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System Identification and Distributed Control for Multi-rate Sampled Systems

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Abstract: System outputs with different sampling times may cause difficulties in subspace identification to obtain an accurate model when some system variables are sampled at faster rate. This identification problem is solved by dividing the multi-rate sampled system into different subsystems, and multi-rate distributed control is proposed to control such system by using the identified model.

Keywords: Distributed control; Multi-rate sampled system; System identification; Lifted model; Model predictive control.

1. INTRODUCTION

The controlled variables of chemical processes may have different sampling times, creating multi-rate sampled systems. For example, concentration measurements in Tennessee Eastman (TE) challenge problem (Downs and Vogel, 1993) are sampled every 6 or 15 minutes, while others measurements are continuous variables. Simply applying subspace identification to a multi-rate sampled system at basic rate (the greatest common factor of different sampling rates) may yield poor prediction results for the variables with large sampling times. A better approach for identification of multi-rate systems is based on a lifted model, in which the system inputs and outputs with slower sampling rates are lifted to a basic (fastest) rate, which will generate larger dimensions of inputs and outputs for system models. Lifted model can be used in control in two ways: direct use of lifted model in subspace predictive control (Qin et al., 2009), and to convert the lifted model to the basic-rate model which is used in regular model predictive control (MPC) (Li, 2001). The limitation of the first approach is the increase in dimensionality of the model due to lifting, and the worst situation happens when some of the variables have much slower sampling rates compared to the fast rate. For the second approach, the inaccuracy of the lifted model may cause noticeable error of basic-rate model when extracting it from the lifted model.

We propose a new approach to solve the control problem of multi-rate sampled systems that still utilizes the lifted model, but we leverage the advantages of distributed control techniques. The key is to let the local controllers communicate with each other and generate the optimized inputs sequences, which guarantee the global stability and (sub)optimality, similar to the concepts of distributed MPC (DMPC) and feasible cooperative MPC (FC-MPC) (Venkat et al., 2007). Extension of DMPC to nonlinear systems, such as nonlinear DMPC based on Lyapunov-based MPC, follows similar information exchange mechanism as FC-MPC (Liu et al., 2009).

For a multi-rate sampled system, the system outputs will be assigned to different subsystems based on their sampling times (in each subsystem, all the controlled variables have the same sampling time, but it is not necessary to have all the variables that have the same sampling time in a single subsystem). Then, only input lifting (to basic rate) is required when identifying the lifted model of one subsystem, with the output unchanged, which would further reduce the dimensions of the subsystem models. For the model of a subsystem, the influences from neighbor subsystems also need to be considered in order to develop distributed control. Thus the system identification should also include the related inputs of neighbor subsystems as inputs to this subsystem. Subspace identification will be utilized to obtain the state space model for each subsystem. After the distributed models under different sampling times are available, DMPC is proposed to deal with the control problem for the multi-rate system.

2. LINEAR DISTRIBUTED MODEL PREDICTIVE CONTROL

For a linear system described by the state space model

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) \end{aligned} \qquad (1)$$

the quadratic objective function with prediction horizon N can be written as

$$\Phi(k) = \sum_{t=k}^{k+N-1} (y(t|k)^T Q y(t|k) + u(t|k)^T R u(t|k)) + y(k+N|k)^T \overline{Q_f} y(k+N|k) \qquad (2)$$

In distributed control, the system is divided to subsystems, and each subsystem should consider the inputs from other subsystems. The linear model for subsystem i is

$$x_i(k+1) = A_i x_i(k) + B_i u_i(k) + \sum_{j \neq i}^{M} W_{ij} u_{ji}(k)$$
$$y_i(k) = C_i x_i(k) \quad (3)$$

where u_{ji} is a subset of u_j (the inputs in subsystem j) and it includes all the inputs that would affect y_i (outputs in subsystem i) in u_j, and W_{ij} is the corresponding input matrix of u_{ji}.

The local objective function of subsystem i is given in the classical quadratic form

$$\Phi_i(x_i(k), \mathbf{u}_i) := \sum_{j=0}^{N-1} (x_i(k+j)^T C_i^T Q_i C_i x_i(k+j) + u_i(k+j)^T R_i u_i(k+j)) + x_i(k+N)^T C_i^T \overline{Q_i} C_i x_i(k+N) \quad (4)$$

where Q_i, R_i, S_i, $\overline{Q_i}$ are the weights of outputs, inputs, the change of inputs, and the outputs at final stage, respectively.

The overall objective function is the weighted sum of the local objective functions, and the local optimization is to minimize the overall objective function

$$\min_{u_i} \sum_{m=1}^{M} w_m \Phi_m([\mathbf{u}_1, ..., \mathbf{u}_{i-1}, \mathbf{u}_i, \mathbf{u}_{i+1}, ..., \mathbf{u}_M]; x_m(k)) \quad (5)$$

by assuming the inputs from other subsystems to be constant. In (5), $u_j, j \neq i$ is the previous optimization results, which can be from previous iteration or previous time step; w_m is the weight with $\sum_{m=1}^{M} w_m = 1$. There could be multiple iterations in each time step, and the local controllers communicate with each other and update the newly generated optimized inputs sequence $\mathbf{u}_i$ after each iteration. It is proved to be stable for any number of iterations, and also it is proved that when the number of iteration approach infinity, optimality could be achieved similar to centralized MPC (Venkat et al., 2007).

3. DISTRIBUTED MULTI-RATE SYSTEM IDENTIFICATION

Consider a system with basic (fastest) sampling rate T, and a subsystem with sampling time nT with basic-rate state space model described in (1). The lifted model is:

$$x(k+n) = \underline{A} x(k) + \underline{B}\ \underline{u}(k)$$
$$y(k) = C x(k) \quad (6)$$

where

$$\underline{A} = A^n$$
$$\underline{B} = [A^{n-1}B\ A^{n-2}B\ \dots\ B]$$
$$\underline{u}(k) = [u(k)^T\ u(k+1)^T\ \dots\ u(k+n-1)^T]^T \quad (7)$$

where the underline indicates the lifted form. The lifted model describes the relation between the current state and the state after nT, which is the sampling time of the controlled variables. It considers all the inputs between two consecutive sampling times at time interval of T, which is the basic rate of the whole discrete-time system.

For the distributed multi-rate sampled system, the lifted model for the distributed model (3) is:

$$x_i(k+n_i) = \underline{A_i} x_i(k) + \underline{B_i}\ \underline{u_i}(k) + \sum_{j \neq i}^{M} \underline{W_{ij}}\ \underline{u_{ji}}(k)$$
$$y_i(k) = C_i x_i(k) \quad (8)$$

where $\underline{W_{ij}}$ is the lifted form of W_{ij} (similar to $\underline{B}$ in (7)).

The above model for each subsystem could be obtained with standard subspace identification techniques. Pseudo random ternary sequence (PRTS) signals are sent to manipulated variables (MV) to stimulate the process (Juricek et al., 2001), and model identification is carried out after the data generation procedure is completed. To model a subsystem, its measurements at their sampling time are taken as outputs, and both its own inputs and the inputs of other subsystems at basic rate between two consecutive sampling times are taken as inputs. Regular distributed state space model and subspace identification are utilized for subsystems whose controlled variables have basic-rate sampling time.

4. MULTI-RATE DISTRIBUTED CONTROL

Using the model (either normal state space model or lifted model) for each subsystem, distributed control allows each local controller to optimize its own objective function and determine its own control actions, while the communications and iterations between local controllers ensure a suboptimal solution of the global objective function and also guarantee the stability of the control system.

4.1 Multi-rate Model Predictive Control

Consider the use of lifted model on a multi-rate centralized system, in which the sampling time of the controlled variables are integer times of basic rate, and its controller is assumed to update at basic-rate. Although this makes no sense for improving performance, it provides a baseline for assessing the performance of the distributed case.

For regular linear MPC with quadratic objective function in (2), the objective function could be converted to a quadratic form under the constraints of (1) as (Muske and Rawlings, 1993):

$$\Phi(k) = (u^N(k))^T H u^N(k) + 2(u^N(k))^T G x(k) \quad (9)$$

The first challenge for the multi-rate system would be the selection of an appropriate prediction horizon for the lifted model. At time k, the prediction horizon should include all sampling times in the range k to $k+N-1$. The prediction horizon is determined by $N^m = floor((N-2)/n+2)$.

Assuming that the first sampling time is at k_0, define $s(k) = floor((k-k_0)/n) * n + k_0$, which stands for the sampling time of current value. Then, using prediction horizon N for regular MPC, the objective function at $s(k)$ would be:

$$\Phi^s(s(k)) = \sum_{j=0}^{N^m-1}(y(k+jn|k)^T Q^m y(k+jn|k) + \underline{u}(k+jn|k)^T R^m \underline{u}(k+jn|k)) + y(k+N^m n|k)^T \overline{Q_f^m} y(k+N^m n|k) = (u^{N^m n})(s(k))^T H^m u^{N^m n}(s(k)) + 2(u^{N^m n}(s(k)))^T G^m x(s(k)) \quad (10)$$

where the superscript s stands for sampling time, Q^m, R^m, and $\overline{Q_f^m}$ are the penalty matrices for outputs, inputs, and terminal outputs, respectively, for the multi-rate system described by the lifted model. $u^{N^m n}$ is the input sequence with the length of $N^m n$.

Conduct the following partition:

$$H^m = \begin{bmatrix} H_{11}^m & H_{12}^m & H_{13}^m \\ H_{21}^m & H_{22}^m & H_{23}^m \\ H_{31}^m & H_{32}^m & H_{33}^m \end{bmatrix}$$
$$G^m = \begin{bmatrix} H_1^m \\ H_2^m \\ H_3^m \end{bmatrix}$$
$$u^{N^m n} = \begin{bmatrix} u_1^{N^m n} \\ u_2^{N^m n} \\ u_3^{N^m n} \end{bmatrix} \quad (11)$$

where $u_1^{N^m n} = [u(s(k)), \cdots, u(k-1)]$, $u_2^{N^m n} = [u(k), \cdots, u(k+N-1)]$, and $u_3^{N^m n} = [u(k+N), \cdots, u(s(k)+N^m n-1)]$. $u_1^{N^m n}$ contains inputs from the most recent sampling time to the current time (not included), $u_2^{N^m n}$ contains inputs that start at the current time (included) and last for the time of control horizon, and $u_3^{N^m n}$ contains the rest of the inputs. The decomposition of H^m and G^m corresponds to the partition of $u^{N^m n}$. At time k, $u_1^{N^m n}$ is known, $u_2^{N^m n}$ is the control sequence for multi-rate MPC (the decision variables), and $u_3^{N^m n}$ is a zero vector. Then,

$$\Phi^s(s(k)) = (u_2^{N^m n})^T H_{22}^m u_2^{N^m n}(s(k)) + 2(u_2^{N^m n}(s(k)))^T (G_2^m x(s(k)) + \frac{1}{2}(H_{12}^{mT} + H_{21}^m) u_1^{N^m n}(s(k))) + constant \quad (12)$$

Define the objective function for the multi-rate MPC at time k:

$$\Phi(k) = \sum_{t=k}^{k+N-1}(y_{samp}(t|k)^T Q y_{samp}(t|k) + u(t|k)^T R u(t|k)) + y_{samp}(k+N|k)^T \overline{Q} y_{samp}(k+N|k) \quad (13)$$

where the sampled value of outputs is evaluated instead of their real values obtained from the simulation with the model or process data collected at high frequency, which is the major difference from the regular MPC. Now let $Q = Q^m/n$, $R = R^m$, $\overline{Q_f} = \overline{Q_f^m}$, and also set $u_3^{N^m n}$ be zero vector as above, then

$$\Phi(k) = \Phi^s(s(k)) + constant \quad (14)$$

therefore, since $u^N \equiv u_2^{N^m n}$,

$$u^N(k) = arg \min_{u^N} \Phi(k) = arg \min_{u_2^{N^m n}} \Phi^s(s(k)) \quad (15)$$

Then, (15) converts the control problem of the multi-rate sampled system to a control problem at its sampling times.

4.2 Multi-rate Distributed Model Predictive Control

The objective function for subsystem i is defined as

$$\Phi_i(k) = \sum_{t=k}^{k+N-1}(y_{samp,i}(t|k)^T Q_i y_{samp,i}(t|k) + u_i(t|k)^T R_i u_i(t|k)) + y_{samp,i}(k+N|k)^T \overline{Q_{f,i}} y_{samp,i}(k+N|k) \quad (16)$$

similarly to (13), where subscript i denotes the ith subsystem, and the objective function would be minimized subject to constraints of the system model (8). When generating H_i^m and G_i^m in (10), let $[u_i^T, u_{1i}^T, \ldots, u_{(i-1)i}^T, u_{(i+1)i}^T, \ldots, u_{Mi}^T]^T$ and $[B_i, W_{i1}, \ldots, W_{i(i-1)}, W_{i(i+1)}, \ldots, W_{iM}]$ be the input vector and its corresponding system matrix (for the multi-rate case, both are changed to the form of lifted model), and let the input penalty matrix be $diag([R, 0_{1i}, \ldots, 0_{(i-1)i}, 0_{(i+1)i}, \ldots, 0_{Mi}])$, where 0_{ji} is the square zeros matrix with the number of rows (or columns) equal to the length of u_{ji}.

The overall objective function is the weighted sum of local objective functions:

$$\Phi = \sum \omega_i \Phi_i, \qquad with \quad \Sigma\omega_i = 1 \quad (17)$$

The centralized solution can be obtained from:

$$u^N(k) = [u_1^N(k)^T, \cdots, u_M^N(k)^T]^T = arg \min_{u^N} \sum \omega_i \Phi_i(k) = arg \min_{u^N} \sum \omega_i \Phi^s(s(k)) = arg \min_{u^N} (u^N(k)^T H^c(k) u^N(k) + 2u^N(k)^T G^c(k) x(k)) \quad (18)$$

The centralized quadratic optimization matrices H^c and G^c are calculated by summation of corresponding local optimization matrices in (12).

The next step is to decompose the overall objective function back to local objective functions. For subsystem i, let $u_i^N = \Lambda_i u^N$, then Λ_i is a matrix with the corresponding columns of the identity matrix. Define $\overline{u_i^N}$ to be the inputs of other subsystems, $\overline{\Lambda_i}$ to be the other columns of the identity matrix. Assume that the current states and inputs of other subsystems are known. Then, the overall objective function is defined as the new local objective function:

$$\Phi_i^l(k) = u_i^N(k)^T \Lambda_i H^c(k) \Lambda_i^T u_i^N(k) + 2u_i^N(k)^T [\Lambda_i \frac{1}{2}(H^c(k) + H^{cT}(k)) \overline{\Lambda_i}^T \overline{u_i^N}(k) + \Lambda_i G^c(k) x(k)] \quad (19)$$

that can be optimized locally by local controllers, and the input or output constraints can be enforced to the local optimization, which is the same as standard MPC.

With the distributed optimization above, the multi-rate distributed MPC uses the same communication and cooperation structure as the other distributed MPC algorithms in literature such as Venkat et al. (2007), in which multiple iterations may be conducted in each time step to improve the optimality. The stability in the sense of Lyapunov is guaranteed if Φ is positive-definite, and

$$\begin{aligned}\Phi_q(k+1) &\leq \Phi_0(k+1)\\ &= \Phi_q(k) - \sum_{i=1}^{M}(x_i^T(k)Q_i x_i(k)\\ &\quad + {u_{i,q}^N}^T(k|k)(1)R_i u_{i,q}^N(k|k)(1))\\ &\leq \Phi_q(k)\end{aligned} \tag{20}$$

where q indicates the number of iterations.

5. CASE STUDY

To demonstrate the effectiveness of the proposed method, two case studies are presented. The first is a linear system artificially sampled in multi-rate, and the second is the classical TE challenge problem.

5.1 Distillation Column

A distillation column model (Venkat et al., 2007), with two manipulated variables (vapor boilup flowrate and reflux flowrate, respectively) and two controlled variables (temperatures of tray 27 and 7), is used to develop a system with two subsystems (described by basic-rate system matrices), each subsystem has one input and one output.

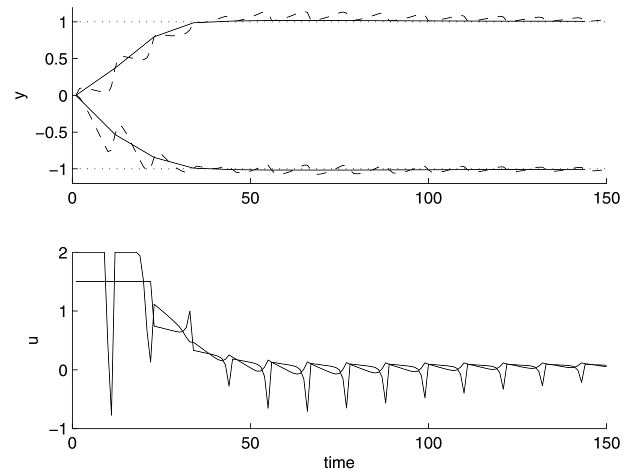
Different sampling times are artificially assigned to controlled variables, and the lifted model is constructed for each subsystem by using (7). Pretending that only the lifted models are known, and this system is controlled by distributed control. The control objective is to make target tracking with a new setpoint $y_t = [1, -1]^T$, under the hard constraints for inputs $-1.5 \leq u_1 \leq 1.5$ and $-2 \leq u_2 \leq 2$.

Before examining the performance of multi-rate distributed control, the multi-rate centralized control is studied to evaluate MPC by lifted model. Make a centralized model for centralized control:

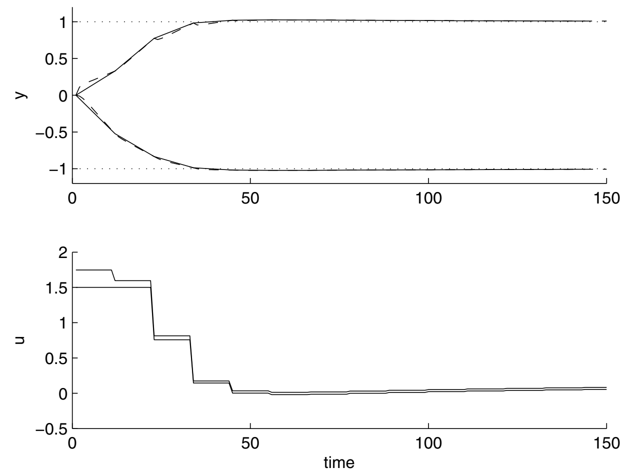
$$A_c = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \qquad B_c = \begin{bmatrix} B_1 & W_{12} \\ W_{21} & B_2 \end{bmatrix} \qquad C_c = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} \tag{21}$$

Set the sampling time to $n_c = 11$, and construct the lifted model for the centralized control system, using MPC with quadratic objective function. The results in Fig. 1(a) indicate that the basic-rate response (dashed line) has many fluctuations because the lifted model has no information about the intermediate status other than at sampling points. The solid line shows the sampled value, and a smooth response is obtained as expected. For the input side, the periodical saw-tooth shape has the same period as sampling, and it is an aggressive behavior for this system with fast dynamics.

The fluctuations for both outputs and inputs are always observed for control with lifted model, however it may not be so severe in systems with slower dynamics. In case such fluctuations are not desirable, zero-order hold can be added as the constraints for the inputs, and the stability is still ensured since (20) still holds. The zero-order hold for inputs can be realized by adding the following equality constraints:



(a) Without zero-order hold.



(b) With zero-order hold.

Fig. 1. Results of centralized multi-rate control. For outputs, the dashed line shows basic-rate response, and the solid line shows the measurements at sampling times.

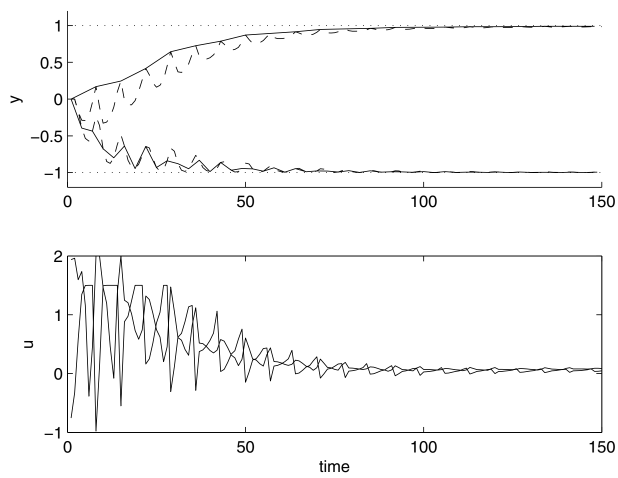
$$\begin{bmatrix} Z & & 0 \\ & \ddots & \\ 0 & & Z \end{bmatrix} u^N = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} \tag{22}$$

The number of Z in the diagonal of the left matrix is N^m, and the dimension of Z is $(n-1) \times n$, where n is the sampling steps.

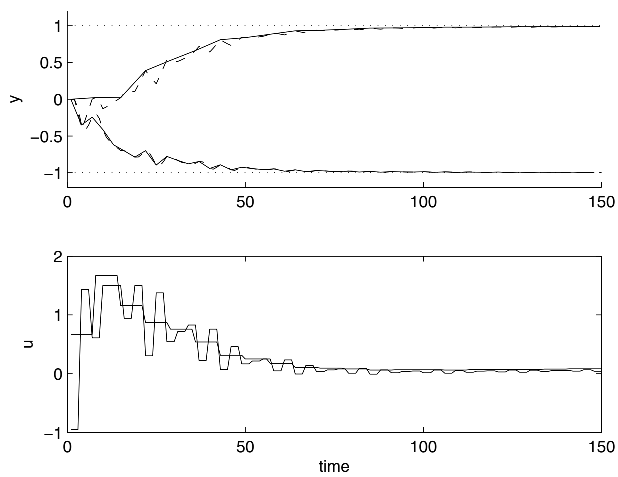
$$Z = \begin{bmatrix} 1 & -1 & 0 & \dots & 0 \\ 1 & 0 & -1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \dots & -1 \end{bmatrix} \tag{23}$$

The control results for centralized control with zero-order hold (Fig. 1(b)) show a smooth response.

To illustrate distributed control, the sampling time of two subsystems are set to $n_1 = 3$ and $n_2 = 7$. Take 10 iterations (maximum q equals to 10) at each sampling time. The results with and without zero-order hold (Fig. 2) are similar but worse responses are obtained in both cases compared to centralized control, and even the zero-order hold constrained case shows some fluctuations. These results are mainly because of the different sampling times of two subsystems, and not distributed control, since 10 iterations could generate results that are close enough to the centralized control, from the study of basic-rate distributed control on this system.



(a) Without zero-order hold.



(b) With zero-order hold.

Fig. 2. Results of distributed multi-rate control. For outputs, the dashed line shows basic-rate response, and the solid line shows the sampling measurements.

The formulation of zero-order hold constraints is different from the centralized control case. Denote the equality matrix in (22) as Ξ, and make the same partition as (11),

$$\Xi = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} \\ \Xi_{21} & \Xi_{22} & \Xi_{23} \\ \Xi_{31} & \Xi_{32} & \Xi_{33} \end{bmatrix} \quad (24)$$

then the constraints become:

$$\Xi_{22} u_2^{N^m n} = -\Xi_{11} u_1^{N^m n} \quad (25)$$

and $u_2^{N^m n}$ is actually the decision variable in the optimization described in (15), while $u_1^{N^m n}$ is known.

A comparison of distributed multi-rate control with zero-order hold, without zero-order hold, and centralized basic-rate control is given in Fig. 3, in which only sampled values are plotted. The centralized basic-rate control has the best control results, and control without constraints generates slightly better response compared to the constrained case as expected, at the expense of more fluctuations.

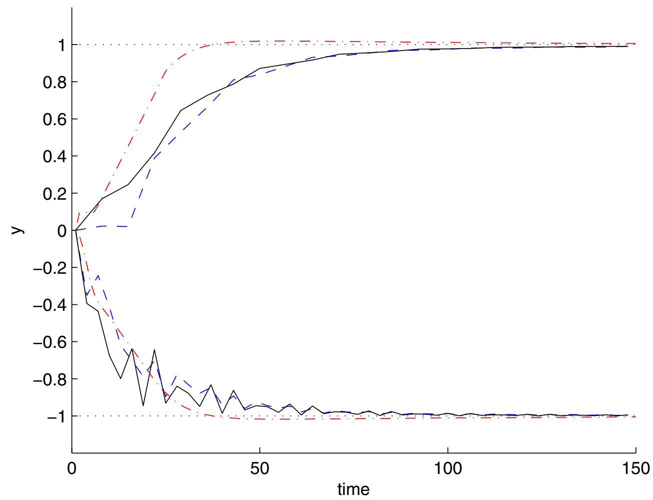


Fig. 3. Comparison of different control method. The solid black line is the response of distributed multi-rate control without zero-order hold, the dashed blue line is the response of distributed multi-rate control with zero-order hold, and the dash-dotted red line is the response of centralized control for *basic rate*.

5.2 *Tennessee Eastman Challenge Problem*

The main target in the TE challenge problem is to maintain the product flowrate and composition (Downs and Vogel, 1993). The process contains three main operation units: a reactor, a separator, and a stripper. It has total 41 measured variables and 12 MVs, four reactants labeled as A, C, D, and E, two products labeled as G and H.

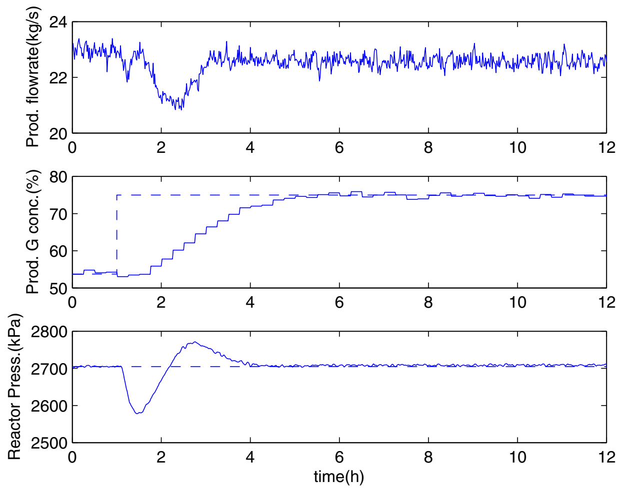
Since product concentration is directly related to control objective, G concentration in product and the product flowrate are chosen as the controlled variables (CV). Also the reactor pressure is very sensitive to changes in process operations, which may cause safety issues, so it is considered as another controlled variable. For the MVs, several factors affect the concentration of G in the product. The feed of D is the material that forms product G, and E forms product H and thus will also influence the ratio of G, therefore, D and E feed are chosen as two MVs. Moreover, the reactor pressure control is tricky because it is too sensitive to a couple of variables, including D feed and E feed. When we conduct the concentration control and D and E feeds change, the reactor pressure will be affected significantly. Initially, the purge flowrate was picked as MV for the reactor pressure, however two problems were raised: 1. The dynamic between purge flowrate and reactor pressure is not fast enough to cover dramatic changes in reactor pressure. 2. The maximum purge flowrate cannot compensate very large changes in reactor pressure. Thus another MV (the reactor temperature) is added to help reactor pressure control. The response of reactor pressure to changes in reactor temperature is fast, but large temperature changes could cause instability and this drawback must be dealt with carefully during control.

For distributed control, the system is partitioned to 2 subsystems according to their sampling rates. The reason is that subspace identification yields poor prediction if all the CVs are sampled at fast rate while applying zero-order hold on the controlled variable with slower sampling rate. Product flowrate and reactor pressure are in subsystem 1, and G concentration in product is in subsystem 2. The MVs are also classified: D feed is in subsystem 2 because it is the main MV that controls G concentration in product, and the other 3 MVs are in subsystem 1.

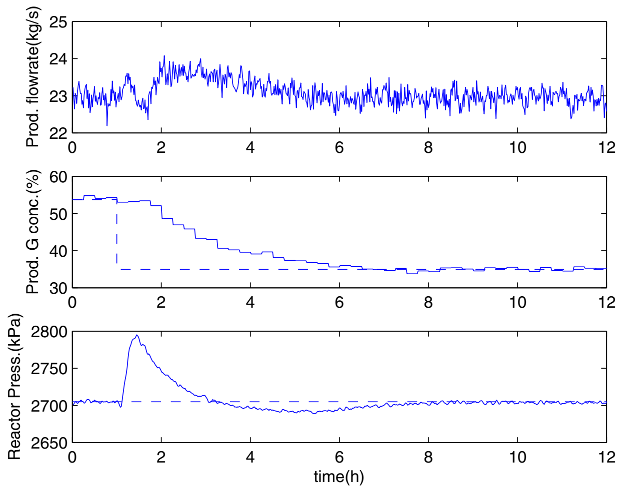
The system model is obtained by subspace identification, then multi-rate control is applied with the identified model. Illustrative results (Fig. 4) show the effects of changes in setpoint of G concentration in product from 50% to 75% and 35%. Offset-free control is applied on the reactor pressure due to the nonlinearity of reactor pressure with respect to the MVs. The identified model and the proposed method track the setpoint change successfully.

6. CONCLUSIONS

In this paper we proposed a system identification and distributed control method to control multi-rate sampled systems, which is challenging to regular subspace identification. The key idea is to convert the multi-rate control problem to a distributed control problem, where lifted model is used to deal with the subsystems with slower sampling rates, and the local MPC based on lifted model is converted to a standard form to fit in the distributed control structure. The proposed method is illustrated with an artificial multi-rate linear system and the TE challenge problem. The results indicate that the proposed method improves the control of multi-rate sampled systems.



(a) Product G concentration to 75%.



(b) Product G concentration to 35%.

Fig. 4. Distributed control for target tracking, product G concentration is changed to 75% and 35%.

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