its neighbors. A four-order simulation example demonstrates the effectiveness of the algorithm.

Keywords: Multi-agent systems, Distributed control, Consensus, High-order, Backstepping, Fuzzy logic systems

1. INTRODUCTION

Cooperative control of multi-agent systems has received increasing attention by the fact that many benefits can be obtained when a single complicated agent is equivalently replaced by multiple simpler agents. Numerous results have been obtained to solve a variety of multi-agent cooperative control problems (Vicsek 1995, Jadbabaie et al. 2003, Olfati-Saber and R.M. Murray 2002, 2004, W. Ren and R.W. Beard 2005, 2008). The control theory of multi-agent systems can be applied in many practical engineering applications such as cooperative control of unmanned ground/air/underwater vehicles, distributed sensor networks, aggregation and rendezvous control, attitude alignment of spacecraft and so on.

Among the existing works mentioned above, most of them studied only the first- and second-order dynamics. Recently, some researchers turned to focus on the distributed cooperative control problems of the networked high-order systems. Ren et al. (2006) showed a matrix approached based framework for high-order multi-agent systems. Consensus of high-order integrators multi-agent systems with time-delays and switching topologies were studied by Jiang et al. (2010) and Yang et al. (2011). Coordination of high-order linear systems with disturbances was investigated by Mo et al.(2011). Discrete-time high-order linear multi-agent systems was considered by Lin et al. (2011), and there also many results for the general high-order linear time-invariant (LTI) systems. As for the consensus of multiple high-order nonlinear systems, Dong et al. (2011) considered the tracking control problem. In term of cooperative adaptive control for high-order nonlinear uncertain multi-agent systems, the challenge is to make sure that the control for the nonlinearities and uncertainties are also in the distributed manner. That is, they are allowed to depend only on locally available information about the agent and its neighbors. Due to the challenges in designing cooperative control laws for distributed systems, it is not straightforward to extend the results for first- and second-order systems to that with higher-order dynamics. In these issues, the unknown dynamics can be considered under the neural network(NN) control framework (Zhang et. al 2012) or adaptive fuzzy control framework. Backstepping control approaches with adaptive fuzzy control can provide a systematic methodology of solving control problems for a larger class of unknown nonlinear systems (Tong et al. 2009a, 2009b, and Huo et al. 2012), where fuzzy logic systems(FLS) are used to approximate unknown nonlinear functions, and the backstepping design technique is applied to construct adaptive controllers and the adaptation parameter laws.

In this paper, a distributed recursive design approach is proposed to archive consensus of multiple high-order nonlinear systems with uncertainties. The main works of this paper include: 1) First, the agent dynamics are extended to general higher-order nonlinear systems in the Brunovsky form, which include first- and second-order systems as special cases. 2) Second, we propose a systematical distributed fuzzy logic systems and backstepping framework for multi-

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agent systems control. And a robust adaptive fuzzy control law is proposed under the distributed backstepping framework. The subsequent sections are organized as follows: In section 2, the control problem is formally stated and the background as well as necessary preliminaries concerning the control problem are given. In section 3, the cooperative control laws are proposed relying on backstepping method and adaptive fuzzy control approaches. The unknown nonlinear functions are dealt by fuzzy logic systems, and the external disturbances are addressed by applying robust adaptive control method. In section 4, a four-order simulation example is provided to demonstrate the performance of the proposed control laws. The last section concludes this paper.

2. PROBLEM STATEMENT AND PRELIMINARIES

2.1 Problem statement

Consider a group of m ($m \geq 2$) agents with non-identical dynamics distributed on a undirected communication network $\mathcal{G}$. The dynamics of the j-th agent is described in the nonlinear Brunovsky form

$$\dot{x}_{ij} = x_{(i+1),j} \tag{1}$$

$$\dot{x}_{nj} = u_j + f_j(x_j) + \zeta_j(t) \tag{2}$$

for $i = 1, \ldots, n-1$, where $x_{ij} \in \mathbf{R}$ is the i-th state of the j-th agent; $x_j = [x_{1j}, \ldots, x_{nj}]^T \in \mathbf{R}^n$ is the state vector of agent j; $f_j(x_j) : \mathbf{R}^n \rightarrow \mathbf{R}$ is locally Lipschitz with $f_j(0) = 0$, and it is assumed to be unknown; $u_j \in \mathbf{R}$ is the control protocol of the j-th system; $\zeta_j(t) \in \mathbf{R}$ is an external disturbance, which is unknown but bounded.

Assumption 1. The external disturbances $\zeta_j(t)$ are unknown and bounded, that is, $|\zeta_j(t)| \leq \overline{\zeta}_j$ with $\overline{\zeta}_j$ being a known constant.

Assumption 2. The communication graph $\mathcal{G}$ is fixed and connected.

The aim of this paper is to design a control law for the j-th system based on its own local states information when the communication topology is fixed and connected, such that

$$|x_{1j} - x_{1l}| \rightarrow 0, \text{ as } t \rightarrow \infty \text{ for } j, l = 1, \ldots, m. \tag{3a}$$

$$x_{ij} \rightarrow 0, \text{ as } t \rightarrow \infty \text{ for } i = 2, \ldots, n. \tag{3b}$$

2.2 Graph Theory

A team of m high-order nonlinear systems labeled as system 1 to m are considered. The communication topology among the m systems is assumed to be bi-directional, and the interactions among the nodes are represented by a undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V}$ is a set of the indices of the systems and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of edges that describe the communications between the agents. If $(p, j) \in \mathcal{E}$, then p is neighboring to j, meaning system j can obtain information form system p. $\mathcal{A}$ is a weighted adjacency matrix with nonnegative adjacency elements a_{pj}. Moreover, it's assumed that $a_{pp} = 0$. If the state of system p is available to system j, then system p is said to be a neighbor of system j. The neighbor set of node v_j is denoted by $\mathcal{N}_j$, where $j \notin \mathcal{N}_j$ (Mesbahi and Egerstedt 2010).

2.3 Fuzzy Logic Systems on Graph

Since the nonlinear functions $f_j(x_j)$ in (2) are unknown, in this paper, based on the fuzzy logic systems (FLS), the unknown function $f_j(x_j)$ can be approximated by $\hat{f}_j(x_j)$, where $\hat{f}_j(x_j) = \theta_f^T \phi_f(x)$, and $\phi_j(x_j) = [\phi_{1j}(x_j), \ldots, \phi_{nj}(x_j)]^T$ is a regressive vector. The knowledge base for FLS can be divided into some fuzzy IF-THEN rules and a fuzzy inference engine. By using product inference, center-average, and singleton fuzzifier (Wang 1994), the output of the j-th fuzzy logic system can be expressed as

$$f_j(x_j) = \frac{\sum_{l=1}^{N} \overline{f}_{lj} \prod_{i=1}^{n} \mu_{F_{ij}^l}(x_{ij})}{\sum_{l=1}^{N} [\prod_{i=1}^{n} \mu_{F_{ij}^l}(x_{ij})]} \tag{4}$$

where $x_j = (x_{1j}, \ldots, x_{nj})^T$ and f_j are the FLS input and output, respectively; $\overline{f}_{lj} = \max_{y \in R} \mu_{G_j^l}(f_j)$; F_{ij}^l and G_j^l are the fuzzy sets associating with the fuzzy functions $\mu_{F_{ij}^l}(x_{ij})$ and $\mu_{G_j^l}(y)$; N is the rule number of IF-THEN.

Define the fuzzy basis functions for the j-th system as

$$\phi_{lj} = \frac{\prod_{i=1}^{n} \mu_{F_{ij}^l}(x_{ij})}{\sum_{l=1}^{N} [\prod_{i=1}^{n} \mu_{F_{ij}^l}(x_{ij})]} \tag{5}$$

Denoting $\theta_j^T = [\overline{y}_{1j}, \ldots, \overline{y}_{Nj}] = [\theta_{1j}, \ldots, \theta_{Nj}]$ and $\phi_j(x_j) = [\phi_{1j}(x_j), \ldots, \phi_{Nj}(x_j)]$, then FLS (4) can be rewritten as

$$f_j(x_j) = \theta_j^T \phi_j(x_j) \tag{6}$$

Lemma 1. (Wang 1994): Let $f_j(x_j)$ be a continuous function defined on a compact set Ω. Then for any constant $\varepsilon_j > 0$, there exists an FLS (6) such that

$$\sup_{x_j \in \Omega} |f_j(x_j) - \theta_j^T \phi_j(x_j)| \leq \varepsilon_j \tag{7}$$

3. DISTRIBUTED CONTROL LAWS DESIGN

The extension of adaptive backstepping control to distributed multiple high-order dynamics is not straightforward. We need to definite a set of new variable for virtual control design in distributed manner.

Definition 1. We define a set of new variable $z_{*j} = [z_{1j}, z_{2j}, \ldots, z_{nj}]^T$ with the aid of backstepping technique as follows

$$z_{1j} = x_{1j} \tag{8}$$

$$z_{ij} = x_{ij} - \alpha_{ij}, 2 \leq i \leq n \tag{9}$$

where $j = 1, \ldots, m$. α_{ij} is the virtual control which is to be elaborately designed through recursive backstepping method.

In the first step, α_{2j} is used to denote the first virtual controller of system j. Using (1) for (8), we can derive that

$$\dot{z}_{1j} = z_{2j} + \alpha_{2j} \tag{10}$$

Consider the error variable $z_{1j} = x_{1j}$ of the first-order subsystem of (1)-(2), and choose the Lyapunov function candidate V_1 as follows

$$V_1 = \frac{1}{2} z_{1*}^T z_{1*} \tag{11}$$

where $z_{1*} = [z_{11}, z_{12}, \dots, z_{1m}]^T$.

Taking the time derivative of V_1 and following (9) and (10), we can obtain

$$\dot{V}_1 = \sum_{j=1}^{m} z_{1j}(z_{2j} + \alpha_{2j}) \tag{12}$$

We design the first distributed virtual controller α_{2j} as

$$\alpha_{2j} = -\sum_{l\in\mathcal{N}_j} a_{jl}(z_{1j} - z_{1l}) \tag{13}$$

where $\mathcal{N}_j$ denotes the neighbor set of the j-th agent and no global information states are included in α_{2j}. With the aid of Eqn. (13), (10) can be written as

$$\dot{z}_{1j} = -\sum_{l\in\mathcal{N}_j} a_{jl}(z_{1j} - z_{1l}) + z_{2j} \tag{14}$$

and $\dot{V}_1$ can be written as

$$\dot{V}_1 = -z_{1*}^T L z_{1*} + \sum_{j=1}^{m} z_{1j} z_{2j} \tag{15}$$

In the second step, by considering eqn.(9) and the second order of the eqn.(1), it can be obtained that

$$\begin{aligned} \dot{z}_{2j} &= x_{3j} - \dot{\alpha}_{2j} \\ &= z_{3j} + \alpha_{3j} - \frac{\partial \alpha_{2j}}{\partial x_{1j}} x_{2j} - \sum_{l\in\mathcal{N}_j} \frac{\partial \alpha_{2j}}{\partial x_{1l}} x_{2l} \end{aligned} \tag{16}$$

Remark 1. α_{3j} is treated as a virtual controller for a high-order subsystem which would be designed to guarantee the consensus of the first-order and the second-order subsystems for the multiple high-order systems. That is, the virtual controller α_{3j} is to be designed such that $\lim_{t\to\infty}(z_{1j} - z_{1l}) = 0$ and $\lim_{t\to\infty}(z_{2j} - z_{2l}) = 0$ for $1 \le j, l \le m$.

Hence, choose the second Lyapunov function candidate V_2 as

$$V_2 = V_1 + \frac{1}{2} z_{2*}^T z_{2*} \tag{17}$$

where $z_{2*} = [z_{21}, z_{22}, \dots, z_{2m}]^T$. Taking the time derivative of V_2 with respect to (15) and (16), we can get

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + \sum_{j=1}^{m} z_{2j}\dot{z}_{2j} \\ &= -z_{1*}^T L z_{1*} + \sum_{j=1}^{m} z_{1j} z_{2j} + \sum_{j=1}^{m} z_{2j} \Bigg[z_{3j} + \alpha_{3j} \\ &\quad - \frac{\partial \alpha_{2j}}{\partial x_{1j}} x_{2j} - \sum_{l\in\mathcal{N}_j} \frac{\partial \alpha_{2j}}{\partial x_{1l}} x_{2l} \Bigg] \end{aligned} \tag{18}$$

In order to ensure that the time derivative of Lyapunov function V_2 is negative definite, an appropriate distributed virtual control α_{3j} should be designed. α_{3j} is designed as

$$\alpha_{3j} = -z_{1j} - c_{2j} z_{2j} + \frac{\partial \alpha_{2j}}{\partial x_{1j}} x_{2j} + \sum_{l\in\mathcal{N}_j} \frac{\partial \alpha_{2j}}{\partial x_{1l}} x_{2l} \tag{19}$$

where c_{2j} is the design parameter, satisfying $c_{2j} > 0$.

Remark 2. Note that α_{3j} only contains its own state information and neighbors' information without using any global information generally. The two items $-\frac{\partial \alpha_{2j}}{\partial x_{1j}} x_{2j}$ and $-\sum_{l\in\mathcal{N}_j} \frac{\partial \alpha_{2j}}{\partial x_{1l}} x_{2l}$ in eqn.(18) are directly canceled by the design of α_{3j}. Furthermore, the item $-z_{1j}$ in α_{3j} is designed to make sure that the item $\sum_{j=1}^{m} z_{1j} z_{2j}$ in eqn.(18) can be eliminated. And the item $-c_{2j} z_{2j}$ in eqn.(19) is designed to ensure the negative definite of the eqn.(18). The item $\sum_{j=1}^{m} z_{2j} z_{3j}$ in eqn.(20) will be handled in the third step by choosing an apropriate virtual controller α_{4j}.

Therefore, by substituting (19) into (18), $\dot{V}_2$ can be rewritten as follows

$$\dot{V}_2 = -z_{1*}^T L z_{1*} - z_{2*}^T \mathrm{diag}(c_{2*}) z_{2*} + \sum_{j=1}^{m} z_{2j} z_{3j} \tag{20}$$

where $c_{2*} = [c_{21}, c_{22}, \dots, c_{2m}]^T$.

In step i, where $1 \le i \le n-1$. Follow the design procedure which is similar to the first and second step, it can be obtained that

$$\begin{aligned} \dot{z}_{ij} &= x_{(i+1)j} - \dot{\alpha}_{ij} \\ &= z_{(i+1)j} + \alpha_{(i+1)j} - \sum_{k=1}^{i-1} \frac{\partial \alpha_{ij}}{\partial x_{kj}} x_{(k+1)j} \\ &\quad - \sum_{k=1}^{i-1} \sum_{l\in\mathcal{N}_j} \frac{\partial \alpha_{ij}}{\partial x_{kl}} x_{(k+1)l} \end{aligned} \tag{21}$$

In (21), the virtual controller $\alpha_{(i+1)j}$ which can guarantee the consensus of the multiple i-rank $(1 < i < n-1)$ subsystems would be designed such that $\lim_{t\to\infty}(z_{kj} - z_{kl}) = 0$ for $1 \le j, l \le m$ and $1 \le k \le n-1$, with the aid of the Lyapunov function

$$V_i = V_{i-1} + \frac{1}{2} z_{i*}^T z_{i*} \tag{22}$$

Note that V_{i-1} can be designed in the $i-1$ step by the recursive method. Taking the time derivative of V_i with considering V_{i-1} in step i and (21), we can get

$$\begin{aligned}\dot{V}_i &= \dot{V}_{i-1} + \sum_{j=1}^{m} z_{ij}\dot{z}_{ij} \\ &= -z_{1*}^T L z_{1*} - \sum_{j=2}^{i-1} z_{j*}^T \mathrm{diag}(c_{j*}) z_{(i-1)*} \\ &\quad + \sum_{j=1}^{m} z_{(i-1)j} z_{ij} + \sum_{j=1}^{m} z_{ij}\Bigg[-\sum_{k=1}^{i-1}\frac{\partial \alpha_{ij}}{\partial x_{kj}} x_{(k+1)j} \\ &\quad + z_{(i+1)j} + \alpha_{(i+1)j} - \sum_{k=1}^{i-1}\sum_{l\in\mathcal{N}_j}\frac{\partial \alpha_{ij}}{\partial x_{kl}} x_{(k+1)l}\Bigg] \quad (23)\end{aligned}$$

Choose the virtual controller $\alpha_{(i+1)j}$ as

$$\begin{aligned}\alpha_{(i+1)j} &= -z_{(i-1)j} - c_{ij} z_{ij} + \sum_{k=1}^{i-1}\frac{\partial \alpha_{ij}}{\partial x_{kj}} x_{(k+1)j} \\ &\quad + \sum_{k=1}^{i-1}\sum_{l\in\mathcal{N}_j}\frac{\partial \alpha_{ij}}{\partial x_{kl}} x_{(k+1)l} \quad (24)\end{aligned}$$

where c_{ij} is the design parameter and satisfy $c_{ij} > 0$. Substituting (24) into $\dot{V}_i$, it is obtained that

$$\dot{V}_i = -z_{1*}^T L z_{1*} - \sum_{j=2}^{i} z_{i*}^T \mathrm{diag}(c_{j*}) z_{j*} + \sum_{j=1}^{m} z_{ij} z_{(i+1)j} \quad (25)$$

where $c_{i*} = [c_{i1}, c_{i2}, \ldots, c_{im}]^T$.

In the last step, fuzzy logic systems is used to approximate the unknown functions $f_j(x_j)$ of the multi-agent systems (2). Define the minimal approximation error $\varepsilon_j = f_j(x_j) - f_j(x_j|\theta_j^*)$, where $f_j(x_j|\theta_j^*) = \theta_j^{*T}\phi_j(x_j)$, and θ_j^* is the optimal fuzzy parameter vector. It is assumed that $|\varepsilon_j| \le \bar{\varepsilon}_j$, where $\bar{\varepsilon}_j$ is a positive constant. Based on the FLS (4)-(6), the unknown function $f_j(x_j)$ can be approximated by $\hat{f}_j(x_j) = \hat{\theta}_j^T\phi_j(x_j)$, where $\hat{\theta}_j$ is the estimation of θ_j, and $\phi_j(x_j) = [\phi_{1j}(x_j), \ldots, \phi_{nj}(x_j)]^T$ is a regressive vector.

By applying the results in the 1 to $n-1$ steps, it can be obtained that

$$\begin{aligned}\dot{z}_{nj} &= u_j + f_j(x_j) + \zeta_j - \dot{\alpha}_{nj} \\ &= -\sum_{k=1}^{n-1}\frac{\partial \alpha_{nj}}{\partial x_{kj}} x_{(k+1),j} - \sum_{k=1}^{n-1}\sum_{l\in\mathcal{N}_j}\frac{\partial \alpha_{nj}}{\partial x_{kl}} x_{(k+1),l} \\ &\quad + f_j(x_j|\theta_j^*) + \varepsilon_j + \zeta_j + u_j \quad (26)\end{aligned}$$

Hence, choose the n-th Lyapunov function candidate V_n as

$$V_n = V_{n-1} + \frac{1}{2} z_{n*}^T z_{n*} + \frac{1}{2}\sum_{j=1}^{m}\tilde{\theta}_j^T \Gamma_j^{-1}\tilde{\theta}_j \quad (27)$$

where $\tilde{\theta}_j = \theta_j^* - \hat{\theta}_j$ is the fuzzy parameter error vector.

Taking the time derivative of V_n with respect to (25) and (26), we obtain

$$\begin{aligned}\dot{V}_n &= \dot{V}_{n-1} + \sum_{j=1}^{m} z_{nj}\dot{z}_{nj} + \sum_{j=1}^{m}\tilde{\theta}_j^T \Gamma_j^{-1}\dot{\tilde{\theta}}_j \\ &= -z_{1*}^T L z_{1*} - \sum_{j=2}^{n-1} z_{j*}^T \mathrm{diag}(c_{j*}) z_{j*} + \sum_{j=1}^{m} z_{(n-1)j} z_{nj} \\ &\quad + \sum_{j=1}^{m} z_{nj}\Bigg[u_j + f_j(x_j|\theta_j^*) + \varepsilon_j - \sum_{k=1}^{n-1}\frac{\partial \alpha_{nj}}{\partial x_{kj}} x_{(k+1)j} \\ &\quad - \sum_{k=1}^{n-1}\sum_{l\in\mathcal{N}_j}\frac{\partial \alpha_{nj}}{\partial x_{kl}} x_{(k+1)l} + \zeta_j\Bigg] + \sum_{j=1}^{m}\tilde{\theta}_j^T \Gamma_j^{-1}\dot{\tilde{\theta}}_j \quad (28)\end{aligned}$$

We choose the adaptation laws:

$$\dot{\hat{\theta}}_j = \Gamma_j z_{nj}\phi_j \quad (29)$$

where Γ_j is positive definite matrices, Note that z_{nj} only contains the local information. And the distributed control law is

$$\begin{aligned}u_j &= -z_{(n-1)j} - c_{nj} z_{nj} + \sum_{k=1}^{n-1}\frac{\partial \alpha_{nj}}{\partial x_{kj}} x_{(k+1)j} \\ &\quad + \sum_{k=1}^{n-1}\sum_{l\in\mathcal{N}_j}\frac{\partial \alpha_{nj}}{\partial x_{kl}} x_{(k+1)l} - \hat{\theta}_j^T\phi_j(x_j) \\ &\quad - \bar{\epsilon}_j \mathrm{sign}(z_{nj}) \quad (30)\end{aligned}$$

where $\bar{\epsilon}_j \ge \bar{\varepsilon}_j + \bar{\zeta}_j$.

Using (29) and (30) for (28), we can derive

$$\begin{aligned}\dot{V}_n &= -z_{1*}^T L z_{1*} - \sum_{i=2}^{n} z_{i*}^T \mathrm{diag}(c_{i*}) z_{i*} \\ &\quad + \sum_{j=1}^{m} z_{nj}[f_j(x_j|\theta_j^*) - \hat{f}_j(x_j) - \bar{\epsilon}_j \mathrm{sign}(z_{nj}) \\ &\quad + \varepsilon_j + \zeta_j] + \sum_{j=1}^{m}\tilde{\theta}_j^T \Gamma_j^{-1}\dot{\tilde{\theta}}_j \\ &= -z_{1*}^T L z_{1*} - \sum_{i=2}^{n} z_{i*}^T \mathrm{diag}(c_{i*}) z_{i*} \\ &\quad + \sum_{j=1}^{m} z_{nj}[\theta_j^{*T}\phi_j(x_j) - \hat{\theta}_j\phi_j(x_j) - \bar{\epsilon}_j \mathrm{sign}(z_{nj}) \\ &\quad + \varepsilon_j + \zeta_j] + \sum_{j=1}^{m}\tilde{\theta}_j^T \Gamma_j^{-1}\dot{\tilde{\theta}}_j \\ &\le -z_{1*}^T L z_{1*} - \sum_{i=2}^{n} z_{i*}^T \mathrm{diag}(c_{i*}) z_{i*} + \sum_{j=1}^{m} z_{nj}(\varepsilon_j + \zeta_j) \\ &\quad - \sum_{j=1}^{m} z_{nj}\tilde{\theta}_j^T\phi_j(x_j) + \sum_{j=1}^{m}\tilde{\theta}_j^T \Gamma_j^{-1}\dot{\tilde{\theta}}_j - \sum_{j=1}^{m}\bar{\epsilon}_j|z_{nj}| \\ &\le -z_{1*}^T L z_{1*} - \sum_{i=2}^{n} z_{i*}^T \mathrm{diag}(c_{i*}) z_{i*} + \sum_{j=1}^{m}(\bar{\varepsilon}_j + \bar{\zeta}_j)|z_{nj}| \\ &\quad - \sum_{j=1}^{m}\bar{\epsilon}_j|z_{nj}| \le 0 \quad (31)\end{aligned}$$

Theorem 1. Consider the multiple nonlinear systems described by (1)-(2), when the communication topology of the systems is fixed and connected, choose the control law given by (30) and the adaptation law (29) for system j, where $1 \leq j \leq m$, then it guarantees that the control objective (3) holds, that is the consensus of high-order nonlinear uncertain systems can be reached asymptotically. In the control laws, all the required information is local.

Proof. By the above design procedure, define the Lyapunov function candidate as (27), then we get (31). By Barbalat's lemma, $\lim_{t\to\infty} z_{1*}^T L z_{1*} = 0$ and $\lim_{t\to\infty} z_{l*} = 0$ for $2 \leq l \leq m$. Furthermore, since L is a Laplacian matrix, we have the results according to the control objective.

Remark 2. The proposed control law contains the sign function, thus leading to control chattering. This situation can be remedied by smoothing out the control discontinuity in a thin boundary layer neighboring the switching surface. To do this, the sign function in the control law (30) can be replaced by a saturation function.

4. SIMULATION

In this section, an example is given to show the effectiveness of the proposed distributed adaptive fuzzy control laws. Consider a 4-node undirected graph described in Fig.1. Note that the communication graph $\mathcal{G}$ satisfies Assumption 2. And the corresponding adjacent weights between agents are assumed to be 1, and all the others are 0.

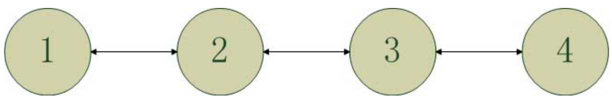

Fig.1. Communication graph $\mathcal{G}$ of the multi-agent system

Consider the following four-order uncertain nonlinear multi-agent systems:

$$\begin{aligned}
\dot{x}_{1j} &= x_{2j} \\
\dot{x}_{2j} &= x_{3j} \\
\dot{x}_{3j} &= x_{4j} \\
\dot{x}_{4j} &= u_j + f_j(x_{1j}, x_{2j}, x_{3j}, x_{4j}) + \zeta_j
\end{aligned}$$

with

$$\begin{aligned}
\dot{x}_{41} &= u_1 + 0.2\sin(x_{11} + x_{41}) + 0.3\sin(t/5) \\
\dot{x}_{42} &= u_2 + (x_{12} + x_{22} - 1)^2 + 0.3\sin(t/5) \\
\dot{x}_{43} &= u_3 + 0.3\cos(x_{13} + x_{23}) + 0.3\sin(t/5) \\
\dot{x}_{44} &= u_4 + 0.2\sin(x_{14} + x_{24}) + \cos(t)
\end{aligned}$$

Let the initial state information and the disturbances of the systems be:

$x_{1j}(0) = [1, 2, 1, -0.5]^T$, $x_{2j}(0) = [-0.5, 1, 3, -1]^T$, $x_{3j}(0) = [1.5, -1, 2, 3]^T$, $x_{4j}(0) = [2, -1, 2, 1]^T$, $\zeta_j(0) = [0.2, 0.2, 0.3, 0.4]^T$.

Define fuzzy membership as follows:

$\mu_{F_4^l}(x_{1j}, x_{2j}, x_{3j}, x_{4j}) = \exp[-(x_{1j} - 3 + l)^2/2] \times \exp[-(x_{2j} - 3 + l)^2/2] \times \exp[-(x_{3j} - 3 + l)^2/2] \times \exp[-(x_{4j} - 3 + l)^2/2], l = 1, \ldots, 5.$

We obtain fuzzy basis functions as follows:

$$\phi_{4p}(x_{1j}, x_{2j}, x_{3j}, x_{4j}) = \frac{\exp[\frac{-(x_{1j}-3+p)^2}{2}] \times \ldots \times \exp[\frac{-(x_{4j}-3+p)^2}{2}]}{\sum_{n=1}^{5} \exp[\frac{-(x_{1j}-3+n)^2}{2}] \times \ldots \times \exp[\frac{-(x_{4j}-3+n)^2}{2}]},$$

where $p = 1, \ldots, 5$.

The FLSs can be expressed in the following form:

$\hat{f}_j(x_j|\theta_j) = \hat{\theta}_j^T \phi_j(x_j)$

where $\hat{\theta}_j^T = [\hat{\theta}_{1j}, \hat{\theta}_{2j}, \hat{\theta}_{3j}, \hat{\theta}_{4j}, \hat{\theta}_{5j}]$, and

$\phi_j(x_j) = [\phi_{1j}(x_j), \phi_{2j}(x_j), \phi_{3j}(x_j), \phi_{4j}(x_j), \phi_{5j}(x_j)]$.

with the initial state information:

$\hat{\theta}_1(0) = [0.01, 0.02, 0.01, 0.01, 0.01]^T$,
$\hat{\theta}_2(0) = [0.1, -0.01, 0.02, 0.05, 0.02]^T$,
$\hat{\theta}_3(0) = [0.3, 0.2, -0.3, 0.4, 0.3]^T$,
$\hat{\theta}_4(0) = [-0.06, 0.03, 0.07, 0.1, -0.02]^T$.

The consensus control laws can be obtained by Theorem 1 in which the control parameters are chosen as $\Gamma_1 = 2.9, \Gamma_2 = 2.8, \Gamma_3 = 2.2, \Gamma_4 = 2.1; c_{21} = 1, c_{31} = 1, c_{41} = 1, c_{22} = 1.2, c_{32} = 1.1, c_{42} = 1, c_{23} = 1.1, c_{33} = 1, c_{43} = 1, c_{24} = 1, c_{34} = 0.8, c_{44} = 1$.

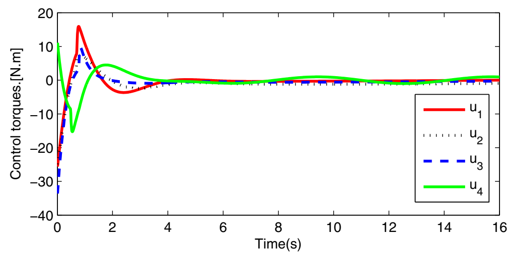

Fig.2. Response of u_j for $1 \leq j \leq 4$

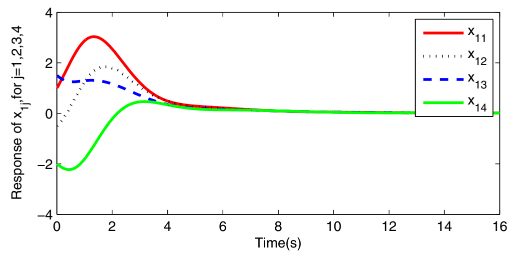

Fig.3. Response of x_{1j} for $1 \leq j \leq 4$

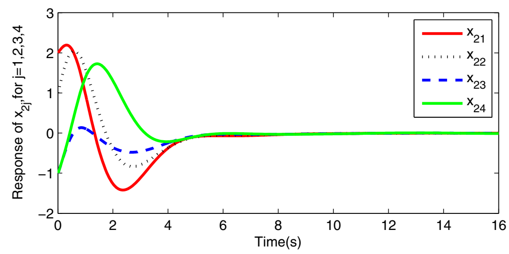

Fig.4. Response of x_{2j} for $1 \leq j \leq 4$

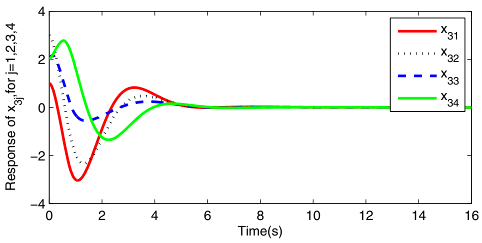

Fig.5. Response of x_{3j} for $1 \leq j \leq 4$

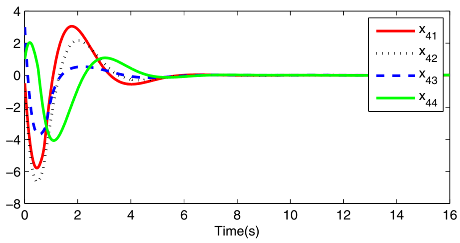

Fig.6. Response of x_{4j} for $1 \leq j \leq 4$

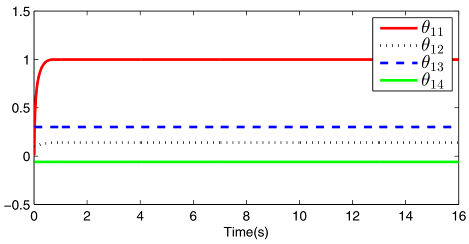

Fig.7. Response of θ_{1j} for $1 \leq j \leq 4$

Fig.2 shows the the control torque of each agent by the distributed control law in (31). Fig.3-6. shows the time histories of state trajectories for each agent. From Fig.3, it can be seen that, under the control torque which are shown in Fig.2, the consensus is achieved. Fig.7 shows the the response of θ_{1j}. These figures demonstrate the efficiency of the proposed algorithm in guaranteeing consensus despite the presence of complex unknown dynamics.

5. CONCLUSION

This paper considered the cooperative consensus control problem of networked high-order nonlinear systems with distinct unknown dynamics and bounded external disturbances. The nonlinearities are only assumed to be locally Lipschitz. A robust adaptive fuzzy control algorithm was proposed under the distributed backstepping framework. The proposed algorithm is completely distributed in the sense that, the controller for each agent only uses information of itself and its neighbors.

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