Teaching Model-based Fault Detection and Isolation using Project-based Learning on a Three-tank System *

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Abstract: Model-based fault detection and isolation is nowadays a well established and mature field. It starts to become part of the curricula of graduate or post-graduate students. However, the lack of good teaching materials makes difficult the teaching/learning process to students and professors. This paper shows how project-based learning methodology has been used to organise the labs of the fault diagnosis course using a real set-up based on a three-tank system. Observer based methods for fault detection and structured residuals for fault isolation will be introduced to the students from a practical point of view by means of a set of exercises that intend students achieve a set of learning objectives.

Keywords: Fault detection and isolation, observers, parity methods, project-based learning

1. INTRODUCTION

Control and system theory is nowadays a common subject in many engineering curricula. Students learn from the very basics of system modeling to the most recent advanced control algorithms. Unfortunately, although the field is mature and very important it is not as common as control and system theory topics. This is due to the fact that still nowadays there is a gap between academia and industry that make that automatic fault diagnosis systems are not so widespread as expected according to the development of the field. In part, this can be explained why engineering students are not aware about the existence of fault diagnosis methodologies.

Among the different fault diagnosis methodologies, model-based fault diagnosis is the most well theoretical developed and closely related to the students with the usual background on systems and control theory. For this reason, model-based diagnosis is the core of the course *Diagnosis and Fault Tolerant Control* in the *Automatic Control and Robotics Master* at Universitat Politècnica de Catalunya (UPC).

Model-based Fault Detection and Isolation (FDI) of dynamic systems is based on the use of models to check the consistency of observed behaviors. This consistency check is based on computing the difference between the predicted value from the model and the real value measured by the sensors. Then, this difference, known as *residual*, will be compared with a threshold value (zero in the ideal case). When the residual is bigger than the threshold, it is

determined that there is a fault in the system. Otherwise, it is considered that the system is working properly. Fault detection is followed by the fault isolation procedure which will distinguish a particular fault from others. Whilst a single residual is sufficient to detect faults, a set (or a vector) of residuals is required for fault isolation [Gertler, 1998]. If a fault is distinguishable from other faults using one residual set, then it can be said that this fault is isolable. Methods that conform the FDI basis are: parity methods [Gertler, 1998], observer methods [Chen and Patton, 1999] and parameter estimation methods [Isermann, 2006]. In the proposed exercises, parity and observer based methods will be applied and compared in the proposed case study.

In this course, in parallel with the theoretical materials, students get a more practical engineering point of view of fault diagnosis field by means of a small project based on a real set-up (a three-tank system) following a Project-Based Learning (PBL) methodology. The learning activities combine conceptual developments, simulation and experimental works. This paper presents some of the materials and proposed learning lab activities regarding fault diagnosis part of the course.

The structure of the paper is the following: in Section 2 the proposed case study used to present the model-based fault diagnosis teaching material is introduced. In Section 3, fault detection is presented, while fault isolation is presented in Section 4. Then, the learning exercises, the results obtained in simulation and in the real lab set-up and supporting material are presented in Section 5, and 6, respectively. Finally, in Section 7, conclusions close the paper.

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2. THE EXPERIMENTAL SETUP

2.1 Set-up description

The learning activities will be proposed using a three-tank system as testbed. Tanks systems, both in three or four-tanks versions, have been widely used for teaching an research purposes [Johansson, 2000, Dormido et al., 2008] due to its interesting dynamic behaviors and relatively simple structure. An scheme of the concrete plant that is used in this work is shown Figure 1.

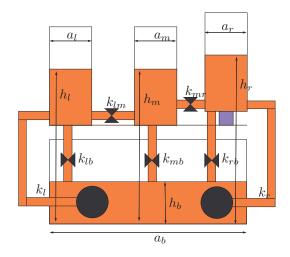


Figure 1. Three tank plant schematic

This plant is used both in simulation and real experiments. The real plant can be easily changed in both parametric and structural manner as follows:

- The connection between the different tanks consists of various pipes that can be connected or disconnected easily.
- Some of the pipes can modify their section.
- The level sensors may be moved slightly.
- Finally, one of the tanks can be moved vertically.

All these changes allow to create different types of fault scenarios that the students should address in the different learning exercises proposed.

2.2 Mathematical model

The model of the three tank system can be written down as:

$$\dot{h}_{l} = \frac{1}{a_{l}} \left(-k_{lb} s \left(h_{l} - h_{b} \right) - k_{lm} s \left(h_{l} - h_{m} \right) + k_{l} u_{l} \right) \tag{1}$$

$$\dot{h}_{m} = \frac{1}{a_{m}} \left(k_{lm} s \left(h_{l} - h_{m} \right) - k_{mr} s \left(h_{m} - h_{r} \right) \right) \tag{2}$$

$$\dot{h}_r = \frac{1}{a_r} \left(k_{mr} s \left(h_m - h_r \right) - k_{rb} s \left(h_r - h_b \right) + k_r u_r \right)$$
 (3)

$$\dot{h}_b = \frac{1}{a_b} \left(k_{lb} s \left(h_l - h_b \right) + k_{rb} s \left(h_r - h_b \right) \right) \tag{4}$$

where h_l , h_m , h_r and h_b correspond to the liquid level of the left, middle, right and bottom tanks respectively (measured as shown in Figure 1), a_l , a_m , a_r and a_b correspond to the cross section area of left, middle, right

and bottom tanks respectively, u_l and u_r are the voltages applied to left and right pumps respectively, k_l and k_r are the left and the right pumps gain respectively, k_{lb} , k_{lm} , k_{mr} , k_{lb} and k_{rb} are the different pipe coefficients; and $s(x) = \text{sign}(x) \sqrt{|x|}$. The nominal parameters values, experimentally obtained, are presented in Table 1. This parameter set corresponds to the nominal case, so the fault detection and isolation schemes will be designed according to these parameters.

$k_{lm} (\mathrm{cm}^2/\mathrm{s})$	$k_{lb}(\mathrm{cm}^2/\mathrm{s})$	$k_{mr}(\text{cm}^2/\text{s})$	$k_{rb}(\mathrm{cm}^2/\mathrm{s})$
15.86	7.04	10.48	6.93
$k_{mb}(\mathrm{cm}^2/\mathrm{s})$	$k_l(\text{cm}^3/\text{s}\cdot\text{V})$	$k_r(\text{cm}^3/\text{s}\cdot\text{V})$	$a_l(\mathrm{cm}^2)$
0.00	6.04	6.04	64.00
$a_m(\mathrm{cm}^2)$	$a_r(\mathrm{cm}^2)$	$a_b(\mathrm{cm}^2)$	$vol(\mathrm{cm}^3)$
64.00	64.00	1044.00	15990.00

Table 1. Model Parameters

The system (1)-(4) is a fourth order non-linear model which can be reduced to a third order one in $\mathbf{x} = [h_l, h_m, h_r]$ and $\mathbf{u} = [u_l, u_r]$ by using the fact the liquid volume is preserved:

$$vol = a_b h_b + a_l h_l + a_m h_m + a_r h_r, (5)$$

so, the system can be written as:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}) \tag{6}$$

$$\mathbf{y} = \mathbf{h}\left(\mathbf{x}, \mathbf{u}\right) \tag{7}$$

As most mature techniques for fault detection and isolation for dynamical system has been developed for linear systems [Gertler, 1998, Blanke et al., 2006], and the course is centered in these techniques, a linear model for this system should be obtained. When working around an equilibrium point $(\mathbf{x}_o, \mathbf{u}_o)$ the system behavior can be linearized following [Roubal et al., 2010] as follows:

$$\dot{\chi} = \mathbf{A}\chi + \mathbf{B}\mu \tag{8}$$

$$\gamma = \mathbf{C}\chi + \mathbf{D}\mu \tag{9}$$

with $\mathbf{A} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}}|_{(\mathbf{x}_o, \mathbf{u}_o)}$, $\mathbf{B} = \frac{\partial \mathbf{f}}{\partial \mathbf{u}}|_{(\mathbf{x}_o, \mathbf{u}_o)}$, $\mathbf{C} = \frac{\partial \mathbf{h}}{\partial \mathbf{x}}|_{(\mathbf{x}_o, \mathbf{u}_o)}$ and $\mathbf{D} = \frac{\partial \mathbf{h}}{\partial \mathbf{u}}|_{(\mathbf{x}_o, \mathbf{u}_o)}$ so that: $\mathbf{x} \approx \mathbf{x}_o + \boldsymbol{\chi}$, $\mathbf{y} \approx \mathbf{y}_o + \boldsymbol{\gamma}$ and $\mathbf{u} \approx \mathbf{u}_o + \boldsymbol{\mu}$. Most experiments during the learning activities presented in this paper will be performed around the equilibrium point defined by $\mathbf{x}_o = (h_{o,l}, h_{o,r}, h_{o,m}) = (41, 39, 40.39)$ cm and $\mathbf{u}_o = (u_{o,l}, u_{o,r}) = (4.35, 8.76)$ V.

For this particular point of interest, a discrete-time system with $T_s=1\mathrm{s}$ is obtained as follows

$$\chi(k+1) = \widetilde{\mathbf{A}}\chi(k) + \widetilde{\mathbf{B}}\mu(k) \tag{10}$$

where $\chi = [\chi_l, \chi_m, \chi_r]$, $\mu = [\mu_l, \mu_r]$. Since all the tank levels (states) are measured matrix $\tilde{\mathbf{C}}$ is the identity.

Note that this system can equivalently be written down in input/oup ut form as: $^{\rm 1}$

$$\gamma(k) = \begin{bmatrix} M_{ll}(q) & M_{lr}(q) \\ M_{ml}(q) & M_{mr}(q) \\ M_{rl}(q) & M_{rr}(q) \end{bmatrix} \mu(k)$$
 (11)

¹ where q^{-1} stands for the delay operator.

3. MODEL-BASED FAULT DETECTION

3.1 Model-based fault detection

In order to present basic model-based fault detection theory, let us consider a linear dynamic system in discretetime that can be described by the following input-output relationship without considering faults, disturbances and noise:

$$y(k) = M(q) u(k) = \frac{V(q)}{W(q)} u(k)$$
 (12)

where: u(k) is the input, y(k) is the output, M(q) is the transfer function with numerator V(q) and denominator W(q) in terms of the classical q-operator.

As discussed in the introduction, the principle of modelbased fault detection is to test whether the measured inputs and outputs from the system are consistent with the model of the faultless system. If the measurements are inconsistent with the model of the faultless system, the existence of a fault is proved. The residual usually describes the consistency check between the predicted, $\hat{y}(k)$ and the real behavior, y(k), as:

$$r(k) = Q(q)(y(k) - \hat{y}(k)) \tag{13}$$

where Q(q) is a filter that could be included to improve fault detection (for example, to decouple model uncertainty and noise) and/or isolation properties of the residual vector (for example, to decouple residuals from some faults).

The fault detection task consists in deciding if a residual given by Eq. (13) is violated at a given instant or not generating a fault signal s_i according to:

$$s_i = \begin{cases} 0, & \text{if } |r_i(k)| < \tau_i \text{ (no fault)} \\ 1, & \text{if } |r_i(k)| \ge \tau_i \text{ (fault)} \end{cases}$$
 (14)

where τ_i is the threshold associated to the *i*-th residual.

3.2 Parity and observer based methods

According to Gertler [1998], given a system described by Eq. (12), a general form of the predicted behavior is

$$\hat{y}(k) = G_u(q)u(k) + G_y(q)y(k) \tag{15}$$

including as special cases Meseguer et al. [2010]: ARMA form (simulation: $G_u(q) = M(q)$, $G_y(q) = 0$) and the MA form (prediction: $G_u(q) = V(q)$, $G_y(q) = 1 - W(q)$). Then, using Eq. (13) the following parity equation forms can be introduced:

$$r(k) = y(k) - G_u(q)u(k) - G_y(q)y(k)$$
 (16)

ARMA residuals are based on the simulation of the system behavior expecting to be zero in the absence of noise, disturbances and faults. However, in general this is not true, because the actual system and its modelled behavior are not initialized identically and since there are non modeled (or unstable) dynamics. It is generally possible to force the convergence of the model simulation adding a Luenberger observer scheme

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + L(y(k) - \hat{y}(k))$$

$$\hat{y}(k) = C\hat{x}(k)$$
(17)

where L is the observer gain, designed to stabilize the matrix A-LC and guarantee desired performance.

The predicted output of the observer given by Eq. (17), expressed in observer canonical form, is generated using the model given by Eq. (15) considering that

$$G_{u}(q) = C (qI - A + LC)^{-1} B = \frac{V(q)}{W(q) + H(q)}$$

 $G_{y}(q) = C (qI - A + LC)^{-1} L = \frac{H(q)}{W(q) + H(q)}$ (18)

where

$$H(q) = \sum_{i=1}^{n} l_i q^{-i}$$
 (19)

and $l_i, i = 1, \dots, n$ are the observer gains.

Looking at the observer expression (17), it possible to see that the two parity equation forms (ARMA and MA) can be obtained as special cases through appropriate selection of the observer gain Meseguer et al. [2010]. Taking observer gain equal to zero, L=0 (or $H\left(q\right)=0$), no correction is introduced, then the observer residual expression given by Eq. (18) will transform into the ARMA parity equation form. On the other hand, assuming that all states are measured and taking the observer gain satisfying: LC=A (or $W\left(q\right)+H\left(q\right)=1$), the MA parity equation form is obtained.

4. MODEL-BASED ISOLATION

4.1 Background

In the literature, two different approaches to constructing residual sets with the desired isolability properties can be found. One approach is based on designing a vector of structured residuals [Gertler, 1998]. Each residual is designed to be sensitive to a subset of faults, whilst remaining insensitive to the remaining faults. An alternative way of achieving the isolability of faults is to design a vector of directional residuals [Gertler, 1998], which lies in a fixed and fault-specified direction in the residual space, in response to a particular fault. The fault isolation problem consists in determining which of the known fault directions, called fault signatures, the generated residual vector lies the closest to. This is done as follows:

The actual fault signature of the system $\mathbf{s}(k) = [s_1(k),$ $s_2(k), \ldots s_n(k)$, obtained as a result of the fault detection phase (see (14)) is provided to the fault isolation module which will try to isolate the fault and give a diagnosis. The actual fault signature is compared against the theoretical Fault Signature Matrix FSM that binary codifies the influence of a fault in a set of considered faults $f_1, f_2, \dots f_{n_f}$ on every residual in the a set of considered residuals $r_1, r_2, \dots r_{n_r}$. This matrix has as many rows as residuals and as many columns as considered faults. An element FSM_{ij} of this matrix being equal to 1 means that the j^{th} fault appears in the expression of the i^{th} residual. Otherwise it is equal to 0. Assuming classical FDI fault hypotheses, i.e, single faults and no-compensation (exoneration), fault isolation will consist in looking for a column of the FSM that matches the actual fault signature $\mathbf{s}(k)$. Therefore, this classic approach in the FDI-community is also known as Column Reasoning Gertler [1998].

Table 2. List of components of case-study system

Component	EAR
Tank l	$c_1: \dot{h_l} = \frac{1}{a_l} \left(q_{plb} - q_{lb} - q_{lm} \right)$
Tank m	$c_2: h_m = \frac{1}{a_m} (q_{lm} - q_{mr})$
Tank r	$c_3: \dot{h_r} = \frac{1}{a_r} \left(q_{prb} - q_{rb} + q_{mr} \right)$
Pipe lb	$c_4: \ q_{lb} = k_{lb}s\left(h_l - h_b\right)$
Pipe lm	$c_5: q_{lm} = k_{lm} s (h_l - h_m)$
Pipe mr	$c_6: q_{mr} = k_{mr}s \left(h_m - h_r\right)$
Pipe rb	$c_7: q_{rb} = k_{rb}s \left(h_r - h_b \right)$
Pump lb	$c_8: q_{plb} = k_l u_l$
Pump rb	$c_9: q_{prb} = k_r u_r$
Level sensor l	$c_{10}: y_l = h_l$
Level sensor m	$c_{11}: y_m = h_m$
Level sensor r	$c_{12}: y_r = h_r$

4.2 Structural analysis

Structural analysis is the analysis of the structural properties of models, i.e., properties which are independent of the actual values of the parameters Blanke et al. [2006]. It only represents the links between the variables and the parameters which result from the model and are thus independent of the form under which this model is expressed (quantitative or qualitative). The links are presented by a graph upon which the analysis of the structure will be performed.

From the structural analysis point of view, the model of the system is considered as a set of constraints which apply to a set of variables among which a subset have known values:

- the sensors which are present in the process together with the control variables, give the subset of those variables whose values are known.
- the set of constraints is given by models of the components which constitute the system. The term constraint refer to the fact that components impose elementary analytical relations (EAR) between the values of variables according to their corresponding physical laws.

The system proposed as a case study for this paper in Section 2 consists of the components enumerated in Table 2. The structural graph associated to this set of components is presented in Fig 2.

From the set of constraints and the set of known variables, the basic tool of structural analysis is to apply a matching algorithm that associates unknown system variables with the system constraint from which they can be calculated. Unknown variables that can not be matched, can not be calculated. A complete matching over the unknown variables identifies the computations to be done in order to express each of them as a function of the known variables. Applying the matching algorithm to the set of constraints presented in Table 2, proposed by Blanke et al. [2006], generates a set of structured residuals. The FSMcorresponding to these residuals when considering faults in level sensors $(f_{y_l}, f_{y_m} \text{ and } f_{y_r})$ and pumps $(f_{u_l} \text{ and } f_{u_r})$ is presented in Table 3. It can be noticed from this table that sensor faults are not isolable. In order to isolate sensor fault, the set of structured residuals can be transformed as described in next section.

Table 3. Fault signature-matrix of case-study system

Residual	f_{y_l}	f_{y_m}	f_{y_r}	f_{u_l}	f_{u_r}
r_1	1	1	1	1	0
r_2	1	1	1	0	0
r_3	1	1	1	0	1

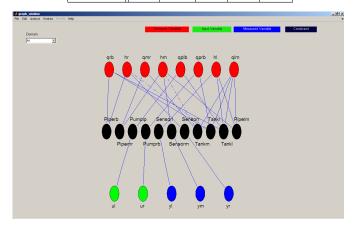


Figure 2. Structural analysis graph.

4.3 Designing a structured set of residuals

A set of structured residuals can be designed using the method proposed by Gertler [1998], where each individual residual is designed to be sensitive to a single fault whilst remaining insensitive to the rest of faults.

For this purpose, let us consider the residuals obtained using structural analysis approach presented in Section 4.2 expressed in computational form as in (16). Assuming additive faults f(k) affecting those residuals and the transfer function S(q) describing the residual fault sensitivity to each f(k), residuals (16) can be rewritten in internal form as Gertler [1998]

$$r(k) = S(q)f(k) \tag{20}$$

Then, structured residuals can be transformed using the filter Q(q) leading to a set of transformed residuals

$$r_t(k) = Q(q)r(k) = Q(q)S(q)f(k)$$
(21)

that present the desired fault response. The filter Q(q) is designed by imposing the transformed residuals present the desired fault response as

$$r_t(k) = Z(q)f(k) \tag{22}$$

where Z(q) is desired fault transfer-function matrix. Comparing Equation (21) and (22), the design equation for the filter Q(q) can be derived:

$$Q(q)S(q) = Z(q) \tag{23}$$

Assuming that the number of residuals n_r is equal to the number of considered faults n_f (i.e., S(q) is a square fault transfer matrix), the solution of design equation (23) can be found by direct inversion:

$$Q(q) = Z(q)S^{-1}(q) (24)$$

Table 4. Structured fault signature-matrix of case-study system

Residual	f_{y_l}	f_{y_m}	f_{y_r}	f_{u_l}	f_{u_r}
r_l	1	0	0	1	1
r_m	0	1	0	1	1
r_r	0	0	1	1	1

what requires that $rank(S(q)) = n_r = n_f$.

By imposing that Z(q) is a diagonal fault transfer function matrix, the transformed set of structured residuals obtained in Section 4.2 leads to a bank of dedicated observers. That scheme uses a separate observer for each monitored sensor being only sensitive to a sensor fault while insensitive to the rest.

5. PROPOSED EXERCISES AND SUPPORTING MATERIAL

5.1 Exercices

Exercise 1: Fault Detection using Observer and Parity Equations Students will obtain a set of primary residuals directly from the input/output model (11) obtained from the linearised state-space representation of the system by using the parity equation and observer approaches introduced in Section 3. Residuals will be evaluated in different fault scenarios (leaks, sensor and actuator faults) in order to see the different sensitivities to the fault of the different approaches. In particular, the observer gain will be changed from L=0 (ARMA parity equation) to L=A (MA parity equations). Section 6 presents the results in case of sensor faults.

Exercise 2: Structural Analysis The system is composed of the components presented in Table 2. The equation that describes the dynamics of each component is also presented. From this set of equations and using the tool SaTool [Blanke and Lorentz, 2006], students will perform the structural analysis of the system. As a result of this analysis, the structure of the system presented in Figure 2 will be obtained. Using the matching algorithm available in SaTool, the set of structured residuals will be obtained as well as the fault signature matrix regarding the set of considered faults. Finally, fault detectability and isolability analysis will be carried out.

Exercise 3: Fault Isolation using Structured Residuals Analysing the set of residuals obtained in Exercise 2, the fault signature matrix presented in Table 3 will be validated in simulation. The students should notice that not all the faults are isolable with the residuals obtained using the structural analysis approach. But transforming this set of residuals by means of the appropriate transformation as described in Section 4.3, a new set of residuals able to isolate the desired set of faults can be derived. This new set of residuals will result in a set of residuals that can be implemented as a bank of observers, each one dedicated to monitor a particular sensor fault.

5.2 Supporting material

Since the seminal book of Patton et al. [1989], that was the only reference for many years, several books have appeared

[Gertler, 1998, Chen et al., 1998, Blanke et al., 2006, Isermann, 2006, 2011, Ding, 2008]. From these books, maybe the book of Isermann [Isermann, 2006] is the only one that offers the widest perspective since presents a wide range of fault diagnosis methods ranging from model-based to signal and knowledge-based methods.

6. RESULTS USING SIMULATED AND REAL SET-UP

The FDI methods designed in the proposed exercises are tested either in simulation using MATLAB/SIMULINK as well as in the real set-up presented in Section 2. The simulator is based on the non-linear equations (4) with nominal parameters in Table 1 in order to represent, as close as possible, the real behaviour of the three tank system proposed as case study.

Figure 3 presents the fault detection results when a fault of 3cm in the sensor measuring the level of the third tank is introduced. This figure shows how the fault detection performance varies when changing the observer gain. It can be noticed that the simulation approach is the most persistently sensitive independently of time. However, in the case of the two other approaches (prediction (MA) and observation (ARMA)) sensitivity evolves with time (dynamic response depending of residual poles) being the maximum value at the fault apparition time but decreases as time elapses. This produces that the predicted output tends to follow the faulty system output ("fault following effect") and the fault indication could not persistently indicate the fault presence Meseguer et al. [2010]. This can cause problems in the fault isolation module if several residuals should be activated at the same time in order to isolate a fault.

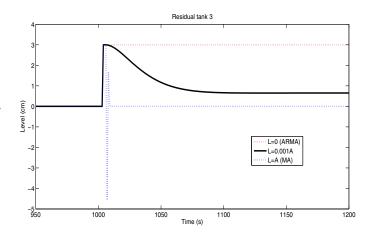


Figure 3. Effect of observer gain in Residual 3.

Figures 4-5 present fault detection results in the real set-up and in simulation using the simulator presented above for the same fault obtained with the set of residuals provided by the bank observers described in Section 4.3 when a offset fault of -4cm appears in the third sensor. These figures show that only the third residual is activated. This is in concordance with what is described in Section 4.3 where each residual generated to be sensitive to a single sensor fault according to the FSM matrix presented in Table 4.

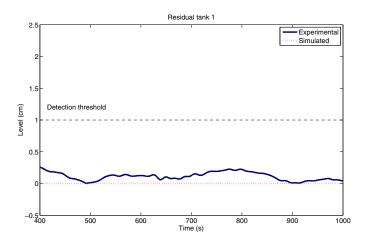


Figure 4. Real Residual 1.

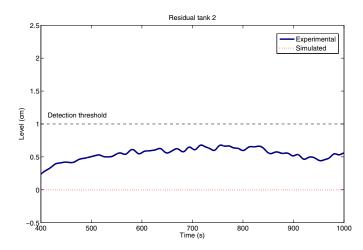


Figure 5. Real Residual 2.

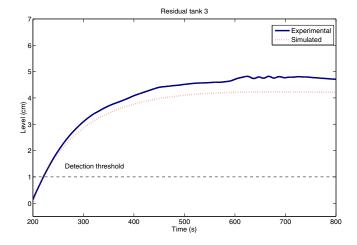


Figure 6. Real Residual 3.

7. CONCLUSIONS

In this paper, an introduction to model-based FDI learning activities based on PBL has been proposed using a three tank system in simulation and using a real set-up. Classical fault detection methods observers and parity methods are

recalled and applied to the three tanks system. Relation between the two methods is also discussed from the analytical and performance point of view. Fault isolation is addressed using structural analysis and structured residuals are used to generate a set of residuals that allow fault isolation. Finally, a set of exercises proposed to the students are described as well as some of the results obtained using the three tank system in simulation and experimentally.

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