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Differential GPS aided Inertial Navigation: a Contemplative Realtime Approach

Sheng Zhao, Yiming Chen, Haiyu Zhang, Jay A. Farrell

Dept. of Electrical Eng., U. of California, Riverside, 92521, USA (e-mail: shzhao, yichen, hzhang, farrell@ee.ucr.edu)

Abstract: Extended Kalman Filtering (EKF) based GPS-INS has poor performance when there are strong nonlinearities and the measurements are corrupted by noise or outliers. This paper proposes a Contemplative Realtime (CRT) framework for tightly coupled Differential GPS aided inertial navigation. This method guarantees low latency for real-time application and meanwhile enhances the accuracy and reliability of navigation. To realize these improvements, a Bayesian optimization based smoothing is combined with conventional filtering in an efficient way. For demonstration purpose, this CRT framework is implemented on an automotive vehicle with differential pseudorange and IMU measurements. The implementation result shows that compared to standard EKF method, the proposed approach could provide accurate vehicle navigation with robustness.

Keywords: Global Positioning System, Inertial Navigation, State Estimation, Smoothing

1. INTRODUCTION

Global Positioning System aided Inertial Navigation System (GPS-INS) is widely applied for navigation purpose on aircrafts, land vehicles, marine surface vehicles and other platforms (see Diesel (1991); Farrell et al. (2000); Bevly et al. (2006); Elkaim et al. (2008)). Due to the complimentary sensor properties, this GPS-INS integration has proven its efficiency under the framework of standard Extended Kalman Filter (EKF) (see Farrell (2008)). Especially, when carrier phase measurements and differential sources are available, the centimeter level accuracy can be achieved by applying Real Time Kinematic (RTK) technique (see Farrell et al. (2000)). However, conventional GPS-INS still has limitations, among which is that the performance of the EKF significantly depends on initial conditions and nonlinearities (see Dellaert and Kaess (2006)). This is due to the fact that previous improper EKF linearization points cannot be corrected at later times.

To overcome this inconsistence problem of the EKF method, a smoothing approach has attracted considerable attention in the Simultaneous Localization and Mapping (SLAM) research community (see Dellaert and Kaess (2006); Kaess et al. (2008)). A key point of smoothing in the SLAM context is keeping the complete robot trajectory in the estimation, so all the useful information over the time window can be considered to reach statistically optimality. By exploring the sparsity of the involved matrices and introducing incremental solution, the smoothing approach is claimed to be fast and efficient (see Kaess et al. (2008)). However, for real-time inertial vehicle navigation and control applications, where temporal latency can cause severe issues, the computational cost is still fairly considerable.

In this paper, a novel tightly coupled GPS-INS framework is proposed for accurate and reliable vehicle navigation. The proposed approach, which is referred as a Contemplative Realtime (CRT) method, guarantees not only realtime performance, but also precision and robustness, by combining filtering, smoothing and outlier rejection in an efficient way. In the smoothing, a *Maximum-a-Posteriori* (MAP) optimization is formed by considering all the information over an extended duration window. Statistical testing procedures are introduced to detect, identify and remove erroneous GPS measurements during the smoothing process. The measurement redundancy in the outlier detection is improved, since a whole window of measurements are included. For demonstration, this CRT GPS-INS is implemented with differential compensated pseudorange and IMU measurements on an automotive vehicle.

In the literature, the most related work is Indelman et al. (2013) in which the GPS measurements are used in a loosely coupled way. In contrast, this paper evaluated the performance of the tightly coupled GPS/INS in a smoothing framework which has not been reported in the literature. The implementation result shows that the proposed framework enhances navigation accuracy and reliability significantly.

The overall GPS-INS positioning accuracy will be determined by the GPS performance. However, faulty GPS measurements can be caused by many factors. For example, drastic atmospheric variations or multipath effects could delay the signal. Receiver Autonomous Integrity Monitoring (RAIM, see Hewitson et al. (2004); Bhatti (2007)) is a set of techniques to check the consistency of measurements relying on measurement redundancy. Conventional RAIM only uses measurements from one epoch, which cannot always guarantee enough available satellites. One way to fix this RAIM 'hole' is to integrate external aiding like INS. Several papers have considered such extended RAIM (eRAIM, see Hewitson and Wang (2010)) using linearized propagation methods over two measurement times. The methods herein extend these ideas over $K > 1$ epochs while addressing the full nonlinear solution.

This paper is organized as follows. Section 2 presents the problem statement. Section 3 introduces the CRT approach. Section 4 presents the details of the *Maximum-a-Posteriori* problem. Section 5 gives a brief discussion of reliable removal of faulty data in the CRT framework. Section 6 shows the experimental results with vehicle data.

2. PROBLEM STATEMENT

This section introduces the background and notation of differential GPS aided inertial navigation. The main INS and GPS references for this presentation are Farrell (2008) and Misra and Enge (2001), respectively, which should be consulted for additional information.

2.1 Aided Inertial Navigation

Let $\boldsymbol{x} \in \mathbb{R}^n$ denote the 6-DOF rover state vector:

$$\boldsymbol{x} = \left[{}^G\boldsymbol{p}_I^\top \; {}^G\boldsymbol{v}_I^\top \; {}^I_G\bar{q}^\top \; {}^I\boldsymbol{b}_g^\top \; {}^I\boldsymbol{b}_a^\top\right]^\top \tag{1}$$

where ${}^G\boldsymbol{p}_I$ and ${}^G\boldsymbol{v}_I$ are the rover position and velocity represented in the global frame, ${}^I_G\bar{q}$ is the quaternion that represents the rotation from the global frame to the IMU body frame, and ${}^I\boldsymbol{b}_g$ and ${}^I\boldsymbol{b}_a$ are the IMU gyro and accelerometer biases represented in the IMU frame.

The kinematic equations for the rover state are

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t)), \tag{2}$$

where $\boldsymbol{f} : \mathbb{R}^n \times \mathbb{R}^m \mapsto \mathbb{R}^n$ and $\boldsymbol{u} \in \mathbb{R}^m$ is the vector of accelerations and angular rates. The function $\boldsymbol{f}$ is accurately known (see Chapter 11 in Farrell (2008)).

Given a distribution for the state vector initial condition $\boldsymbol{x}(0) \sim N(\hat{\boldsymbol{x}}(0), \boldsymbol{P}(0))$ and measurements $\tilde{\boldsymbol{u}}$ of $\boldsymbol{u}$, an Inertial Navigation System (INS) propagates an estimate of the vehicle state between aiding measurement times as a solution of

$$\dot{\hat{\boldsymbol{x}}}(t) = \boldsymbol{f}(\hat{\boldsymbol{x}}(t), \tilde{\boldsymbol{u}}(t)), \tag{3}$$

where $\hat{\boldsymbol{x}}(t)$ denotes the estimate of $\boldsymbol{x}(t)$.

Due to initial condition errors, system calibration errors, and measurement noise, the state estimation error $\delta\boldsymbol{x}(t) = \boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t)$ develops over time. The dynamics and stochastic properties of this estimation error are well understood.

When aiding measurements

$$\tilde{\boldsymbol{z}}(t) = \boldsymbol{h}(\boldsymbol{x}(t)) + \boldsymbol{n}_z(t) \tag{4}$$

are available, various methods are available to use the initial state, inertial measurements, and aiding measurement information to estimate the vehicle state vector (Farrell (2008); Kay (1993)). The $\boldsymbol{n}_z(t)$ is the measurement noise.

2.2 DGPS Pseudorange Measurements

Throughout this article, double differenced GPS measurements are considered. For simplicity of notation, it is assumed that DGPS approach completely removes all common-mode errors (e.g., ionosphere, troposphere, satellite clock and ephemeris errors). The code (pseudorange) measurements for the $i-$th satellite at time t can be modeled as

$$\bar{\boldsymbol{\rho}}^i = \bar{R}^i + c\boldsymbol{\delta}t_r + \bar{\boldsymbol{n}}^i_\rho, \tag{5}$$

where $\bar{R}^i = \|\boldsymbol{p} - \boldsymbol{p}^i\|_2$ is the geometric distance between the vehicle position $\boldsymbol{p} \in \mathbb{R}^3$ and $\boldsymbol{p}^i \in \mathbb{R}^3$ is the $i-$th satellite position, $c\boldsymbol{\delta}t_r \in \mathbb{R}$ is the receiver clock bias, $\bar{\boldsymbol{n}}^i_\rho$ is the measurement noise. Subtracting all satellite's measurements by a common satellite's measurement will remove the clock bias $c\boldsymbol{\delta}t_r$. The new measurement model after double difference is:

$$\boldsymbol{\rho}^i = R^i + \boldsymbol{n}^i_\rho, \tag{6}$$

where $R^i = \bar{R}^i - \bar{R}^c$, $\boldsymbol{\rho}^i = \bar{\boldsymbol{\rho}}^i - \bar{\boldsymbol{\rho}}^c$ and $\boldsymbol{n}^i_\rho = \bar{\boldsymbol{n}}^i_\rho - \bar{\boldsymbol{n}}^c_\rho$.

Thus, the code measurement in eqn. (6) conforms with the standard measurement model represented in eqn. (4).

2.3 Technical Problem Statement

This paper considers vehicle state estimation problem using inertial and DGPS pseudorange measurements. Without loss of generality, we consider the problem on the time interval $[t_0, t_K]$, which contains K GPS measurement time instants, where K can be designer specified, time varying, or data dependent. The technical problem can be stated as follows.

For a system described by eqn. (2), we have

- an initial distribution for the state $\boldsymbol{x}(t_0)$,
- IMU measurements $\mathbf{U} = \{\tilde{\boldsymbol{u}}(\tau_k)\}_{k=0}^{\kappa}$,
- DGPS code measurements $\mathbf{Y} = \{\mathbf{Y}(t_j)\}_{j=1}^{K}$, where $\mathbf{Y}(t_j) = \{\boldsymbol{\rho}^i(t_j)\}_{i=1}^{m_j}$.

where $t_0,\ t_j \in (\tau_0, \tau_\kappa]$ and $m_j = m(t_j)$ is the total number of available satellites at t_j. For convenience of notation later, let $\{t_k\}_{k=0}^K = \{t_0\} \cup \{t_j\}_{j=1}^K \subset [t_0, t_K]$, such that the set of times is ordered and non-repeating. This interval of time will be referred to as the *CRT window*.

Then, we have the *objective*: Estimate the optimal state trajectory $\mathbf{X} = \{\boldsymbol{x}(t_k)\}_{k=0}^K$ with the given sensor measurements $\mathbf{U}$, $\mathbf{Y}$ and the prior state density $p_{\boldsymbol{x}}(\boldsymbol{x}(t_0))$.

3. CONTEMPLATIVE REALTIME APPROACH

This paper develops a Contemplative RealTime (CRT) approach proposed in Ramanandan et al. (2011); Chen et al. (2013) for the GPS INS data scenario described in Section 2.3. The CRT approach has both real-time and contemplative aspects. The real-time state estimate is required for control and planning purposes, without latency. The contemplative aspects are intended to enhance accuracy and reliability and are inspired by recent research in the field robotics literature, see e.g.: Dellaert and Kaess (2006); Kaess et al. (2008); Indelman et al. (2013); Li and Mourikis (2013).

A typical measurement scenario over the CRT window is depicted in Fig. 1. The contemplative process starts when $t = t_K$. At t_K, all IMU, GPS, and other measurements are available for the time interval $[t_0, t_K]$. A prior trajectory $\hat{\boldsymbol{x}}(\lambda)$ for $\lambda \in [t_0, t_K]$ is also available. Starting from this prior trajectory, the CRT algorithm will contemplate the available information to reliably and

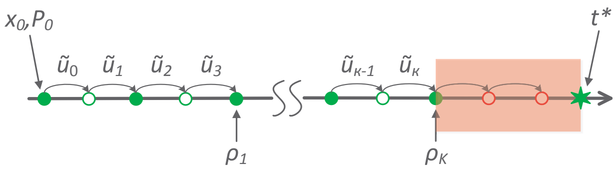

Fig. 1. Measurement timeline for one CRT cycle.

accurately compute the state trajectory over the CRT window using optimization shown in Dellaert and Kaess (2006); Kuemmerle et al. (2011) and statistical hypothesis testing methods discussed in Baarda (1968); Hewitson et al. (2004). This contemplative process ends at a time $t^* > t_K$, ideally providing an optimal trajectory estimate $\hat{\boldsymbol{x}}^*(\lambda)$ for $\lambda \in [t_0, t_K]$ from which the effects of sensor faults have been removed. For the computation time interval $t \in [t_K, t^*]$, the real-time estimate of the realtime state estimate $\hat{\boldsymbol{x}}(t)$ is maintained by the INS using the IMU data and starting from the prior estimate of $\boldsymbol{x}(t_K)$. At $t = t^*$, $\hat{\boldsymbol{x}}(t_K)$ is corrected to the result of the CRT contemplative process and propagated through time using the IMU data and eqn. (3) to provide an improved estimate of $\boldsymbol{x}(t)$ at the present time. At some time $t \geq t^*$, the CRT window can be redefined and the process can repeat indefinitely.

Fig. 1 depicts a typical CRT window. The dots on the time-line indicate IMU measurement times τ_k. The state transition between these times is constrained by the kinematic model of eqn. (3) and the IMU data $\mathbf{U}$. Additional constraints are imposed by the initial state estimate, and GPS measurements $\mathbf{Y}$. The initial condition $(\boldsymbol{x}_0, \mathbf{P}_0)$ constraints are shown above the time-line. The GPS measurement constraints $(\boldsymbol{\rho})$ are depicted below the time-line. Each of these constraints is quantified by a probability density that enables a Bayesian problem formulation. While Fig. 1 depicts all GPS measurements occurring at the IMU measurement time, unaligned measurements can be addressed by interpolation, and unknown latencies can be calibrated by the methods in Li and Mourikis (2014, to appear).

The main contribution of this article is the derivation and implementation of this Bayesian approach, particularly for the tightly coupled DGPS/INS application.

4. MAP ESTIMATION

Given the sensor measurements $\mathbf{U}$, $\mathbf{Y}$ and the prior state density $p_{\boldsymbol{x}}(\boldsymbol{x}(t_0))$, our objective is to compute the state trajectory $\mathbf{X} = \{\boldsymbol{x}(t_k)\}_{k=0}^{K}$ that maximizes the joint probability density $p(\mathbf{X}, \mathbf{Y}, \mathbf{U})$. Due to the use of the prior density at t_0, this is a *Maximum-A-Posteriori* (MAP) estimation problem.

The algorithm below builds upon the methods and notation standard for INS implementations, see e.g., Farrell (2008). For convenience, a very brief review of the method and notation are presented in the appendix .

4.1 Theoretical Solution

For the derivation, we make the reasonable and standard assumptions that $\boldsymbol{\omega}_u \sim \mathcal{N}(\mathbf{0}, \mathbf{Q})$ is the IMU measurement noise, $\boldsymbol{n}_\rho \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_\rho)$ and $\boldsymbol{x}(t_0) \sim \mathcal{N}(\boldsymbol{x}_0, \mathbf{P}_0)$. With various modifications, the derivation goes through for other distributions.

Using standard methods (see Chapters 10-11 in Kay (1993)) and assuming that $\boldsymbol{x}(t_0)$, $\boldsymbol{\omega}_u$ and $\boldsymbol{n}_\rho$ are all independent, the MAP estimate of $\mathbf{X}$ is

$$\hat{\mathbf{X}} = \arg\max_{\mathbf{X}} \Big(p_{\delta\boldsymbol{x}_0}(\boldsymbol{x}(t_0) - \boldsymbol{x}_0) \\ p_{\boldsymbol{\omega}_u}(\mathbf{X}_+ - \boldsymbol{\phi}(\mathbf{X}, \mathbf{U})) p_{\boldsymbol{n}_\rho}(\mathbf{Y} - \boldsymbol{h}(\mathbf{X})) \Big), \tag{7}$$

where $\mathbf{X}_+ = \{\boldsymbol{x}(t) \text{ for } t = t_1, \ldots, t_K\}$, $\delta\boldsymbol{x}_0 = (\boldsymbol{x}(t_0) - \boldsymbol{x}_0) \sim \mathcal{N}(\mathbf{0}, \mathbf{P_0})$ and the operator $\boldsymbol{\phi}$ is defined in eqn. (15) in the appendix.

Direct maximization of eqn. (7) is complicated by various factors. First, each $\boldsymbol{x}(t_k) \in \mathbb{R}^n$ where in most applications $n \geq 15$. Second, both the kinematic model $\boldsymbol{f}$ and the measurement model $\boldsymbol{h}$ are nonlinear.

With the Gaussian noise assumptions, maximization of eqn. (7) is equivalent to minimization of

$$\begin{aligned}\|\mathbf{v}\|^2_{\mathbf{W}} = {} & \|\boldsymbol{x}(t_0) - \boldsymbol{x}_0\|^2_{\mathbf{P}_0} \\ & + \sum_k \|\boldsymbol{\phi}(\boldsymbol{x}(t_k), \mathbf{U}_k) - \boldsymbol{x}(t_{k+1})\|^2_{\mathbf{Q}_k} \\ & + \sum_j \sum_i \|\boldsymbol{h}^i_j(\boldsymbol{x}(t_j)) - \boldsymbol{\rho}^i(t_j)\|^2_{{}^i_j\mathbf{R}_\rho}\end{aligned} \tag{8}$$

where $\|\mathbf{v}\|^2_{\mathbf{W}} = \mathbf{v}^\top \mathbf{W}^{-1} \mathbf{v}$ is the squared Mahalanobis distance with the matrix $\mathbf{W}$ and the operator $\boldsymbol{\phi}$ is defined in the appendix, and $\boldsymbol{h}^i_j(\cdot)$ denotes the measurement equation for the i-th satellite at time j. The vector $\mathbf{v}$ is the concatenation of each of the vectors summed in the right-hand side of eqn. (8) and $\mathbf{W}$ is the postive definite block diagonal matrix formed by the positive definite submatrices ${}^i_j\mathbf{R}_\rho$, $\mathbf{P}_0$ and $\mathbf{Q}_k$.

Let $\mathbf{\Sigma}_W$ be the square root of $\mathbf{W}^{-1}$: $\mathbf{W}^{-1} = \mathbf{\Sigma}_W^\top \mathbf{\Sigma}_W$. We can write $\|\mathbf{v}\|_{\mathbf{W}} = \|\boldsymbol{r}\|_2$ where $\boldsymbol{r} \triangleq \mathbf{\Sigma}_W \mathbf{v}$. Use the same notation for ${}^i_j\mathbf{R}_\rho$, $\mathbf{P}_0$ and $\mathbf{Q}_k$ we got:

$$\boldsymbol{r}(\mathbf{X}) = \begin{bmatrix} \mathbf{\Sigma}_{\mathbf{P}_0}(\boldsymbol{x}(t_0) - \boldsymbol{x}_0) \\ \hdashline \mathbf{\Sigma}_{\mathbf{Q}_0}(\boldsymbol{\phi}(\boldsymbol{x}(t_0), \mathbf{U}_0) - \boldsymbol{x}(t_1)) \\ \vdots \\ \mathbf{\Sigma}_{\mathbf{Q}_{K-1}}(\boldsymbol{\phi}(\boldsymbol{x}(t_{K-1}), \mathbf{U}_{K-1}) - \boldsymbol{x}(t_K)) \\ \hdashline \mathbf{\Sigma}_{{}^1_1\mathbf{R}_\rho}(\boldsymbol{h}^1_1(\boldsymbol{x}(t_1)) - \boldsymbol{\rho}^1(t_1)) \\ \vdots \\ \mathbf{\Sigma}_{{}^{m_K}_K\mathbf{R}_\rho}(\boldsymbol{h}^{m_K}_K(\boldsymbol{x}(t_K)) - \boldsymbol{\rho}^{m_K}(t_K)) \end{bmatrix}. \tag{9}$$

The $\boldsymbol{r}(\mathbf{X})$ is composed of the prior, the IMU and the GPS information which are separated by dash lines. With this notation, the minimization problem of eqn. (8) reduces to a nonlinear least square optimization

$$\min_{\mathbf{X} \in \mathbb{R}^{n(K+1)}} \|\boldsymbol{r}(\mathbf{X})\|^2_2. \tag{10}$$

Standard iterative methods (e.g. Gauss-Newton, gradient descent, Levenberg-Marquardt) are applicable.

The Gauss-Newton method is used in this paper and at each iteration we solve the following equation:

$$\mathbf{J}\delta\boldsymbol{x} = -\boldsymbol{b} \tag{11}$$

where $\boldsymbol{b} \triangleq \boldsymbol{r}(\hat{\mathbf{X}})$ and $\mathbf{J}$ is the Jacobian matrix of $\boldsymbol{r}(\mathbf{X})$ evaluated at $\hat{\mathbf{X}}$. The inverse covariance weighted Jacobian matrices in $\mathbf{J}$ can be demonstrated well-defined.

When solving for $\delta\boldsymbol{x}$ in eqn. (11), to save the computation as shown in Kuemmerle et al. (2011), we form the equation: $\boldsymbol{\Lambda}\delta\boldsymbol{x} = \boldsymbol{\eta}$, where $\boldsymbol{\Lambda} = \mathbf{J}^\top\mathbf{J}$, $\boldsymbol{\eta} = -\mathbf{J}^\top\boldsymbol{b}$. This equation can be solved efficiently by the Cholesky factorization as discussed in Dellaert and Kaess (2006).

4.2 Marginalization

Due to the computational resource constraints, we cannot save the entire past history of states in the CRT window. The Schur complement is employed to marginalize out the oldest states from the CRT window and generate a new prior state density, see Dong-Si and Mourikis (2011).

5. RELIABLE REMOVAL OF FAULTY DATA

Receiver Autonomous Integrity Monitoring (RAIM, Hewitson et al. (2004)) is a set of techniques to detect, identify and remove GNSS receiver outlier measurements. Traditionally, in the navigation community, RAIM is designed assuming only one outlier occurs and that there is enough measurement redundancy to detect and identify the source. The proposed CRT approach, which enhances the redundancy by incorporating a window of IMU and GPS data, can be expected to enable multiple outlier detection, identification, and removal. This outlier rejection scheme, which enhances the robustness of vehicle GPS-INS significantly, could make critical contributions as necessary for life-safety applications. The key technique in standard RAIM is *outlier detection and identification*. This section considers the detection and identification, removal procedures within the CRT framework. Interested readers may find more details in Hewitson et al. (2004).

Suppose that the MAP optimization in eqn. (10) finally converges to a optimal estimate $\mathbf{X}^*$. The generalized *a-posteriori* variance factor test evaluated as

$$\hat{\sigma}_0^2 = \frac{\|\boldsymbol{r}(\mathbf{X}^*)\|_2^2}{M-N}, \tag{12}$$

can be used to detect outlier. In this expression, $M = n(1+K) + \sum_{j=1}^{K}(m_j - 1)$ is the total number of residuals (constraints) and $N = n(K+1)$ is the dimension of $\mathbf{X}$. Note that $\sum_{j=1}^{K}(m_j - 1)$ is the total number of double-differenced GPS measurements. The degrees-of-freedom $(M - N)$ can be considered as the index of the measurement redundancy. For conventional GNSS-only RAIM which uses one epoch measurements, the redundancy is $(m_j - 4)$, which requires at least five satellites to be available. For the proposed CRT framework, the measurement redundancy is $M - N = \sum_{j=1}^{K}(m_j - 1)$, which indicates that it has enhanced detectability against faulty data.

The final step of outlier detection is to test the above variance factor against the two-tailed Chi-square test limits with respect to a significance level α,

$$\frac{\chi^2_{1-\alpha/2,M-N}}{M-N} \leq \hat{\sigma}_0^2 \leq \frac{\chi^2_{\alpha/2,M-N}}{M-N}. \tag{13}$$

If the test succeeds, $\mathbf{X}^*$ is finalized as the smoothing result and the real-time part will use it to reinitialize. If not, outlier identification executes by testing each residual with the w-test. Once the source of the outlier is identified, the corresponding measurement will be removed and the procedure continues until no additional outliers are identified.

Fig. 2. Vehicle trajectory (red). The yellow and blue markers show the start and end points, respectively.

When outlier identification completes, the outliers are removed and the MAP optimization is formed with the cleaned-up measurement set, then the detect-identify-remove procedure repeats.

6. EXPERIMENTAL RESULTS

This section uses 200Hz MEMS IMU and 1Hz Differential GPS data collected on a vehicle. The navigation system is implemented in C++ with multi-threading and reports the vehicle state in realtime at the IMU sampling rate. The CRT window is chosen to be 10 sec. In this experiment, the vehicle stays stationary while pointing north at the beginning. After about 20 seconds the vehicle accelerated north. The trajectory of vehicle is illustrated in Fig. 2.

This section compares the following real-time estimators:

(1) CRT using double-differenced code measurements;
(2) EKF using double-differenced code measurements;
(3) EKF using integer-resolved double-differenced carrier phase measurements.

The definition of the state vector is the same for all cases. In addition, we post-process the data through an off-line batch smoother for the entire trajectory using integer-resolved double-differenced carrier phase and code measurements. This post-processed trajectory will be considered as the ground truth to which the other three estimators are compared to determine error statistics.

Firstly, we demonstrate the capability of the proposed estimator to initialize yaw, without a compass. For a stationary vehicle, yaw is unobservable from GPS measurements. When the vehicle accelerates, yaw becomes observable. For the EKF, the yaw needs to be initialized close to the true value to satisfy the EKF small error assumption; otherwise the EKF may diverge. For EKF implementations, yaw may be initialized via magnetometer, or other means. The CRT estimator optimally initialize the yaw using the data in the CRT window. Everything is done naturally within the smoothing framework and it is also optimal with respect to the pre-defined noise model. To demonstrate this, the initial yaw is set to have an error of 180 degrees. Fig. 3 shows that, when the vehicle accelerates (near $t = 20s$), the yaw rapidly converges toward the correct value.

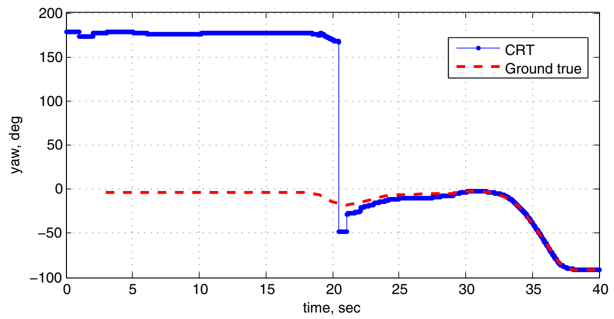

Fig. 3. Yaw estimated by the CRT at the beginning of the trajectory. The yaw is initialized naturally as the vehicle starts to accelerate at around 20 sec.

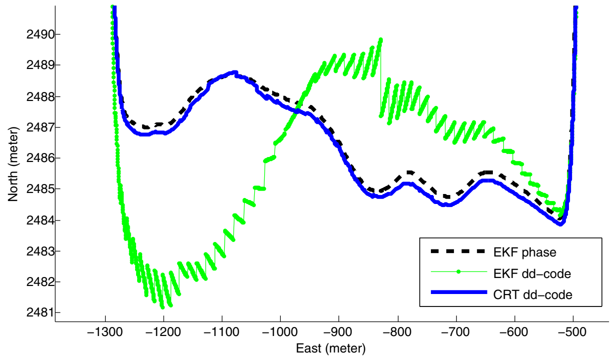

Fig. 4. EKF and CRT position accuracy comparison for one segment of the trajectory.

Secondly, the proposed estimator significantly improves the state estimate accuracy. Fig. 4 and Fig. 5 compare the estimated trajectory error from the three estimators mentioned above. Fig. 4 shows a segment of the estimated trajectories. The trajectory of the CRT approach using double-differenced code is very near the trajectory of the EKF using double-differenced integer-resolved phase. The EKF using double-differenced code has significantly larger errors. The distribution of the norm of the position error in the horizontal plane (north-east) is shown in Fig. 5. The Fig. 5 use the state estimate data at 1 Hz (324 data points in total for this trajectory). The error statistics clearly show that the proposed estimator has the position estimate error in the decimeter level, while the EKF using the same code measurements has the error in the meter level. This large accuracy improvement is gain by leveraging a window of measurements. The performance of the proposed estimator is already very close to that of the EKF using phase measurement (centimeter level). However, to achieve the centimeter accuracy, the integer ambiguity needs to be resolved in realtime.

The performance of the CRT and EKF approaches have also been evaluated on the following two testing trajectories:

- Test2: 600 sec driving in mostly open sky environment.
- Test3: 500 sec driving in an area where a portion of the trajectory has partial GPS signal blockage due to trees and buildings along the road.

The statistical comparison of the position error in the north and east directions over the entire trajectory for three testing trajectories is given in Table. 1. The Test1 trajectory is the one shown in Fig. 2. It is clear to see from the table that the CRT approach consistently outperforms the EKF approach by keeping the position error at the decimeter level while the EKF tends to have position errors in meters.

The proposed CRT method also improves the estimate of the other variables in the 6DOF state vector (eqn. (1)). Fig. 6 shows the estimated accelerometer bias from the EKF and CRT method for the test2 trajectory. It is obvious that the accelerometer bias estimate converges significantly faster in the CRT approach. The fast convergence of the IMU bias estimate is the key to maintaining high precision navigation performance.

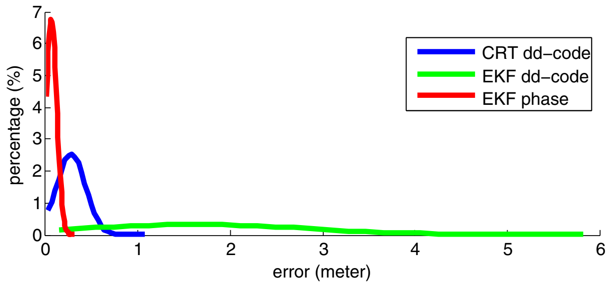

Fig. 5. Distribution of horizontal position error.

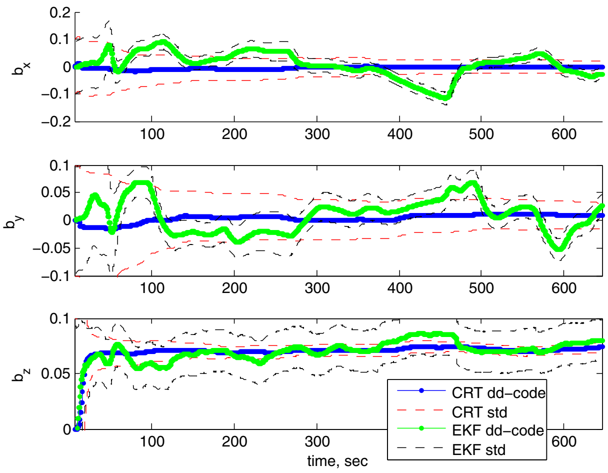

Fig. 6. The comparison of the estimated accelerometer bias with the $\pm 1\sigma$ bound. The unit of bias is m/s^2.

7. DISCUSSION AND FUTURE WORK

This paper has presented a novel GPS-INS framework which is referred as a CRT estimator. Using the proposed framework, we demonstrated that yaw initialization can be done naturally and correctly without a magnetometer or ad-hoc methods as may be required for EKF approaches. In addition, the state estimate accuracy is enhanced significantly with respect to an EKF that uses exactly the same measurements. In addition, we envision that the proposed framework has the potential to allowing enhanced outlier rejection to ensure the robustness of the estimator. Interesting future work directions include development, implementation, and test of the outlier rejection approach, and incorporation of carrier phase and doppler measurements.

Table 1. Comparison of position error statistics. Mean and standard deviation are denoted as μ and σ (unit is in meter). Double differenced code measurements are used in both estimators.

		Test1	Test2	Test3
North:	CRT:	$\mu = -0.016,\ \sigma = 0.18$	$\mu = -0.174,\ \sigma = 0.27$	$\mu = 0.003,\ \sigma = 0.31$
	EKF:	$\mu = 0.162,\ \sigma = 1.69$	$\mu = -0.097,\ \sigma = 1.08$	$\mu = 0.040,\ \sigma = 1.10$
East:	CRT:	$\mu = 0.164,\ \sigma = 0.21$	$\mu = -0.081,\ \sigma = 0.19$	$\mu = 0.220,\ \sigma = 0.41$
	EKF:	$\mu = 0.326,\ \sigma = 1.08$	$\mu = 0.034,\ \sigma = 0.67$	$\mu = 0.629,\ \sigma = 1.39$

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APPENDIX I: INS REVIEW

For any initial state $\boldsymbol{x}(\tau_k)$, the solution to (2) for $t \in [\tau_k, \tau_{k+1}]$ is

$$\boldsymbol{x}(t) = \boldsymbol{x}(\tau_k) + \int_{\tau_k}^{t} \mathbf{f}(\mathbf{x}(\tau), \mathbf{u}(\tau))\mathbf{d}\tau. \tag{14}$$

While nature solves (14) in continuous time, the INS only has IMU and aiding measurements at discrete time instants; therefore, the INS numerically solves

$$\begin{aligned}\hat{\boldsymbol{x}}(\tau_{k+1}) &= \boldsymbol{\phi}\big(\hat{\boldsymbol{x}}(\tau_k), \hat{\boldsymbol{u}}(\tau_k)\big)\\ &= \hat{\boldsymbol{x}}(\tau_k) + \int_{\tau_k}^{\tau_{k+1}} \mathbf{f}(\hat{\mathbf{x}}(\tau), \hat{\mathbf{u}}(\tau))\mathbf{d}\tau.\end{aligned} \tag{15}$$

The result of the numeric integration of (15) is the INS state estimate of $\hat{\boldsymbol{x}}(\tau_{k+1})$ given $\hat{\boldsymbol{x}}(\tau)$ and $\hat{\boldsymbol{u}}(\tau)$. The numeric integration repeats to propagate the state measurements between the times of aiding measurements. The aiding measurement times can be unequally spaced in time without causing any complications.

Let $\hat{\mathbf{U}}_j = \{\hat{\boldsymbol{u}}(\tau_k),\ \tau_k \in [t_j, t_{j+1}]\}$, then eqn. (15) can be called recursively to compute $\hat{\boldsymbol{x}}(t_{j+1})$ from $\hat{\boldsymbol{x}}(t_j)$ and $\mathbf{U_j}$, denote this as

$$\hat{\boldsymbol{x}}(t_{j+1}) = \boldsymbol{\phi}\big(\hat{\boldsymbol{x}}(t_j), \hat{\mathbf{U}}_j)\big). \tag{16}$$

At the same time, nature is integrating eqn. (14) which it denoted as

$$\boldsymbol{x}(t_{j+1}) = \boldsymbol{\phi}\big(\boldsymbol{x}(t_j), \mathbf{U}_j)\big). \tag{17}$$

The linearized error growth model is

$$\delta\hat{\boldsymbol{x}}(t_{j+1}) = \boldsymbol{\Phi}_j \delta\hat{\boldsymbol{x}}(t_j) + \boldsymbol{\omega}_j \tag{18}$$

where $\boldsymbol{\omega}_j \sim N(0, \mathbf{Q}_j)$ and $\boldsymbol{\Phi}_j = \frac{\partial\boldsymbol{\phi}(\boldsymbol{x})}{\partial\boldsymbol{x}}\Big|_{(\hat{\boldsymbol{x}}(t_j), \hat{\mathbf{U}}_j)}$. The symbol $\boldsymbol{\omega}_u$ will be used to represent $\{\boldsymbol{\omega}_j\}_{j=1}^{K}$. The INS provides both $\mathbf{Q}_j$ and $\boldsymbol{\Phi}_j$, see Section 7.2.5.2 in Farrell (2008).