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Continuum Evolution of a System of Agents with Finite Size

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Abstract: In this paper, considered is the evolution of a multi agent system (MAS) where every agent of the MAS is a ball in R^n with radius ε. Evolution of the MAS occurs with $n+1$ agents, called leader agents, moving independently, with rest of the agents of the MAS, called follower agents, updating their positions through communication with $n+1$ local neighboring agents with weights of communication of follower agents assigned based only on the initial positions of agents of the MAS. When weights of communications are all positive, it is assured that final formation of the MAS is a homogenous transformation of its initial configuration. During transition however, the follower agents will deviate from the state specified by the homogenous transformation. This deviation from the state corresponding to that of the homogenous transformation during transition from the initial configuration to the final configuration can be controlled by imposing a limit on leaders' velocities. This velocity limit depends on (i) the norm of the network matrix (specified based on initial positions of the agents), (ii) maximum allowable deviation from the state of homogenous transformation, and (iii) a control parameter g. Thus, if the velocities of the leader agents don't exceed an assigned maximum value, deviation of followers from the state of homogenous transformation is limited to a maximum value throughout the transient motion.

Keywords: Homogenous Transformation, Local Inter-Agent Communication, Asymptotic Convergence, Deviation from Desired Formation.

1. INTRODUCTION

Formation control in a network has many applications in areas such as gaming, terrain mapping, transportation engineering, formation flight, etc. (*Murray, 2008*). Common approaches for formation control of MAS are leader-follower (*Consolinia et. al, 2008; Vidal et. al, 2004*), virtual structures (*Ren and Beard, 2004; Wang and Schuab, 2011*), behavioral based (*Balch and Arkin., 1998*), artificial potential function (*Roussos and Kyriakopoulos, 2010; Gerdes and Rossetter, 2001*), consensus algorithm (*Olfati-Saber et. Al, 2004; Olfati-Saber et. Al, 2007; Qu, 2009; Chebotarev P., 2010*), PDE based (*Ghods and krstic, 2012; Frihauf P. and Krstic, 2010; Frihauf P. and Krstic, 2011; Kim et. al., 2008*) and containment control (*Ji et. al, 2008; Cao and Ren, 2009; Cao et. al, 2011; Wang et. Al, 2012; Lin et. al, 2013*). Consensus algorithm, containment control and PDE are the interesting approaches that apply Laplacian control to issue a global coordination under local communication. Although, asymptotic convergence of an initial formation to a desired final formation are assured by choosing positive weights of communication in these three methods, inter-agent collision avoidance and boundedness of the followers during transition are not necessarily guaranteed.

Recently, the authors proposed a new continuum based approach for formation of MAS evolving in an n-D space (*Rastgoftar and Jayasuriya, 2012; Rastgoftar and Jayasuriya, 2013 a, b, c and d; Rastgoftar and Jayasuriya, 2014 a, b and c*) to address the aforementioned issues. We showed how inter-agent collision can be avoided by choosing weights of communication consistent with the initial configuration of the swarm. We considered MAS as a continuum which transforms under a specific deformation mapping, called a homogenous transformation. It has been shown that homogenous transformation of a MAS, in an *n-D* space, can be prescribed based on position vectors of $n+1$ agents of the MAS, called leader agents, that evolve independently. In (*Rastgoftar and Jayasuriya, 2012; Rastgoftar and Jayasuriya, 2013 a and b; Rastgoftar and Jayasuriya, 2014 a*), designed is a homogenous transformation based solely on leaders' positions thereby defining the evolution of the rest of the agents of the MAS, called followers. Consequently, MAS evolution is achieved with zero inter agent communication. In (*Rastgoftar and Jayasuriya, 2013 a, b, c and d; Rastgoftar and Jayasuriya, 2014 a and b*), evolution of a MAS is prescribed based on independent evolution of leader agents, while follower agents update their positions through local communication with $n+1$ neighbouring agents. It is assured that final formation of the MAS is a homogenous transformation of the initial configuration.

Although, in (*Rastgoftar and Jayasuriya, 2013 a, b, c and d; Rastgoftar and Jayasuriya, 2014 a*) it is guaranteed that final formation is a homogenous transformation of the initial configuration, the follower agents of the MAS deviate from the reference state defined by the homogenous transformation during transition. In this paper, we address this limitation by treating every follower agent of the MAS as a ball, in *n-D* space, whose maximum deviation from the desired state, prescribed by a homogenous transformation,

has a limit. It results in velocities of the leader agents being limited to a maximum value. We develop a formulation for maximum velocities of leader agents depending on (i) maximum allowable deviation of follower agents from state of homogenous transformation, (ii) the norm of the network matrix, and (iii) a control parameter g which will be defined in the sequel.
The paper is organized as follows: In section 2 basics of homogenous transformation protocol is described. In section 3, dynamics of MAS evolution under homogenous transformation protocol is formulated. Design of leaders' paths subjected to maximum velocities of the leader agents is presented in section 4. Simulations of formation control of a MAS moving in the plane are presented in section 5 followed by concluding remarks in section 6.

2. COMMUNICATION PROTOCOL

Let a MAS consist of N agents evolving in R^n, where agents *1, 2, …, n+1* are considered the *leader* agents that evolve independently, such that their positions, $\boldsymbol{r}_i(t)$, satisfy the following rank condition:

$$\forall t \geq t_0, \text{Rank}\,\{\boldsymbol{r}_1(t), \boldsymbol{r}_2(t), \dots, \boldsymbol{r}_{n+1}(t)\} = n \tag{1}$$

and rest of the agents called *follower* agents, numbered *n+1, n+2, …, N,* update their positions through local communication with *n+1* neighbouring agents. The communication topology and weights of communication considered for follower agents are given below:

2.1 Communication Topology

Communication topology of MAS evolution under homogenous transformation protocol is shown by a graph $G = \mu \cup \partial\mu$, where *n+1* nodes belonging to boundary $\partial\mu$ of G, represent leader agents and rest of the nodes of G belong to the sub-graph μ representing follower agents. In this regard, every leader agent is attached to one of the followers by an arrow terminating on the follower. This implies that every leader agent evolves independently, but is tracked by at least one of the nearby follower agents of the MAS. Every node belonging to sub-graph μ is connected to *n+1* agents belonging to G which in turn implies that every follower agent communicates with *n+1* neighbouring agents. It is noted that, communication between two follower agents is shown by a simple edge which indicates bi-communication between those agents. A sample communication graph of a MAS consisting of 10 agents moving in a plane is shown in Fig. 1.

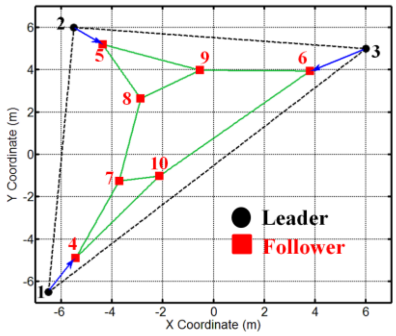

Fig. 1. Sample communication graph of a MAS moving in a plane

2.2 Weights of Communication

Suppose that agent i communicates with agents i_1, i_2, …, i_{n+1} to update its position, with $\boldsymbol{r}_{i_d}(t)$, called the desired transient position of agent *i*, when the set of transient position vectors $\{\boldsymbol{r}_{i_d}(t),\ \boldsymbol{r}_{i_1}(t), \boldsymbol{r}_{i_2}(t), \dots, \boldsymbol{r}_{i_{n+1}}(t)\}$ is a homogenous transformation of initial positions of the set of agents *i,* i_1, i_2, …, i_{n+1}.
In (*Rastgoftar and Jayasuriya, 2013 a, b, c and d; Rastgoftar and Jayasuriya, 2014 a and b*), it was shown that $\boldsymbol{r}_{i_d}(t)$ can be expressed as a linear combination of position vectors $\boldsymbol{r}_{i_1}(t), \boldsymbol{r}_{i_2}(t), \dots, \boldsymbol{r}_{i_{n+1}}(t)$,

$$\boldsymbol{r}_{i_d}(t) = \sum_{k=1}^{n+1} w_{i,i_k} \boldsymbol{r}_{i_k}(t) \tag{2}$$

where $w_{i,i_k}(k = 1, 2, \dots, n+1)$ called *weights of communication* which are constants obtained by solving the following set of *n+1* linear algebraic equations:

$$\begin{bmatrix} X_{1_{i_1}} & X_{1_{i_2}} & \cdots & X_{1_{i_{n+1}}} \\ X_{2_{i_1}} & X_{2_{i_2}} & \cdots & X_{2_{i_{n+1}}} \\ \vdots & \vdots & & \vdots \\ X_{n_{i_1}} & X_{n_{i_2}} & \cdots & X_{n+1_{i_{n+1}}} \\ 1 & 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} w_{i,i_1} \\ w_{i,i_2} \\ \vdots \\ w_{i,i_n} \\ w_{i,i_{n+1}} \end{bmatrix} = \begin{bmatrix} X_{1_i} \\ X_{2_i} \\ \vdots \\ X_{n_i} \\ 1 \end{bmatrix}. \tag{3}$$

In eqn. (3), X_{k_j} denotes the *k-th* (*k=1,2,…,n*) coordinate of the initial position vector of agent *j* (*j=i,* i_1, i_2, …, i_{n+1}), where position vector of agent *j* is expressed with respect to a basis $\{\boldsymbol{e}_1, \boldsymbol{e}_2, \dots, \boldsymbol{e}_n\}$.

Remark 1: In eqn. (3), the (*n+1*)*x*(*n+1*) coefficient matrix must be invertible to obtain unique weights of communication. In (*Rastgoftar and Jayasuriya, 2013 a, b, c and d; Rastgoftar and Jayasuriya, 2014 a and b*), we showed that this condition is satisfied, if

$$\text{Rank}\,\{\boldsymbol{r}_i(t_0), \boldsymbol{r}_{i_1}(t_0), \boldsymbol{r}_{i_2}(t_0), \dots, \boldsymbol{r}_{i_{n+1}}(t_0)\} = n. \tag{4}$$

2.3 Weight Matrix W

An (*N-n-1*)*xN* matrix W is called a weight matrix which has elements given by:

$$W_{ij} = \begin{cases} w_{i+n+1,j} \neq 0 & if\ \ i+n+1 \sim j \\ -1 & if\ \ i+n+1 = j \\ 0 & Otherwise \end{cases}. \tag{5}$$

We note that the symbol "~" denotes adjacency between follower agent $i+n+1$ (*i=1, 2, …, N-n-1*) and agent *j.* With the weight matrix W partitioned as:

$$W = [B_{(N-n-1)\times(n+1)} \ \vdots \ A_{(N-n-1)\times(N-n-1)}], \tag{6}$$

the following properties of the weight matrix are apparent:

1- W is zero-sum row stochastic, i.e. sum of every row of W is zero.
2- All diagonal elements of A are *-1*.
3- Although, A is not necessarily symmetric, when $A_{ij} \neq 0$, then, $A_{ji} \neq 0$, and if $A_{ij} = 0$, then, $A_{ji} = 0$.

Remark 2: In (*Rastgoftar and Jayasuriya, 2013 a, b, c and d; Rastgoftar and Jayasuriya, 2014 a*), we proved that if weights of communication are all positive, then, partition A of the weight matrix W is negative definite.

3. MAS EVOLUTION DYNAMICS

In this section, a single integrator kinematic model is utilized for MAS evolution under the proposed communication protocol. For leader agents (*i=1, 2, ..., n+1*), trajectories are chosen such that (i) rank condition (1) is satisfied, and (ii) no two agents of the MAS get closer than minimum distance $d = 2\gamma$ ($\gamma > 0$ is a radius defining a safe zone), during MAS evolution. Before proceeding further, we rewrite $\boldsymbol{r}_{i_d}$ of the follower agents, in a more general form as:

$$\boldsymbol{r}_{i_d} = \sum_{j=1}^{N} w_{i,j}\boldsymbol{r}_j \tag{7}$$

where

$$\begin{cases} w_{i,j} \neq 0 & if\ i \sim j \\ w_{i,j} = 0 & Otherwise \end{cases} \tag{8}$$

and

$$\sum_{j=1}^{N} w_{i,j} = 1. \tag{9}$$

Remark 3: In this paper, we assume that all weights of communications are positive, and the partition A of the weight matrix W (See eqn. (6).) is negative definite.

Suppose that follower agent i (*i=n+2, n+3, ..., N*) updates its position according to

$$\dot{\boldsymbol{r}}_i = \boldsymbol{u}_i \tag{10}$$

where $\boldsymbol{u}_i$ is the velocity control input, chosen as follows:

$$\boldsymbol{u}_i = g\left(\boldsymbol{r}_{i_d} - \boldsymbol{r}_i\right) \tag{11}$$

and g is a positive control parameter.

Substituting for $\boldsymbol{r}_{i_d}$ in eqn. (11) from eqn. (7), leads to

$$\dot{\boldsymbol{r}}_i = g\left(\sum_{j=1}^{N} w_{i,j}\boldsymbol{r}_j - \boldsymbol{r}_i\right). \tag{12}$$

Equation (12) is the row *i-n-1* of the following dynamic equation:

$$\dot{Z} = g(AZ + BU) \tag{13}$$

where A and B are partitions of W, $U = [\boldsymbol{r}_1 \ \cdots \ \boldsymbol{r}_{n+1}]^T$ denotes positions of the leader agents, and $Z = [\boldsymbol{r}_{n+2} \ \cdots \ \boldsymbol{r}_N]^T$ denotes positions of follower agents. As mentioned in Remark 3, positive weight of communication results in a negative definite matrix A that assures asymptotic convergence of initial formation of the MAS to a final configuration which is a homogenous map of the initial formation.

4. MAS EVOLUTION DYNAMICS

In this section, considered is the design of trajectories of the leader agents where their initial and final positions are given. First, we formulate homogenous transformation of an *n-D* manifold based on position vectors of *n+1* points of the manifold. The paths of the leaders are to be designed such that (i) leaders don't collide with obstacles, and (ii) any two follower agents of the MAS don't get closer than a minimum distance $d = 2\gamma$ during MAS evolution.

4.1 Designing Paths of Leaders

In (*Rastgoftar and Jayasuriya, 2013 a, b, c and d; Rastgoftar and Jayasuriya, 2014 a and b*), we have shown that state of homogenous transformation of the transient configuration of the MAS can be formulated based on leaders' positions. Let $\boldsymbol{r}_{i_{HT}}(t)$ denote position of a follower agent i (*i=n+2, n+3, ..., N)* at time t, where the transient configuration of the MAS is a homogenous map of the initial formation, then,

$$\boldsymbol{r}_{i_{HT}}(t) = \sum_{k=1}^{n+1} \alpha_{i,k}\boldsymbol{r}_k(t). \tag{14}$$

It is noted that $\boldsymbol{r}_k(t)$ is the position of leader agent k (*k=1, 2, ..., n+1*) and $\alpha_{i,k}$s are the constant weights specified by eqn. (3) based on initial positions of follower agent i and leaders *1, 2, ..., n+1* i.e. the weight w_{i,i_k} is denoted by $\alpha_{i,k}$ where agents i_1, i_2, …, and i_{n+1} are all leader agents. Furthermore, $\alpha_{i,k}$ is is at row *i-n-1* and column k of $(-A^{-1}B)$. It is noted that

$$\sum_{k=1}^{n+1} \alpha_{i,k} = 1. \tag{15}$$

As shown in Fig. 2, for the case of MAS evolution in a 2-D domain, three leader agents *1, 2,* and *3* are located at the vertices of a triangle, called *leading triangle*, and followers are distributed inside the leading triangle. As seen, every follower is a disk with radius ε which is located inside a circular domain with radius $\gamma > \varepsilon$ called a *safe zone.* Our desire is that none of the follower agents leave the safe zone, throughout their evolution under local inter agent communication. In other words, deviation from state of homogenous transformation, $\left\|\boldsymbol{r}_{i_{HT}} - \boldsymbol{r}_i\right\|$, is not larger than $\delta = \gamma - \varepsilon$, or

$$\left\|\boldsymbol{r}_{i_{HT}} - \boldsymbol{r}_i\right\| \leq \delta = \gamma - \varepsilon. \tag{16}$$

Furthermore, safe zone of every follower agent must not be penetrated by other agents. This requires the distance between any two agents of the MAS to be greater than 2γ during MAS evolution.

4.2 Maximum Deviation of Followers from the State of Homogenous Transformation

We consider the follower agents to be disks with radius ε, where, they are all distributed initially in the interior of the leading line segment, of leading triangle or points of the leading tetrahedron, for *1-D*, *2-D*, and *3-D* MAS evolution, respectively. This implies that weights of communication of follower agents with respect to leaders are all positive. Also, leaders evolve in such a way that no two agents of the MAS get closer than 2γ, from the state prescribed by the homogenous transformation.

When followers evolve under local communication however, they will deviate from the state of homogenous transformation, during transition. Thus, an upper limit for the maximum deviation of follower agents is desired. The remark 4 followed by a theorem specify the maximum velocity for the leader agents in order to assure that deviation of every follower agent from the state of homogenous transformation does not exceed δ during MAS evolution.

Remark 4: If follower agents are initially placed (i) at the interior points of a leading segment with two leaders at the ends, for *1-D* MAS evolution, or (ii) inside a leading triangle whose vertices are occupied by three leader agents, for *2-D* MAS evolution, or (iii) interior points of a leading tetrahedron whose vertices are located by four leader agents, for *3-D* MAS evolution, then, initial weights of communication with respect to leaders, $\alpha_{i,k}$s, are all positive i.e. $(-A^{-1}B)$ is a positive matrix. Now, let

$$max\{\|\dot{\boldsymbol{r}}_1\|, \|\dot{\boldsymbol{r}}_2\|, \dots, \|\dot{\boldsymbol{r}}_{n+1}\|\} = V, \quad (17)$$

so,

$$\left\|\frac{d}{dt}(\boldsymbol{r}_{i_{HT}})\right\| = \left\|\sum_{k=1}^{n+1} \alpha_{i,k}\dot{\boldsymbol{r}}_k(t)\right\| \le \sum_{k=1}^{n+1} \alpha_{i,k}\|\dot{\boldsymbol{r}}_k(t)\|$$

$$\le V\sum_{k=1}^{n+1} \alpha_{i,k} = V. \quad (18)$$

Theorem: *For MAS evolution in an n-D space where the total number of agents (leaders and followers) is N and V denotes the maximum of magnitude of velocity of all leader agents, if every follower agent updates its position according eqn. (11) with a positive control parameter g, then, the deviation of follower agents from the state of homogenous transformation is not greater than*

$$\delta = \frac{\sqrt{N-n-1}\,\|A^{-1}\|}{g}V \quad (19)$$

during MAS evolution. Therefore, if every follower agent is a ball with radius ε*, then, it does not leave the safe zone which is a ball with radius* $\gamma = \varepsilon + \delta$ *centered at* $\boldsymbol{r}_{i_{HT}}(t)$*.*

Proof: Let the dynamics of the MAS (eqn. (13)) be pre-multiplied by A^{-1}, then

$$-A^{-1}\dot{Z} = g(-Z - A^{-1}BU) = g(Z_{HT} - Z) \quad (20)$$

where $Z_{HT} = [\boldsymbol{r}_{n+2_{HT}} \ \cdots \ \boldsymbol{r}_{N_{HT}}]^T = -A^{-1}BU$ denotes positions of the followers under state of homogenous transformation. Now, if eqn. (20) is pre-multiplied by $-A$, it is simplified to

$$\dot{Z} = -gA(Z_{HT} - Z). \quad (21)$$

Let us define $E = Z_{HT} - Z$ as the transient error, then the error dynamics is obtained to be

$$\dot{E} - gAE = \dot{Z}_{HT}. \quad (22)$$

Therefore, transient error

$$E(t) = e^{gAt}E(0) + \int_0^t e^{gA(t-\tau)}\dot{Z}_{HT}d\tau. \quad (23)$$

Since, weights of communication are specified based on initial configuration, so,

$$Z_{HT}(0) = Z(0) \quad (24)$$

and $E(0) = Z_{HT}(0) - Z(0) = 0$. Considering remark 4, we conclude $\|\dot{Z}_{HT}\| \le V$,

$$E(t) = \int_0^t e^{gA(t-\tau)}\dot{Z}_{HT}d\tau \le \left\|\int_0^t e^{gA(t-\tau)}\dot{Z}_{HT}d\tau\right\| \le \frac{\sqrt{N-n-1}}{g}\|A^{-1}\|\|\dot{Z}_{HT}\| \le \frac{\sqrt{N-n-1}\,\|A^{-1}\|}{g}V. \quad (25)$$

This implies that maximum deviation from state of homogenous transformation is

$$\|\boldsymbol{r}_{i_{HT}} - \boldsymbol{r}_i\| \le \delta = \frac{\sqrt{N-n-1}\,\|A^{-1}\|}{g}V. \blacksquare$$

Remark 5: Although above theorem assigns a limit δ for deviation of each follower from state of homogenous transformation, however, this deviation does not vanishes unless leaders settle. One way to reduce deviation of followers from state of homogenous transformation during transition is to designing leaders' velocities under a model that we call *Hill-Valley*. Let a leader is supposed to move on a path connecting both given initial and final positions. We design several *Hill* and *Valley* stations on the desired path where (i) the magnitude of the leader's velocity is V at the hills and zero at the valleys and (ii) tangential acceleration is zero at any hill point. Thus, length of the path connecting two consecutive hills can be represented by a polynomial as a function of time satisfying mentioned end point velocity and acceleration conditions. It is noted that we don't consider Hill-Valley model in this paper to design leaders' trajectories.

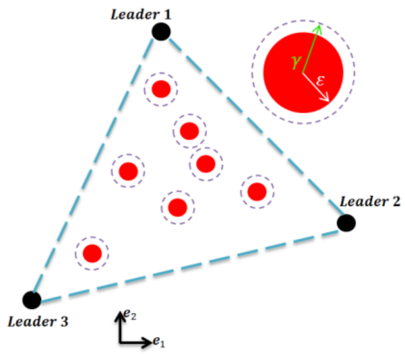

Fig. 2. State of homogenous transformation of a MAS moving in a plane

Remark 6: As it is obvious from eqn. (19), reducing deviation from state of homogenous transformation is achieved by decreasing leaders' velocities and $\|A^{-1}\|$, and increasing control parameter g. However, increasing g is costly and it is more efficient to reduce $\|A^{-1}\|$ by raising weights of communication with respect to the leader agents, for the followers connected to the leader agents. For example, for the network that is shown in Fig. 1, $w_{4,1}$, $w_{5,2}$, and $w_{6,3}$ are required to be chosen appropriately large, to reduce $\|A^{-1}\|$. As seen in Table 1, $w_{4,1}$, $w_{5,2}$, and $w_{6,3}$are considered to be *0.7*.

5. SIMULATION RESULTS

In this section, two scenarios are simulated. In the first example, leader agents of the MAS move in a plane with the same velocities. In the second case, MAS is deformed to pass through a narrow channel. In both examples, MAS consists of 10 agents evolving in a plane, where agents *1, 2,* and *3* are the leaders and agents *4, 5, ..., 10* are the followers. Also, every follower agent is considered to be a disk with diameter $2\varepsilon = 5\ cm$. For both case studies, positions of follower agents are updated according to eqn. (11), where control parameter g is 25. Furthermore, Communication topology of MAS evolution is shown in Fig. 1.

Initial Distribution of the MAS: Leader agents 1, 2, and 3 are initially located at the vertices of triangle *P,* at $(-6.5, -6.5)$, $(-5.5, 6)$, and $(6,5)$, respectively. Furthermore, weights of communication of follower agents are chosen as listed in Table 1. Based on chosen values of weight ratios, initial position of follower agents are calculated as

$$Z_0 = -A^{-1}BU_0 \quad (26)$$

where

$$U_0 = [-6.5\boldsymbol{e}_1 - 6.5\boldsymbol{e}_2 \quad -5.5\boldsymbol{e}_1 + 6\boldsymbol{e}_2 \quad 6\boldsymbol{e}_1 + 5\boldsymbol{e}_2]^T$$

denotes initial positions of the leader agents and

$$Z_0 = [\boldsymbol{r}_4(0) \quad \dots \quad \boldsymbol{r}_{10}(0)]^T$$

denotes initial positions of the follower agents. Initial positions of the followers are also listed in Table 1.

Table 1. Weights of communication of follower agents

	Weights of Communication			Initial Position	
				X (m)	*Y (m)*
F4	$w_{4,1}=0.70$	$w_{4,7}=0.15$	$w_{4,10}=0.15$	-5.4278	-4.8900
F5	$w_{5,2}=0.70$	$w_{5,8}=0.15$	$w_{5,9}=0.15$	-4.3598	5.1931
F6	$w_{6,3}=0.70$	$w_{6,9}=0.15$	$w_{6,10}=0.15$	3.8002	3.9443
F7	$w_{7,4}=0.40$	$w_{7,8}=0.36$	$w_{7,10}=0.34$	-3.7164	-1.2494
F8	$w_{8,5}=1/3$	$w_{8,7}=1/3$	$w_{8,9}=1/3$	-2.8688	2.6411
F9	$w_{9,5}=0.31$	$w_{9,6}=0.42$	$w_{9,8}=0.27$	-0.5300	3.9796
F10	$w_{10,4}=0.35$	$w_{10,6}=0.29$	$w_{10,7}=0.36$	-2.1356	-1.0174

Initial distribution of the MAS is shown in Fig. 1, where follower and leader agents are illustrated by squares and disks, respectively. Therefore, $\|A^{-1}\|=3.5775$ is obtained from initial positions of the agents.

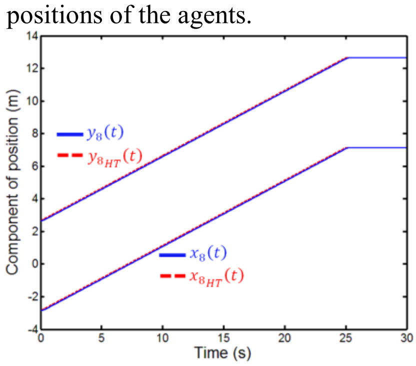

Fig. 3. x and y coordinates $\boldsymbol{r}_8(t), \boldsymbol{r}_{8_{HT}}(t)$

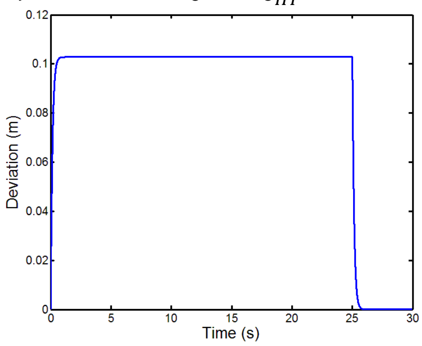

Fig. 4. Deviation from state of homogenous transformation, $\|\boldsymbol{r}_8(t)-\boldsymbol{r}_{8_{HT}}(t)\|$

5.1 Case Study 1: Rigid Body Translation

In this example, leader trajectories are as follows:

$$\text{Leader 1:}\begin{cases} x_1(t)=-6.5+0.4t & 0\le t\le 25 \\ x_1(t)=3.5 & t>25 \\ y_1(t)=-6.5+0.4t & 0\le t\le 25 \\ y_1(t)=3.5 & t>25 \end{cases}, \quad (27)$$

$$\text{Leader 2:}\begin{cases} x_2(t)=-5.5+0.4t & 0\le t\le 25 \\ x_2(t)=4.5 & t>25 \\ y_2(t)=6+0.4t & 0\le t\le 25 \\ y_2(t)=16 & t>25 \end{cases}, \quad (28)$$

$$\text{Leader 3:}\begin{cases} x_3(t)=6+0.4t & 0\le t\le 25 \\ x_3(t)=16 & t>25 \\ y_3(t)=5+0.4t & 0\le t\le 25 \\ y_3(t)=15 & t>25 \end{cases}. \quad (29)$$

Clearly, $v_1=v_2=0.4m/s$ and so $V=0.4\times\sqrt{2}m/s$. With the control parameter $g=25$, δ is obtained according to eqn. (19) as follows:

$$\delta=\frac{\sqrt{N-n-1}\|A^{-1}\|}{g}V=\frac{\sqrt{10-3}\times 3.5775}{25}\times 0.4\times\sqrt{2}=0.2142m.$$

As all leader agents have the same velocity, final formation of the MAS is a homogenous transformation of the initial configuration. In Fig. 3, x and y coordinates of $\boldsymbol{r}_i(t)$ and $\boldsymbol{r}_{i_{HT}}(t)$, associated with follower agent 8 are depicted. Moreover, in Fig. 4, deviation from the state of homogenous transformation, $\|\boldsymbol{r}_8(t)-\boldsymbol{r}_{8_{HT}}(t)\|$, is illustrated for the follower agent 8.

5.2 Case Study 2: Passing through a Narrow Channel

Shown in Fig. 5, leader agents are initially located at the vertices of triangle P and followers are distributed inside P. Final formation of the MAS is desired to be Q that is a homogenous transformation of initial configuration. For MAS to pass through the narrow channel and reach the configuration Q, a contraction is required.

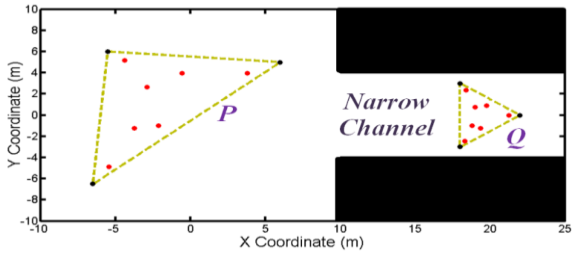

Fig. 5 Initial and final formations of the MAS; Motion field

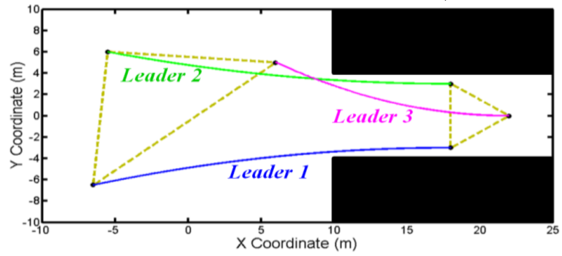

Fig. 6 Trajectories of leader agents

Trajectories of leader agents are depicted in Fig. 6. Furthermore, shown in Fig. 7 are velocities of leaders along their trajectories.

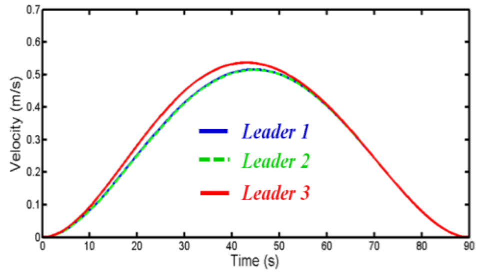

Fig. 7 Velocities of the leaders along the trajectories

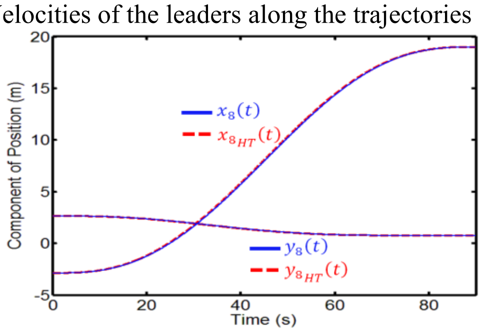

Fig. 8 x and y coordinates $\boldsymbol{r}_8(t), \boldsymbol{r}_{8_{HT}}(t)$

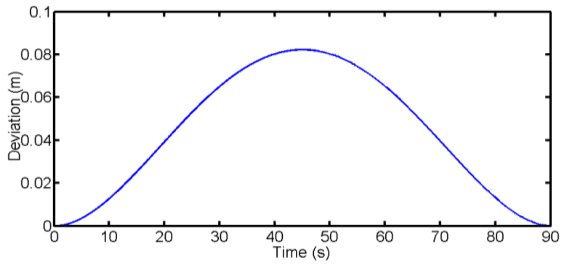

Fig. 9 Deviation from state of homogenous transformation, $\|\boldsymbol{r}_8(t)-\boldsymbol{r}_{8_{HT}}(t)\|$

It is noted that, leaders settle in $90s$, where leader agent *3* reaches the maximum velocity $V=0.5367m/s$. Thus, according to eqn. (19), maximum deviation of followers from the state of homogenous transformation is not greater than:

$$\delta=\frac{\sqrt{N-n-1}\|A^{-1}\|}{g}V=\frac{\sqrt{7}\times 3.5775}{25}\times 0.5367=0.2032\,m.$$

x and y coordinates of actual position vector and state of homogenous transformation of follower 8 are shown in Fig. 8. Moreover, deviation from state of homogenous transformation, $\|\boldsymbol{r}_8(t)-\boldsymbol{r}_{8_{HT}}(t)\|$, is shown in Fig. 9.

6. CONCLUSION

MAS evolution in an n-D space, based on a homogenous transformation driven communication protocol, is formulated, where every follower agent is considered a ball with radius $\varepsilon>0$, that communicates with n+1 local agents to update its position. In order to avoid inter-agent collision, we developed a limit for maximum velocity of leader agents depending on a control parameter g, $\|A^{-1}\|$, and maximum allowable deviation of follower agents. A key approach for reducing deviation from state of homogenous transformation is to increase the minimum eigenvalue of the network, through increasing weights of communication with respect to leaders, associated with follower agents that track leaders.

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