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Communication and Control Protocols for Load Networks in the Smart Grid ⋆

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Abstract: As pointed out in Baillieul and Antsaklis (2007), in the three decades since researchers at Bosch GmbH launched the technology of networked control systems for automobiles, there has been an explosion of interest in both the theory and deployment of real-time networks of devices. This interest is especially apparent in much of the current research on the smart grid. In what follows, recent work to illuminate the challenges and benefits of various communication and control protocols within pools of networked energy consuming devices and energy providers is discussed. We compare what can be possibly achieved in demand side management under two different protocols between the smart building operator [SBO] and the distributed energy consuming appliances. In the first protocol carrying the complete appliances' state information, the SBO is able to design a control law to reduce consumption uncertainty and to guarantee consumers' satisfaction. In the second protocol that communicates binary appliances' information, the scheduling performance reflects lower consumer utility and greater consumption uncertainty. In addition, we discuss the optimal energy reserve purchasing strategy for the SBO that strikes a balance between having excess capacity and having energy deficiency. Numerical simulation illustrates the protocols.

Keywords: Load networks, smart grid, demand response, direct load control, communication and control protocol

1. INTRODUCTION

The realization of the full potential of the *Smart Grid* heavily relies on information exchange between distributed nodes in this electric networked control system. These nodes include communication enabled local appliances at the lower level, the SBO with control authority at the middle level, and the ISO who manages the grid at the higher level. To achieve the promise of demand side management, intermittent interactions among the three levels are needed, and in a growing body of literature, a variety of control and optimization solutions have been proposed that include the reduction of peak consumption and uncertainty in Lu et al. (2005), Samadi et al. (2013), the provision of dynamic ancillary services in Alizadeh et al. (2012), Caramanis et al. (2012), etc. Parallel to these solutions, a fundamental problem is to understand how different protocols for information exchange facilitate the operation of the smart grid. This motivates us to consider the networked information aspects of the smart grid, and more specifically communication protocols needed to maximally realize the potential of grid-friendly appliances in providing demand response.

The contribution of this paper is to establish a rough hierarchy of the effectiveness of different control and communication strategies based on recent work reported in [Zhang and Baillieul (2012), Zhang and Baillieul (2013)]. We focus our attention on heating and cooling appliances since these have the greatest capacity for demand side regulation response. A particular control protocol called the Packetized Direct Load Control [PDLC] is proposed based on the concept of an *energy packet* that is distributed to local appliances. The PDLC framework can be used on top of two communication protocols that contain vastly different levels of information exchange between the SBO and the distributed appliances. The baseline case is one in which grid friendly appliances communicate essentially complete information about their states–including the ambient temperature of their thermal loads and their status of operation (time spent since becoming idle or time spent since commencing operation). The alternative protocol communicates information that is much courser. In this case the appliances provide only binary information to the SBO – a 0 if the appliance does not need to change its operating state (from either operational or idle) or a 1 if it senses the need to change its operating state. After reviewing the optimal performance of both protocols, we present a comprehensive comparison of the two protocols that explains the different roles of the SBO, the contrast in terms of the loss of consumer utility, and the increased system uncertainty due to an essential information loss in passing information from complete to binary. We further show that when the SBO is constrained to binary informa-

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tion, there is a trade off for the SBO to choose the proper value of energy packet length δ where a small value of δ will reduce the mean time for appliances to get energy packets with the potential of increased system uncertainty, and vice versa. Furthermore, we propose an optimization framework for the SBO to determine the optimal amount of energy reserves to purchase from the ISO when binary information presents. This strategy minimizes the sum of a penalty for excess capacity $Ex(m, \delta)$ and for capacity deficiency $De(m, \delta)$ when the energy reserve level m (packets) and choice of packet length δ are provided. In the course of our derivation, we show monotonicity properties of the excess capacity and the capacity deficiency when the SBO either changes the value of m or the value of δ. These monotonicity properties guarantee the existence of the optimal strategy with purchase $m^\star(\delta)$ that balances the two costs for a given packet length δ. We also note that the optimal energy reserve strategy $m^\star(\delta)$ tends to increase when δ increases.

The paper proceeds as follows. Sec. 2 formally defines energy packet and briefly reviews two control/communication protocols between the SBO and the distributed appliances. Sec. 3 compares the system uncertainty and implicit model assumptions of the two systems. Sec. 4 proposes an optimal energy reserve purchasing strategy that strikes the balance between having too much and too little capacity. Sec. 5 presents simulations that verify the theoretical results. Sec. 6 concludes.

2. COMMUNICATION BETWEEN THE SBO AND THE DISTRIBUTED APPLIANCES

We discuss the control performance that can be achieved when the communication channels are established between the SBO and the distributed appliances. Each appliance is connected with a communication channel that can transmit the state information regulated by the appliance. All the information is then sent to the SBO who is in charge of the energy scheduling of the building with the authority to exercise direct control of the on/off switch of appliance once the consumers provide the comfort settings. Unlike traditional energy distribution, the SBO would dispatch energy to all appliances in the form of *energy packets* which are defined as follows.

Definition 1. An *energy packet* for a given appliance is a fixed time interval δ during which electricity is consumed at the appliance's rated power with its nominal voltage and current.

The electricity consumption of an appliance with binary intermittent operating state can be view as a consecutive consumption of energy packets, see Fig. 1. The corresponding direct load control [DLC] protocol based on the energy packet is therefore called packetized direct load control [PDLC]. In this protocol an appliance will consume one energy packet if authorized, and the SBO can re-authorize additional energy packets after time δ based on the information received within δ. We will use the control of air conditioners [AC] as an illustrative example for the rest of the paper. The control of the AC can be easily extended to other thermostatic appliances.

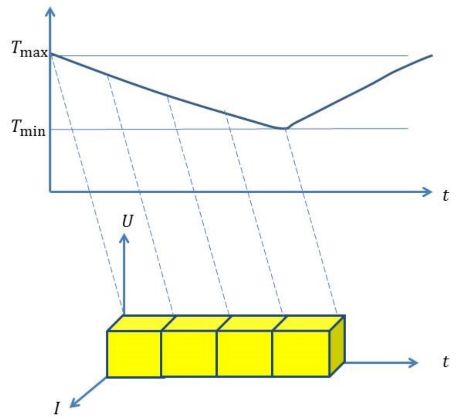

Fig. 1. Electricity consumption in the operating cycle can be viewed as consecutive needs of the energy packets.

2.1 PDLC with Complete State Information

We provide a baseline control result that can be maximally achieved under an ideal communication protocol. To be specific, we assume the communication channel enables the SBO to acquire real time temperature information from the sensors that are installed around the AC. The SBO plans to provide a certain number of energy packets over time to serve the needs of all appliances in the building. There are two challenges for the SBO: 1) Determine the *right* amount of energy to purchase based on the information collected from all appliances, and 2) consume at the energy level that has been purchased with minimum deviation and guarantee consumers' comfort.

As for AC temperature control, each consumer i will provide a preferred set point T^i_{set}, and the objective for successful temperature regulation is to control the temperature within a small temperature range Δ^i with $T^i_{set} \in \Delta^i$. We use the model in Ihara and Schweppe (1981) to represent the evolution of the temperature $T(t)$

$$\frac{d}{dt}T(t) = \frac{T_{out} - T(t) - T_g u}{\tau}, \tag{1}$$

where T_{out} is the outside temperature, T_g is the temperature gain when the AC operates, τ is the effective thermal time constant, and u is binary specifying the on/off operating status. Denoting the total number of appliances by N, we show that the PDLC protocol can determine the right number of energy packets that need to be provided to the network of appliances.

Proposition 1. If the SBO purchases and schedules a fixed number $m = (NT_{out} - \sum_{i=1}^{N} T^i_{set})/T_g$ of packets, then the average room temperature converges to the average set point $\lim_{t\to\infty} \sum_{i=1}^{N} T^i(t) = \sum_{i=1}^{N} T^i_{set}$ where $T^i(t)$ is the temperature at time t for appliance i, staring from any initial conditions $T^i(0), i = 1, \ldots, N$. The system reaches the steady state thermal equilibrium [SSTE].

In addition, the SBO can schedule exactly m packets at any time to guarantee that all consumers' room temperatures can be controlled to lie within their designated comfort bands by properly choosing δ.

Proposition 2. Assuming that the system reaches the SSTE at time $t^\star$ and that $T^i(t^\star) \in \Delta^i, \forall i$, then there exists a $\delta > 0$ such that if the fixed number of m packets are allocated for time $t \geq t^\star$, then all consumers' preferred temperatures can be properly controlled to satisfy $T^i(t) \in \Delta^i$ for all $t \geq t^\star$.

Both Proposition 1 and 2 are proved in Zhang and Baillieul (2012). From a practical point of view, the SBO may not be able to acquire real time complete information from all appliances due to consumer privacy issues, inaccurate temperature monitoring, etc. This motivates us to consider a constrained communication scenario. Nevertheless, the above results on PDLC establish an important baseline of optimal performance that can be achieved by an SBO in a building micro grid.

2.2 PDLC with Binary State Information

We consider next what can be maximally achieved with the same energy level, where m appliances are allowed to operate at the same time in the case that constrained *binary* information is transmitted from local appliances to the SBO. Binary information means that the appliance will send a request signal to the SBO if it wishes to consume energy or a relinquish signal if it does not need energy. The SBO, after receiving the request, will authorize energy if the current number of operating appliances is less than m; otherwise the authorization is delayed until some appliance sends a relinquish signal.

We can model the request and relinquish process as a queuing system with Markov arrivals and departures. Assuming the request rate from an idle AC is λ, and the relinquish rate from an operating AC is μ, then the system can be described as a multi-server (maximum m servers) closed loop queuing system (N appliances), namely an $M/M/m$ queuing system. From standard queuing system theory, we can associate a mean waiting time (MWT) $W_{M/M/m}$ with this system (Kleinrock and Gail (1996)). This means consumers will on average wait for $W_{M/M/m}$ to get energy when their temperatures reach the threshold $T_{\max}$. This can lead to an unfair energy dispatch where some consumers consume electricity continuously but other consumers who are in greater need have to wait to get energy.

The notion of energy packet can be used to guarantee fair energy share. The idea is to share m energy packets authorization at any time t among appliances such that we can reduce the MWT for the appliance pool. In addition, we wish to reduce the total waiting time (TWT) for an appliance to receive the necessary number of packets in an operating duty cycle. When the SBO schedules energy packets with fixed duration δ, the system can be described as a multi-server queuing system with deterministic service time δ and queue re-entries with probability $p(\delta)$, namely an $M/D/m$ system. An appliance will return and request an additional packet if it does not get the desired level of energy within δ. This probability is $p(\delta) = \int_\delta^\infty \mu e^{-\mu t} dt = e^{-\mu\delta}$; otherwise with probability $1 - p(\delta)$ an appliance will switch to the idle operating state. The MWT to get an energy packet for the $M/D/m$ system can be approximated according to Kleinrock and Gail (1996), by

$$W_{M/D/m}(\delta) \cong \frac{W_{M/M/m}}{W_{M/M/1}} W_{M/D/1}(\delta), \qquad (2)$$

where $W_{M/M/1}$ and $W_{M/D/1}(\delta)$ are the MWT for the single server system with the exponential and deterministic servers, respectively. In order to receive the desired amount of energy in an operating cycle, the expected number of energy packets that need to be requested is $E(\delta, n) = 1/[1 - p(\delta)]$. Therefore the TWT to get desired number of energy packets is $T_{M/D/M}(\delta) = W_{M/D/m}(\delta)E(\delta, n)$. We derive the following limits to guarantee that the MWT is reduced while the TWT remains at the same level for binary information PDLC (Zhang and Baillieul (2013)).

Proposition 3. Both $W_{M/D/M}(\delta)$ and $T_{M/D/M}(\delta)$ are monotonically increasing functions of δ. Moreover,

$$\lim_{\delta \to 0} W_{M/D/m}(\delta) = 0, \qquad \lim_{\delta \to 0} T_{M/D/m}(\delta) = W_{M/M/m}. \qquad (3)$$

3. COMPARISONS OF THE TWO PROTOCOLS

3.1 Uncertainty Comparison

We show that the SBO can completely reduce the demand uncertainty around the average consumption level of m packets if (a) it can directly allocate energy with finely quantized packet duration δ, and (b) it has real time temperature information from all the appliances. In the control protocol based on binary information, there is an associated probability distribution $\bar{p}(n, \delta)$ for the number of consuming appliances $n = 0, \ldots, m$ in steady state. This distribution is determined by the steady state probability distribution of the number of appliances $p(x, \delta)$ for $x = 0, \ldots, N$ in the queue. We analyze $p(x, \delta)$ as follows.

When x appliances are in the queue at time t, $k = \min(x, m)$ servers are operating to serve the appliances with packet duration δ. All k appliances will finish the energy packet $[t, t + \delta]$ by the end of the interval and they will independently decide whether to request additional packets. With probability $p(\delta) = e^{-\mu\delta}$, an appliance will request an additional packet and there is no departure in this case. With probability $1 - p(\delta) = 1 - e^{-\mu\delta}$, an appliance will switch into the idle state that results in one departure. Since the probability of departure linearly increases with the departure rate for small value of δ, the departure rate for an appliance is $(1 - p)/\delta$. Therefore the departure rate with k operating appliances is $k[1 - p(\delta)]/\delta$. In addition, the arrival rate that packet requests are received from idle appliances is $(N - x)\lambda$. We can solve for the steady state probability distribution $p(x, \delta)$ based on the departure/arrival rate

$$p(x, \delta)(N - x)\lambda = p(x + 1, \delta)k[1 - p(\delta)]/\delta, \qquad (4)$$

where $k = \min(x, m)$. This yields

$$p(x, \delta) = \begin{cases} p(0, \delta)\binom{N}{x} r(\delta)^x, & x < m \\ p(0, \delta)\binom{N}{x} r(\delta)^x \dfrac{x!}{m^{x-m} m!}, & x \geq m, \end{cases} \qquad (5)$$

where $r(\delta) = \dfrac{\lambda\delta}{1 - e^{-\mu\delta}}$, $m = N\lambda/(\lambda + \mu)$ is the average packet level, and $p(0, \delta)$ is the normalizing factor

$$p(0, \delta) = \left[\sum_{i=0}^{m-1} \binom{N}{i} r(\delta)^i + \sum_{i=m}^{N} \binom{N}{i} r(\delta)^i \frac{i!}{m^{i-m} m!} \right]^{-1}. \qquad (6)$$

This can be used to define

$$\bar{p}(n,\delta) = \begin{cases} p(n,\delta), & n = 0,\ldots,m-1 \\ \sum\limits_{k=m}^{N} p(k,\delta), & n = m. \end{cases} \tag{7}$$

In simulation when we chose a small value of δ, the variance of the number of consuming appliances n approached the variance of the system that does not use energy packets. We state this property formally as follows.

Proposition 4. Let $q(x)$ denote as the probability distribution of the number of appliances x in the queue that are not using energy packets, and let $\bar{q}(n)$ be defined by $q(x)$ similar to (7). Then we have $\lim_{\delta\to 0} \bar{p}(n,\delta) = \bar{q}(n)$ for all n.

Proof. Based on queuing theory in Kleinrock and Gail (1996), $q(x)$ is given by

$$q(x) = \begin{cases} q(0)\binom{N}{x}(\lambda/\mu)^x, & x < m \\ q(0)\binom{N}{x}(\lambda/\mu)^x \dfrac{x!}{m^{x-m}m!}, & x \geq m. \end{cases} \tag{8}$$

Note that $\lim_{\delta\to 0} r(\delta) = \lambda/\mu$. Therefore $p(x,\delta)$ equals to $q(x)$ when $\delta \to 0$ and the Proposition holds. □

In simulation, we also note that the variance of n decreases as δ increases. This property is hard to prove due to the complex structure in (5) and (6). We provide a proof for the special case when $\lambda = \mu$. The result is stated as follows.

Proposition 5. When $\lambda = \mu$, the variance of the number of consuming devices n decreases as δ increases.

Proof. When $\lambda = \mu$, we have $N = m(\lambda+\mu)/\lambda = 2m$. First we prove that for $n = 0,\ldots,m-1, \bar{p}(n,\delta)$ decreases when we increase δ. We take a derivative to determine the change of $\bar{p}(n,\delta)$

$$\begin{aligned} &\frac{d}{d\delta}\bar{p}(n,\delta) = \frac{d}{d\delta}\left[\tbinom{2m}{n} r(\delta)^n / p(0,\delta)\right], \\ &= \left[\frac{d}{d\delta}\tbinom{2m}{n} r(\delta)^n] p(0,\delta) + \tbinom{2m}{n} r(\delta)^n [\frac{d}{d\delta} p(0,\delta)\right] p(0,\delta)^{-2}. \end{aligned} \tag{9}$$

We only need the sign of the derivative to determine the monotonicity of $\bar{p}(n,\delta)$. It can be verified that the sign in (9) equals the following

$$\mathrm{sgn}(\tfrac{d}{d\delta}\bar{p}(n,\delta)) = \sum_{i=0}^{m-1}\tbinom{2m}{i} r(\delta)^i (n-i) + \sum_{i=m}^{2m}\tbinom{2m}{i} r(\delta)^i (n-i)\frac{i!}{m^{i-m}m!}. \tag{10}$$

Since the terms in the second summation are negative and $\dfrac{i!}{m^{i-m}m!} \geq 1$, we have

$$\begin{aligned} &\mathrm{sgn}(\tfrac{d}{d\delta}\bar{p}(n,\delta)), \\ &\leq \sum_{i=0}^{m-1}\tbinom{2m}{i} r(\delta)^i (n-i) + \sum_{i=m}^{2m}\tbinom{2m}{i} r(\delta)^i (n-i), \\ &\leq \sum_{i=0}^{m-1}\tbinom{2m}{i} r(\delta)^i (n-i) + \sum_{i=m+1}^{2m}\tbinom{2m}{2m-i} r(\delta)^i (n-i), \\ &= \sum_{i=0}^{m-1}\tbinom{2m}{i} r(\delta)^i (n-i) + \sum_{i=0}^{m-1}\tbinom{2m}{i} r(\delta)^{2m-i} (n-2m+i), \\ &\leq \sum_{i=0}^{m-1}\tbinom{2m}{i} [r(\delta)^i - r(\delta)^{2m-i}](m-1-i). \end{aligned} \tag{11}$$

The second inequality is derived by dropping the negative term $\binom{2m}{m} r(\delta)^m (n-m)$ and noting that $\binom{2m}{i} = \binom{2m}{2m-i}$. The third inequality is derived by substituting the maximum value of n which is $n = m-1$. When $\delta > 0$, $r(\delta) > 1$, and the term $r(\delta)^i - r(\delta)^{2m-i}$ is negative, the above summation is negative and $\frac{d}{d\delta}\bar{p}(n,\delta) < 0$ for $n = 0,\ldots,m-1$. Therefore $\bar{p}(n,\delta)$ decreases for $n = 0,\ldots,m-1$. Consequently $\bar{p}(m,\delta)$ increases as the sum of probability equals to 1.

From (5) and (7), we know that $\bar{p}(n+1,\delta) \geq \bar{p}(n,\delta)$ for all δ and $n = 0,\ldots,m-1$ when $\lambda = \mu$. Therefore when we increase δ, the probability distribution $\bar{p}(n,\delta)$ will be concentrated more to the state $n = m$ that decreases the variance of n. □

Remark. $\lambda = \mu$ is the scenario when the probability distribution of the number of appliances in the uncontrolled queueing system has the maximum uncertainty (variance and entropy). When $\lambda < \mu$ ($\lambda > \mu$), the average aggregated level of consumption shifts to the right (left) from the half consumption level that reduces the uncertainty of the system.

From the above proposition, we see a trade off between the waiting time performance and the variance of the energy consumption, namely when we decrease the value of packet duration, we can reduce both the MWT and the TWT with a sacrifice of growing the consumption variance. The good news is that this variance is bounded above by the variance generated by the protocol that does not use packetized energy. This means we can always achieve a variance reduction with the control protocol based on energy packets. This discussed further in Sec.5.

3.2 Model Comparison

We will end this section by discussing the two underlying models which the SBO assumes to characterize the behaviours of appliances in the buildings. In the first model (1) is a drift process that results in a deterministic duty cycle. This is different from the second model in which exponential inter-arrival time is assumed. To transform the deterministic model into a stochastic model, we assume that consumers' random behaviors will affect the thermal model by contributing i.i.d Gaussian random variables $w(t) \sim N(0,\sigma^2)$ between each interval $[t, t+dt]$. Therefore the contribution introduced by the randomness between time $[s,t]$ is a Brownian diffusion process. When the outside temperature is far from the set point such as in the hot summer or cold winter, the drift rate $(T_{\text{out}} - T)/\tau$ can be viewed as a constant $(T_{\text{out}} - T_{\text{set}})/\tau$. Hence the stochastic thermal model becomes the following drift-diffusion process

$$dT = (T_{\text{out}} - T_{\text{set}})/\tau dt + \sigma dB(t). \tag{12}$$

The inter-arrival time for an idle appliance to request an energy packet is the first passage time that the temperature rises from $T_{\min}$ to $T_{\min} + \Delta$ which is inverse Gaussian distributed (Karatzas and Shreve (1991)):

$$p(t) = \frac{\Delta}{\sqrt{2\pi\sigma^2 t^3}} \exp\left[-\frac{[\Delta - (T_{\text{out}} - T_{\text{set}})t/\tau]^2}{2\sigma^2 t}\right]. \tag{13}$$

To have a reasonable choice of λ in the Markov model, we need to match the mean arrival time to the mean time of the first passage,

$$\frac{1}{\lambda} = \int_{t=0}^{\infty} tp(t)dt = \frac{\Delta\tau}{T_{\text{out}} - T_{\text{set}}}. \tag{14}$$

Hence we choose $\lambda = \frac{T_{\text{out}} - T_{\text{set}}}{\Delta\tau}$. When the first order information is matched, we consider the proximity of the second order information. The variance of the exponential inter-arrival time model is $(\frac{\Delta\tau}{T_{\text{out}} - T_{\text{set}}})^2$. The variance of the first passage time model is $\frac{\Delta\sigma^2\tau^3}{(T_{\text{out}} - T_{\text{set}})^3}$. Therefore the variance of the two models will be equal when the variance of the noise satisfies $\sigma_\star^2 = \frac{\Delta(T_{\text{out}} - T_{\text{set}})}{\tau}$. If the variance of the noise is around $\sigma_\star^2$, we will see in Sec. 5 that the Markov model can accurately capture the duty cycle of the thermal process. If the variance of the noise is too large or small, then it will corrupt the accuracy of the Markov model with exponential duty cycle time assumption.

4. OPTIMAL PACKET RESERVES FOR THE SBO

In this section we relax the restriction that in the binary information protocol the SBO will purchase the average packet level $m = N\lambda/(\lambda + \mu)$. Instead, we discuss the optimal procurement level $m^\star$ such that the SBO can balance between the risk of purchasing too much energy and the risk of having a long requesting queue. Based on the steady state probability distribution of the queue in (5), the *excess capacity*, which is related with the expected number of packets consumed in steady state, can be defined for a pair of $\{m, \delta\}$.

$$Ex(m, \delta) = \sum_{x=0}^{m-1} p(x, \delta)(m - x). \tag{15}$$

In addition to the expected excess capacity, there is an associated expected capacity deficiency that characterizes the expected number of consumers queued in the system. We define the *capacity deficiency* as follows

$$De(m, \delta) = \sum_{x=m+1}^{N} p(x, \delta)(x - m). \tag{16}$$

An optimal procurement level of energy packets will depend on the value of $Ex(m, \delta)$ and $De(m, \delta)$. The following result describes the dependence $Ex(m, \delta)$, $De(m, \delta)$ on a pair of $\{m, \delta\}$.

Proposition 6. Two properties hold:

(1). The expected excess capacity $Ex(m, \delta)$ is a monotonically increasing function of m for fixed δ, and is a monotonically decreasing function of δ for fixed m.

(2). The expected capacity deficiency $De(m, \delta)$ is a monotonically decreasing function of m for fixed δ, and is a monotonically increasing function of δ for fixed m.

Proof. (1). We first prove that $Ex(m, \delta)$ monotonically increases as a function of m. When the SBO purchases m packets, denote the normalizing constant in (5) by

$$N(m) = \sum_{i=0}^{m-1} \binom{N}{i} r(\delta)^i + \sum_{i=m}^{N} \binom{N}{i} r(\delta)^i \frac{i!}{m^{i-m} m!}. \tag{17}$$

We consider the purchasing level $n = m + 1$. Denote the normalizing constant for this purchase level by $N(n)$, it can be verified that

$$\begin{aligned} &N(n) - N(m) \\ &= \sum_{i=0}^{m} \tbinom{N}{i} r(\delta)^i + \sum_{i=m+1}^{N} \tbinom{N}{i} r(\delta)^i \tfrac{i!}{(m+1)^{i-m-1}(m+1)!} \\ &\quad - \sum_{i=0}^{m-1} \tbinom{N}{i} r(\delta)^i + \sum_{i=m}^{N} \tbinom{N}{i} r(\delta)^i \tfrac{i!}{m^{i-m} m!} \\ &= \sum_{i=m+1}^{N} \tbinom{N}{i} r(\delta)^i \left[\tfrac{i!}{(m+1)^{i-m} m!} - \tfrac{i!}{m^{i-m} m!}\right] < 0. \end{aligned} \tag{18}$$

Denote $p(x, \delta, m)$ as the steady state probability distribution that x appliances are in the queue when the SBO has purchased reserves of m packets from the ISO and authorizes packet duration δ, then $p(0, \delta, m) = N(m)^{-1}$ and $p(0, \delta, n) = N(n)^{-1}$. We have

$$p(0, \delta, m) < p(0, \delta, n). \tag{19}$$

From (5) and (19) we have

$$p(x, \delta, m) < p(x, \delta, m + 1), \quad \forall x < m. \tag{20}$$

Based on the definition of excess capacity

$$\begin{aligned} &Ex(m, \delta) - Ex(n, \delta) \\ &= \sum_{x=0}^{m-1} p(x, \delta, m)(m - x) - \sum_{x=0}^{m} p(x, \delta, m + 1)(m + 1 - x) \\ &< \sum_{x=0}^{m-1} p(x, \delta, m + 1)(m - x) - \sum_{x=0}^{m} p(x, \delta, m + 1)(m + 1 - x) \\ &= - \sum_{x=0}^{m-1} p(x, \delta, m + 1) - p(m, \delta, m + 1) < 0. \end{aligned} \tag{21}$$

The above inequality indicates that the excess capacity increases as the SBO increases the purchasing level m.

We next prove that $Ex(m, \delta)$ monotonically decreases as a function of δ. We investigate how the steady state probability distribution in (5) will change as a function of δ. We take the derivative with respect to δ and focus on the sign of the derivative to get

$$\begin{aligned} &\text{sgn}[\tfrac{d}{d\delta} p(x, \delta)] \\ &= \text{sgn}[\sum_{i=0}^{m-1} \tbinom{N}{i} (x - i) r^i(\delta) + \sum_{i=m}^{N} (x - i) r^i(\delta) \tfrac{i!}{m^{i-m} m!}] \end{aligned} \tag{22}$$

It can be verified that $\text{sgn}[\frac{d}{d\delta} p(0, \delta)] < 0$ and that $\text{sgn}[\frac{d}{d\delta} p(N, \delta)] > 0$. Since the value in the square bracket in (22) is a monotonically increasing function of x, the sign in (22) only changes once. There exists a value $x^\star$ such that

$$\text{sgn}[\frac{d}{d\delta} p(x, \delta)] \begin{cases} \le 0, & x \le x^\star \\ \ge 0, & x \ge x^\star + 1 \end{cases}. \tag{23}$$

Now consider a slight decrease of packet length from δ to $\bar{\delta}$, which results in a change in the probability distribution from $p(x, \delta)$ to $p(x, \bar{\delta})$ for each state x with corresponding probability change $\Delta(x) = p(x, \delta) - p(x, \bar{\delta})$. We have

$$\Delta(x) \begin{cases} \le 0 & x \le x^\star \\ \ge 0 & x \ge x^\star + 1 \end{cases} \tag{24}$$

In addition to the conservation of probability we have,

$$\sum_{x=0}^{N} \Delta(x) = 0. \tag{25}$$

The change of the excess capacity is given by

$$\begin{aligned} Ex(m, \delta) - Ex(m, \bar{\delta}) &= \sum_{x=0}^{m-1} (m - x)(p(x, \delta) - p(x, \bar{\delta})) \\ &= \sum_{x=0}^{m-1} (m - x)\Delta(x). \end{aligned} \tag{26}$$

When $x^\star \geq m$, $\Delta(x)$ in the above equation is less than zero. It is clear that $Ex(m,\delta)$ will decrease when δ increases. When $x^\star \leq m-1$, based on (26) we have

$$\begin{aligned} & Ex(m,\delta) - Ex(m,\bar{\delta}) \\ = & \sum_{x=0}^{x^\star}(m-x)\Delta(x) + \sum_{x=x^\star+1}^{m-1}(m-x)\Delta(x) \\ < & (m-x^\star)\sum_{x=0}^{m-1}\Delta(x) + (m-x^\star-1)\sum_{x=x^\star+1}^{m-1}\Delta(x) \\ = & -(m-x^\star)\sum_{x=m}^{N}\Delta(x) - \sum_{x=x^\star+1}^{m-1}\Delta(x) \\ < & \ 0, \end{aligned} \tag{27}$$

where the equality above is from (24) and the third equality is from (25). Therefore the excess capacity decreases as we increase δ for fixed m.

(2) We first prove that $De(m,\delta)$ monotonically decreases as a function of m. We investigate how $p(x,\delta)$ changes when the number of packets being served changes from m to $m+1$. From the previous proof, we know that $p(x,\delta)$ will decrease when $x \leq m$. When $x \geq m+1$, similar to the derivation in (22) we have

$$\begin{aligned} & \mathrm{sgn}[p(x,\delta,m) - p(x,\delta,m+1)] \\ = & \ \mathrm{sgn}[m^{m-x}/p(0,\delta,m+1) - (m+1)^{m-x}/p(0,\delta,m)] \end{aligned} \tag{28}$$

It can be verified that the sign only changes once as we increase x.

Defining $\bar{\Delta}(x) = p(x,\delta,m) - p(x,\delta,m+1)$, then there exists a $x^\star$ such that

$$\bar{\Delta}(x)\begin{cases} \leq 0 & x \leq x^\star \\ \geq 0 & x \geq x^\star + 1. \end{cases} \tag{29}$$

Repeating the proof in the first part of this proposition, it can be seen that capacity deficiency $De(x,\delta)$ decreases as m increases for a fixed δ.

We next prove that the $De(m,\delta)$ monotonically increases as a function of δ. From the definition of capacity deficiency

$$\begin{aligned} & De(m,\delta) - De(m,\bar{\delta}) \\ = & \textstyle\sum_{x=m+1}^{N}(x-m)(p(x,\delta) - p(x,\bar{\delta})), \\ = & \textstyle\sum_{x=m+1}^{N}(x-m)\Delta(x). \end{aligned} \tag{30}$$

When $x^\star \leq m$, $\Delta(x)$ in the above equation is greater than zero. It is clear that $De(m,\delta)$ will increase when δ increases. When $x^\star \geq m+1$, using a derivation similar to above based on (24), (25), and (30) we have

$$\begin{aligned} & De(m,\delta) - De(m,\bar{\delta}) \\ = & \sum_{x=m+1}^{x^\star}(x-m)\Delta(x) + \sum_{x=x^\star+1}^{N}(x-m)\Delta(x) \\ > & (x^\star-m)\sum_{x=m+1}^{x^\star}\Delta(x) + (x^\star+1-m)\sum_{x=x^\star+1}^{N}\Delta(x) \\ = & -(x^\star-m)\sum_{x=1}^{m}\Delta(x) + \sum_{x=x^\star+1}^{N}\Delta(x) \\ > & \ 0. \end{aligned} \tag{31}$$

This indicates that the capacity deficiency increases when we increase δ for fixed m. □

Fig. 2 illustrates graphically the statement of the proposition in terms of the change of $Ex(m,\delta)$, $De(m,\delta)$ as a function of m and δ. Since for a given δ, $Ex(m,\delta)$ ($De(m,\delta)$) increases (decreases) with m, there must exist an optimal packet procurement level $m^\star(\delta)$ to balance the cost of this two value for a given δ; see the last subplot in Fig. 2. We note that the unit of the two values is packet in both cases. It is reasonable to directly minimize the sum of the two values to achieve the optimal packet reserve level. We propose that the SBO can determine $m^\star(\delta)$ by solving the following optimization problem

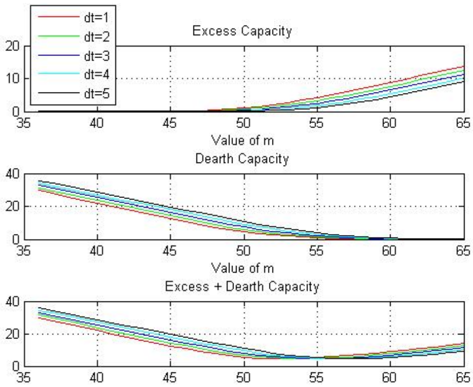

Fig. 2. Monotonicity properties of the excess capacity and capacity deficiency when $m \in [36,65]$, $\delta = \{1,\ldots,5\}$ with $N = 100$, $\lambda = \mu$. Average packet level 50.

$$m^\star(\delta) = \arg\min_{m\in[1,N]} T(m,\delta) = Ex(m,\delta) + De(m,\delta). \tag{32}$$

where $T(m,\delta)$ stands for the total cost of the system. This is an integer optimization problem that needs us to search inside the feasible space of m. A good initial point of m is the average consumption level and we increase m till the cost starts increasing to get $m^\star$. In addition, the optimal purchasing level $m^\star$ can change when the SBO chooses a different value of δ, and that $m^\star(\delta)$ is a non-decreasing function of δ. It is easy to see that the two curves with packet duration δ_1, δ_2 ($\delta_1 < \delta_2$) must cross at one point m^*. This is because $T(m,\delta)$ is dominated by $De(m,\delta)$ when m is small and dominated by $Ex(m,\delta)$ when m is large. (It can be shown that the $Ex(m,\delta)$ between different value of δ are negligible for small values of m, and that $De(m,\delta)$ between different value of δ are negligible for large values of m.) It can be seen that $m^\star(\delta_1) \leq m^*$ and that $m^\star(\delta_2) \geq m^*$. Therefore the optimal purchasing level does not decrease as δ increases.

In practice, the SBO may behave conservatively or aggressively when determining the optimal energy reserve level. It can add different weight into the optimization of (32) which results in a shift of $m^\star$ either to the left with aggressive purchase strategy or conservative purchase strategy. When the price of energy is high during a high demand period, the cost of holding excess capacity can be high with the result of a lower reserve of energy packets. On the other hand, if consumers have inelastic utility preferences that penalize the cost of waiting to the SBO, then reserving a large number of packets can be a good choice.

5. SIMULATION ILLUSTRATION

Fig. 3 shows the trade off between reducing the consumption uncertainty and the MWT and that we cannot expect to reduce both when we vary the packet length δ. However, the increase of δ results in a significant increase in the MWT, but an insignificant decrease of the consumption variance. This indicates that the effectiveness of decreasing the consumption uncertainty by increasing the value of packet length δ is limited. In order to essentially decrease the uncertainty of the aggregated consumption, we need more than binary information.

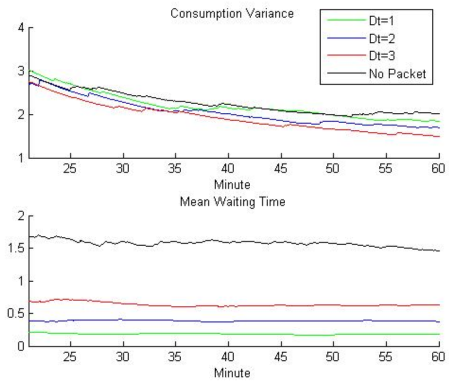

Fig. 3. Trade off illustration between the reduction of the consumption uncertainty and the MWT. With limited binary state information, we cannot expect to decrease both simultaneously by varying δ.

The duty cycle distributions differ when we use a stochastic thermal model and the Markov model that are inverse Gaussian distributed and exponential distributed, respectively. However, when the variance of the noise is moderate, the Markov model can accurately capture the essence of the inter-arrival times; see Fig. 4. We simulate the first 1000 inter-arrival times based on the stochastic thermal model as well as the Markov model. The samples are then used to draw the probability density function [PDF] and the cumulative density function [CDF] of the inter-arrival time with the blue curve being the distribution for the stochastic thermal model, and the red curve being the Markov model. We note that when the variance of the noise is too large (small), then the thermal model inter-arrival time distribution becomes flat (deterministic). This can result in an inaccurate Markov model that may invalidate the assumption of exponential duty cycles.

6. CONCLUSION

The paper has addressed the effectiveness of the PDLC solution when either complete or binary information is exchanged between the SBO and the local appliances in a smart building microgrid. It has been shown that when complete information is exchanged, the SBO can guarantee a high level of consumer utility while minimizing uncertainty. The loss of utility and increase in uncertainty when only binary information is exchanged have been characterized. We have further shown that our framework for packetized energy distribution admits an optimal packet reserve strategy to balance the cost of having having too much or too little reserve capacity.

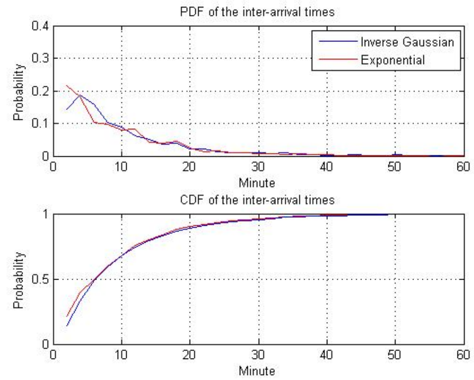

Fig. 4. Compare the inter-arrival time generated by the inverse Gaussian distribution and the exponential distribution. With moderate value of the noise, the Markov model can accurately capture the inter-arrival characteristic of the stochastic thermal model.

REFERENCES

Alizadeh, M., Li, X., Wang, Z., Scaglione, A., and Melton, R. (2012). Demand-side management in the smart grid: Information processing for the power switch. *Signal Processing Magazine, IEEE*, 29(5), 55–67. doi: 10.1109/MSP.2012.2192951.

Baillieul, J. and Antsaklis, P. (2007). Control and communication challenges in networked real-time systems. *Proceedings of the IEEE*, 95(1), 9–28. doi: 10.1109/JPROC.2006.887290.

Caramanis, M.C., Paschalidis, I.C., Cassandras, C.G., Bilgin, E., and Ntakou, E. (2012). Provision of regulation service reserves by flexible distributed loads. In *Decision and Control (CDC), 2012 51st IEEE Conference on*, 3694–3700.

Ihara, S. and Schweppe, F. (1981). Physically based modeling of cold load pickup. *Power Apparatus and Systems, IEEE Transactions on*, PAS-100(9), 4142–4150. doi:10.1109/TPAS.1981.316965.

Karatzas, I.A. and Shreve, S.E. (1991). *Brownian motion and stochastic calculus*, volume 113. Springer.

Kleinrock, L. and Gail, R. (1996). *Queueing Systems, Problems and Solutions.* John Wiley & Sons, Inc.

Lu, N., Chassin, D., and Widergren, S. (2005). Modeling uncertainties in aggregated thermostatically controlled loads using a state queueing model. *Power Systems, IEEE Transactions on*, 20(2), 725–733. doi: 10.1109/TPWRS.2005.846072.

Samadi, P., Mohsenian-Rad, H., Wong, V., and Schober, R. (2013). Tackling the load uncertainty challenges for energy consumption scheduling in smart grid. *Smart Grid, IEEE Transactions on*, 4(2), 1007–1016. doi: 10.1109/TSG.2012.2234769.

Zhang, B. and Baillieul, J. (2012). A packetized direct load control mechanism for demand side management. In *Conference on Decision and Control (CDC), 2012 51st IEEE Conference on*, 3658–3665.

Zhang, B. and Baillieul, J. (2013). A novel packet switching framework with binary information in demand side management. In *Conference on Decision and Control (CDC), 2013 52nd IEEE Conference on.*