Preprints of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Efficiency analysis of a continuously variable transmission with linear control for a series hybrid electric vehicle

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Abstract: This paper investigates the performance of a continuously variable transmission (CVT) using an intuitive control in a hybrid electric vehicle (HEV) of a series architecture under varying driving conditions. The detailed dynamic vehicle simulation model employed in this study enables an in-depth evaluation of the CVT as compared to a fixed transmission (FT) that utilises a fixed final drive ratio between the motor and wheels. The investigation uses three distinct driving cycles to show that the CVT offers reduction in motor energy consumption of up to 9.38%. Also, the particular impacts of the transmission systems are understood by analysing the energy flows from the Permanent Magnet Synchronous Motor (PMSM).

Keywords: continuously variable transmission, automotive control, hybrid electric vehicle, series architecture, permanent magnet motors, efficiency enhancement

1. INTRODUCTION

Amid growing concerns of climate change and scarcity of fossil fuel, both manufacturers and regulators are increasingly recognizing that business as usual within the automotive industry is not an option. Consequently, there has been a general trend towards electrification of vehicles that has seen the rise in the presence of hybrid electric vehicles (HEVs). Simultaneously there has been an increase in the use of mechatronic systems, including for the transmission system. What has commonly been a hydraulic or mechanical system is receiving competition from electromechanical continuously variable transmission (CVT) systems, which have shown promising performance.

However, existing literature has primarily studied the use of CVTs for parallel hybrid vehicles (Bowles et al., 2000; Won et al., 2005; Lee and Kim, 2005) where it often doubles it's role by acting as the power split device between the engine and the motor. However, this paper investigates the effectiveness of a CVT in a series HEV where only the motor drives the wheels. Consequently a more robust and intuitive control can be designed while still delivering improved performance. Also, for a series powertrain the performance of the CVT is independent from the choice of supervisory control system (SCS) for the powertrain energy management, as the motor (which the CVT is connected to) provides all the propulsion in the vehicle. This is in contrast to a parallel HEV where the wheels are mechanically connected to both the engine and the motor and thus the performance of a CVT would be strongly dependent on the choice of SCS. Thus this study is able to isolate the performance of the CVT from the SCS. The results also have relevance for the performance of a CVT for a battery electric vehicle (BEV) where the motor is also the only means of driving the wheels.

The paper is organised as follows. The paper first introduces the vehicle model used in this investigation in Section 2, including the implementation of a CVT and its control. Simulations of multiple driving cycles are conducted and the results are presented and discussed in Section 3. Finally, conclusions are drawn in Section 4.

2. VEHICLE MODEL

The vehicle model used in this investigation is described in (Evangelou and Shukla, 2012), with some modifications. It represents a mid-sized passenger car with a hybrid powertrain of series architecture, as shown in Fig. 1, and uses physics-based equations as far as possible and appropriate. The components of the vehicle have been dynamically modelled in a modular fashion and successfully capture realistic transient behaviour during driving.

As is shown in Fig. 1, the car is driven solely by the Permanent Magnet Synchronous Motor (PMSM) that is mechanically connected to the wheels of the car via a transmission system. The motor is powered by a Lithium-ion battery and a turbocharged 2.0L diesel engine, mechanically coupled to a Permanent Magnet Synchronous Generator (PMSG). Between these sources and the PMSM, there is power electronics (inverter, rectifier and a bi-directional DC-DC converter) to interface the three branches.

This paper is particularly interested in the transmission system and its impact on the performance of the 3-phase PMSM that is used in the vehicle. The implementation of these components in the model is shown in Fig. 2.

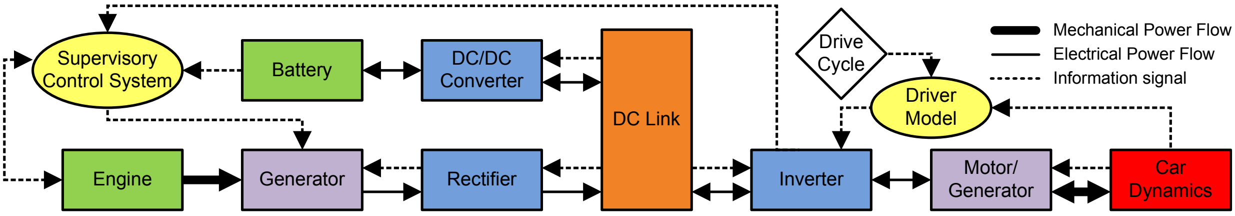

Fig. 1. Overview of the architecture of the modelled series HEV, with the transmission included in the car block.

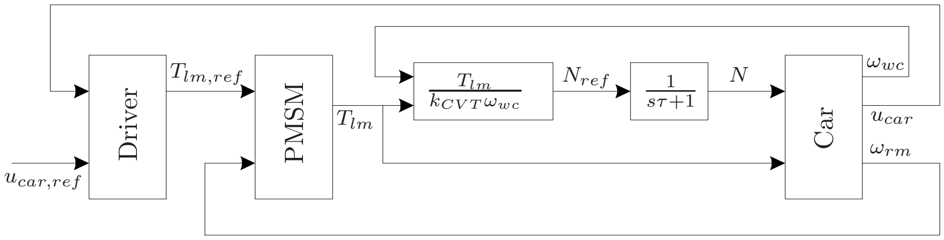

Fig. 2. The diagram illustrates the interaction between the driver, PMSM, CVT and car model. The driver model reads the reference car speed $u_{car,ref}$ to provide the PMSM with the reference load torque. The CVT control uses load torque T_{lm} and wheel speed ω_{wc}, with $k_{CVT} = 0.5$, to determine the final gear reduction ratio N.

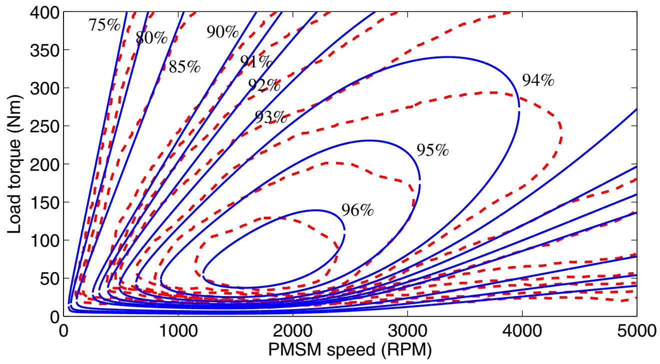

Fig. 3. Steady-state power efficiency map of PMSM for variations in load torque, T_{lm}, and rotor speed, ω_{rm}, for the experimental results (red dashed line) and for the PMSM model used (blue solid line). The efficiency contours are in the range 75%-96%.

2.1 PMSM

The PMSM can be modelled by converting the standard 3-phase equations into the rotor d-q reference using Park's Transformation as described by (Pillay and Krishnan, 1989). This method reduces the fundamental PMSM dynamics to the following coupled differential equations:

$$\frac{di_{dm}}{dt} = (v_{dm} - R_m i_{dm} + \omega_{sm} L_{qm} i_{qm})/L_{dm} \qquad (1)$$

$$\frac{di_{qm}}{dt} = (v_{qm} - R_m i_{qm} - \omega_{sm}(L_{dm} i_{dm} + \lambda_{fm}))/L_{qm} \quad (2)$$

where i_{dm} and i_{qm} are the d- (direct) and q- (quadrature) axis components of stator current, v_{dm} and v_{qm} are the d- and q-axis components of stator voltage, R_m is the stator resistance, L_{dm} and L_{qm} are the d- and q-axis stator inductances, λ_{fm} is the flux linkage due to the permanent magnets, and ω_{sm} is the inverter frequency defined by $\omega_{sm} = p_m \omega_{rm}$ in which p_m is the number of pole pairs per phase in the stator and ω_{rm} is the rotor speed. The electromagnetic torque produced by the motor is then expressed as

$$T_{em} = \frac{3}{2} p_m \left(\lambda_{fm} i_{qm} + (L_{dm} - L_{qm}) i_{dm} i_{qm}\right). \qquad (3)$$

This torque is applied on the rotor shaft that is connected to the car transmission, thereby driving the car forward. The rotor dynamics and the transmission system are thus both included inside the car block in Fig. 1. The model also includes a generalised friction torque T_{fm} acting on the motor shaft (Evangelou and Shukla, 2012), such that the overall motor load is given by $T_{lm} = T_{em} - T_{fm}$. The car block is modelled using VehicleSim (formerly known as Autosim (Anon., 1998)) and imported to Simulink, which is the main platform used for modelling all the components. Under steady state operation (1) − (3) reduce to a set of algebraic equations that can be used to calculate the efficiency contours as shown in Fig. 3. For validation purposes this figure shows also experimentally measured efficiency contours provided by the manufacturer of the specific electrical machine used in our study.

2.2 CVT

The transmission in a series HEV behaves similarly to a transmission in a conventional powertrain as there is a single power source driving the wheels (unlike most modern HEVs that use a power-split architecture). However, as the PMSM can operate quite efficiently across a wide range of motor speeds, the benefit of being able to vary the gear ratio is not as significant as for an internal combustion engine (ICE). Thus, conventionally, a series HEV has been fitted with a fixed transmission (FT) to reduce mechanical complexity and maintain high transmission efficiency (modelled as 96.7% in this work). In contrast, a CVT has a higher mechanical complexity and according to (Miller, 2003) it typically has an average efficiency between 89% and 94%, which matches the experiments of (Bonsen et al., 2004) as well. In this paper a toroidal CVT is modelled with an average efficiency of $\eta_{tr} = 93\%$, as suggested by (Heath, 2007), and the efficiency is implemented by reducing the load torque on the transmission shaft accordingly.

The main function of the CVT is to continuously optimize the motor speed, which is given by:

$$\omega_{rm} = N\omega_{wc}. \tag{4}$$

The total gear ratio N includes the reduction by the CVT as well as from the differential gear. The CVT typically operates in the range $3.117:1 - 0.427:1$ (Anon., 2011) while the differential gear is fixed at 3.42. This enables the CVT to achieve values of N in the range $10.66 - 1.42$. In addition, the dynamic response of the modelled CVT is characterised by a first order lag with a time constant τ, of 200 ms as used by (Kim and Kim, 2002).

As the wheel speed varies continuously with the vehicle speed, it requires the CVT to control N such that the motor operates optimally in terms of its speed and torque profile. This set of optimal operating points can be defined as a linear relationship between motor speed and load torque, corresponding to a straight line passing through the origin and the point of peak efficiency in Fig. 3. This line is given by:

$$T_{lm} = k_{CVT}\omega_{rm}. \tag{5}$$

This line of operation (with $k_{CVT} = 0.5$) is not only a good approximation for the most efficient mode of operation but is also practical to implement. Furthermore, the linear increase of both motor speed and load torque with respect to required power facilitates smooth operation.

3. SIMULATION RESULTS

This section will present and discuss simulation results of the vehicle model. It will first introduce an appropriate selection of driving cycles. Thereafter the performance of the transmission systems will be benchmarked to determine the effectiveness of the CVT.

3.1 Driving cycles

To meaningfully run a simulation, the model requires a predefined driving profile in terms of vehicle speed. This input driving cycle is then read as $u_{car,ref}$ (as shown in Fig. 2), which the driver model uses to send the appropriate signal to the motor to ensure that the desired vehicle speed is tracked. The choice of predefined driving profile is flexible, but the results presented in this paper have primarily been generated for two distinct American driving cycles, as shown in Fig. 4: firstly, the NYCC (New York City Cycle) that is an urban driving cycle with a low-speed profile and frequent stop-and-go characteristics; and secondly, the HWFET (Highway Fuel Economy Test) which represents highway driving with a sustained high-speed profile. These will be used to demonstrate vehicle performance under two extreme types of driving.

To complement these driving cycles we will also make use of the FTP-75 driving cycle as shown in Fig. 5. It is a high-speed urban driving cycle and thus represents combined characteristics of the two former driving cycles, and is more representative of typical driving. This selection of driving cycles will enable us to distinguish the performance of the vehicle under varying driving conditions.

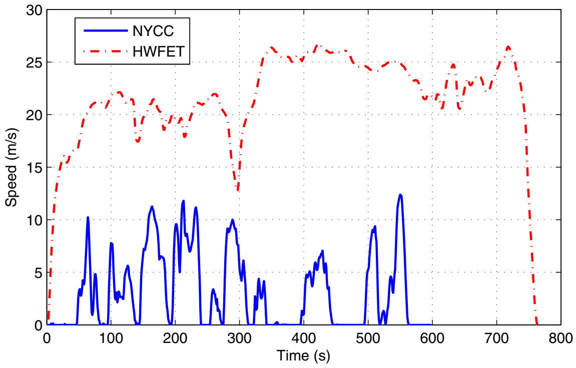

Fig. 4. Speed profiles of the NYCC and HWFET driving cycles.

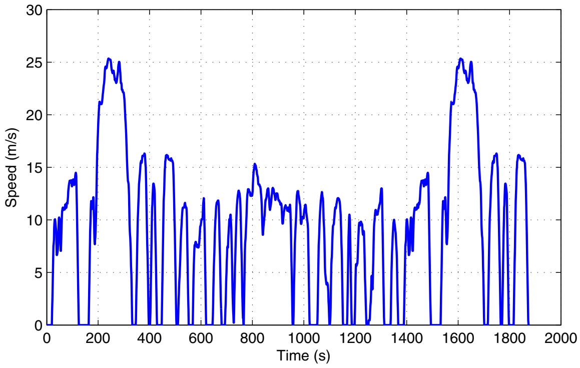

Fig. 5. Speed profile of the FTP-75 driving cycle.

3.2 Instantaneous efficiency

As has been described, the CVT is able to continuously optimise the final drive ratio N to ensure efficient operation of the motor. To investigate this behaviour, simulations are run to measure the motor and transmission efficiency for both the NYCC and HWFET driving cycles, using both CVT and FT (where N is chosen to be 4, which is shown later to be the most suitable choice). The collected data is presented in Figs. 6 and 7.

It can firstly be seen in both plots that the final drive ratio of the CVT (N_{CVT}) varies significantly compared to the constant value of the FT (N_{FT}) and changes continuously, even over small windows of time. Assuming N_{CVT} is following the optimal value of final drive ratio, it indicates the limitation of using a single value persistently as FT does. Accordingly, the efficiency (considering both motor and transmission system) is observed to be quite sensitive to the final drive ratio. In the case of the CVT the efficiency is very steady at a high level (apart from instances where the motor and vehicle stop completely) compared to that of the FT, as N_{CVT} adapts continuously to optimise performance. The FT on the other hand suffers from a lower efficiency as the optimal N differs from the chosen fixed N_{FT} value of 4. This behaviour clearly illustrates the benefit of the CVT.

However, it is noteworthy that the FT system has a higher efficiency at some points. That is when the optimal N value happens to be very close to the chosen N_{FT}. Operation in this region means that both the CVT and

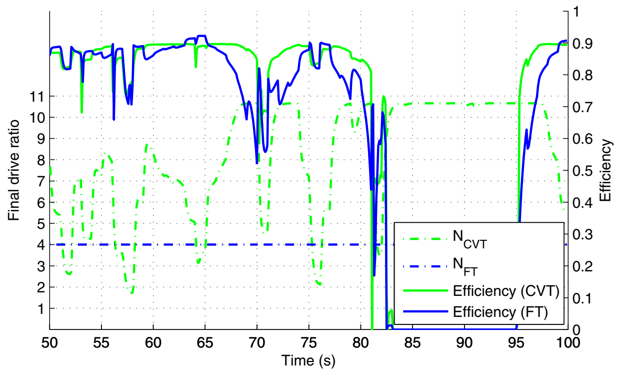

Fig. 6. Variations of the final drive ratio N, and consequent changes in efficiency (of motor and transmission system combined), for vehicle systems using either CVT or FT when simulating the urban driving cycle NYCC.

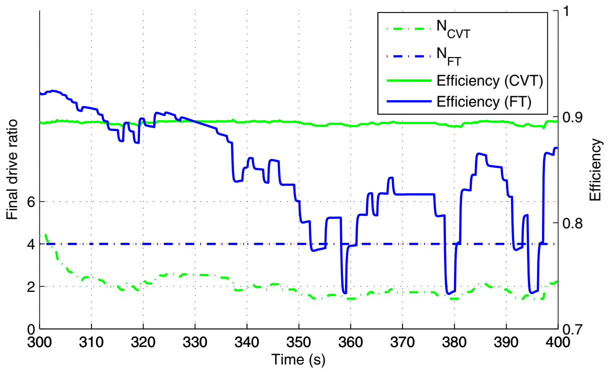

Fig. 7. Variations of the final drive ratio N, and consequent changes in efficiency (of motor and transmission system combined), for vehicle systems using either CVT or FT when simulating the extra-urban driving cycle HWFET.

the FT cases are enjoying the motor performing efficiently, but as the CVT has only a transmission efficiency of 93% compared to 96.7% of the FT, the latter ends up with a higher overall efficiency.

These observations hold for both the NYCC and the HWFET driving cycles. However, there are some differences as well. Generally the NYCC is a more erratic driving cycle in the sense that urban driving involves more changes over time compared to highway driving. Consequently, the significant changes in NYCC will favour the CVT more than the HWFET compared to a well-tuned FT. Another key difference exists in that urban driving typically requires high N values due to sharp accelerations at low speeds, while highway driving typically requires low N values due to cruising at high speeds. As a single fixed N has to be chosen for the FT, the performance will have to be compromised for both urban and highway driving. Thus while the N_{FT} value of 4 might be the optimal value for mixed driving, it is clearly suboptimal in these two specific cases: it is too low for NYCC but too high for HWFET.

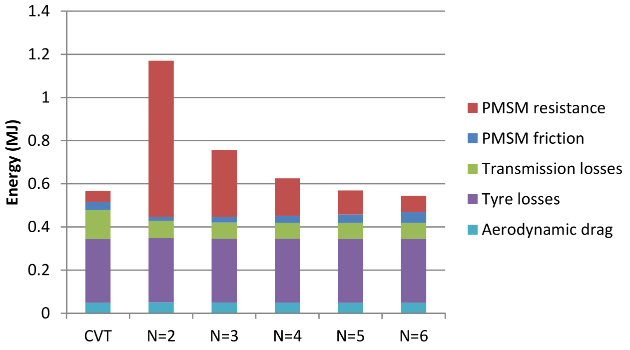

Fig. 8. Comparison of energy consumption with varying values of N for FT as well as the use of CVT, when simulating the urban driving cycle NYCC.

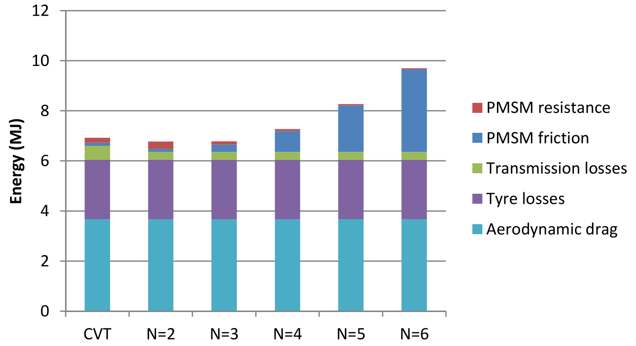

Fig. 9. Comparison of energy consumption with varying values of N for FT as well as the use of CVT, when simulating the extra-urban driving cycle HWFET.

3.3 *Overall energy*

To make a more effective comparison in overall efficiency as dependent on the transmission system it is insightful to also look at the overall energy consumption in a driving cycle rather than only analysing instantaneous efficiencies. Simulations were run for all three driving cycles with CVT as well as FT, with N_{FT} values ranging from 2 to 6. The total energy required by the motor mainly originates from the following five elements: motor resistance, motor friction, transmission losses, tyre losses and aerodynamic drag. The tyre losses are mainly due to rolling resistance but there is a contribution of slip losses as well.

The results for the NYCC and HWFET driving cycle are shown in Figs. 8 and 9 respectively. The most striking difference arises in the correlation between the N value and energy consumption. The NYCC case consumes less energy as the N_{FT} value increases, while the HWFET case consumes more energy. As was discussed previously with respect to Figs. 6 and 7, this trend is expected as urban driving often involves low speeds that are best performed using a high N value while the high speeds prefers lower N values. If the transmission losses are ignored, it is also clear that the CVT outperforms the FT for any choice of N_{FT} values for NYCC as well as HWFET. However, the higher transmission losses of the CVT eliminate this advantage as compared to high N_{FT} for the former driving cycle and low N_{FT} values for the latter. Nevertheless, if a single common N_{FT} value of 4 is chosen for both the driving cycles, then

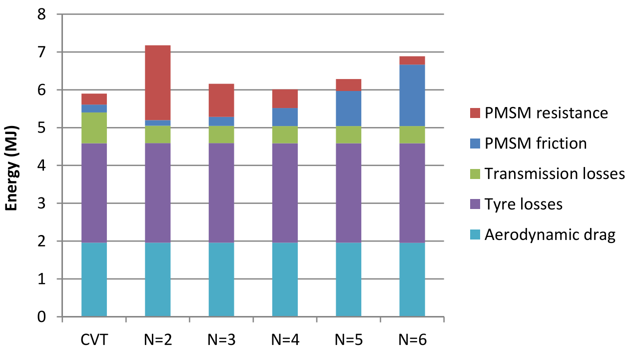

Fig. 10. Comparison of energy consumption with varying values of N for FT as well as the use of CVT, when simulating the combined driving cycle FTP-75.

the CVT outperforms the FT by 9.38% for NYCC and by 4.70% for HWFET in terms of overall motor load.

Figure 10 shows the results for the FTP-75 driving cycle, which consists of both low-speed and high-speed elements. Thus, the very low N_{FT} value case suffers heavily during the low-speed parts while the high N_{FT} value case suffers heavily from the high-speed parts. Therefore, the results appear almost like a superposition of the urban and highway results from before, producing a U-shaped trend (although distorted) with a minimum at a N_{FT} value of 4. This result suggests that the vehicle model should use this particular N value in case it uses a FT system, although it is a compromise between optimal urban and optimal highway driving. It is here that the CVT can truly demonstrate its usefulness by fully enjoying the flexibility to adapt the N value depending on driving conditions. Consequently, the CVT outperforms the FT for any selection of final drive ratio for this driving cycle. The CVT reduces the motor losses by 48.3% and reduces overall motor load by 1.80% relative to the best FT case.

It is also evident that the energy spent on propulsion is independent of transmission system, as the tyre losses and aerodynamic drag are dependent only on the speed profile of the vehicle. Consequently, it is highly dependent on the choice of driving cycle. As could be expected, the propulsion energy when driving the NYCC driving cycle is primarily spent on overcoming tyre losses, while the HWFET driving cycle has a large component of aerodynamic drag, which becomes only significant at high speeds.

While the plot (in Fig. 8) showing the energy for NYCC might suggest that the motor losses are significantly higher than the “useful” output energy when compared to HWFET, this is mainly due to the fact that during regenerative braking the propulsion energy is subtracted while the losses are still added. Also, the aerodynamic losses are significantly higher at higher speeds. Consequently, urban driving can expect quite low propulsion energy relative to the losses.

Another noteworthy feature is the balance between motor losses through resistance and friction. The nature of these losses are such that friction losses increase with higher rotor speed (and thus with a higher gear ratio) while the resistive losses decrease. If the rotor speed can be used at its optimal point, by optimizing the gear ratio, the magnitude of the friction and resistance losses can be expected to be similar. When the motor is driven by a lower-than-optimal N value, then the motor suffers from slightly lower friction losses but significantly higher resistance losses, and vice versa for higher-than-optimal N values. Consequently, in a driving cycle such as NYCC where optimal N is typically quite high, the resistance losses are a lot more significant than friction losses for N value between 2 and 6, and the opposite is true for HWFET. The FTP-75 driving cycle follows the same pattern but as the optimal N is at 4, it is possible to observe both the high resistance and the high friction type of operation at each end of the gear ratio spectrum. Note also that the balance between resistance and friction losses is almost even for the CVT for all driving cycles as it has low losses in both categories continuously.

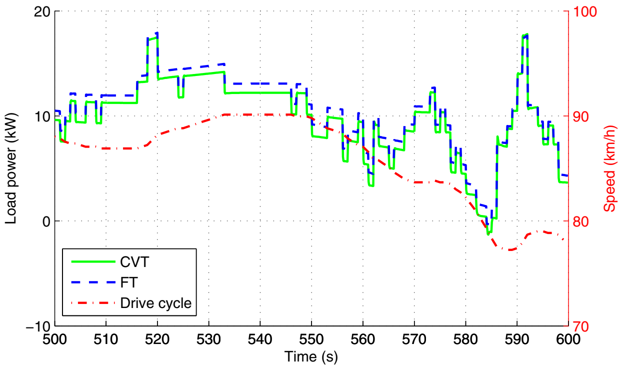

Fig. 11. Comparison of motor load requirements depending on the choice of transmission system (with $N_{FT} = 4$). The varying load requirement is shown for a window of the HWFET driving cycle, which is also shown.

It is important to emphasise that while the energy flow in the motor and transmission are directly affected by the choice of transmission, they are completely independent from the choice of supervisory control system (SCS) due to the series topology of the powertrain. The scope of the SCS is to make appropriate decisions of how to use the battery and engine-generator set to meet the load demanded by the motor. Thus, the choice of transmission affects the energy efficiency from the motor to the wheels while the SCS affects the energy efficiency from the energy sources to the motor. Figure 11 shows the effect of transmission choice on the instantaneous load required by the motor through a driving cycle. Notably, the motor typically requires less power when using the CVT and is also able to generate more power during regenerative braking.

4. CONCLUSIONS

A CVT with an intuitive control has been investigated, comparing its performance to an FT in a series HEV for the NYCC, HWFET and FTP-75 driving cycles. Simulations showed that the use of FT leads to very high PMSM friction losses during low-speed driving and very high PMSM resistance losses during high-speed driving, while a CVT keeps both losses at a low level for any type of driving. This is accomplished by operating the PMSM close to its optimal point in terms of rotor speed and torque. Although these significant reductions of motor losses (48.3%) are almost eliminated by the increase in transmission losses, the overall energy consumption of the motor is shown to be reduced by up to 9.38%, thus leading to significant fuel savings. The results are mainly limited by the simplistic modelling of the CVT, in particular the constant efficiency, which will be addressed in future work.

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