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Integral Sliding Mode Control of Small Satellite Attitude Motion by Purely Magnetic Actuation

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Abstract: This paper deals with the purely magnetic attitude control problem. Nonlinear equations of attitude motion under environmental disturbances are presented. It is observed that the environmental disturbances affecting the control system appear as unmatched uncertainties. The controlled dynamic system can be represented in regular state-space form, which allows appropriate controller design. Attenuation of the disturbance effects on the steady-state behavior of the attitude angles is still an important problem in small satellite missions. Therefore, the integral sliding mode control method is used to solve the purely magnetic attitude control problem. The control torque vector at the output of the controller acts on the spacecraft after successive manipulations in magnetic actuation and interaction steps. The magnetic attitude control system is designed by using Lyapunov's direct method in the framework of sliding mode control theory. The performance of the resulting control system is evaluated through realistic simulations, and it is seen that the integral sliding mode controller has a superior steady-state performance with respect to the nominal controller.

1. INTRODUCTION

Since the beginning of the eighties, small satellites have been increasingly preferred for space missions. These platforms are restricted in terms of size, mass, and power, so they require attitude control actuators that are smaller in size, weigh less, and consume less energy than the conventional ones. Consequently, the attitude control problem of small satellites has become a topic of interest, on which many solutions have been proposed in literature. One of them is the purely magnetic attitude control method, which enables controlling the attitude in three axes by using only electromagnetic actuation.

The magnetic actuators are highly suitable for small satellites employed in missions with relatively low pointing accuracy requirements because they are low mass electromagnetic coils or rods fitting small volumes easily, and they require less energy during the nominal operation mode. They are free of degradation thanks to their non-mechanical structure. In addition, they are driven by controlled currents, which makes them much more suitable than mechanical momentum exchange devices for use with discontinuous control algorithms such as sliding mode control.

Although the magnetic torquers have been considered as auxiliary actuators since 1961, it has been also worked on benefiting from them as primary actuators in the last twenty five years. One of the first important papers that deal with purely magnetic attitude control was published in 1989 (Musser *et al.*). In that work, an infinite-time horizon linear quadratic regulator is proposed. In a Ph.D. thesis dated to 1996, many linear and nonlinear control laws, one of which is based on sliding mode control method, are designed to control a small satellite in low Earth orbit by using only three mutually perpendicular magnetic actuators (Wisniewski, 1996). In that study, the controller is designed based on a continuous reaching law, which eventually leads to the loss of disturbance rejection capability of the sliding mode controller (Wisniewski, 1998). Another example of application of sliding mode control method to purely magnetic attitude control problem can be found in (Wang *et al.*). Two more of the fewer nonlinear solutions to purely magnetic attitude control problem are given in (Lovera *et al.*, 2001) and (Lovera *et al.*, 2004), where nearly global asymptotic stability is achieved by designed nonlinear controllers. In a similar work (Gravdahl), uniformly global stability result is obtained for the nonlinear problem. (Bolandi *et al.*) provides the problem with a solution also by using sliding mode control method. In a recent attempt to solve the problem, a nonlinear sliding manifold and a second-order sliding mode controller are used (Janardhanan *et al.*). In (Sofyalı *et al.*), it is shown that the nonlinear attitude dynamics can be stabilized asymptotically by using the classical discontinuous reaching law on the contrary to the result in (Wisniewski, 1996), and a new modified discontinuous sliding mode controller is designed, which is shown to be superior to the continuous one.

To the best knowledge of the authors, there has been only one attempt in (Das *et al.*) to propose solutions to the nonlinear robust attitude control problem by using only electromagnetic actuation, where by using neural network approach, the stabilization is achieved for the nonlinear system with parameter uncertainties and under disturbance effects. Variable structure controllers have the theoretical potential to provide that problem with solutions insensitive to external disturbances and model parameter uncertainties.

In this study, nonlinear equations of attitude motion subject to environmental disturbances are presented because the considered mission phase is the attitude acquisition, in which the spacecraft are carried from an initial state that is quite distant from the equilibrium state. It is observed that the environmental disturbances affecting the control system appear as unmatched uncertainties. The controlled dynamic system can be represented in regular state-space form, which allows appropriate controller design. Integral sliding mode control method is employed to attenuate the environmental disturbance effects on the steady-state behaviour of the attitude angles, which is an important problem in small satellite missions. The control torque vector produced by the controller acts on the spacecraft after successive manipulations in magnetic actuation and interaction steps. The magnetic attitude control system is designed in the framework of sliding mode control theory. The performance of the resulting control system is evaluated through simulations employing realistic disturbance models.

2. SPACECRAFT ATTITUDE DYNAMICS

2.1 State Equations of Attitude Motion

For a rigid spacecraft in a circular orbit, the controlled and disturbed rotational motion can be described in state-space as follows:

$$\dot{\vec{x}}(t)=\vec{f}(\vec{x})+b(t)\vec{u}(\vec{x})+\vec{d}(t). \tag{1}$$

Here, the state vector is $\vec{x}=\begin{bmatrix}\vec{q} & q_4 & \vec{\omega}^{B/A}\end{bmatrix}^T$ consisting of the quaternion vector $\vec{q}=\begin{bmatrix}q_1 & q_2 & q_3\end{bmatrix}^T$, the scalar quaternion component q_4, and $\vec{\omega}^{B/A}$, the 3x1 angular velocity vector of the satellite's body (principal) reference system B with respect to the orbit reference system A (Fig. 1). The quaternions define the orientation of B with respect to A (Wie). In Fig. 1, N is the Earth-centred inertial (ECI) reference system

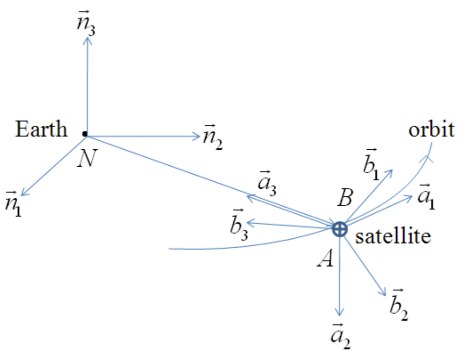

Fig. 1. Reference axis systems A, B, N (Wie)

The terms on the right-hand side of (1), which can be easily derived from kinematic and dynamic attitude equations in (Wie), are the 7x1 nonlinear system vector

$$\vec{f}(\vec{x})=\begin{bmatrix}\frac{1}{2}\left(\vec{\omega}^{B/A}q_4-\vec{\omega}^{B/A}\times\vec{q}\right)\\ -\frac{1}{2}\left(\vec{\omega}^{B/A}\cdot\vec{q}\right)\\ -J^{-1}\begin{bmatrix}\left(\vec{\omega}^{B/A}-n\vec{a}_2\right)\\ \times J\left(\vec{\omega}^{B/A}-n\vec{a}_2\right)\end{bmatrix}\dots\\ \dots+3n^2J^{-1}\left(\vec{a}_3\times J\vec{a}_3\right)\dots\\ \dots+n\left(\vec{a}_2\times\vec{\omega}^{B/A}\right)\end{bmatrix}, \tag{2}$$

the 7x3 control matrix (Calloni *et al.*)

$$b(t)=\begin{bmatrix}\vec{0}_{1x3}\\ \vec{0}_{1x3}\\ \vec{0}_{1x3}\\ \vec{0}_{1x3}\\ \dfrac{J^{-1}\tilde{B}^T(t)\tilde{B}(t)}{\left\|\vec{B}(t)\right\|^2}\end{bmatrix}, \tag{3}$$

which is explicitly dependent on time and where $\tilde{B}(t)$ is the skew-symmetric matrix corresponding to the local geomagnetic field vector $\vec{B}(t)=\begin{bmatrix}B_1(t) & B_2(t) & B_3(t)\end{bmatrix}^T$, the 3x1 control vector $\vec{u}(\vec{x})$ evaluated by the proposed control law, and the 7x1 disturbance vector

$$\vec{d}(t)=\begin{bmatrix}\vec{0}\\ 0\\ J^{-1}\vec{T}_d(t)\end{bmatrix}. \tag{4}$$

While $\vec{a}_2=\begin{bmatrix}2\left(q_1q_2+q_3q_4\right)\\ 1-2\left(q_3^2+q_1^2\right)\\ 2\left(q_3q_2-q_1q_4\right)\end{bmatrix}$ and $\vec{a}_3=\begin{bmatrix}2\left(q_1q_3-q_2q_4\right)\\ 2\left(q_2q_3+q_4q_1\right)\\ 1-2\left(q_1^2+q_2^2\right)\end{bmatrix}$, $J=\begin{bmatrix}J_1 & 0 & 0\\ 0 & J_2 & 0\\ 0 & 0 & J_3\end{bmatrix}$ is the diagonal inertia matrix. $\vec{T}_d(t)$ is the 3x1 environmental disturbance torque vector consisting of the components due to the aerodynamic drag and the solar radiation pressure, which are represented in the simulation environment by realistic mathematical models that take satellite's attitude and geometry into account.

2.2 System Analysis

The rank of the control matrix (3) is equal to 2, which implies that the system is underactuated. This is in coincidence with the physical interpretation of the magnetic torque production law (Martel *et al.*)

$$\vec{T}_{mc}(t) = \vec{M}(\vec{x},t) \times \vec{B}(t) = \left[\frac{\vec{B}(t) \times \vec{u}(\vec{x})}{\left\| \vec{B}(t) \right\|^2} \right] \times \vec{B}(t), \tag{5}$$

where $\vec{T}_{mc}(t)$ is the 3x1 magnetic control torque vector and $\vec{M}(\vec{x},t)$ is the 3x1 magnetic control moment vector having the magnetic moment components produced by the three magnetic actuators as its elements. Fortunately, the underactuated direction continuously changes with respect to the body reference system when the spacecraft move along the orbit. If the change is quasi-periodic, the purely magnetic control approach becomes adequate for three-axis regulation of attitude dynamics. This fact makes the engineering application of the purely magnetic attitude control method possible (Wisniewski, 1996).

The integral sliding mode control method requires that the considered system is affine (a nonlinear system with right-hand side as linear function of the control input u) (Utkin *et al.*, 2009), and the system's time-independent control matrix has a rank equal to the control input number m (Utkin *et al.*, 2009). On the considered problem, the control matrix is time-dependent, and accordingly, the uncontrolled direction varies continuously with respect to the body reference system B, which is the frame the control moment and torque are produced in. This fact provides the control system with the property of being instantaneously underactuated. It may be asserted that the difference between the properties of being underactuated and being instantaneously underactuated allows the application of the integral sliding mode control method to the purely magnetic attitude control problem.

There is no such a $\vec{\gamma}(t)$ vector satisfying the following relation

$$\vec{d}(t) = b(t)\vec{\gamma}(t) \tag{6}$$

because the matrix

$$C_B(t) \triangleq \frac{\tilde{B}^T(t)\tilde{B}(t)}{\left\| \vec{B}(t) \right\|^2} = \frac{1}{B_1^2 + B_2^2 + B_3^2} \begin{bmatrix} B_2^2 + B_3^2 & -B_1 B_2 & -B_1 B_3 \\ -B_2 B_1 & B_3^2 + B_1^2 & -B_2 B_3 \\ -B_3 B_1 & -B_3 B_2 & B_1^2 + B_2^2 \end{bmatrix} \tag{7}$$

in (3) has no inverse. Therefore the matching condition is not satisfied (Utkin *et al.*, 2009). That means that disturbances enter the system at different points with inputs thus they are described as unmatched.

If a system can be divided into two blocks, the first one with row number of n-m including no control terms and the second one with m rows including the control inputs, it means that the system is in so-called regular form (Utkin *et al.*, 2009). In our case, it can be concluded from (1) together with (2)-(4) that the considered system is in regular form.

3. MAGNETIC INTEGRAL SLIDING MODE CONTROLLER

3.1 Integral Sliding Mode Control

The integral sliding mode control method is introduced in (Utkin *et al.*, 1996) to remove the reaching mode that exists in the conventional sliding mode control method. In the reaching mode preceding the sliding mode, the invariance with respect to the disturbances and parametric uncertainties is not valid. Under the integral sliding mode control, the sliding mode starts at the initial time instant t_0, and the invariance is guaranteed for the whole closed-loop control process. The order of the motion in integral sliding mode is equal to the original system's order n whereas the motion in conventional sliding mode has an order equal to n-m. It theoretically provides the system under perturbations with the same performance as under no perturbations.

When first proposed, the integral sliding mode control method required that the perturbations acting on the considered system are matched (Utkin *et al.*, 2009). However, approaches enabling its application on systems with unmatched perturbations have been developed in the last ten years. As asserted in (Rubagotti *et al.*, 2011), it is possible to reject matched disturbances completely and to avoid the amplification of unmatched disturbances by defining a proper sliding manifold and guaranteeing sliding mode. As a result, the effect of disturbances on the system can be kept at a minimum level.

In this study, the approach presented in (Rubagotti *et al.*, 2011) is applied on the purely magnetic attitude control problem by benefiting from the fact that the closed-loop system is in regular form in the light of (Utkin *et al.*, 2009) and (Rubagotti *et al.*, 2010).

The vectorial control signal is divided into two parts:

$$\vec{u}(\vec{x}) = \vec{u}_0(\vec{x}) + \vec{u}_1(\vec{x}). \tag{8}$$

Here, $\vec{u}_0(\vec{x})$ is the law designed to regulate the system that is not disturbed as desired and is named as nominal control law. $\vec{u}_1(\vec{x})$ is the discontinuous control law responsible to keep the system in sliding mode by counteracting to disturbances (Utkin *et al.*, 2009, Rubagotti *et al.*, 2010, 2011).

It is necessary to assume that the disturbance vector is bounded for $t \geq t_0$:

$$\left|T_d\right|_i(t)\Big| \le \left\|\vec{T}_d(t)\right\|_\infty = \sup_\tau \left\{\max_i \left[\left|T_d\right|_i(\tau)\Big|\right]\right\} \; ; i=1,2,3 \cdot \quad (9)$$

The sliding manifold with the dimension number of $m=3$ is defined as follows:

$$\vec{s}(\vec{x}) = \vec{g}(\vec{x}) - z(\vec{x}) \cdot \quad (10)$$

As aforementioned, in this method, $\vec{s}\left[\vec{x}(t_0)\right] = \vec{s}(\vec{x}_0) = \vec{s}_0 = \vec{0}$ holds, and the sliding vector is forced to be equal to zero for $t \geq t_0$. Here, $\vec{g}$ is equal to the sliding vector defined in the conventional sliding mode control method. $\vec{z}$ is the integral term, and it is obtained by the integration of the equation

$$\dot{\vec{z}}(\vec{x}) = \dot{\vec{g}}(\vec{x}) = \frac{\partial \vec{g}}{\partial \vec{x}} \dot{\vec{x}} \equiv G(\vec{x})\left[\vec{f}(\vec{x}) + b(t)\vec{u}_0(\vec{x})\right], \quad (11)$$

which is derived from the equation (10) that holds for $t \geq t_0$. The result of the integration is

$$\vec{z}(\vec{x}) = \vec{g}\left[\vec{x}(t_0)\right] + \int_{t_0}^{t} G(\vec{x})\left[\vec{f}(\vec{x}) + b(\tau)\vec{u}_0(\vec{x})\right] d\tau \cdot \quad (12)$$

Here, $G(\vec{x})$ is the mxn Jacobian matrix. Although it is generally assumed to be dependent on the state vector, it can be a constant in accordance with the definition of $\vec{g}$ (Rubagotti *et al.*, 2010, 2011).

Regarding systems in regular form, it is claimed in (Rubagotti *et al.*, 2011) that the most proper selection for $\vec{g}$ is a linear function of the state vector: $\vec{g}(\vec{x}) = G\vec{x}$.

The constant Jacobian matrix is defined as $G = \left[0_{mx(n-m)} \quad N_{mxm}\right]$, where N is any full-rank matrix (Rubagotti *et al.*, 2011). Note that the $mx(n-m)$ block corresponding to the first block with $n-m=4$ rows, which is defined in the previous section, is taken as zero to avoid possible amplification of disturbances entering that block. However, in the considered problem, there is no disturbance entering the first block as seen from (4). Therefore, the selection of a $\vec{g}$ vector that leads to a G matrix with nonzero elements in its first $n-m$ columns is allowable.

It is shown in (Vadali) through the minimization of a quadratic performance index consisting of the quaternions and the angular velocities that the sliding motion on the selected manifold

$$\vec{g} = \vec{\omega}^{B/A} + K_q\vec{q} \;\; ; \;\; K_q = \begin{bmatrix} k_q & 0 & 0 \\ 0 & k_q & 0 \\ 0 & 0 & k_q \end{bmatrix} \;, \; k_q > 0 \quad (13)$$

is optimal. Here, k_q is the sliding manifold design parameter. The resulting Jacobian matrix is as follows:

$$G = \begin{bmatrix} k_q & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & k_q & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & k_q & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \quad (14)$$

3.2 Controller Design

The employed nominal control law

$$\vec{u}_0 = \vec{u}_{eq} - K_g\vec{g} \;\; ; \;\; K_g = \begin{bmatrix} k_g & 0 & 0 \\ 0 & k_g & 0 \\ 0 & 0 & k_g \end{bmatrix} \;, \; k_g > 0 \quad (15)$$

is first proposed in (Wisniewski, 1996). Here, k_g is the nominal controller design parameter, and the equivalent control vector is

$$\begin{aligned} \vec{u}_{eq} &= \left(\vec{\omega}^{B/A} - n\vec{a}_2\right) \times J\left(\vec{\omega}^{B/A} - n\vec{a}_2\right) \\ &\quad - \frac{1}{2} JK_q\left(\vec{\omega}^{B/A} q_4 - \vec{\omega}^{B/A} \times \vec{q}\right) \\ &\quad - 3n^2\left(\vec{a}_3 \times J\vec{a}_3\right) - nJ\left(\vec{a}_2 \times \vec{\omega}^{B/A}\right) \end{aligned} \cdot \quad (16)$$

The discontinuous control term can be taken as

$$\vec{u}_1 = -K_{ss} sgn(\vec{s}) \;\; ; \;\; K_{ss} = \begin{bmatrix} k_{ss} & 0 & 0 \\ 0 & k_{ss} & 0 \\ 0 & 0 & k_{ss} \end{bmatrix} \;, \; k_{ss} > 0 \quad (17)$$

with k_{ss} being the controller design parameter.

The resulting control vector is

$$\vec{u}(\vec{x}) = \vec{u}_0(\vec{x}) + \vec{u}_1(\vec{x}) = \vec{u}_{eq} - K_g\vec{g} - K_{ss} sgn(\vec{s}) \cdot \quad (18)$$

The control term $\vec{u}_1$ is designed by using Lyapunov's direct method. The following Lyapunov function candidate is chosen:

$$V = \frac{1}{2}\left[\vec{s} \cdot (J\vec{s})\right] = \frac{1}{2}\left[\vec{s}^T J\vec{s}\right] \cdot \quad (19)$$

Its time derivative along the state trajectories can be written as

$$\dot{V} = \vec{s}^T \left[J\dot{\vec{\omega}}^{B/A} + JK_q\dot{\vec{q}} - JG\left(\vec{f} + b\vec{u}_0\right)\right], \quad (20)$$

which leads to

$$\dot{V} = -\vec{s}^T \left[C_B K_{ss} sgn(\vec{s}) - \vec{T}_d \right] \tag{21}$$

after appropriate substitutions and simplifications.

There are three marginal cases for the matrix C_B, which occurs when $\vec{B}$ is directed instantaneously along the *i*th (*i* = *1,2,3*) principal body axis of the spacecraft. Then C_B becomes diagonal with its *i*th element equal to zero while the other two are equal to one. For this particular case, it can be concluded from (21) and (17) that k_{ss} has to be infinite to guarantee that $\dot{V} < 0$. However, further investigation of the control torque production equation (5) indicates that the component of $\vec{T}_{mc}$ that is responsible for disturbance rejection $\vec{T}_{mc}\big|_1 = \vec{M}_1 \times \vec{B} = \left[\frac{\vec{B} \times \vec{u}_1}{\|\vec{B}\|^2} \right] \times \vec{B}$ lies in the plane defined by the other two body axes. This fact reduces the requirement on k_{ss} to

$$k_{ss} > \left\| \vec{T}_d(t) \right\|_\infty \quad ; \; i = 1,2,3 \tag{22}$$

because at that moment the disturbance counteraction occurs only along the other two body axes.

For the general case of $\vec{B} = \begin{bmatrix} B_1 & B_2 & B_3 \end{bmatrix}^T$, $\vec{T}_{mc}\big|_1$ has components along all of the three body axes. k_{ss} is required to be infinite when $B_2 = -B_3$, $B_3 = -B_1$, or $B_1 = -B_2$, however it can be shown that the component of $\vec{T}_{mc}\big|_1$ along the *i*th (in respective order: *i* = *1,2,3*) axis also becomes zero then. This means that the physical interpretation given above for the marginal cases is also valid for the general case. While $\vec{B}$ changes its orientation with respect to the body reference system along the orbit, there are moments when high k_{ss} values are required to counteract the disturbance torques. Even at those moments, the uniform ultimate boundedness of the system is maintained, which means that the stability condition $\dot{V} \le 0$ is satisfied.

4. SIMULATION RESULTS

The spacecraft model used in simulations belongs to the satellite Oersted of Denmark, and the properties of the satellite are found in (Wisniewski, 1996). The model consisting of satellite's three principal moments of inertia is $J_1; J_2; J_3 = 2.904;\ 3.428;\ 1.275\ \text{kgm}^2$.

The simulations are carried out along Oersted's orbit. The geomagnetic field vector is obtained by the highly accurate spherical harmonic model, IGRF. Oersted's mean motion is equal to 6.02×10^{-2} deg/s while its orbital period is equal to $T = 5.98 \times 10^3\ \text{s} = 99.6\ \text{min}$.

The initial Euler angles, which are the respective rotation angles around the three body axes and can be calculated from the four quaternion components, and the initial angular velocities are $[\phi \;\; \theta \;\; \psi]^T\big|_{t=t_0} = [180° \;\; 0° \;\; 0°]^T$ and $\vec{\omega}^{B/A}\big|_{t=t_0} = [0°/\text{s} \;\; 0°/\text{s} \;\; 0°/\text{s}]^T$, respectively. The initial state is the stationary upside down state. The equilibrium state is represented by $[\phi \;\; \theta \;\; \psi]^T\big|_{equilibrium} = [0° \;\; 0° \;\; 0°]^T$ and $\vec{\omega}^{B/A}\big|_{equilibrium} = [0°/\text{s} \;\; 0°/\text{s} \;\; 0°/\text{s}]^T$.

The sliding manifold design parameter and the nominal controller design parameter respectively have the values $k_q = 2.5 \times 10^{-3}$ rad/s and $k_g = 1 \times 10^{-3}$ Nms/rad. The value of the controller design parameter k_{ss} is selected to be higher than $\left\| \vec{T}_d(t) \right\|_\infty = 2.222 \times 10^{-7}$ Nm according to (22) as $k_{ss} = 2.3 \times 10^{-7}$ Nm.

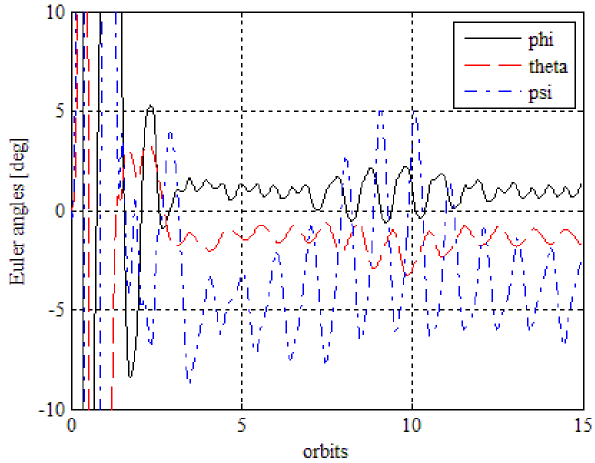

Fig. 2. Euler angles by nominal controller (15)

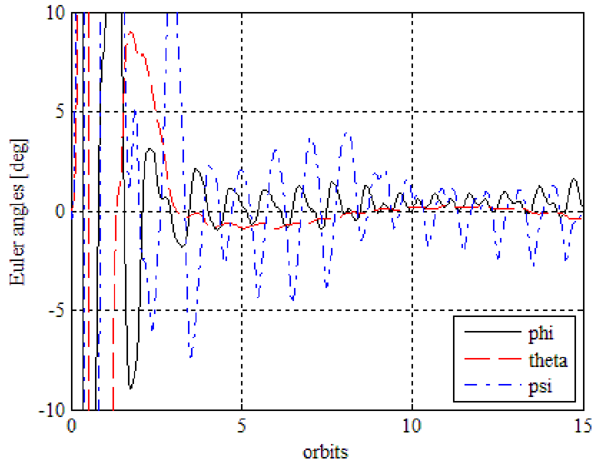

Fig. 3. Euler angles by integral sliding mode controller (18)

To be able to evaluate the superiority of the integral sliding mode controller to the nominal controller, the time responses of Euler angles and angular velocities are presented in Fig. 2-3 and in Fig. 4-5, respectively. Fig. 3 shows that the integral effect reduces steady-state errors for each of the three Euler angles by forcing them to oscillate around the horizontal axis rather than an axis with an offset as seen in Fig. 2. The used spacecraft model corresponds to a configuration that is gravity-gradiently stable in terms of rolling and pitching motions. Thus the integral sliding mode control method's superior performance is best observed on the yawing motion.

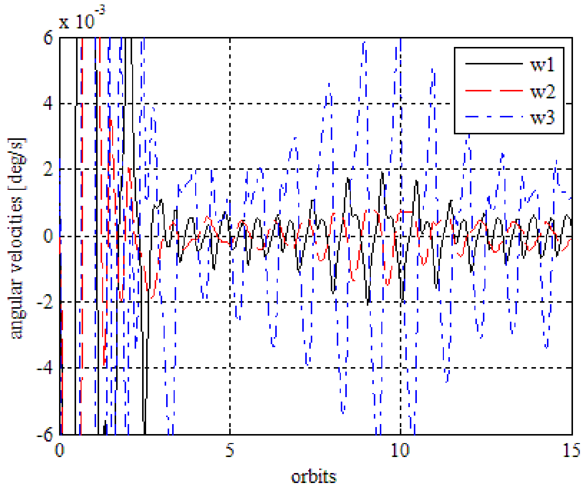

Fig. 4. Angular velocities by nominal controller

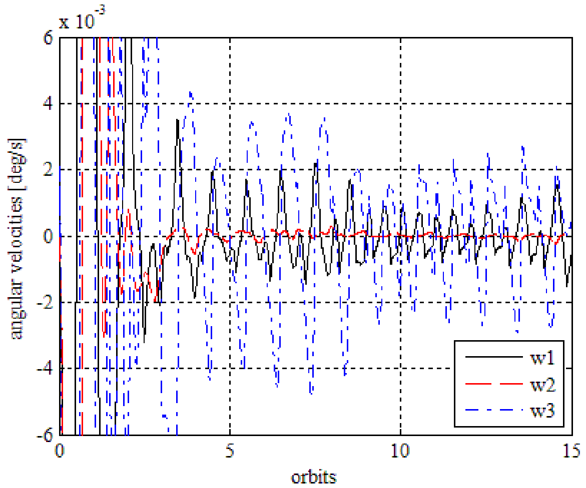

Fig. 5. Angular velocities by integral sliding mode controller

The comparison of Fig. 4 and Fig. 5 shows that the steady-state oscillation band is made narrower by the integral sliding mode control law. On the other hand, oscillations with high frequency and low amplitude appear in the angular velocity responses (Fig. 5).

It is seen from Fig. 6 that the components of the sliding vector defined in (10) can be kept in the vicinity of zero.

The magnetic control moment components produced by the nominal and integral sliding mode controllers are given in Fig. 7 and Fig. 8, respectively.

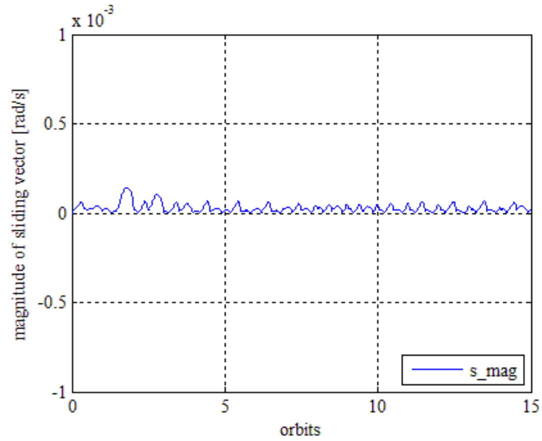

Fig. 6. Sliding vector's magnitude by integral sliding mode controller

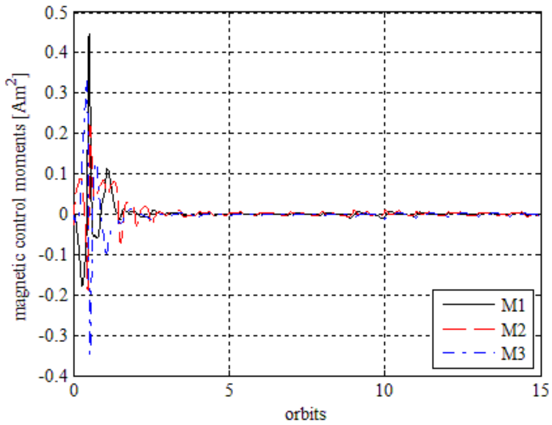

Fig. 7. Magnetic control moments by nominal controller

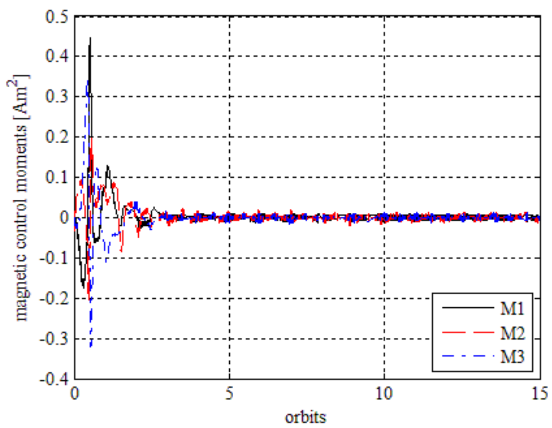

Fig. 8. Magnetic control moments by integral sliding mode controller

Signals in both figures are far from the saturation limit of Oersted's magnetic actuators, which is ±20 Am^2 (Wisniewski, 1996). The effect of the discontinuous control term is observed as chattering in Fig. 8. Chattering is not an important issue for the considered problem because the actuators are driven by current, which's direction can be changed with high frequency by proper circuit design. The oscillations in the angular velocity responses emerging from chattering in the inputs have negligible amplitudes, however their possible degenerating effects especially on optical sensing can be eliminated by low-pass filtering.

5. CONCLUSIONS

The designed integral sliding mode controller increases the steady-state performance of the nominal controller under the effect of environmental disturbances. The available approach specific to systems in regular form and with unmatched uncertainties is applied to the considered problem with an allowable modification in the structure of the Jacobian matrix G.

The complete rejection of the disturbances cannot be achieved because there is difference between the orientations of the control torque vector computed by the controller and the applied control torque vector. This is due to the inclusion of C_B in the control matrix, which mathematically models the effects of magnetic actuation and interaction on the controller output. To solve this fundamental problem specific to the purely magnetic attitude control problem, there seems to be a need for novel approaches. A related study is currently carried out by the authors.

A source of model uncertainty in the attitude control problem of a rigid spacecraft is the fact that, in reality, the principal body axes mostly do not coincide with the body axes that are orthogonal to the body surfaces. Therefore, modelling the spacecraft with a diagonal inertia matrix may lack accuracy. For further studies, model uncertainty should be taken into consideration.

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